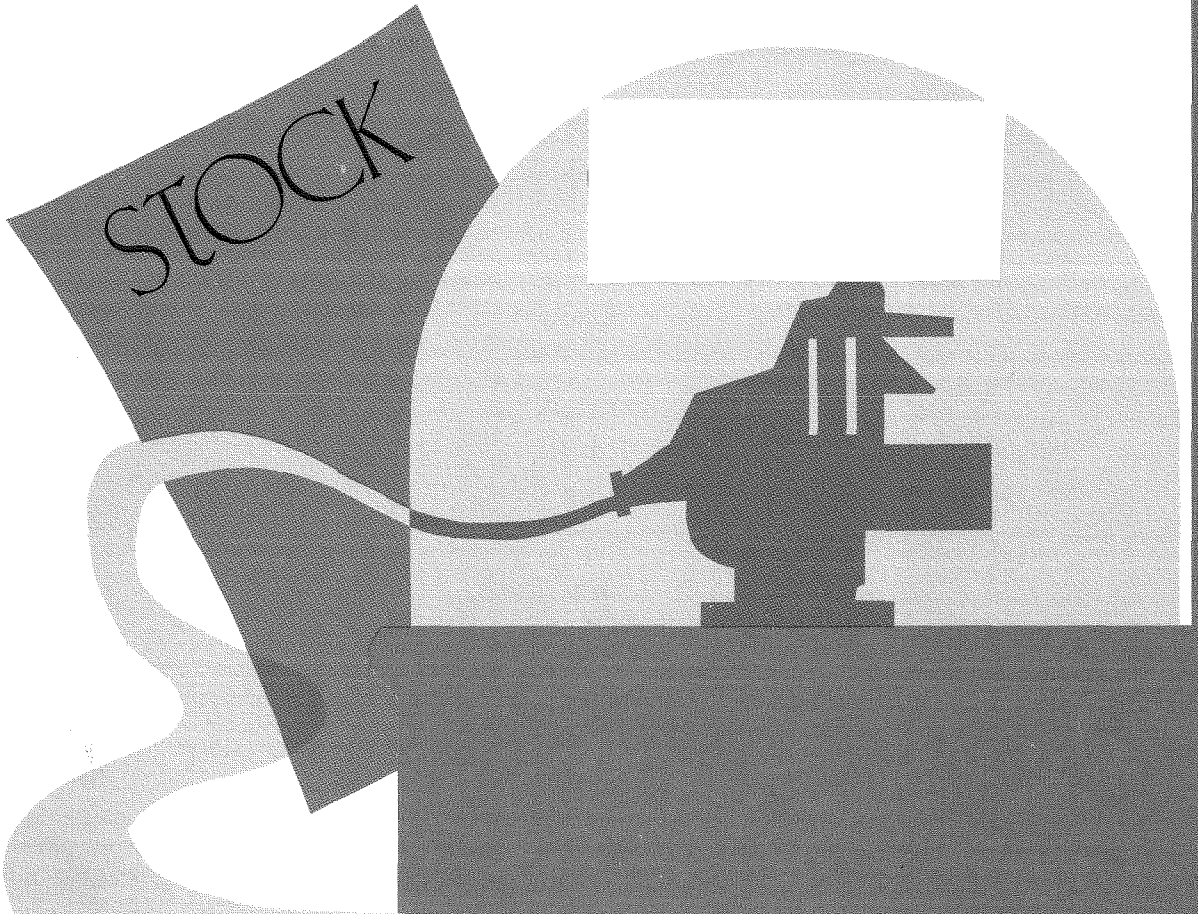


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# Using T-Bill Futures to Gauge Interest-Rate Expectations

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Trading in three-month Treasury-bill futures began on January 6, 1976. Six contracts were traded originally: March, June, September, and December of 1976, and March and June of 1977. When each contract matured, trading began in a new contract dated three months beyond the most distant contract previously traded. More recently trading has been conducted in eight contracts.

The details of this market and its uses in various types of hedging, speculative, and tax-motivated transactions have been fully described elsewhere.<sup>1</sup> The purpose of this paper is to provide an analysis of the link between the futures market and the spot market in Treasury bills of varying maturities, and to examine the policy significance of the interest-rate expectations incorporated in the T-bill futures.

In the first section of the paper, it is shown that the spot and futures Treasury bill markets are closely linked in practice; profitable arbitrage opportunities between the two markets rarely exist, at least for the nearest futures maturity traded at any given point in time. (Only this maturity is examined in this paper.)

In the second section the issue of liquidity, or

term, premiums is examined. Studies of the term structure of interest rates have generally found that longer-term securities on average have higher yields than shorter-term securities. This finding is of importance in its own right, but it also implies that a term premium must be subtracted from a futures rate if that rate is to be interpreted as the market expectation of the future spot rate at the maturity of the futures contract. This rather technical issue is treated at some length, because it is of great importance in assessing the significance of yields in the futures market.

From evidence presented in previous studies of the term structure, and from new evidence on the futures market, it is argued that part of the observed term premiums may reflect transactions costs rather than risk aversion. The conclusion reached is that, since transaction costs in the futures market are almost non-existent, it is probably not necessary to make any allowance for term premiums when using futures rates to gauge market expectations of future spot rates.

In the last section, the policy implications of market interest-rate forecasts are explored. The major issue concerns the significance of differences between market forecasts and policy-makers' forecasts of interest rates.

## I. Relationships Between Spot and Futures T-Bill Markets

At the present time, eight contracts are traded in the Treasury-bill futures market. In August, 1977, for example, trading was conducted in futures for September and December, 1977; March, June, September and December of 1978; and March and June of 1979. Government security futures other than bills are also available. When yields on these securities get out of line

with yields in the futures market, profitable risk-free arbitrage transactions are possible.

Only for short maturities, however, is it possible to find a perfect match of maturities in the spot and futures markets. For example, from March 24 through June 22, 1977, spot bills due June 23 and September 22 and June futures provided instruments with exactly matching maturities. Settlement on the June futures took place on June 23, and required delivery of the September 22 bill—a 91-day bill on June 23—on all June futures contracts still open. If held to maturity,

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an investment in the combination package of the June 23 spot bill and a long position in June futures had identical characteristics to an investment in the September 22 spot bill. The two investments should, therefore, have identical yields—except for possible differences in transactions costs should the investor desire to sell out before maturity. The yield differences are limited, however, by the possibility of arbitrage between the two markets.

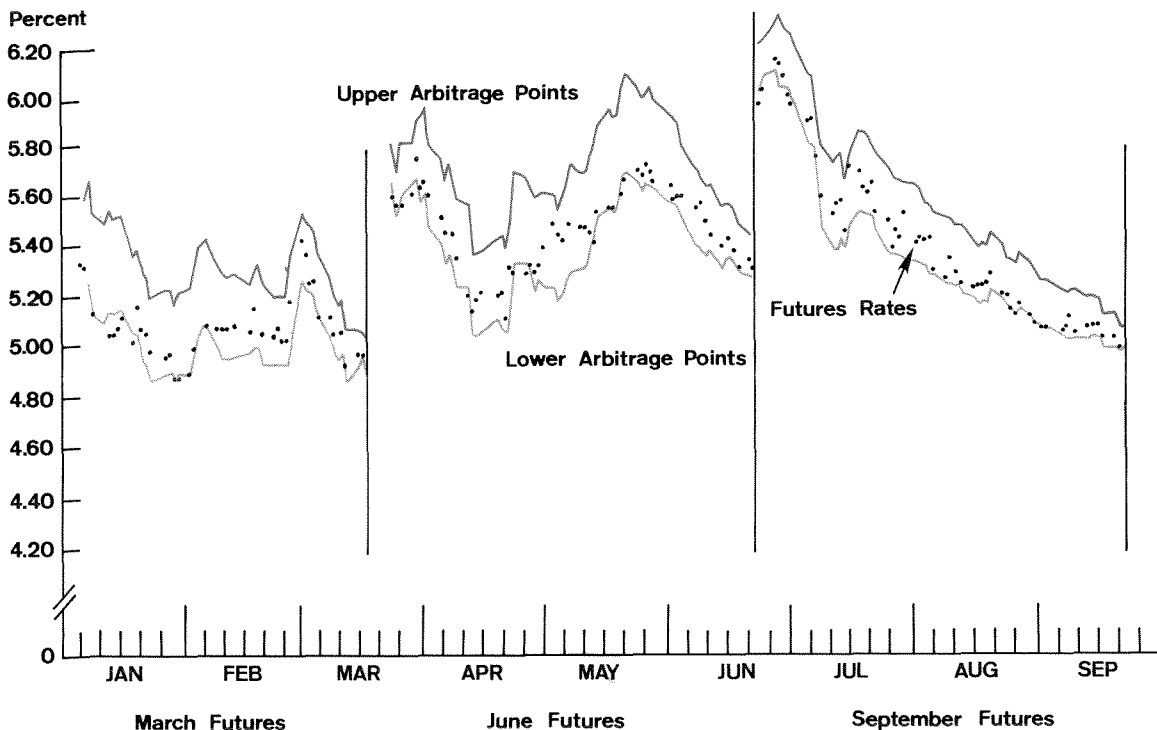
Arbitrage opportunities for futures maturities other than the nearest one are not quite risk-free because the maturities do not quite match. For example, between December 23, 1976 and March 22, 1977, arbitrage involving June 1977 futures had to be based on bills dated September 20 and June 23; the September 22 bill was not issued until March 24.

In studying the completeness of arbitrage, we may limit the investigation to the nearest maturity futures contract, so as to avoid the need for extra assumptions concerning arbitrage when maturities do not quite match. In examining arbitrage, we may proceed as if the spot bill yields

are fixed; the problem is the determination of the range of bill futures yields such that risk-free arbitrage profits are possible considering the explicit transactions costs involved. The range will be defined in terms of an upper critical point,  $F^U$ , above which substitution of the short spot bill and a long futures position for the long spot bill will be profitable; and a lower critical point,  $F^L$ , below which substitution of the long spot bill and a short futures position for the short spot bill will be profitable. Although we will be determining upper and lower critical points for the futures rate given the spot bill yields, we could just as well have determined upper and lower critical points for either spot bill given the yield on the other bill and the futures yield.

In the derivations below it is assumed that bills are infinitely divisible, and all calculations are per \$100. In fact, the discreteness of bills and of futures contracts—each futures contract is for \$1 million face value of bills—prevents arbitrage from being profitable precisely at these critical points. However, the critical points derived under the perfect divisibility assumption provide

Chart 1



benchmarks against which the market may be judged with respect to the exhaustion of arbitrage opportunities.

Suppose that an  $n+91$ -day bill is owned, where  $n$  is the number of days to the maturity of the nearest futures contract. If the futures yield is high enough, the investor can raise his rate of return over the  $n+91$ -day horizon by selling the  $n+91$ -day bill and using the proceeds to buy an  $n$ -day bill and a long position in a futures maturing in  $n$  days. What futures yield will be high enough to make this substitution profitable?

Each  $n+91$ -day bill is worth  $P_{n+91,t}^b$  at time  $t$ , where  $P^b$  is the dealer's bid price—the price at which investors other than dealers can sell the bill. By the definition of the banker's discount yield—the quotation method used in the bill market—we have

$$P_{n+91,t}^b = 100 - \frac{n+91}{360} R_{n+91,t}^b$$

where  $R^b$  is the bid yield, in percent, on the banker's discount basis. In the arbitrage transaction being examined, enough  $n+91$ -day bills are sold to buy the  $n$ -day bills required to provide the cash needed in  $n$  days to settle the maturing long futures position. The cash requirement at time  $t$  also includes the futures market commission—\$60 per contract—and the futures market margin requirement—\$1500 per contract. Since each contract is for \$1 million face value of bills,

the commission and margin amount to only \$0.006 and \$0.15, respectively, per \$100 of face value.<sup>2</sup>

Working backwards, in  $n$  days the amount needed to settle the long position in the futures market will be

$$q_{n,t} = 100 - \frac{91}{360} F_{n,t} \quad \text{where}$$

$F_{n,t}$  is the yield at time  $t$  on the futures contract maturing in  $n$  days. However, when the futures contract matures, the \$1500 per contract margin will be returned, and so the net cash requirement per \$100 in  $n$  days is  $q_{n,t} - 0.15$ .

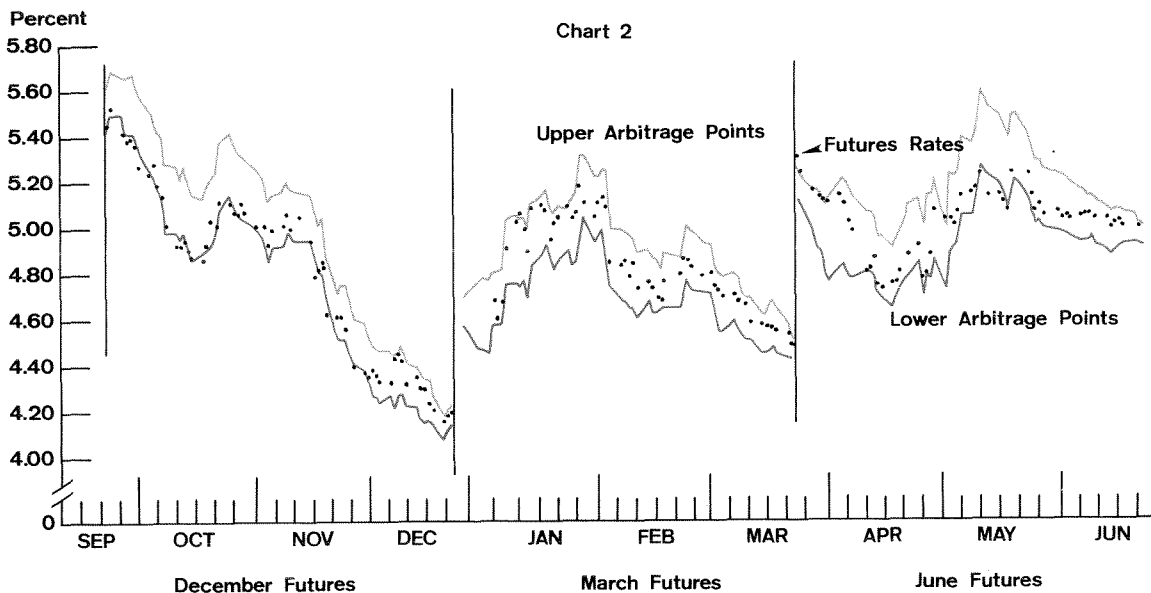
Each  $n$ -day bill will be worth 100 upon maturity in  $n$  days; thus a  $(q_{n,t} - 0.15)/100$  fractional  $n$ -day bill must be purchased at time  $t$  to provide the cash needed at time  $t+n$ . For investors other than dealers, the purchase price of an  $n$ -day bill is the dealers' asked price,  $P_{n,t}^a$ , which is related to the asked yield by

$$P_{n,t}^a = 100 - \frac{n}{360} R_{n,t}^a$$

Thus, the cash needed at time  $t$  is that required to buy the fractional bill at the price of  $P_{n,t}^a$  per bill plus the amount needed for the futures contract margin requirement and commission, or \$0.15 and \$0.006 per \$100. Thus, the total cash requirement at time  $t$  is

$$\left( \frac{q_{n,t} - 0.15}{100} \right) P_{n,t}^a + 0.15 + 0.006.$$

The cash requirement at time  $t$  is to be raised by selling a fractional part,  $X$ , of the  $n+91$ -day



bill already owned. If this fraction is less than one, then the arbitrage operation will be profitable. The purchase of the  $n$ -day bill and the futures contract package will produce \$100 in  $n+91$  days. Simply holding the  $n+91$ -day bill will also produce \$100 in  $n+91$  days. Thus, if the arbitrage transaction requires that a fraction less than one of the  $n+91$ -day bill be sold, then the fraction  $1-X$  of an  $n+91$ -day bill will be a risk-free arbitrage profit.

From these considerations, the fraction,  $X$ , of  $n+91$ -day bills selling at price  $P_{n+91,t}^b$  sold must be such that

$$X P_{n+91,t}^b = \left( \frac{q_{n,t} - 0.15}{100} \right) P_{n,t}^a + 0.15 + 0.006.$$

Dividing through by  $P_{n+91,t}^b$  defines  $X$ ; arbitrage is profitable if  $X < 1$ , or in yield terms,

$$(1) \quad F_{n,t} > \left( 1 - \frac{n}{36000} R_{n,t}^a \right)^{-1} \left[ \frac{n+91}{91} R_{n+91,t}^b - \frac{n}{91} R_{n,t}^a + \frac{360}{91} (0.006) + \frac{n}{91} (0.0015 R_{n,t}^a) \right].$$

The right-hand side of the inequality (1) defines the upper critical point for profitable arbitrage. The expression has been written so that the components due to explicit transactions costs in the futures market—the terms involving 0.006 and 0.0015—may be clearly identified.

It may also be noted that without the two futures market transaction-cost terms, the right-hand side of (1) defines the implicit forward rate of interest in the term structure calculated from the bid yield on the  $n+91$ -day bill and the asked yield on the  $n$ -day bill. In the example being discussed, the implicit forward rate is the rate of interest that would have to be earned on a 91-day bill to be issued at time  $t+n$ , so that the total yield over  $n+91$  days would be the same on an  $n+91$ -day bill and on an  $n$ -day bill with the pro-

ceeds invested on maturity in a 91-day bill. The yield on a 91-day bill is, of course, unknown before the bill is issued, but the investor can (if desired) lock in a known yield by buying a bill futures contract. He can also lock in that yield implicitly by buying an  $n+91$ -day bill, provided he is willing to lock in the package combination of the equivalent of an  $n$ -day bill and the 91-day bill to be issued at time  $t+n$ .

From a similar line of reasoning, the lower critical point may be defined. A risk-free arbitrage opportunity exists if

$$(2) \quad F_{n,t} < \left( 1 - \frac{n}{36000} R_{n,t}^b \right)^{-1} \left[ \frac{n+91}{91} R_{n+91,t}^a - \frac{n}{91} R_{n,t}^b - \frac{360}{91} (0.006) - \frac{n}{91} (0.0015 R_{n,t}^b) \right].$$

The right-hand side of (2) defines the lower critical point for profitable arbitrage.

The critical points defined by (1) and (2) have been calculated from daily data for the period from January 6, 1976 to June 23, 1977, and plotted as solid lines in Charts 1 and 2.<sup>3</sup> The futures quotes are plotted as dots in the charts.

The charts suggest that profitable arbitrage opportunities rarely exist, and when they exist are small in magnitude. This finding is especially significant because only explicit costs were included in the calculation of the arbitrage points—no allowance was made, for example, for the labor time of the arbitrageur—and perfect divisibility was assumed.

Two other features stand out in the charts. First, there appears to be a tendency for the futures rate to fall closer to the lower than the upper arbitrage point, especially in the first month plotted for each contract. Second, there seems to be a tendency for the futures rate to fall in the last month of trading for each contract. These observations are directly related to the nature of term premiums in interest rates for securities of various maturities.

## II. Term Premiums and Bid-Asked Spreads<sup>4</sup>

It is now generally agreed that longer-term securities have systematically higher yields than shorter-term securities, the differences being labeled "term premiums," or "liquidity premiums."

The existence of term premiums had been widely assumed, and so recent empirical findings have seemed to confirm the theoretical expectation that risk aversion would cause longer-term secu-

rities to sell at higher yields on the average than shorter-term securities.

To this author's knowledge, however, the relationship of transactions costs to term premiums has never been carefully investigated. The data used in previous studies of the term structure have consisted either of points drawn free-hand through yield observations—the Durand and Treasury *Bulletin* yield curves—or means of bid and asked yields. Given the significant size of bid-asked spreads—especially for short-term securities—it is clear that transactions costs need to be examined carefully.

The second and third columns of Table 1 suggest that transactions costs may be related to estimated term premiums. These two columns are reproduced from Tables 5-3 and 6-12 in Richard Roll's study of the Treasury bill market.<sup>5</sup> (The other column in Table 1 will be discussed later.) The sharp drop in Roll's estimated marginal term premium—the average difference between the one-week implicit forward rate  $m$  weeks in the future and the one-week spot rate realized in  $m$  weeks—between the 13- and 14-week maturities appears to be suspiciously related to the sharp increase in the mean spread between the same two maturities. Before discussing this issue further, however, a review of some of the a priori arguments concerning term premiums will prove helpful.

As a matter of *arithmetic*, a given change in yield to maturity produces a larger change in the price of a longer-term security than in the price of a shorter-term security. As a matter of *fact*, long-term yields do not fluctuate as much as short-term yields, but the relative variability of long-term and short-term yields is such that the prices of long-term securities nevertheless fluctuate more than the prices of short-term securities; thus, the capital values of long-term securities are subject to more interest rate risk. If we assume that investors are risk averse, we would expect that the average yield on long-term securities will have to be higher to compensate investors for the greater risk.

Another argument suggesting the probable existence of term premiums depends on transactions costs. Consider the situation faced by a firm that temporarily has excess cash which it will

**Table 1**  
**Bid-Asked Spreads and Term Premiums**

Weeks to Maturity	Mean Spread <sup>a</sup>	Term Premium	
		Marginal <sup>b</sup>	Average <sup>c</sup>
1	.2336	0	0
2	.1762	.00704	.00352
3	.1486	.0555	.0208
4	.1288	.168	.058
5	.1121	.291	.104
6	.0993	.323	.141
7	.0893	.347	.170
8	.0813	.383	.197
9	.0753	.445	.224
10	.0695	.427	.245
11	.0649	.396	.258
12	.0580	.414	.271
13	.0424	.562	.294
14	.0843	.0403	.276
15	.0835	.0696	.262
16	.0831	.142	.254
17	.0822	.175	.250
18	.0810	.189	.246
19	.0788	.256	.247
20	.0762	.262	.248
21	.0734	.296	.250
22	.0710	.305	.252
23	.0681	.310	.255
24	.0620	.328	.258
25	.0555	.365	.262
26	.0415	NA	NA

NA: Not Available.

<sup>a</sup> Weighted (by number of observations) averages of mean spreads for March, 1959-December, 1961 and January, 1962-December, 1964 reported in Roll, R., *The Behavior of Interest Rates*, Table 5-3.

<sup>b</sup> For March, 1959-December, 1964, from Roll, Table 6-12.

<sup>c</sup> For maturity  $m$ , mean of marginal term premiums for maturities 1, 2, ...,  $m$ .

need in  $m$  days. The firm could buy an  $m$ -day Treasury bill, which would mature just when the cash was needed.<sup>6</sup> Alternatively, the firm could buy a longer-term security and then sell it in  $m$  days. A firm that is not risk averse would compare the yield on the  $m$ -day bill with the expected yield over  $m$  days from buying an  $n$ -day bill, where  $n$  is larger than  $m$ , and selling it after  $m$  days. This yield would have to be calculated from the asked price of the  $n$ -day bill and the expected bid price of an  $n$ - $m$  bill in  $m$  days.

Letting  $P_{k,t}$  be the price at time  $t$  of a bill with  $k$  days to maturity, the continuously compound-

ed yield to maturity is

$$R_{k,t} = \frac{365}{k} (\log 100 - \log P_{k,t}).$$

The expected continuously compounded holding-period yield  ${}_nH_{m,t}^*$  from buying an  $n$ -day bill at the asked price  $P_{n,t}^a$  and selling it  $m$  days later at the expected bid price  $P_{n-m,t+m}^{b*}$  is

$$\begin{aligned} {}_nH_{m,t}^* &= \frac{365}{m} (\log P_{n-m,t+m}^{b*} - \log P_{n,t}^a) \\ &= \frac{n}{m} R_{n,t}^a - \frac{n-m}{m} R_{n-m,t+m}^{b*} \end{aligned}$$

The firm needing cash in  $m$  days will buy an  $m$ -day bill rather than an  $n$ -day bill if  $R_{m,t}^a > {}_nH_{m,t}^*$ . Using the definition of the bid-asked yield spread  $S_k$  on a bill with  $k$  days to maturity as the difference between the bid and asked yields, this inequality yields the expression

$$(3) \quad R_{m,t}^a > \frac{n}{m} R_{n,t}^a - \left(\frac{n-m}{m}\right) (R_{n-m,t+m}^{a*} + S_{n-m,t+m}^*).$$

A particularly convenient interpretation of inequality (3) arises for  $n = 2m$ . In this case, we have

$$(4) \quad R_{m,t}^a > 2R_{2m,t}^a - (R_{m,t+m}^{a*} + S_{m,t+m}^*).$$

Suppose interest rates on particular maturities are not expected to change so that  $R_{m,t+m}^{a*} = R_{m,t}^a$ , and suppose that bid-asked spreads for given maturities are constant over time so that  $S_{m,t+m}^* = S_m$ .<sup>7</sup> Then we can write (3) as

$$(5) \quad R_{m,t}^a > R_{2m,t}^a - \frac{1}{2} S_m.$$

Letting  $R_{k,t} = \frac{1}{2} (R_{k,t}^b + R_{k,t}^a)$

$$= R_{k,t}^a + \frac{1}{2} S_{k,t},$$

(5) may be written in terms of yields defined as the means of bid and asked yields.

$$(6) \quad R_{m,t} > R_{2m,t} - \frac{1}{2} S_{2m}.$$

Since bid-asked mean yields have typically been employed in term structure studies (including Roll's), (6) is in a form that relates the present argument to previous work. The typical finding that short-term rates are on the average below long-term rates is consistent with (6) provided that the yield differential is not excessive compared to the spread. The average difference between the yield to maturity on an  $m$ -week bill and the yield on a one-week bill is the average of the marginal term premiums for maturities 2, 3, ...,  $m$ . This average term premium, calculated from Roll's estimates of marginal term premiums, is reported in Table 1. Using these estimates of average term premiums for various maturities and the estimated spreads in Table 1, inequality (6) is found to hold for maturities of 1, 2, 9, 10, 11, and 12 weeks but not for maturities of three through eight weeks.

It is interesting to note that Roll found the hypothesis of market efficiency well-supported except for maturities of 4 to 8 weeks.<sup>8</sup> For these maturities yields seem to be too low, on the average. We may conjecture, however, that the apparent anomaly would disappear with a fuller accounting of transactions costs.

A few numbers will provide a feel for the magnitudes involved. From inequality (6), the yield on a four-week bill is too low by about 10 basis points according to Roll's evidence. (A basis point is .01 percent.) Comparing the two sides of inequality (3) and using the fact that  $R_m^{a*} = R_m^a - \frac{1}{2} S_m$ , this 10 basis point discrepancy makes the right-hand side of (3) larger than the left-hand side by about 20 basis points. The firm with cash to invest for four weeks could, therefore, have a 20 basis points advantage on the average from investing in an eight-week bill (which would be sold after four weeks) rather than in a four-week bill.<sup>9</sup> These yields, however, are all expressed at annual rates. The yield advantage per four weeks is only 4/52 of 20 basis points, or about \$154 per million of invested funds. It is easy to imagine that the extra transactions costs from buying an eight-week bill and selling it four weeks later as compared to simply buying a four-week bill and holding it to maturity would exceed \$154 per million of funds invested.

The analysis of the transaction-cost effect in

depressing yields on very short-term bills is, however, only indirectly relevant to the issue of the size of term premiums in bill futures-market quotes. We need to know whether the term premium should be subtracted in order to interpret the futures quotes as reflecting market expectations of future spot rates on three-month bills; the fact that a one-week bill has an average yield below that on a 13-week bill is not directly relevant to this issue.

The transaction-cost argument suggests that yields on very short-term bills could be depressed without there being any noticeable impact on longer-term bills. For example, in comparing the yield from holding a 13-week bill to maturity with the yield from holding a 26-week bill for 13 weeks, the bid-asked yield spreads are small enough, and the 13-week holding period long enough, so that there is little room for the average 13-week bill yield to fall below the average 26-week bill yield. From inequality (6) and the mean spread on 26-week bills (Table 1), the effect would be only two basis points.

Nevertheless, the transaction-cost effect on very short-term bills can affect term premiums (as estimated in previous studies) because of the way in which implicit forward rates are calculated. To understand the argument, consider first the expression defining the implicit forward rate of interest on a 13-week loan to begin  $m$  weeks in the future, calculated from the yields to maturity on spot bills with  $m$  and  $m+13$  weeks to maturity. Using continuously compounded yields,

$$(7) \quad {}_{13}r_{m,t} = \left(\frac{m+13}{13}\right) R_{m+13,t} - \frac{m}{13} R_{m,t},$$

where  ${}_{13}r_{m,t}$  is the implicit forward rate as of time  $t$  on a 13-week loan to begin in  $m$  weeks and  $R_{k,t}$  is the yield to maturity on a spot bill with  $k$  weeks to maturity. On the average, the yield on a  $k$ -week bill exceeds the yield on a one-week bill by the average term premium  $\bar{L}_k$ . Thus, on the average we have

$$(8) \quad {}_{13}r_{m,t} = \left(\frac{m+13}{13}\right) (R_1 + \bar{L}_{m+13}) - \frac{m}{13} (R_1 + \bar{L}_m) \\ = R_1 + \left(\frac{m+13}{13}\right) \bar{L}_{m+13} - \frac{m}{13} \bar{L}_m$$

$$= R_1 + \frac{1}{13} \sum_{j=m+1}^{m+13} L_j.$$

The third line in equation (8) is derived from the definition

$$\bar{L}_k = \frac{1}{k} \sum_{j=1}^k L_j.$$

Each  $L_j$ , it may be recalled, is the marginal term premium—the amount by which the implicit forward rate on a one-week loan to mature  $k$  weeks in the future exceeds the realized spot rate on a one-week loan maturing  $k$  weeks in the future.

The summation term in (8) contains 13  $L_j$ 's. If the  $L_j$ 's were nondecreasing so that  $L_{j+1} \geq L_j$ , then

$$\frac{1}{13} \sum_{j=m+1}^{m+13} L_j \geq \frac{1}{13} \sum_{j=1}^{13} L_j = \bar{L}_{13}.$$

In this case the implicit forward rate  ${}_{13}r_{m,t}$  would be an upward biased estimate of  $R_{13} = R_1 + L_{13}$ .

Roll's estimates of the  $L_j$ , however, are not nondecreasing for all  $j$ . When the summation term in (8) is calculated using Roll's estimates it is found that  ${}_{13}r_m$  is an upward biased estimate of  $R_{13}$  for  $m$  from 1 to 7 weeks but a downward biased estimate for  $m$  from 8 to 12 weeks. The maximum size of the downward bias is about four basis points and the maximum size of the upward bias is about two basis points. While the size of the upward bias is very small based on Roll's estimates, the phenomenon may help to explain the appearance in the charts of a decline in the implicit forward rates underlying the arbitrage points in the last month of trading of a futures contract.

McCulloch provides another term-structure study of direct relevance to this issue.<sup>10</sup> Using somewhat different estimation methods than Roll and a sample period from March 1951 to March 1966, McCulloch reports estimates of the term premium attached to implicit forward 13-week rates various periods in the future (Table 2). If these estimates are taken at face value, 10 to 20 basis points should be subtracted from implicit forward rates for 13-week bills one or more months in the future to obtain market expecta-



**Table 2**  
**McCulloch Estimates of Term Premiums**  
**in 13-week Implicit Forward Rates**

Bill to be Issued in:	Term Premium	
	Free Form Estimates	Exponential Form Estimates
1 month	0.10	0.09
2 months	0.15	0.14
3 months	0.16	0.17
6 months	0.12	0.21
9 months	0.11	0.22
1 year	0.13	0.22
2 years	0.12	0.22

tions of future spot rates on 13-week bills.

These estimates of term premiums are above those relevant for the bill futures market if the argument on transaction costs is accepted, because transaction costs affect implicit forward rates calculated from spot bills of varying maturities but not the bill futures market. If this argument is correct, quotes in the futures market should generally be below the corresponding implicit forward rates.

This hypothesis was tested by calculating the mean futures rate and the mean implicit forward rate over the three-month period preceding the maturity date of the six futures contracts maturing between January, 1976 and June, 1977 (Table 3).<sup>11</sup> In every case the mean of the rate on a given futures contract is below the corresponding mean of the implicit forward rate calculated from bid-asked mean rates. The means of the up-

per and lower arbitrage points are also reported, although it is obvious from the charts that the futures rate almost always lies between the two arbitrage points.

A test of the statistical significance of the results in Table 3 is reported in Table 4. The test has been confined to the first 20 observations in each of the periods listed in Table 3, since there is much more interest in market forecasts of the bill rate a few months in the future than in forecasts a few weeks in the future. For the first 20 trading days in each period, the difference between the futures rate and the implicit forward rate was calculated; the means and standard deviations of these differences appear in Table 4 along with the statistic for testing the statistical significance of the mean difference. The mean difference is negative for all periods. Using a one-tailed t-test, the mean differences for the first, second, and fourth periods are significant at the .001 level, the third period at about the .02 level, the fifth period at almost the .05 level, and the last period at about the .15 level. From these results for the individual periods, it is obvious that, in the pooled sample for the six periods combined, the mean is statistically different from zero at a very high level of statistical significance.

The evidence suggests that yields on very short maturities are depressed by the existence of transaction costs. Investors depress the return on very short-term bills when they attempt to obtain a return on balances invested for only a few weeks' time. The return is apparently slightly lower than can be explained by the bid-asked spreads on longer-term bills, but not by much.

**Table 3**  
**Means of Futures Rates, Implicit Forward Rates,**  
**and Arbitrage Points for Selected Periods**

Period	Futures Contract	Futures Rate	Implicit Forward Rate		Arbitrage Point		
			Bid- Asked		Lower	Upper	
			Mean	Bid			Asked
1/6/76-3/17/76	March 1976	5.10	5.17	5.21	5.13	5.02	5.32
3/24/76-6/23/76	June 1976	5.48	5.54	5.57	5.50	5.38	5.69
6/24/76-9/22/76	Sept. 1976	5.42	5.45	5.49	5.42	5.34	5.57
9/23/76-12/22/76	Dec. 1976	4.84	4.95	4.98	4.92	4.84	5.05
12/23/76-3/23/77	March 1977	4.82	4.85	4.87	4.82	4.73	4.96
3/24/77-6/22/77	June 1977	5.05	5.11	5.12	5.10	4.99	5.23

**Table 4**  
**Futures and Implicit Forward Rates**  
(Differences, first 20 observations each period)

Differences	1/6/76-	3/24/76-	6/24/76-	9/23/76-	12/23/76-	3/24/77-
	3/17/76	6/23/76	9/22/76	12/22/76	3/23/77	6/22/77
Mean, $\bar{X}$	-0.1345	-0.0505	-0.0285	-0.1590	-0.0405	-0.0205
Standard Deviation, S	0.0788	0.0511	0.0584	0.0397	0.1079	0.0894
Test Statistic, $ \bar{X}/S\sqrt{20} $	7.63	4.42	2.18	17.91	1.68	1.03

The term premiums involved, however, do not in any event extend very far into the yield structure. Beyond maturities of about 13 weeks, the average term structure is essentially flat.

Quotes on the nearest maturity in the bill fu-

tures market can, therefore, be interpreted for all practical purposes as the market's unbiased estimates of the future spot rates on 13-week bills. The policy significance of this finding will now be explored.

### III. Policy Implications of T-Bill Futures

The evidence discussed above shows that for the nearest bill futures maturity there is a close correspondence between the futures rate and the implicit forward rate calculated from spot rates. If this finding also applies to the other bill futures maturities—and in this section it will be assumed that the finding does apply to all maturities—then it is clear that the opening of the bill futures market did not provide policymakers with much new information. Nevertheless, the futures rates, by displaying investors' expectations of future spot rates on 13-week bills explicitly, have focused attention on these expectations in a way implicit forward rates never did.

Since the start of trading in bill futures in January, 1976 the rates on more distant futures have always been higher than the rates on near futures; investors have been expecting spot bill rates to rise over time. As of this writing—early April, 1978—realized bill rates have been almost always below prior expectations as measured by rates on the more distant futures contracts. For example, on January 30, 1976 the futures rates for March, June, September, and December, 1976 were 4.89, 5.33, 5.64, and 5.86 percent, respectively.<sup>12</sup> The realized bill rates on the maturi-

ty of these futures were 4.97, 5.32, 5.01, and 4.25 percent, respectively. For a more recent example, on September 30, 1976 the futures rates for December, 1976, March, June, September, and December, 1977, and March, 1978 were 5.37, 5.71, 6.07, 6.44, 6.77, and 7.10 percent, respectively, whereas the realized spot rates were 4.25, 4.52, 5.00, 5.85, 5.96, and 6.22 percent, respectively.

If the findings in the previous section apply to all futures maturities, then the differences between the futures rates and the realized spot rates over the last two years reflect genuine expectational errors rather than term premiums attached to the futures rates. A variety of interpretations of these expectational errors is possible.

One starting point would be a hypothesis concerning the relationship between economic activity and inflation on the one hand and the spot bill rate on the other. It is generally argued that higher levels of economic activity add to the demand for funds to finance business inventories, purchases of consumers' durables, and so forth, and so tend to raise interest rates. Higher rates of inflation also tend to raise interest rates. Expectational errors, therefore, could have occurred if

economic activity and the inflation rate had been below investors' anticipations. This explanation seems not very satisfactory, however, because the performance of the economy over the past two years has, if anything, been slightly stronger than earlier forecasts had suggested likely.

Another possible explanation of expectational errors emphasizes the influence of government policy on interest rates. In the short run, accelerated money growth probably tends to depress interest rates, and slower money growth to raise interest rates. If money growth is higher than anticipated, interest rates will tend to be lower than anticipated. Similarly, since government budget deficits require financing, smaller-than-anticipated budget deficits will tend to lead to lower-than-anticipated interest rates. Interpretation of the interest-rate effects of monetary policy is complicated, however, by the fact that higher money growth in the long-run raises the rate of inflation and, therefore, raises interest rates. It is not known exactly where the dividing line in time lies between the short-run effect of depressing interest rates and the long-run effect of raising interest rates.

The explanation for recent expectational errors that emphasizes errors in anticipating government policy fits the facts better than the explanation based on the performance of the economy. Money growth on the  $M_1$  definition was higher in 1976 than in 1975, and higher in 1977 than in 1976; on the  $M_2$  definition, money growth was higher in 1976 than in 1975, but lower in 1977 than in 1976.<sup>13</sup> And the total government-budget deficit—federal, state and local government combined—has been lower than anticipated by many observers because of below-budget federal spending and surprisingly large state-and-local budget surpluses.<sup>14</sup>

A third explanation—one consistent with much recent discussion—is that the demand for money may have declined over the past several years. Especially on the  $M_1$  definition, money growth in 1975 and 1976 was much slower than would have been anticipated given the observed changes in income and interest rates. Or, viewed another way, interest rates were much lower than would have been anticipated given the observed growth in  $M_1$  and income. From the point of

view of a bill futures market participant in early 1976, the consensus forecast for income growth and the Federal Reserve's announced money growth targets implied, from the existing evidence on money demand relationships, higher interest rates than were in fact realized.

While this brief discussion may or may not be a correct analysis of the interest rate expectational errors of the past two years, it serves to introduce the nature of the problem faced by policymakers in interpreting the interest rate forecasts incorporated in T-bill futures rates. The key problem faced by policymakers is that of assessing the significance of market interest rate forecasts that differ from the policymaker's own forecasts.

Suppose, for example, that T-bill futures rates are higher than policymakers' forecasts of future interest rates. One possibility is that the market is anticipating a higher level of economic activity and/or a higher inflation rate than policymakers are anticipating. It is especially important to consider this possibility, because the market forecasts incorporated in bill futures rates reflect more than simply the interest-rate guesses of speculators. Firms may enter the bill futures market on the basis of their anticipated cash flows arising, for example, from the expected effects of current plans or commitments to accumulate inventories.

This type of activity in the bill futures market is similar to that in commodity futures markets; the wheat futures price, for example, reflects expected demands for wheat by bakeries and supplies of wheat by farmers. Trading in this market, therefore, reflects the impact of current decisions—bread supply commitments by bakeries and planting decisions by farmers—that will affect wheat supplies and demands and, therefore, wheat prices in the future.

If policymakers' forecasts of interest rates below those in the bill futures market do reflect mistaken forecasts by policymakers of the future strength of aggregate demand, then their decisions may provide for a more expansionary policy than is appropriate. The accuracy of the economic forecasts available to policymakers is not so high that the possibility that high futures rates are forecasting higher levels of economic activity and/or higher inflation can be ignored.

An even more troubling possibility, though, is that rates in the bill futures market may reflect anticipations concerning policy decisions that do not reflect actual policy plans. Failure of policy decisions to ratify private anticipations concerning policy then falsifies one of the assumptions under which business decisions are made and leads to less appropriate business decisions than would otherwise be the case.

To avoid private expectational errors, policymakers must provide clear information, through formal announcements or otherwise, concerning prospective policies. And if statements concerning policy intentions are to be believed, policymakers must in fact determine policy in accordance with those announced intentions. If policies typically do not reflect previously announced policy intentions, then statements of policy intent will simply not be believed. Business planning will be subject to unnecessary uncertainty, but so also will policy planning. To interpret current economic data in such a situation, policymakers will have to guess what businessmen are guessing the policymakers will do.

An apparently easy solution to this problem would be for policymakers to make clear announcements of their policy plans and then to ensure that these plans are realized. Under this approach, however, policy could not be adjusted in a flexible and timely manner when economic conditions change unexpectedly. The policy dilemma is clear. To encourage sound and sensible business planning, policymakers need to make their plans clear and must realize their plans to retain credibility. But policy plans should, presumably, be adjusted from time to time to reflect changing economic conditions.

Different policy analysts place differing degrees of emphasis on the relative importance of realizing policy plans and of retaining policy flexibility. Unfortunately, there is no simple way of determining how to strike a balance between

those two goals. What can be done, though, is to broaden the concept of the announced policy plan by making clear the nature of the policy responses to be expected under specified conditions. It is well understood, for example, that the Federal Reserve will intervene heavily to stabilize money markets disrupted by a spectacular bankruptcy such as the Penn-Central failure in 1970, even if such intervention produces a temporary surge in money growth far above what had been planned.

But it is important to distinguish between specific intervention of this type and a more generalized intervention to cushion interest-rate increases. An excellent example of the benefits of *not* cushioning interest-rate increases occurred in April 1977, when M<sub>1</sub> increased at a 20-percent annual rate (since revised to 14 percent). That episode raised fears in the markets that the Federal Reserve was permitting money to expand at a rate far above its announced policy intentions. By permitting short-term interest rates to rise sharply at that time—the 13-week bill rate went from 4.57 percent in the week ending April 1 to 5.06 percent in the week ending May 27—the Federal Reserve convinced the markets that money growth would not be permitted to continue at clearly excessive rates.

While the rate on 13-week bills was rising in May 1977 rates on the more distant bill futures fell. Comparing weekly average rates for the week ending April 1 to weekly average rates for the week ending May 27, the September 1977 futures went from 5.88 to 5.65, the March 1978 futures from 7.03 to 6.62, and the September 1978 futures from 7.83 to 7.22. In this situation, expanding the rate of money growth even further to hold down the rate on 13-week bills might very well have led to heightened fears of future inflation which would have raised rates in the futures market.

## IV. Summary and Conclusions

The evidence reviewed in this paper demonstrates that the Treasury-bill futures market is closely linked to the spot market in Treasury bills. Unexploited arbitrage opportunities between the two markets rarely exist.

A key question is whether term premiums must be subtracted from T-bill futures rates to convert those rates into market forecasts of future spot rates on Treasury bills. A review of evidence on term premiums from previous studies suggests that very short-term bills trade at lower yields than longer-term bills on the average but that much, and perhaps all, of the average yield differential probably reflects the extra transactions costs from selling longer-term bills before maturity compared to holding very short-term bills to maturity. Because transactions costs in trading bill futures are so very small, futures rates were hypothesized to be slightly lower than the forward rate implicit in the yields on spot bills of various maturities. This hypothesis is supported by the evidence presented in this paper.

What is the policy significance of the new market in Treasury bill futures? The existence of these explicit market interest-rate forecasts emphasizes the need for policymakers to understand the reasons for discrepancies between their own interest-rate forecasts and market interest-rate forecasts. If, at some point in time, rates in the bill futures market are based on forecasts of a stronger and/or more inflationary economy than projected by policymakers, and if the market is correct, then there is a danger that policymakers

will determine a more expansionary policy than is appropriate for the needs of the economy.

Market interest-rate forecasts may also reflect forecasts of policies that differ from those that policymakers are actually planning. This possibility emphasizes the importance of policymakers making their plans known and maintaining credibility by ensuring that announced policy plans are realized. However, strict adherence to policy plans makes it difficult for policy to be adjusted flexibly in response to changing circumstances.

While there is no easy solution to this dilemma, the problems raised can be eased by including in the concept of a policy plan an understanding of the policy adjustments required by certain contingencies. For example, permitting temporarily high money growth to cushion market disruptions caused by a major bankruptcy, such as the Penn-Central failure, need not imply that long-run plans for money growth will not be realized.

Although the accuracy of the bill futures rates as predictors of future spot rates was not discussed in detail, it is clear that futures rates, even if unbiased, are not especially accurate forecasts. For this reason the policy significance of these interest rate forecasts ought not to be exaggerated. However, the policymakers' own forecasts of interest rates are not very accurate either. Unless policymakers have solid evidence that their own forecasts are more accurate than market forecasts, they cannot afford to ignore the T-bill futures market.

### FOOTNOTES

1. See, for example: Albert E. Burger, Richard W. Lang, and Robert H. Rasche, "The Treasury Bill Futures Market and Market Expectations of Interest Rates," **Federal Reserve Bank of St. Louis Review**, June, 1977; Wallace H. Duncan, "Treasury Bill Futures—Opportunities and Pitfalls," **Federal Reserve Bank of Dallas Review**, July, 1977; Paul L. Kasriel, "Hedging Interest Rate Fluctuations," **Business Conditions** (Federal Reserve Bank of Chicago), April, 1976; and Linda Snyder, "How to Speculate on the World's Safest Investment," **Fortune**, July, 1977.

2. The calculations discussed below are based on the assumptions that the \$60 commission is paid when the futures position is taken and that the \$1500 margin is put up in cash. In fact, the commission may in some cases be paid when the futures position is covered and the margin requirement may be satisfied by putting up interest-bearing securities. In addition, futures price fluctuations may lead to a requirement that additional cash or securities be added to the margin account or may permit some

cash or securities to be withdrawn from the margin account. Because the amounts involved are so small, these considerations would have a negligible effect on the arbitrage calculations presented below and so are ignored.

3. The data base consists of closing bid and asked yields on bills, and closing futures quotes—all from the **Wall Street Journal**.

4. This section is somewhat technical and may be skipped by the reader primarily interested in the policy implications of the bill futures market.

5. Richard Roll, **The Behavior of Interest Rates** (New York: Basic Books, 1970).

6. Treasury bills, of course, do not mature every day. The firm wanting to invest in a maturing bill would have to select the existing bill with maturity best matching the firm's predicted cash needs. The following analysis ignores the fact that purchase of a bill with more than  $m$  days to maturity permits the firm to keep its

funds invested right to the day its cash needs arise, since an existing bill can be sold on any business day.

7. Rather than interpreting equation (4) as applying to a time when rates are not expected to change, the rates in (4) may be interpreted as the means of the rates over a long sample period in which there is no overall trend in the level of rates. The means of  $R_{m,t+m}^a$  and  $R_{m,t}^a$  differ only by virtue of one observation at each end of the sample.

8. See Roll, p. 116.

9. That the holding period yield advantage is greater than the discrepancy in yields to maturity can be seen readily from the fact that the yield to maturity,  $R_n$ , on an  $n$ -week bill is the weighted average of the yield over the first  $m$  weeks and the yield over the remaining  $n-m$  weeks. If the latter yield is below  $R_n$ , then the former yield must be above  $R_n$ .

10. J. Huston McCulloch, "An Estimate of the Liquidity Premium,"

*Journal of Political Economy*, 83 (February, 1975), 95-119.

11. The yields in Table 3 are bankers' discount yields. The implicit forward rates were calculated with due regard for discounting considerations.

12. The two longer futures contracts, March and June 1977, were not actively traded in the first several months after the futures market opened.

13. Measuring money growth from December of one year to December of the next,  $M_1$  growth was 4.1 percent in 1975, 6.1 percent in 1976, and 7.7 percent in 1977, while  $M_2$  growth was 8.5, 11.4, and 9.2 percent, respectively.

14. See Edward M. Gramlich, "State and Local Budgets the Day after It Rained: Why Is the Surplus So High," in Arthur M. Okun and George L. Perry, eds., *Brookings Papers on Economic Activity*, 1978:1, 191-214.

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