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Index Numbers and the Measurement of Real GDP

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The measures of real GDP and inflation are aggregates of many individual prices and quantities. These variables are measured using fixed-weight indexes, which can give a misleading impression of price and output changes in a particular year if the structures of output and relative prices are different from those in the base year. This measurement problem adds to the uncertainties facing policymakers.

These ambiguities result from the definitions of output and inflation in use. This article describes alternative measures of growth and inflation that have a stronger theoretical basis and avoid these ambiguities. Operational versions of these measures will be introduced by the Bureau of Economic Analysis in 1992. These new measures will remove one source of uncertainty facing policymakers.

The Bureau of Economic Analysis (BEA), a division of the Commerce Department, is responsible for preparing and publishing estimates of the gross domestic product (GDP), the most comprehensive measure of our economy's total output.[1] Most commentators take it for granted that these BEA estimates of GDP represent objective measures of the nation's output. They assume, in other words, that there is a "correct" measure of output that could be computed exactly if sufficient information were available and that the GDP data issued by the BEA represent the best available estimate of this "correct" measure. In fact, however, these measures of real GDP are subject to an inherent arbitrariness known as the "index number problem."

This problem arises because the nation's total output consists of a huge number of individual goods and services. Measures of real GDP are constructed as an aggregate of these separate components and so depend on the method of aggregation used and the weights assigned to the individual components. Last December, the BEA released revised GDP estimates that, among other changes, altered these weights. These revised data suggest that the cyclical downturn in the winter and spring of 1990-91 was somewhat more severe than reported earlier.

Measures of the average price level encounter the same problem. Price index numbers, such as the GDP fixed-weight price index or the consumer price index, are weighted averages of the prices of individual goods and services. When the prices of some items change more than those of others, the value of such an index depends on the weights attached to these prices.[2]

This article discusses a number of issues raised by these measurement problems. It examines the extent to which existing methods of data construction might introduce systematic biases into the numbers. Because of the arbitrariness inherent in existing measures of output and prices, a number of alternative procedures are described that have a stronger theoretical basis. The BEA plans to introduce one such alternative approach to measuring output and prices in 1992.

The plan of the paper is as follows. In Section I the index number problem is described and illustrated. Sections II and III explain two alternative approaches to measuring the nation's output and price level that avoid the arbitrariness of the existing measures. In the first of these, the focus is on GDP as an indicator of the "standard of living" of the typical consumer, while the second emphasizes the "productivity" of the representative firm in converting factors of production ("inputs") into final products ("outputs"). Section IV discusses the recent benchmark revisions to the national accounts and describes alternative measures of GDP growth and inflation that the BEA plans to introduce later in 1992. These alternative measures are based on the theory of index numbers discussed in Sections II and III. Since these alternative indexes will be forms of a "chain index," this section also includes a brief discussion of this type of index number. Section V concludes.

I. Pitfalls in Measuring the Nation's Output

The nation's total output includes a vast array of different goods and services. The nominal gross domestic product (GDP) measures the aggregate of these individual components, with each item valued at the price at which it was sold to its final purchaser.[3] Thus, GDP may be viewed as the weighted sum of its component commodities, with their current prices serving as weights. Specifically, nominal GDP at date s may be written as:

$$NOMGDP_s = (p_{1s}q_{1s}) + (p_{2s}q_{2s}) + \dots + (p_{Ns}q_{Ns})$$

(1)

$$GDP_s = \sum_{n=1}^{N} p_{ns}q_{ns} \,.$$

It is natural to use prices as weights since, in a competitive, private enterprise economy, the amounts paid for commodities are good indicators of their usefulness (at the margin) to their purchasers. However, if the average level of prices increases (or decreases) over time, the change in nominal GDP includes the effects of this price change and so does not provide an accurate measure of the growth in real output.

A measure of real output may be obtained by valuing the output of each commodity at the price existing in some (arbitrarily selected) base year rather than at the price buyers actually paid. Operationally, the BEA calculates its estimates of real GDP at date s in date t prices by deflating each component of nominal GDP by the change in the price of that component from date t to date s:

$$\text{(2) REALGDP}_s = \left[\frac{(p_{1s}q_{1s})}{(p_{1s}/p_{1t})}\right] + \dots + \left[\frac{(p_{Ns}q_{Ns})}{(p_{Ns}/p_{Nt})}\right]$$

$$= \sum_{n=1}^{N} \frac{(p_{ns}q_{ns})}{(p_{ns}/p_{nt})}$$

$$= \sum_{n=1}^{N} p_{nt}q_{ns}$$

This means that the growth rate of real GDP from date s to date $s+1$ is a weighted average of the growth rates of its components:

$$\frac{\text{REALGDP}_{s+1} - \text{REALGDP}_s}{\text{REALGDP}_s} = \left[\frac{q_{1s+1}-q_{1s}}{q_{1s}} \cdot \frac{p_{1t}q_{1s}}{\sum_{i=1}^{N} p_{it}q_{is}}\right] + \dots$$

$$\text{(3)} \qquad \dots + \left[\frac{q_{Ns+1}-q_{Ns}}{q_{Ns}} \cdot \frac{p_{Nt}q_{Ns}}{\sum_{i=1}^{N} p_{it}q_{is}}\right]$$

$$= \sum_{n=1}^{N} \left[\frac{q_{ns+1}-q_{ns}}{q_{ns}} \cdot \frac{p_{nt}q_{ns}}{\sum_{i=1}^{N} p_{it}q_{is}}\right]$$

The weights, $p_{nt}q_{ns}/\sum_{i=1}^{N} p_{it}q_{is}$, are given by the expenditure shares of each component in GDP calculated at the *base-year* prices. This means that if the base period is changed, the weights, *and hence the measured growth rate of real GDP*, also will change. Between 1985 and 1991, real GDP was calculated with 1982 as the base year, but last December this was changed to 1987.

This procedure also means that real growth in a particular year is in many cases measured using relative prices ruling in the distant future or past. The most recent measures of real growth and inflation during the 1930s, for example, use the relative prices ruling a half-century later. The significant changes in relative prices over this period may introduce large biases into the data.

In constructing its estimates of real GDP, BEA breaks down nominal GDP (excluding the federal government) into 811 components, each of which is deflated separately by an appropriate price index (Young 1988, Table 5). Purchases of goods and services by the federal government are divided into no fewer than 17,000 components! Equations (2) and (3) show that not only the level but also the growth rate of measured real GDP depend on which year's prices are used in the process of aggregating the outputs of these 17,811 separate components.

As discussed in the accompanying Box, changing the base to a later date usually reduces the estimate of long-run

BOX

An Example of the Index Number Problem

For a simple illustration of the effect of a change in the base date on the measurement of real GDP, consider a hypothetical economy producing only two commodities, bread and wine. The top panel of the table shows the prices, quantities produced, and current-dollar values of these two goods in four successive years. Nominal GDP in this simple economy is the total value of the two goods. The middle panel of the table shows measures of real GDP in this economy using each of the four years as a base year. These are calculated by multiplying the quantities of each good by its price in the base year and summing the resulting values. Finally, the bottom panel shows the corresponding annual growth rates of real GDP. Over the four years, real GDP increases 102.9 percent when the base is year 1, but 95.8 percent when the base is year 4.

In this example, selecting a later year as the base period produces a lower growth rate than selecting an earlier year. This result arises because the good with the smaller increases in output over the four-year period (bread) was selected as the one with the larger increases in price. This feature of the example corresponds to the observation that buyers tend to substitute away from goods and services with the largest price increases and toward those with the smallest increases. As a result, the sectors of the economy that experience the largest increase in prices tend to be those with the smallest increases in real output. Since sectors are weighted by relative prices, moving to a later base date tends to increase the weights given to sectors with below average increases in output and to decrease the weights given to those with above average output growth. As a result, a later base date tends to produce lower estimates of average growth.[a]

The Index Number Problem in a Simple Economy

Data

Year	Price of Bread	Price of Wine	Quantity of Bread	Quantity of Wine	Value of Bread	Value of Wine	Nominal GDP
Y1	7	6	15	23	105	138	243
Y2	8	6	17	35	136	210	346
Y3	10	7	18	50	180	350	530
Y4	13	9	19	60	247	540	787

Levels of Real GDP

Year	Year 1 Base	Year 2 Base	Year 3 Base	Year 4 Base
Y1	243	258	311	402
Y2	329	346	415	536
Y3	426	444	530	684
Y4	493	512	610	787

Growth Rates of Real GDP

Year	Year 1 Base	Year 2 Base	Year 3 Base	Year 4 Base
Y1 to Y2	35.4	34.1	33.4	33.3
Y2 to Y3	29.5	28.3	27.7	27.6
Y3 to Y4	15.7	15.3	15.1	15.1
Y4 to Y1	102.9	98.4	96.1	95.8

[a]In terms of equation (3) in the text, components of GDP with weights, $p_{nt}q_{ns}/\sum_{i=1}^{N} p_{it}q_{is}$ that become larger when a later base date is chosen tend also to be those with low growth rates (for which $(q_{ns+1}-q_{ns})/q_{ns}$ is small).

real GDP growth. This is because buyers substitute away from goods and services with larger than average price increases in favor of items with smaller than average gains. As a result, sectors of the economy that grow slowly tend also to be those that have the largest price increases, and so have larger weights in real GDP if a later base date is chosen. Conversely, sectors that grow rapidly are generally those with the smallest price increases and so have smaller weights in real GDP if the base date is later.

The inverse relation between changes in sectoral prices and outputs implies that most relative price changes are the result of changes in costs on the *supply* side rather than of taste changes on the *demand* side. If most relative price changes were due to demand shifts, one would observe that the sectors with the largest increases in prices also would be those with the greatest increases in sales. Historically, this has not been the case, implying that supply shifts were more important than demand shifts in changing relative prices.

An example of this effect is that between 1977 and 1990, real GDP increased at an annual rate of 2.7 percent when measured in 1982 dollars but only 2.5 percent in 1987 dollars (see *Survey of Current Business* 1991). A major portion of the difference may be traced to the computer industry. The output of computers increased very rapidly during this period, while their prices fell sharply. As a result of the price decline, the measured contribution of this industry to overall growth is smaller if it is weighted by 1987 prices than if 1982 prices are used.

Similar revisions occurred on earlier occasions when the base date was changed (see *Survey of Current Business* 1976 and 1985). When the base date was shifted from 1972 to 1982, the estimated average annual growth rate of real GDP between 1972 and 1984 was reduced by 0.4 percentage points. This also was due largely to the changed weighting of the computer industry. The change in the base from 1958 to 1972 lowered the average annual growth rate from 1958 to 1974 by 0.2 percentage points. In this case, the main cause was the decreased weight assigned to the auto industry. Auto prices rose less than average prices and auto sales increased more than total GDP over this period.

Is There a "Correct" Measure of Real GDP?

The fact that a change in the base date produces a different measure of real GDP growth suggests that there is an arbitrary element to these measures that can never be fully eliminated. Whereas *nominal GDP* is an aggregate of transactions that actually occurred, *real GDP* is a statistical construct that represents the sum of a set of fictional transactions. Hence, nominal GDP could, in principle, be measured exactly if we had full and complete information from the original transactors, but there may be no clearly "correct" measure of real GDP, even with unlimited data. For analogous reasons, there may be no measure of the average level of prices that is obviously "correct".

A branch of microeconomic theory known as the *economic theory of index numbers* suggests that this conclusion may be too pessimistic. This theory indicates that if we are prepared to define precisely what we mean by a "correct" measure of GDP, it is possible to derive index-number formulae that measure the quantity and price of GDP with no arbitrary element. Initially, this theory was applied to the problem of defining a price index that would measure the "cost of living." Later it was extended to the definition of other price and quantity indexes.

II. Measuring the "Cost" and "Standard" of Living

Consider first the problem of measuring changes in the "cost of living." Suppose that in a particular base period, the representative consumer faces a given set of prices and buys a certain bundle of goods and services. In a subsequent period, she faces a different set of prices and chooses a different bundle of commodities. The problem is to determine how much the average price level (or "cost of living") changed between the two periods. The corresponding "quantity" problem is to determine how much larger (or smaller) the second commodity bundle is compared to the first (that is, how much her "standard of living" changed).

One way to measure the change in the average price level is to compute how much the base period commodity bundle would cost at the second-period prices. This is the procedure that underlies both the consumer price index and the fixed-weight GDP price index. These types of measures are known as Laspeyres indexes.[4] The drawback of this procedure is that it does not allow for the fact that the consumer generally can reduce her expenditures in the second period— with no reduction in her satisfaction—by substituting away from commodities that have become relatively dearer in favor of others that have become relatively cheaper.[5] Because the Laspeyres index does not allow for such substitutions, this type of fixed-weight price index has an upward bias as a measure of the cost of maintaining a given level of satisfaction.

Alternatively, one may evaluate how much the second commodity bundle would have cost at base period prices and compute the increase in the cost of this bundle. However, an index number constructed this way, which is known as a Paasche index, tends to understate the increase in the cost of living.[6] This is because the second bundle was not the one that the consumer actually chose in the base period, so computing its cost at the first set of prices overstates the cost of living in that period.

If one knew the consumer's preferences, one could predict what substitutions she would make in order to maintain the same degree of satisfaction in response to any given changes in relative prices. Thus, one could calculate the minimum cost of attaining a particular level of satisfaction at any given set of prices. Changes in this minimum cost over time would provide an exact measure of changes in the "true cost of living," defined not as the cost of buying a particular bundle of goods and services but as the cost of obtaining a particular level of satisfaction. Although this approach has been attempted by some economists (for example, Klein and Rubin 1947-48), it has the disadvantage of requiring a large body of data from which to estimate consumers' responses to changes in the prices they face. The economic theory of index numbers provides an alternative and more economical approach.[7]

The Economic Theory of Index Numbers

This theory begins with the assumption that the quantities of individual goods and services that we observe consumers buying are those that maximize their satisfaction (or utility) given their incomes and the prices they face. The theory then shows that by making certain mathematical assumptions about the form of consumer preferences, one may derive index number formulae that measure changes in the true cost of living (that is, the cost of obtaining a certain level of satisfaction) in terms of the observable prices and quantities of individual goods and services. Index numbers that have this property are said to be "exact."[8] The appeal of this approach is that it is necessary only to specify the form of the functions that describe consumers' preferences and not necessary to know the actual values of their parameters. This follows from the assumption that if the consumer buys a particular bundle of goods and services at a particular set of prices, this means that this bundle maximizes her utility from a given expenditure level (or minimizes the expenditure required to obtain a given utility level). Hence, price and quantity observations provide information about utility levels.

Two exact index number formulae that have been derived and used by advocates of this approach are the Fisher Ideal index and the Törnqvist index. The Fisher ideal measure of the increase in average prices from base period t to period s is the geometric average of the Laspeyres and Paasche price indexes:

$$(4)\quad P_F = \left[\sqrt{\left(\frac{\sum_n p_{ns}q_{nt}}{\sum_n p_{nt}q_{nt}}\right) \times \left(\frac{\sum_n p_{ns}q_{ns}}{\sum_n p_{nt}q_{ns}}\right)} - 1\right]$$

The Fisher Ideal price index exactly represents the consumer's true cost of living if the utility function that describes her preferences at date s is a quadratic function of the form:[9]

$$(5)\quad U_s = \sum_{n=1}^{N}\sum_{m=1}^{N} \alpha_{nm} q_{ns} q_{ms}, \text{ where } \alpha_{nm} = \alpha_{mn}.$$

The Törnqvist measure of the overall price increase is the weighted geometric average of the increases in individual commodity prices, with weights equal to the average expenditure shares in the base period t and the current period s:

$$(6)\quad P_T = \prod_{n=1}^{N}\left(\frac{p_{ns}}{p_{nt}}\right)^{r_n}, \text{ where}$$
$$r_n = \frac{1}{2}\left[\left(\frac{p_{nt}q_{nt}}{\sum_i p_{it}q_{it}}\right) + \left(\frac{p_{ns}q_{ns}}{\sum_i p_{is}q_{is}}\right)\right]$$

The exact price index will be a Törnqvist one if preferences may be described by a translog expenditure function (Diewert 1976). The translog unit expenditure function has the form:[10]

$$(7)\quad \ln e_s = \alpha_0 + \sum_{n=1}^{N}\alpha_n \ln p_{ns} + \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N}\alpha_{nm}\ln p_{ns}\ln p_{ms},$$
$$\text{where } \alpha_{nm} = \alpha_{mn}.$$

In this equation, e_s represents the minimum expenditure that yields a unit level of utility at the prices ruling in period s. This expenditure function imposes fewer restrictions on the structure of consumer preferences than the quadratic utility function.

At first sight, the assumptions on the forms of the utility and expenditure functions that underlie the Fisher and Törnqvist price indexes appear to be rather restrictive. However, it can be shown that a wide range of alternative utility and expenditure functions can be approximated closely by either a quadratic or a translog function.[11] Diewert describes forms of the utility or expenditure function that have this approximation characteristic as "flexible forms" and the corresponding exact index number formulae, such as the Fisher ideal or the Törnqvist, as "superlative" indexes.

By construction, the Fisher ideal price index lies between the Laspeyres and Paasche indexes. It can be shown that this also is true of the Törnqvist measure. For measuring changes in prices over time, there is little to choose between these alternative measures, since in most cases they give very similar results.

If the consumer's nominal income rises by the same amount as the true cost of living, this means that her satisfaction is unchanged. It is natural, therefore, to measure the change in the consumer's real income between two dates by the extent to which the increase in her nominal income exceeds the rise in the true cost of living, since "real income" then will be an indicator of her utility level or standard of living. If a measure of real GDP is constructed by deflating nominal GDP by a true cost of living price index number, the result is a measure of the "quantity" of output that represents changes in the standard of living enjoyed by the representative consumer. In other words, with this definition, an increase in real GDP represents a rise in consumer satisfaction or welfare. This seems to be a sensible way of defining what is meant by the quantity of output when the proportions of individual commodities in the total change over time.

A drawback to defining and measuring real GDP in terms of the standard of living of a representative consumer is that many of the commodities included in the GDP are not consumer goods and do not directly contribute to consumer welfare. An alternative approach that avoids this drawback is to base the measure of real GDP on the production capability of the representative firm rather than the preferences of the representative household.

III. Production-Based Measures of Real GDP

Suppose that, in the base period, a representative firm—with a given technology and set of inputs and facing a given set of output prices—produces a certain bundle of outputs with a certain dollar value. In a later period, facing a different set of output prices, it produces a different bundle of outputs, using a different technology and set of inputs. The problem is to determine how much of the change in the nominal value of the firm's output (that is, in its revenue) is due to a change in the prices of its products and how much to a change in the quantities produced. The microeconomic theory of production may be used to address this problem.

A rise in the firm's revenues represents an increase in the *quantity* of its output if it may be attributed entirely to a change in the inputs it uses or in its technology and not at all to changes in the prices of any of its outputs.[12] Conversely, an increase in revenue that occurs with no change either in the inputs used or in technology, must be due to a change in the prices of its products and represents a rise in the *average price* of its output. Put in more technical terms, a revenue change is an increase in the *quantity* of the firm's output if it represents an outward shift in its production possibility frontier, but is a *price* change if it represents a movement along the frontier.[13]

In the same way as the consumption-based approach relies on the assumption that consumers choose their purchases so as to minimize the cost of obtaining any given level of satisfaction, the production-based approach assumes that firms choose their outputs so as to maximize their revenues given the technology and inputs they have available. This assumption guarantees that the observed quantities of output are those that maximize the firm's revenues given its production possibilities and the prices that it faces. As in the case of the consumption-based approach, it is possible to derive exact output and price indexes by suitably choosing the mathematical form of the function that describes the firm's production possibilities. Production possibilities may be described by either a production function or a revenue function.[14] If the revenue function is assumed to be translog, the corresponding output *price* index will be a Törnqvist index.[15] A similar restriction on the production function implies a Törnqvist output *quantity* index.[16] Somewhat stronger restrictions on the production and revenue functions imply that these price and quantity measures will be Fisher ideal indexes.

If an exact *price* index is constructed, a measure of *real output* is obtained by deflating the nominal value of output using that index. Conversely, if an exact *quantity* index is constructed, the corresponding *price* index is obtained by dividing the nominal value of output by this quantity index. Fisher ideal indexes have the useful technical property that if a Fisher *price* index is used to deflate nominal GDP, the result is a Fisher index of the *quantity* of real GDP, and conversely.[17] Thus, a Fisher price index is an exact measure of the price level, and the corresponding real GDP index is an exact measure of the quantity of output, but at the same time their product is equal to nominal GDP.

Neither the Törnqvist index nor the measures that are currently used by the BEA have this "factor reversal" property. Real GDP currently is measured by a Laspeyres fixed-weight output index and the preferred measure of inflation is the fixed-weight GDP price index, which also is a Laspeyres index. The product of these measures of output and prices is not equal to nominal GDP. The measure of prices obtained by dividing nominal by real GDP (the implicit price deflator) is a poor indicator of inflation because it reflects not only changes in prices but also changes in the composition of GDP. Conversely, the measure of output that would be obtained by dividing nominal GDP by the fixed-weight price index (which might be described as an "implicit output measure") would be a poor measure of real growth since it would reflect not only changes in output but also changes in relative prices. Adoption of Fisher ideal measures of prices and real GDP would avoid these ambiguities.

IV. Recent Changes in the National Income Accounts

The Bureau of Economic Analysis issued revised GDP estimates last December. In the course of this "benchmark" revision, the base date of the estimates was changed from 1982 to 1987.[18] As mentioned earlier, the average rate of real GDP growth from 1977 to 1990 was 0.2 percentage point lower in the revised data. However, in some periods the rebasing caused much larger changes in measured growth. For example, the growth of real GDP was reduced by 0.5 percentage point in both 1987 and 1988 as a result of rebasing, and the decline in real GDP in the cyclical downturn in the winter and spring of 1990-91 appears to have begun earlier and been somewhat more severe when measured at 1987 prices than when measured at 1982 prices. Chart 1 compares the quarterly growth rates from 1975 to 1990 in the pre- and post-benchmark data.[19]

The BEA has indicated that, beginning sometime in 1992, two alternative measures of both real growth and inflation will be published, using forms of the Fisher ideal index. These alternative indexes will eliminate the periodic revisions to measured growth resulting from the effects of rebasing, and will remove the long-run bias in the current measure of real output that results from the use of constant relative prices. In addition, because the Fisher ideal index is based on the economic theory of index numbers, these alternative measures of the economy's total production will have a sounder theoretical basis.[20]

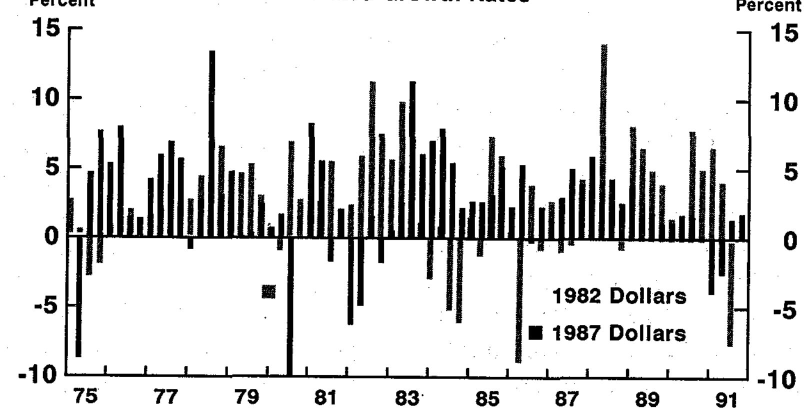
"Chain" Measures of GDP Growth

The planned alternative indexes will be forms of *chain indexes*. A quarterly chain measure of GDP growth is constructed by computing the real growth rate between each successive pair of adjacent quarters, using current relative prices as weights. For several years, the BEA has published chain indexes of GNP growth, but these have attracted little attention. In these indexes, real GNP growth between each pair of adjacent quarters was measured using the relative prices ruling in the first quarter. Thus, these quarterly chain growth rates were Laspeyres indexes. Average growth over longer periods could have been computed by compounding these one-quarter chain growth rates, but in the past the BEA did not do this.

To measure the growth of real GDP in a particular quarter, it makes sense to weight its components by the relative prices prevailing in that quarter rather than in the distant past or future (see Moorsteen 1961). Measures of average growth over longer periods constructed by compounding these chain growth rates would take account of the changes in relative prices and the composition of output that occurred. Hence the measurement bias that results from the use of fixed-weight indexes would be reduced.

The measured average growth rate over a longer period

Chart 1
Gross Domestic Product
1982 and 1987 Dollars
Annualized Growth Rates



computed by compounding quarterly chain growth rates would depend on the (changing) relative prices and composition of output throughout the period. This is because the growth rate between each successive pair of quarters depends on the relative prices and on the composition of output in those quarters. By contrast, a measure of growth calculated directly from the beginning to the end of the period depends only on relative prices and on the composition of output at the beginning and the end. In other words, a growth rate calculated by compounding quarterly chain growth rates is "path-dependent."[21] It represents the average growth rate *during* the period rather than the average growth rate *from the beginning to the end* of the period. In practice, however, the difference is likely to be very small.[22]

New Measures of Growth and Inflation

The new alternative measures of real GDP and the price level to be introduced by BEA combine the features of the Fisher ideal index and the chain approach. The BEA terms these new measures *time-series generalized Fisher ideal (TGFI) indexes*.[23] The TGFI index calculates real growth between benchmark years using the standard Fisher ideal formula. Growth rates in periods between the benchmarks are calculated as the geometric average of the growth rates calculated using the weights in the two benchmark years. Thus, if A and B are benchmark years and t and $t+1$ are years between A and B, the TGFI real growth rate from t to $t+1$ is:

$$(8)\quad \left[\sqrt{\left(\frac{\sum_n q_{nt+1}p_{nA}}{\sum_n q_{nt}p_{nA}}\right)\times\left(\frac{\sum_n q_{nt+1}p_{nB}}{\sum_n q_{nt}p_{nB}}\right)}-1\right].$$

Similarly, the TGFI increase in prices between t and $t+1$ is given by

$$(9)\quad \left[\sqrt{\left(\frac{\sum_n p_{nt+1}q_{nA}}{\sum_n p_{nt}q_{nA}}\right)\times\left(\frac{\sum_n p_{nt+1}q_{nB}}{\sum_n p_{nt}q_{nB}}\right)}-1\right].$$

Direct computation shows that the cumulation of the TGFI growth rates for the periods between A and B is equal to the Fisher ideal measure of growth calculated directly from year A to year B. As a result, the TGFI measure of growth between benchmark years is not path-dependent.[24] The TGFI index also has the factor reversal property that the growth rates of real GDP and the price level from one benchmark year to the next sum to the growth rate of nominal GDP.

An attractive property of chain Fisher ideal indexes is that the measures of real growth and inflation in each quarter incorporate the structure of the economy and relative prices in that quarter and so should give a more accurate indication of current developments. For this reason, these measures might be more valuable to policymakers. We have found, for example, that the chain measure of real GNP growth is a slightly better predictor of changes in the unemployment rate than the standard measure. The TGFI indexes will have similar advantages, since the real growth and inflation measures for each quarter will be based on the relative prices and the structure of output in nearby benchmark years.

BEA plans to construct two alternative TGFI indexes. The first alternative index will use as weights the relative prices and composition of output in the preceding and current years. In terms of equations (8) and (9), years A and B refer to the previous and current years.[25] The BEA describes this index as a "chain-type annual weights" index. The second index, which will be termed a "benchmark-years weights" index will use as weights the relative prices and composition of output in benchmark years five years apart.[26]

A disadvantage of the chain approach (including the TGFI measures) is that it provides a measure of the *growth rate* of real GDP in a given quarter or year, but no unique measure of its *dollar level*. A measure of the level of real GDP can be constructed by multiplying nominal GDP in an arbitrary base year by the compounded chain growth rates. However, the resulting measure of real GDP does not have the easily understood interpretation of the fixed-weight measure now in use. Specifically, it does not measure what the GDP would be if all prices had remained constant since the base year.

A related disadvantage of a GDP measure computed by cumulating a chain index such as the TGFI is that the level of real GDP constructed in this way is not equal to the simple sum of its components (consumption, investment, etc.). Instead, it is a *weighted sum* of these components with weights that change as relative prices vary. Over short periods this might not cause problems, but it could be inconvenient for studying the sources of growth over longer periods.[27] The BEA will avoid this aggregation problem by publishing only index numbers of real GDP and its principal components rather than dollar values. Hence it will not be possible to study the decomposition of GDP growth over time using these new measures.

V. CONCLUSION

The measures of real GDP and inflation to which policymakers respond are aggregates of vast numbers of individual prices and quantities. Measuring these macroeconomic variables using fixed-weight indexes adds to the uncertainties facing policymakers, since changes in the base date used in constructing measures of output and prices sometimes alter our perceptions both of the economy's long-run real growth and inflation rates and of its short-run cyclical behavior.

This article has shown that these ambiguities are the result of the particular definitions of output and inflation that are currently in use. The economic theory of index numbers shows that if an increase in total output were defined as a change in the bundle of goods and services produced that either raises the utility level of the representative consumer or increases the revenue of the representative firm with no change in the prices of its outputs, the ambiguities could, in principle, be resolved. These definitions may be made operational by specifying the mathematical form either of the household's utility function or of the firm's production function.

The alternative measures of real GDP and inflation that the BEA soon will introduce appear to be a sharp improvement over those that have been in use since the Census Bureau began constructing national product data on a regular basis in 1947. These new indexes of real GDP and inflation will make use of the economic theory of index numbers discussed in this paper, and so will have a sounder theoretical basis than the current measures. In addition, the alternative data will avoid much of the ambiguity associated with fixed-weight aggregates and will more closely reflect the current structure of the economy, because the price and quantity weights used will be based on conditions in nearby benchmark years. These improvements will remove at least one source of uncertainty facing policymakers.

ENDNOTES

1. Until last December, the BEA focused on gross *national* product rather than gross *domestic* product. GNP measures the output of resources owned by U.S. residents (including output produced abroad using American-owned labor and capital), whereas GDP measures the output produced within the borders of the U.S. (including the output of foreign-owned labor and capital). For purposes of the issues discussed in this article, this distinction is not an important one.

2. It also depends on the type of average used. The existing official price indexes are constructed as weighted *arithmetic* averages of the prices of their components, but index numbers also could be constructed as weighted geometric averages. The Törnqvist index discussed below is an example of one constructed as a geometric average of its components.

3. Measuring the prices of individual items correctly involves a host of difficult problems. For example, when the amount spent on an item increases at the same time as its quality improves, it may be difficult to determine whether its true price has risen or declined. The rising cost of medical care is an example of this problem. To keep its length manageable, this paper will ignore these issues and assume that the price and quantity produced of each individual commodity are measured without error.

4. The Laspeyres measure of the increase in prices from base period t to period s is:

$$P_L = \left[\frac{\sum_{n=1}^{N} p_{ns} q_{nt}}{\sum_{n=1}^{N} p_{nt} q_{nt}} - 1 \right].$$

5. If, for example, chicken has risen in price more than fish, she may obtain the same satisfaction at less cost by consuming less chicken and more fish.

6. The Paasche measure of the increase in prices from base period t to period s is:

$$P_P = \left[\frac{\sum_{n=1}^{N} p_{ns} q_{ns}}{\sum_{n=1}^{N} p_{nt} q_{ns}} - 1 \right].$$

7. For a useful survey of the literature on index numbers, see W.E. Diewert (1987). Diewert has been responsible for much of the recent theoretical development of this branch of economics.

8. In technical terms, the theory requires the mathematical form of the utility function or the expenditure function to be specified. The utility function assigns a utility value to each commodity bundle, such that if the consumer prefers one bundle to another, it will have a higher utility value. The expenditure function specifies the minimum cost of attaining a given utility level as a function of the commodity prices that the consumer faces. It can be shown that either of these functions may be used to represent the consumer's preferences.

9. This was first proved in Konüs and Byushgens (1926).

10. The expenditure function defines the minimum expenditure required to obtain a given level of utility and hence depends on the specified utility level as well as on prices. However, since the measurement of utility is arbitrary, it is convenient to set the reference level of utility at

unity. This causes the terms involving the utility level to drop out of equation (7) since the logarithm of one is zero.

11. Specifically, either of these forms can provide a second order approximation to any twice continuously differentiable linearly homogeneous function.

12. In addition, an increase in the quantity of output that occurs with no increase in the amounts of inputs used must be attributed to a change in technology, and hence represents a rise in *productivity*. The index number methodology discussed in this section also may be used to define exact measures of productivity growth.

13. For more detailed discussions of this issue, see Moorsteen (1961) and Fisher and Shell (1972).

14. The *production function* describes the combinations of outputs and inputs that are feasible for the firm with its given technology. The *revenue function* defines the maximum revenue the firm can obtain from selling (at the output prices it faces) the outputs it can produce with a given set of inputs and a given technology. It can be shown that the firm's production possibilities may be fully described by either a production function or a revenue function.

15. The maximum revenue that the firm can obtain depends on the prices of its outputs and the quantities of inputs it has available. If the firm produces N outputs with prices $p_1 \ldots p_N$ using M inputs $v_1 \ldots v_M$, the translog revenue function is

$$\ln\pi = a_0 + \sum_{m=1}^{M} \beta_m \ln v_m + \frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}\beta_{ij}\ln v_i \ln v_j + \sum_{n=1}^{N}\gamma_n \ln p_n$$

$$+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_{ij}\ln p_i \ln p_j + \sum_{n=1}^{N}\sum_{m=1}^{M}\delta_{nm}\ln p_n \ln v_m \; .$$

This form is "flexible" since it can approximate any arbitrary linearly homogeneous twice-differentiable function.

16. Proofs of these results are given in Diewert (1983). The result with regard to the *output* deflator requires that the *output distance* function be translog in form. The distance function, which may be derived from the production function, measures the distance of the firm's present production possibilities frontier from some base frontier.

17. This can be shown by direct computation. For simplicity, consider the two-commodity case. The increase in nominal GDP from period 0 to period 1 divided by the Fisher ideal measure of the increase in prices is:

$$\frac{\dfrac{(p_{11}q_{11})+(p_{21}q_{21})}{(p_{10}q_{10})+(p_{20}q_{20})}}{\sqrt{\dfrac{(p_{11}q_{10})+(p_{21}q_{20})}{(p_{10}q_{10})+(p_{20}q_{20})}\cdot\dfrac{(p_{11}q_{11})+(p_{21}q_{21})}{p_{10}q_{11})+(p_{20}q_{21})}}} .$$

This expression may be simplified to:

$$\sqrt{\frac{(p_{10}q_{11})+(p_{20}q_{21})}{(p_{10}q_{10})+(p_{20}q_{20})}\cdot\frac{(p_{11}q_{11})+(p_{21}q_{21})}{(p_{11}q_{10})+(p_{21}q_{20})}} \; .$$

This is the Fisher ideal measure of the increase in real output from period 0 to period 1.

18. In addition to altering the base date for measuring constant dollar quantities, this benchmark revision incorporated a number of other procedural changes, including the replacement of GNP by GDP as the primary measure of U.S. output.

19. The benchmark revisions also incorporate new sources of data and some methodological changes. However, in most quarters, the change of base from 1982 to 1987 is the largest source of revisions in the measured GDP growth rate.

20. Since many commentators take it for granted that there is only one "correct" measure of real GDP, the publication of alternative measures of real output may create uncertainty at first.

21. This was first pointed out by Triplett (1988). Note that path dependence occurs even if growth in each individual quarter is measured by an exact index such as a Fisher ideal or Törnqvist index.

22. Between 1982 and 1987, for example, real GNP increased at an average annual rate of 3.76 percent in 1982 prices and 3.54 percent in 1987 prices. Since these are the Laspeyres and Paasche measures of real growth, respectively, the Fisher ideal measure of average growth between these two dates is equal to their geometric mean, or 3.65 percent. The average growth rate calculated by compounding quarterly Fisher ideal chain measures is 3.64 percent.

23. This index was introduced in Young (1988).

24. However, measured growth over shorter or longer periods will be path dependent. For example, if A, B, and C are benchmark years, the direct Fisher ideal measure of growth from A to C will not be equal to the product of growth from A to B and that from B to C.

25. For measuring quarterly real GDP and inflation during the current year, the previous year's weights will be used until the current year is complete.

26. For example, 1982 and 1987 are benchmark years. Quarterly growth and inflation rates between the third quarter of 1982 and the second quarter of 1987 will be calculated using the relative prices and composition of output in 1982 and 1987. Thus, in future benchmark revisions, these data will be unaffected by base-date changes. For quarters after 1987.Q2, the calculations will use weights for 1987 and the most recent complete year. After complete data for 1992 are available, growth between 1987.Q3 and 1992.Q2 will be measured using weights for 1987 and 1992.

27. In the case of a TGFI measure, the weights would remain constant between benchmark years, but would change when moving from one inter-benchmark period to the next.

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Exchange Rate Policy and Shocks to Asset Markets: The Case of Taiwan in the 1980s

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This paper uses a simple theoretical model to show how the credibility of unsterilized intervention policy may affect the pattern of adjustment in the exchange rate, velocity, and asset prices. When the outcome of unsterilized intervention is credible, any degree of exchange rate stability can be achieved at the cost of a sufficiently large, one-time change in the money supply. When the outcome of intervention is not credible, intervention can lead to persistent, and possibly accelerating, changes in exchange rates, the money supply, velocity, and asset prices. Under certain conditions, intervention may even amplify the cumulative change in the exchange rate, rather than reduce it. The model is used to interpret Taiwan's experience with unsterilized exchange rate intervention in the second half of the 1980s.

Over the past decade, international capital mobility in many Pacific Basin economies has increased considerably. This trend has made it more difficult for policymakers to stabilize the foreign value of their currencies. The greater ability of speculators to buy and sell domestic currency in foreign exchange markets has in some cases resulted in unwelcome fluctuations in currency values, in spite of government efforts to limit such fluctuations.

Some progress has been made in understanding the problems of stabilizing the exchange rate in economies with mobile international capital. Research in open economy macroeconomics since the 1960s describes how disturbances to foreign exchange markets and government policies affect exchange rate behavior given certain institutional features of the economy, such as the degree of capital mobility or asset substitutability.

More recently, research has clarified how credibility affects the ability of the government to enforce an exchange rate target. For example, Krugman (1979) shows how government attempts to peg the exchange rate with limited foreign exchange reserves may lead to speculative attack and an abandonment of the peg. Another literature (see Lessard and Williamson 1987) analyzes capital flight in economies that are forced to deal with serious macroeconomic imbalances or that are saddled with large external debt burdens. Such capital flight may impair the government's ability to stabilize the exchange rate. However, these approaches do not necessarily highlight the difficulties that may arise when a well-managed economy (one that faces no foreign exchange reserve constraints, maintains a largely balanced government budget, and has no external debt burden) attempts to stabilize its currency.

This paper draws on the experience of Taiwan in the 1980s to shed some light on these potential difficulties. Due to certain asymmetries in foreign exchange controls, Taiwan had a relatively high degree of capital mobility up to 1987, while it maintained a policy of limiting movements in the exchange rate. Taiwan's relative openness exposed it to disturbances to its foreign exchange markets in the second half of the 1980s that illustrate the difficulties that may arise when a country attempts to stabilize its exchange rate.