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Thomas M. Mertens
Federal Reserve Bank of San Francisco

Tony Zhang
Federal Reserve Board of Governors

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A Financial New Keynesian Model*

Thomas M. Mertens† and Tony Zhang‡

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Abstract

This paper solves a standard New Keynesian model in terms of risk-neutral expectations and estimates it using a cross-section of longer-dated financial assets at a single point in time. Inflation risk premia appear in the theory and cause inflation to deviate from its target on average. We re-estimate the model based on each day's closing prices to capture high-frequency changes in the expected path of the economy. Our estimates show that financial markets reacted to the post-COVID surge in inflation with higher short-run inflation expectations, an increase in the inflation risk premium, and an increase in the long-run neutral real rate, r^*, while long-term inflation expectations remained well anchored. Our model produces long-term inflation forecasts that outperform several standard alternative measures.

*The views expressed here are solely those of the authors and do not necessarily represent those of the Federal Reserve Bank of San Francisco, the Board of Governors of the Federal Reserve, or the Federal Reserve System.

†**Federal Reserve Bank of San Francisco**, 101 Market Street, Mailstop 1130, San Francisco, CA 94105; Thomas.Mertens@sf.frb.org.

‡**Federal Reserve Board of Governors**, Constitution Ave NW & 20th St NW, Washington, DC 20551; tony.zhang@frb.gov.

1 Introduction

The New Keynesian model remains the workhorse model for discussions around monetary policy. For quantitative analysis, the model is typically estimated using macroeconomic time series data, which are published at monthly or quarterly frequencies and with a lag. The low frequency nature of these data limits the ability to estimate time variation in model parameters and to capture changes in the expected path of the economy. Since macroeconomic policies benefit from incorporating the latest available information about the path of the economy, the question arises whether real-time financial market data can inform the New Keynesian model.

In this paper, we show how to study and interpret asset prices through the lens of a textbook New Keynesian model, as in Clarida, Gali and Gertler (1999) and Woodford (2003), and how to extract key model parameters and the expected path of the economy from them. Therefore, we solve the model in terms of risk-neutral expectations that have a direct counterparts in financial market prices. We then show that a subset of the parameters within the model are identified using the cross-section of financial market prices on any given day. The estimation can thus be implemented on a new set of data at any point in time, facilitating event study analysis. Our estimation results show an increase in the long-run neutral real rate (r^*) during the post-COVID episode of high inflation while long-run inflation expectations remained well-anchored. Financial markets started to react to the rise in inflation in the fourth quarter of 2021 with higher expected inflation and an increase in the inflation risk premium.

We use a simple textbook New Keynesian model to cleanly illustrate how the standard three-equation model can be rewritten such that it has direct counterparts in financial market data. Households consume a continuum of differentiated goods, aggregated into a consumption bundle. Their utility increases with consumption and decreases with the amount of labor supplied. Firms produce goods using labor and face costs when adjusting their prices, as in Rotemberg (1982).

We rewrite the non-linear optimality conditions for households and firms in terms of risk-neutral expectations. This reformulation does not require any approximation but rather takes the risk-pricing of households accurately into account. For example, we show that the Euler equation of households directly corresponds to a version of the Fisher equation that links the nominal interest rate to the risk-neutral expectation of inflation and the real interest rate. These same variables appear in the log-linearized version of the model that takes the same form as the standard three-equation New Keynesian model with risk-neutral replacing physical expectations.

Financial markets allow us to observe risk-neutral expectations of inflation, nominal interest rates, and real interest rates through traded financial instruments. We use Treasury bond yields for nominal interest rates and break-even inflation (derived as the difference between nominal and real Treasury yields) for risk-neutral expectations of inflation.

We assume the central bank sets interest rates optimally under discretion. It minimizes a quadratic loss function over deviations of inflation from target and the output gap. The model features "divine coincidence" such that demand shocks can be fully offset and do not enter the solution for inflation, output gaps, and thus consumption. Mark-up shocks, on the other hand, result in a trade-off between inflation and output that leads the central bank to partially offset them.

While the log-linearized equations take the standard form, except with risk-neutral instead of physical expectations, the solution of the model now contains additional terms. These terms reflect inflation risk premia for future mark-up shocks, which affect inflation and output gaps, and translate into term premia for interest rates. Since demand shocks do not enter consumption and thus the stochastic discount factor, there are no risk premia associated with them.

These inflation risk premia create a modest long-term bias in the rate of inflation. As a result, inflation on average hovers around a level different from its target and long-term inflation expectations reflect this bias. Our estimation reveals the median estimate for the absolute value of this bias is about 18 basis points since the Federal Reserve raised interest rates above zero at the end of 2015.

We show analytically that the model directly links a subset of its parameters to risk-neutral expectations that can be identified from financial market data. Inverting this relationship delivers formulae for computing a total of six parameters from expected inflation and forward rates of interest at various horizons. We show that no further parameter can be identified from the set of financial market data for the stochastic process underlying the shocks.

We therefore split up the set of model parameters into two categories. We calibrate one set of parameters that we deem invariant over time. These parameters reflect preferences of households as well as the central bank. All other parameters can vary over time and are estimated from the data. These parameters are the inflation risk premium, the long-run real rate of interest (r^*), and the level and persistence of shocks.

Importantly, our estimation technique identifies the parameters from data available at a single point in time. We collect end-of-day Treasury yields and break-even inflation rates at all available maturities between 1 and 30 years out. Because these data are not necessarily available at all maturities, we compute the full term structure of interest rates and inflation expectations by estimating Nelson-Siegel curves using the raw data. We then estimate the higher frequency parameters using the Nelson-Siegel curves starting from five years out when the transition dynamics have likely waned and our AR(1)-structure of the stochastic process is a good description of actual dynamics. Our estimation then returns a time series of daily parameter values.

The model exhibits a good fit to the underlying data. Using the estimates, the median average fitting error is 1.3 basis points for a given maturity on a given day relative to the Nelson Siegel curves.

The daily estimates reveal interesting patterns over time, in particular during the post-COVID bout of inflation. The estimated inflation risk premium is very small — typically on the order of 1 or 2 basis points. While turning negative during the COVID recession of 2020, it turned positive during the ensuing surge in inflation.

While short-term inflation expectations picked up during this time, long-term inflation expectations remained well-anchored. This result is consistent with a small estimated inflation risk premium and the fact that forward rates of break-even inflation did not drift far from the Federal Reserve's target rate.

Estimates of the long-term neutral real interest rate, r^* fell during our sample period. Before the Great Financial Crisis of 2007-2009, estimates of r^* were about 2.5% while they fell below zero after the COVID recession. Estimates of the neutral rate had recovered by the end of our sample in April of 2023.

Since our estimation requires only a cross-section of financial data, the methodology lends itself to event study analysis. To illustrate this point, we estimate the model on November 9, 2021, a day before the Consumer Price Index (CPI) was announced that surprised with higher readings of price increases. We then re-estimate the model the following day and analyze the changes.

This analysis shows that short-term inflation forecasts were revised upwards with the news. Long-term inflation forecasts rose by much less and, as a result, the persistence of inflation fell.

Lastly, we compare the out-of-sample forecast accuracy for 10-year inflation with alternative measures of inflation expectation. Once we remove risk premia from break-even inflation, our model delivers a superior accuracy in predicting inflation over this time frame relative to all other estimates. We compare our estimates to inflation swaps and survey evidence from the Survey of Professional Forecasters, the Michigan Survey, as well as the ATSIX data set.

We point out three main caveats to our analysis. First, we do not use the full term structure of Treasury yields and break-even inflation rates. We chose the model based on simplicity to cleanly show the innovation in this paper: Solving the model in terms of risk-neutral expectations. The modeling choice comes at the cost of not being able to capture all near-term transition dynamics. Second, we do not explicitly deal with the zero lower bound on interest rates. Since the lower bound affected the pricing of interest rates and inflation in the aftermath of the Great Financial Crisis (see Mertens and Williams (2021)), our estimation might be contaminated by the influence of the zero lower bound. We deal with this issue by filtering some of the resulting noise. Third, our estimation strategy can only reveal a subset of the parameters, as we show in our analysis. We keep all other parameters fixed. However, one could imagine combining our estimation technique of using financial market data with an estimation based on macroeconomic time series to jointly estimate all parameters. We view this estimation as outside the scope of this analysis.

This paper relates to several strands of the literature. First, a substantial body of work has been devoted to estimating the New Keynesian model. Smets and Wouters (2007) showed how a medium-scale DSGE model with several frictions can be estimated from macroeconomic data. And while we cannot do justice to all the valuable contributions in the area, An and Schorfheide (2007), Del Negro, Schorfheide, Smets and Wouters (2007), Sbordone, Tambalotti, Rao and Walsh (2010), Justiniano, Primiceri and Tambalotti (2013), and Cúrdia, Ferrero, Ng and Tambalotti (2015) particularly influenced our thinking. The novelty in this paper relative to these contributions is that we use financial market data instead of macroeconomic time series. The advantage of this data is that it is, albeit with liquidity noise, available at high frequency.

Second, a recent literature has emerged that uses and investigates the link between monetary policy and financial markets. Kiyotaki and Moore (1997), Bernanke et al. (1999), and Brunnermeier and Sannikov (2014) incorporate a financial sector into a macroeconomic framework. Campbell, Pflueger and Viceira (2020), Caballero and Simsek (2020b), Caballero and Simsek (2020a), Bianchi, Lettau and Ludvigson (2022), and Bok, Mertens and Williams (2022) study the asset pricing implications from monetary policy and the real side of the economy. Closely related to this paper is Pflueger (2023) who measures risk of stagflation from nominal and real bonds. In this paper, we stay within the textbook New Keynesian model which we re-write in terms of risk-neutral expectations so that it can be estimated directly from the data. We abstract from noise in financial market trading, as, e.g., in Hassan and Mertens (2017).

Third, our estimation relates to the vast literature on the estimation of the components of the New Keynesian model. Following the seminal work of Laubach and Williams (2003), a series of papers, including Lubik and Matthes (2015), Holston, Laubach and Williams (2017), Johannsen and Mertens (2016), and Del Negro, Giannone, Giannoni and Tambalotti (2017), has presented measures of the natural real rate of interest, r^*. Closest to our measure is Christensen and Rudebusch (2017) who use TIPS markets as well but build a more elaborate model of risk premia. The inflation risk premium is been estimated, among others, by d'Amico, Kim and Wei (2018), Chernov and Mueller (2012), Grishchenko and Huang (2013), Fleckenstein, Longstaff and Lustig (2017), and Andreasen, Christensen and Riddell (2021). A series of papers studies and analyzes inflation expectations under the physical measure, including Faust and Wright (2013), Coibion and Gorodnichenko (2015), Duffee (2018), and Aruoba (2020). Closely related to our paper is Bauer, Pflueger and Sunderam (2022) who use survey data to estimate perceptions about monetary policy rules. Instead of estimating the various parts separately, we extract measures of expectations and risk premia within the general framework of the New Keynesian model.

The remainder of this paper is structured as follows. Section 2 lays out the model, derives the equilibrium and its log-linearized form, and discusses the theoretical implications of the model. Section 3 estimates the model. Section 4 discusses the quantitative results, and section 5 concludes.

2 Model

This section lays out the textbook New Keynesian model that underlies our analysis. Households receive utility from a constant elasticity of substitution (CES) aggregate of a continuum of goods varieties and receive disutility from working. Using this labor as input, firms produce the goods and face Rotemberg adjustment costs when setting prices. The description of the economy results in an IS and a Phillips curve that we derive in terms of risk-neutral expectations. The central bank faces a quadratic loss function over deviations of inflation from target and output gaps and sets interest rates optimally under discretion. Since the setup is standard, we give an incomplete description of the model in the main text and present all remaining details in Appendix A.

2.1 Setup

Time is discrete and there exists a unit mass of households that live forever. In each period, households optimally consume, supply labor N_t, and save so as to maximize expected lifetime utility

$$\sum_{s=t}^{\infty} \beta^s \mathbb{E}_t \left[\frac{C_s^{1-\gamma} - 1}{1-\gamma} - \frac{N_s^{1-\phi} - 1}{1-\phi} \right]. \tag{1}$$

Their preferences are pinned down by the coefficient of relative risk aversion γ, the time preference factor β, and the Frisch elasticity of labor supply determined by ϕ. $\mathbb{E}$ denotes the expectations operator under the physical probability measure, i.e., the $\mathbb{P}$-measure.

Consumption C_t is comprised of a CES aggregate of a unit mass of differentiated goods

$$C_t = \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where i indexes varieties of differentiated goods and $\varepsilon > 1$ governs the elasticity of substitution.

With P_t denoting the accompanying price index to the CES consumption aggregate, households face the budget constraint

$$P_t C_t + \frac{1}{I_t} B_{t+1} \leq B_t + W_t N_t + T_t, \tag{2}$$

where I_t denotes the gross return on risk-free assets, B_t bond payoffs, and W_t wages. T_t refers to transfers from the government that redistribute firm profits to households. We also allow households to trade a complete set of Arrow-Debreu securities. In our model, it is sufficient for them to trade the bond given the state contingent transfers T_t. Market completeness ensures that we have a unique stochastic discount factor that prices any asset.

The first-order condition for households with respect to bond holdings is given by

$$\mathbb{E}_t \left[M_{t+1} I_t \right] = 1, \tag{3}$$

where

$$M_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+1}} \tag{4}$$

denotes the nominal stochastic discount factor.

A unit mass of monopolistically competitive firms produces the differentiated goods according to the production function $Y_t(i) = A_t N_t(i)$. Firms pay Rotemberg adjustment cost when updating their prices:

$$\frac{\eta}{2} \left(\frac{P_t(i) - P_{t-1}(i)}{P_{t-1}} \right)^2 P_{t-1} Y_{t-1}.$$

With Ψ_t denoting the firm's real marginal cost of production, the firm's dynamic optimization problem leads to the first-order condition

$$(1 - \epsilon) + \epsilon \Psi_t + \eta \left(1 - \Pi_t \right) \frac{Y_{t-1}}{Y_t} = \eta \mathbb{E}_t \left[M_{t+1} \right] - \eta \mathbb{E}_t \left[M_{t+1} \Pi_{t+1} \right]. \tag{5}$$

The first-order conditions for households and firms share a common feature: Both equations contain expressions reflecting risk-neutral pricing. In other words, the stochastic discount factor appears in the expectations operator.

The common log-linearization separates expected stochastic discount factors from expectations about other variables. In the following section, we first rewrite the expectations in terms of the risk-neutral measure before log-linearizing.

2.2 Risk-neutral expectations

We follow the standard convention when defining the risk-neutral measure. We therefore start from the pricing equation $\mathbb{E}_t\left[M_{t+1}\widetilde{R}_{t+1}\right] = 1$ for any nominal return $\widetilde{R}_{t+1}$. Dividing both sides by $\mathbb{E}_t[M_{t+1}]$ allows us to write expectations as

$$\hat{\mathbb{E}}_t[\widetilde{R}_{t+1}] \equiv \mathbb{E}_t\left[\frac{M_{t+1}}{\mathbb{E}_t[M_{t+1}]}\widetilde{R}_{t+1}\right] = \int \widetilde{R}_{t+1}(\omega)\frac{M_{t+1}(\omega)h_t(\omega)}{\int M_{t+1}(\omega)h_t(\omega)d\omega}d\omega = \frac{1}{\mathbb{E}_t[M_{t+1}]}, \tag{6}$$

where ω denotes the state of the economy and $h_t(\omega)$ the probability density function over those states. Note that, by definition, the fraction in the third part of the equation (6) is non-negative and integrates to one. It is thus a probability measure: The risk-neutral probability measure or $\mathbb{Q}$-measure.

With the conversion to risk-neutral expectations, the log-linearization of the Euler equation (3) turns into the IS curve

$$x_t = -\frac{1}{\gamma}\left(i_t - \hat{\mathbb{E}}_t[\pi_{t+1}] - r_t^*\right) + \mathbb{E}_t[x_{t+1}] + g_t, \tag{7}$$

where x_t denotes the logarithm of the output gap and g_t a demand shock that follows an AR(1) process. Lower-case symbols thereby denote logarithms of the upper-case analogues. The IS curve takes the typical form of the textbook New Keynesian model except that inflation expectations are replaced by risk-neutral expectations. Expectations about the output gap remain under the physical measure.

Similarly to the optimality condition for households, the first-order condition for firms takes the standard form when log-linearized

$$\pi_t = \lambda x_t + \beta\hat{\mathbb{E}}_t\left[\pi_{t+1}\right] + u_t. \tag{8}$$

Here again, the only difference to the standard model is the expectation operator. u_t denotes a mark-up shock that, just like the demand shock, follows an AR(1) process

$$
\begin{aligned}
g_{t+1} &= \rho_g g_t + \varepsilon_{g,t+1} \\
u_{t+1} &= \rho_u u_t + \varepsilon_{u,t+1}
\end{aligned} \tag{9}
$$

While these changes to the formulation of the model may seem insignificant, they have at least two important consequences. First, as we turn to now, the expectations in these equations have a direct counterpart in financial market data. Second, as we shall see in Section 2.4, the solution to the model contains additional terms reflecting risk premia.

To show that these risk-neutral expectations are observable from financial market prices, we have two alternatives. The Euler equation (3) implies a version of the Fisher equation

$$I_t = \hat{\mathbb{E}}_t \left[\Pi_{t+1}\right] R_t . \tag{10}$$

Taking logarithms on both sides of the equation shows that we can observe risk-neutral expectations of inflation from the difference between nominal and real interest rates, typically referred to as break-even inflation when using the Treasury and the Treasury inflation-protected securities (TIPS) markets.

Alternatively, we can price a fixed-for-floating inflation swap. The rate on the fixed leg is adjusted such that the payoffs have equal values and no up-front payment is exchanged. We apply equation (6) to both the fixed payoff and inflation to obtain the (fixed leg) swap rate between time t and $t+1$

$$S_{t,t+1} = \hat{\mathbb{E}}_t[\Pi_{t+1}]. \tag{11}$$

Risk-neutral expectations about inflation are directly observable from the swap rate. In practice, inflation swap rates and break-even inflation rates have been trading in close alignment during times when both markets were sufficiently liquid.

2.3 Optimal policy

To close the model, a central bank sets the nominal interest rate optimally under discretion. It does so by minimizing an expected loss function of the form

$$\min_{i_t} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\pi_s^2 + \alpha x_s^2\right].$$

The first-order condition for the central bank's optimization problem,

$$x_t = -\frac{\lambda}{\alpha}\pi_t, \tag{12}$$

establishes a direct link between the output gap and the rate of inflation. This first-order condition optimally addresses the trade-off between stabilizing inflation and minimizing the output gap.

Interest rate policy implements this link between output and inflation. Plugging the IS curve (7) and Phillips curve (8) into the central banks first-order condition and solving for the interest rate results in

$$i_t = r_t^* + \left(1 + \frac{\beta\lambda\gamma}{\lambda^2 + \alpha}\right)\hat{\mathbb{E}}_t[\pi_{t+1}] + \gamma\mathbb{E}_t[x_{t+1}] + \frac{\lambda\gamma}{\lambda^2 + \alpha}u_t + \gamma g_t. \tag{13}$$

To see the implications of interest rate policy on inflation and output gaps, we turn to the model's solution in the next section.

2.4 Model solution

The previous sections lead to a slight variant of the familiar three-equation New Keynesian model. The model is fully described by the IS curve (7), the Phillips curve (8), the interest rate rule (13), and the processes for the shocks (9). Note that the first three equations imply the optimality condition for the central bank (12).

Following the same steps to solve the classic formulation of the New Keynesian model, we derive the solution for inflation as

$$\pi_t = \frac{\alpha}{\lambda^2 + \alpha}(\beta\hat{\mathbb{E}}_t[\pi_{t+1}] + u_t). \tag{14}$$

By the central bank's first-order condition, the output gap is inversely proportional to inflation.

As in the standard New Keynesian model, the demand shock is absent from expressions for inflation and output gaps. This result emerges because of the "divine coincidence" that interest rate policy can offset demand shocks from both inflation and output gaps simultaneously. Since output gaps are unaffected by demand shocks, neither consumption nor the stochastic discount factor respond to demand shocks either.

A risk premium emerges when solving equation (14) for inflation as a function of shocks. The output gap depends on mark-up shocks, resulting in a covariance between the variables. We denote the risk premium associated with innovations to the mark-up shock by μ_u such that

$$\hat{\mathbb{E}}_t[u_{t+1}] = \rho_u u_t + \hat{\mathbb{E}}_t[\epsilon_{u,t+1}] = \mu_u + \rho_u u_t. \tag{15}$$

No risk premium is associated with demand shocks since divine coincidence prevents them from spilling over into inflation and output.

With the expression for the risk premium, we obtain a solution for inflation by iterating equation (14) forward

$$\pi_t = \frac{\alpha^2 \beta}{(\lambda^2 + \alpha(1-\beta))(\lambda^2 + \alpha(1-\beta\rho_u))}\mu_u + \frac{\alpha}{\lambda^2 + \alpha(1-\beta\rho_u)}u_t. \tag{16}$$

An interesting special case arises when $\alpha = 0$, i.e., when the central bank only aims at stabilizing inflation and does not take its effect on the output gap into account. Then, the central bank achieves its objective: Inflation is always at target, $\pi_t = 0$, and the inflation risk premium disappears.

In the case when $\alpha > 0$, inflation and inflation expectations exhibit a bias. Taking unconditional expectations of equation (16),

$$\mathbb{E}[\pi_t] = \frac{\alpha^2 \beta}{(\lambda^2 + \alpha(1-\beta))(\lambda^2 + \alpha(1-\beta\rho_u))}\mu_u,$$

shows that inflation is on average away from zero when a risk premium is present. Theory suggests that the inflation risk premium should be positive. High mark-up shocks lead to above-target inflation, low consumption growth and, therefore, a high stochastic discount factor. One caveat, however, is that the presence of a lower bound, which we do not analyze here, might affect this result.

Equipped with the solution in equation (16), we link risk-neutral and physical expectations of inflation with the inflation risk premium so that two estimates imply the third

$$\hat{\mathbb{E}}_t[\pi_{t+1}] - \mathbb{E}_t[\pi_{t+1}] = \frac{\alpha}{\lambda^2 + \alpha(1 - \beta\rho_u)}\mu_u. \tag{17}$$

Intuitively, the risk-neutral expectation increases in both the mark-up shock risk premium, μ_u, and the persistence of the market shock, ρ_u. Appendix A.5 contains additional details.

3 Estimation Strategy and Identification

This section describes our empirical estimation strategy and the data we use in the process. The goal of this paper is to show how we can use financial market data to estimate a subset of the model parameters from data taken at a given point in time. Therefore, we describe the model parameters that can be identified from the cross-section of financial data alone, and we calibrate all other parameters that would require additional data to estimate.

3.1 Identification and Estimation Strategy

Data on risk-neutral expectations of inflation and on nominal interest rates identify a subset of parameters in the model. Consider the following proposition:

Proposition 1 (Parameter identification)
The time-t level of shocks, their persistence, the inflation risk premium, and the long-run neutral real interest rate r^* are identified from information about risk-neutral expectations in the model, conditional on all other parameters. Furthermore, no other parameters are identified from expectations data.

According to Proposition 1, six parameters are exactly identified in the model. These variables include the level and persistence of the shocks (u_t, g_t, ρ_u, ρ_g) and the intercept of the process of inflation and interest rates. These intercepts identify the long-run neutral rate as well as the inflation risk premium (μ_u, r^*).

Given the linear solution for inflation in equation (16), we pin down the three parameters for mark-up shocks from three data points along the term structure of expected inflation. Since the solution for inflation takes the form $\pi_t = \kappa_0 \mu_u + \kappa_1 u_t$ the formulae for the parameters are given by

$$\rho_u = \frac{\hat{\mathbb{E}}_t[\pi_{t+2}] - \hat{\mathbb{E}}_t[\pi_{t+3}]}{\hat{\mathbb{E}}_t[\pi_{t+1}] - \hat{\mathbb{E}}_t[\pi_{t+2}]} \qquad \mu_u = \frac{\hat{\mathbb{E}}_t[\pi_{t+3}] - \kappa_1 u_t \rho_u^3}{\kappa_0 + \kappa_1 \left(\rho_u^2 + \rho_u + 1\right)} \qquad u_t = \frac{\hat{\mathbb{E}}_t[\pi_{t+2}] - \mu_u \left(\kappa_0 + \kappa_1 \left(\rho_u + 1\right)\right)}{\kappa_1 \rho_u^2}.$$

The first equation on the left shows the persistence of the shock depends only on the risk-neutral expectation of inflation. Neither of the coefficients κ_0 and κ_1 are present in the formula. The estimate of the persistence of the shock enters the formulae for the level of the shock and the risk premium. The estimation for demand shocks from interest rate data works analogously.

These formulae show the relevant input data are risk-neutral expectations, which have direct counterparts in financial market data. The term structure of interest rates can be converted into forward rates. And risk-neutral inflation expectations can be obtained from break-even inflation and inflation swaps.

Intuitively, the standard AR(1) structure assumed in the New Keynesian model implies that inflation, on average, reverts to the mean. This mean reverting path is governed by three parameters: The current level, the persistence, and the level it converges to. As a result, the estimated path can identify exactly three parameters per curve.

At the same time, the identification of these six parameters exhausts the information from the term structure of expectations. Even if one were to use additional data points, we would not be able to identify more parameters. For example, if we took risk-neutral expectations at time t for inflation in period $t+4$, $\hat{\mathbb{E}}_t[\pi_{t+4}]$, the model would tell us that it is entirely pinned down by expectations on the shorter end of the term structure:

$$\hat{\mathbb{E}}_t[\pi_{t+4}] = \frac{\hat{\mathbb{E}}_t[\pi_{t+2}]^2 + \hat{\mathbb{E}}_t[\pi_{t+3}]^2 - \left(\hat{\mathbb{E}}_t[\pi_{t+1}] + \hat{\mathbb{E}}_t[\pi_{t+2}]\right)\hat{\mathbb{E}}_t[\pi_{t+3}]}{\hat{\mathbb{E}}_t[\pi_{t+2}] - \hat{\mathbb{E}}_t[\pi_{t+1}]}.$$

While we can estimate r^*, risk premia, persistence parameters, and shocks, the other parameters of the model cannot be estimated using financial variables alone. These parameters are the discount rate, β, the coefficient of relative risk aversion, γ, the weight of the output gap in the central bank's objective, α, and the slope of the Phillips curve, λ. We are therefore left with three options. We can calibrate these other parameters based on estimates in the literature, use the time-series dimension of the financial data, or we introduce additional macroeconomic data at lower frequencies to obtain estimates of these other parameters. We opt for the first and calibrate this remaining set of parameters. This choice is mainly driven by the desire to focus this paper on the high-frequency use of financial data for the New Keynesian model.

Table 1
Calibrated Invariant Parameters

Parameter	Description	Value
β	Discount factor	0.98
γ	Coefficient of relative risk aversion	2
α	Central bank weight on output gap	0.33
λ	Slope of Phillips curve	0.05

Table 1 presents our benchmark calibration. We calibrate the annual discount factor to 0.98. We calibrate the coefficient of relative risk aversion to 2, which implies an intertemporal elasticity of substitution of 0.5. We set the weight of the output gap in the central bank's objective to 1/3 based on Gali (2008). Finally, we calibrate the slope of the Phillips curve to be consistent with the range of estimates from the macroeconomics literature, as summarized in Mavroeidis et al. (2014).

The remaining set of parameters contains variables for which perceptions can change quickly:

$$\{\rho_u, \mu_u, u_t, \rho_g, g_t, r_t^*\}$$

We obtain an estimate of this vector of parameters on each day based on closing prices. While the parameters of the model are constant, the estimate of these parameters may vary day-by-day. We do not put any restrictions on these daily variations.

We estimate this vector of parameters using a four step procedure that attempts to minimize the effect of financial market noise on our parameter estimates over time. In the first step, we estimate ρ_u by minimizing the sum-of-squared errors between the model implied forward break-even rates and the data. After obtaining an estimate of ρ_u, our second step is to estimate μ_u and u_t by minimizing the sum-of-squared errors between model implied break-even curve and the data.

We then repeat the procedure using the nominal yield curve to estimate the remaining three parameters. In the third step, we estimate ρ_g by minimizing the sum-of-squared errors between the model implied forward nominal interest rates and the data. Finally, after obtaining an estimate of ρ_g, we estimate g_t and r_t^* by minimizing the sum-of-squared errors between model implied nominal yields and the data.

3.2 Data

We use daily data on risk-neutral expectations from break-even inflation and the nominal Treasury yield curve. For the nominal yield curve, we obtain Treasury yields from FRED. To minimize the effects of market illiquidity on financial market prices at isolated maturities, we begin by smoothing the cross-section with Nelson-Siegel curves using daily Treasury yield data at 1, 2, 3, 5, 7, 10 and 20 year maturities. We then evaluate the Nelson-Siegel-implied Treasury yield curve at annual maturities between 5 and 20 years and use them in our estimation of ρ_g, g_t and r_t^*. We discuss the focus on long-term yields in the next section when analyzing the model fit.

We measure risk-neutral expectations of inflation using break-even inflation rates, which is the difference in the yield of a nominal Treasury security and a Treasury Inflation-Protected Security (TIPS) of the same maturity. We rely on break-even inflation rates primarily because the main alternative, inflation swaps, displayed low liquidity in the early part of the sample. For instance, Fleming and Sporn (2013) show that average daily inflation swap activity did not hit 100 million dollars until late 2008. During this earlier period, TIPS markets were relatively more active.[1]

Similar to our treatment of the Treasury yields data, we begin by smoothing out the break-even inflation curve by estimating Nelson-Siegel curves. We use daily data on break-even inflation rates

[1]Fleming and Krishnan (2012) show trading activity in the TIPS market for 0 to 10 year maturity bonds averaged 512.8 million dollars per day between March 4, 2005 and March 27, 2008.

at 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 30 year maturities. We then compute the Nelson-Siegel implied break-even inflation curve at all maturities between 5 and 20 years to estimate ρ_u, u_t and μ_u.

We gather end-of-day break-even inflation rates from Bloomberg and Treasury yields from FRED. Our sample period ranges from January 2, 2001 to September 14, 2023. Appendix B.1 contains the list of Bloomberg tickers. For additional details about our estimation procedure, see Appendix B.2.

4 Estimation Results

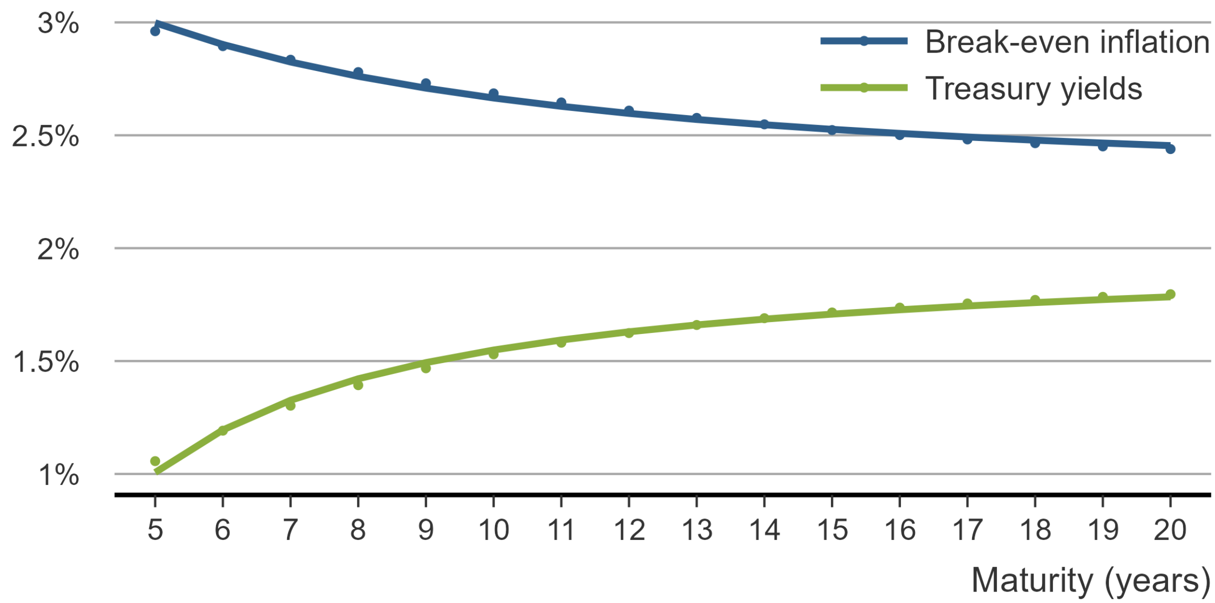
This section presents various components of our estimated model. We begin by discussing the model fit, our estimates of inflation risk premiums and expectations for inflation in the long-run, followed by a discussion of our estimate of r^*. We perform an event study around the October 2021 CPI release on November 10, 2021, which greatly changed perceptions of U.S. inflation in the aftermath of the COVID pandemic. Finally, we conclude this section with a comparison of long-term inflation expectations.

We start with a discussion of the model fit. As stated above, the AR(1) structure of the process limits the shape that inflation expectations can take. It can only produce a reversion from a current level to a long-term average.

We focus on matching the 5 to 20 year maturity data for two reasons. First, the main outcome variables of interest in this paper are medium to longer-run variables. These outcome variables include the long-run real rate, long-run inflation expectations and the medium to long-run policy stance. For these outcome variables, the longer-end of the yield curve is most informative.

Second, the financial New Keynesian model that we derived in Section 2 is not well suited for matching both the short- and long-run behavior in Treasury and break-even yields that we observe in the data. Forward rates in the model essentially follow an AR(1) process. This is not necessarily the case in the data. For example, there could be hump-shaped dynamics when financial market participants expect interest rates to increase for a few more years before reverting to a lower long run level. In this case, the forward yield curve would not be well approximated by an AR(1) process, and our model would not produce a reliable estimate of both the short and long-run dynamics.

Figure 1
Model Fit on November 9, 2021



Notes: This figure presents the model fit on November 9, 2021. The left-hand figure plots the model risk-neutral inflation expectations curve against the data (plotted as dots), and the right-hand figure plots the model implied nominal yield curve against the data (plotted as dots). We use break-even inflation rates to capture risk-neutral expectations of inflation in the data. The nominal yield curve data is derived from Treasury yields.

Thus, we focus our efforts on matching the medium to longer-term in this paper, and leave the estimation of full yield curve to future research.

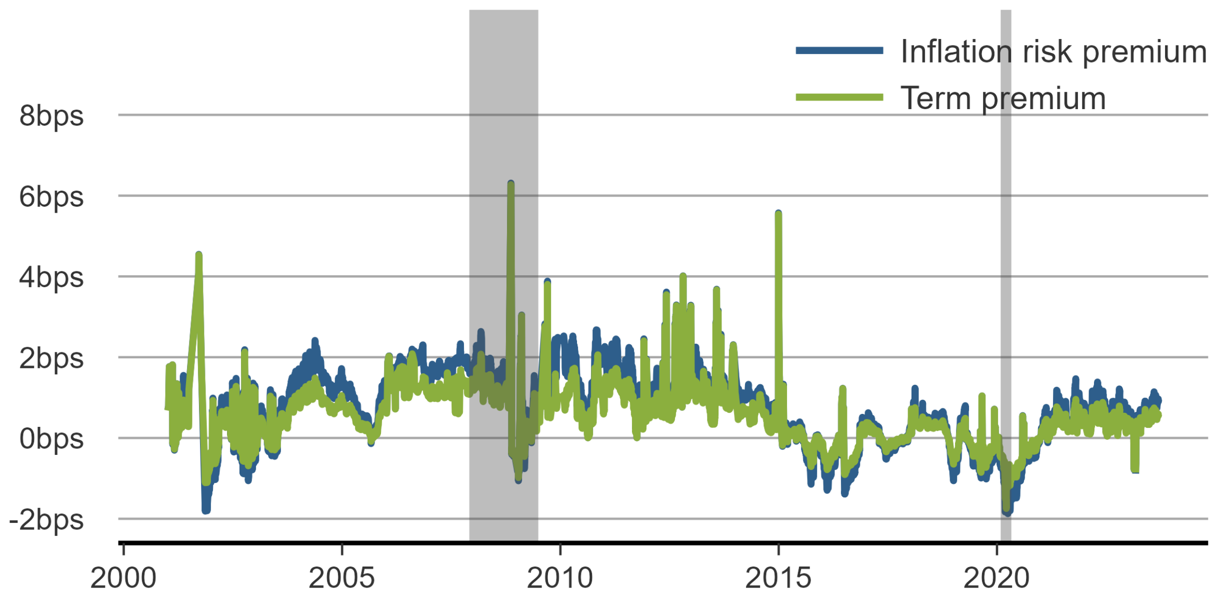
Figure 1 shows the fit of our model on November 9, 2021. The downward sloping blue line plots the model-implied inflation break-even curve against the data (plotted as dots), and the upwards sloping green line plots the model-implied nominal yield curve against the data (plotted as dots). The figure shows the model fits the data well.

The close fit of the model to the data is representative of the results throughout the sample period. In general, the model accurately captures both the break-even inflation curve and the nominal yield curve, and the mean absolute value of the fitting error across all dates and maturities is only 1.3 basis points. These results are a reflection of the tendency of longer-term forward rates to stabilize, particularly during a period of anchored inflation expectations.

4.1 Inflation Risk Premiums and Long-run Expectations

Next, we discuss our estimation of risk premiums. The theory shows that risk premiums impact the economy through at least two channels. First, inflation is on average away from its target when a risk premium is present. We estimate that the median absolute value of the inflation bias was 18 basis points since the Federal Reserve raised interest rates above zero at the end of 2015. Thus, risk premiums can cause inflation to deviate notably from target.

FIGURE 2
INFLATION RISK AND TERM PREMIUMS



Notes: The figure above presents estimates of the five-year inflation risk premium, and the five-year term premium. Estimates are winsorized at the 1 percent level and smoothed over a 10 day window. Grey shading indicates NBER recessions.

Risk premiums also drive a wedge between the risk-neutral expectation of inflation and the physical expectation of inflation. The inflation risk premium is the difference between risk-neutral and physical expectation of inflation, and the term premium is the difference between risk-neutral and physical expectations of nominal interest rates. The five-year inflation risk premium is presented in green in Figure 2. We extract daily estimates of the inflation risk premium through the estimation procedure described above. Since there is some noise due to fluctuating liquidity in financial market prices, we present moving averages for our variables of interest.

The estimated inflation risk premium at the five-year inflation horizon is very small, staying mostly around 2 basis points (bps) or less. This result is not due to our specification of the stochastic discount factor. We use the model to obtain the equations we estimate and let the data speak as to the size of the inflation risk premium.

Our estimates show that the one-year inflation risk premium tended to be larger earlier in the sample period, reaching up to 19 basis points in the years following the Global Financial Crisis. After 2015, inflation risk premiums remained close to zero.

The blue line in Figure 2 presents estimates of the five-year term premium. Given that no risk premium is associated with demand shocks, our estimates of the term premium are very similar to our estimates of the inflation risk premium. Both are largely determined by time series variation in the mark-up risk premium, μ_u.

As a consequence of low inflation risk premia, inflation is close to target on average. In other words, the wedge between the target rate of inflation and the actual rate of inflation on average in equation (16) is small.

The small inflation risk premium implies that physical and risk-neutral inflation expectations are similar in size, as their gap is proportional to the inflation risk premium (see equation (17)).

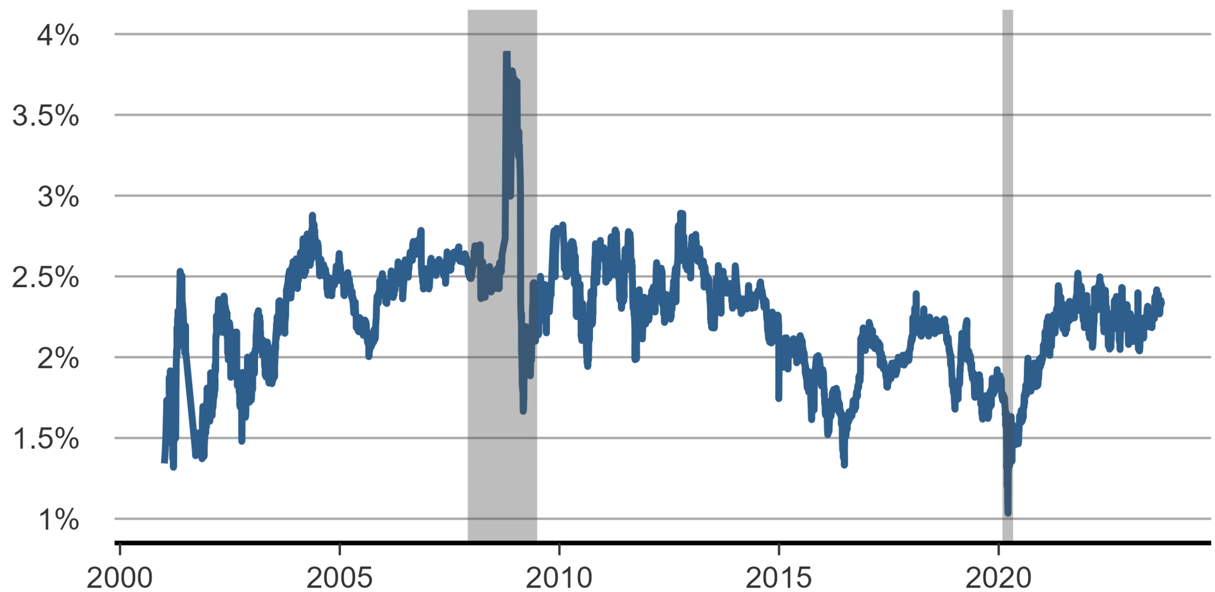
Figure 3 presents the physical expectation of the five-year inflation rate five years ahead, which we interpret as the long-term expectation of inflation. We compute this measure by subtracting the inflation risk premiums from the risk-neutral expectation of the inflation rate from 5 years ahead to 10 years ahead computed using break-even inflation rates. The expression for the inflation risk premium between s years ahead can be expressed as

$$\hat{\mathbb{E}}[\pi_{t+s}] - \mathbb{E}[\pi_{t+s}] = \frac{\alpha\left(\sum_{j=0}^{s-1} \rho_u^j\right)}{\lambda^2 + \alpha(1 - \beta\rho_u)}\mu_u,$$

which is increasing in the risk premium associated with the mark-up shock, μ_u, and the persistence of the mark-up shock, ρ_u. The expression for inflation risk premiums also shows the impact of μ_u on the inflation risk premium grows with the time horizon s, albeit discounted by the shock persistence, ρ_u.

Figure 3 shows that throughout our sample period long-term inflation expectations remained

FIGURE 3
LONG-TERM INFLATION EXPECTATIONS

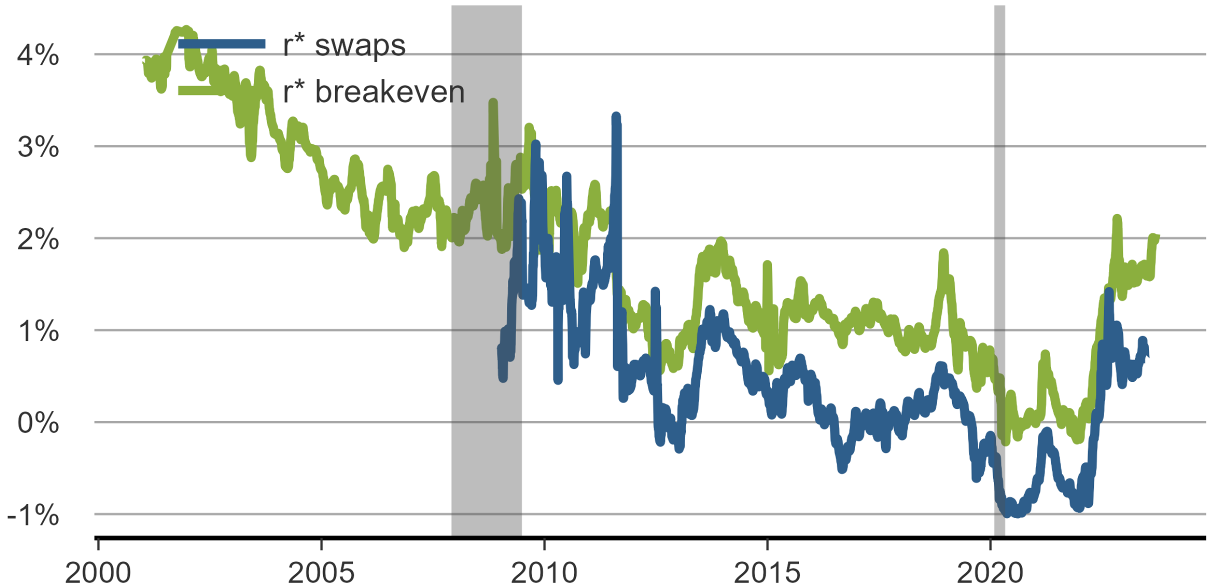


Notes: This figure presents the physical expectation of the average 5 year inflation rate 5 years ahead. In other words, the figure presents the physical expectation of the inflation rate from 5 years ahead to 10 years ahead. Estimates are winsorized at the 1 percent level and smoothed over a 10 day window. Grey shading indicates NBER recessions.

anchored around the Federal Reserve inflation target. Even as annual inflation rates increased to 9 percent in aftermath of the COVID pandemic, long-term inflation expectations never rose above 2.5 percent. Thus, even as the United States experienced its highest level of inflation in 40 years, financial markets expected inflationary pressures to die out over time and inflation to return to a level close to the Federal Reserve's target in the long run. These estimates reflect risk-neutral expectations of the Consumer Price Index (CPI). CPI inflation has tended to run about 30 to 40 bps higher than Personal Consumption Expenditure (PCE) price inflation which the Federal Reserve uses to measure its inflation goals. Taking this into account, CPI inflation expectations at levels of 2.3% or 2.4% thus seem consistent with the Federal Reserve's target level.

The only periods in which long-term inflation expectations deviated notably from the Federal Reserve's 2-percent target were the heights of the Global Financial Crisis of 2007-2009 and the COVID pandemic. These periods are also associated with significant liquidity risk in Treasury markets and estimates should thus be viewed in this light.

Figure 4
Long-run Real Interest Rate



Notes: This figure presents estimates of the natural rate of interest, r^*. Estimates are winsorized at the 1 percent level and smoothed over a 10-day window. Grey shading indicates NBER recessions.

4.2 Long-run Real Interest Rate

Figure 4 presents our estimate of the long-run real interest rate in the economy, r^*. While the estimates of inflation risk premia and inflation expectations derives from the break-even inflation curve alone, estimates of long-run neutral real rate r^* require information from the nominal yield curve as well.

We show two estimates of the neutral real rate. The green line presents our measure of r^* extracted from break-even inflation and Treasury yields consistent with the charts above. This estimate starts at around 4 percent in the early 2000s and gradually declines towards zero during the zero lower bound period. Towards the end of the sample, the $r*$ measure extracted from break-even inflation has rebounded and hovers around 2 percent at the very end.

We present an additional measure of r^* in Figure 4 from swaps (blue line). We estimate this alternate r^* measure with the same procedure as for Treasury curves using inflation swap rates to measure risk-neutral expectations of inflation and overnight indexed swaps (OIS) written on the Federal Funds rate to capture nominal yields. Figure 4 shows that using swaps in our estimation produces a long-run real interest rate series with very similar variation to our benchmark estimate. However, the level of the series estimated from swaps is lower and now hovers around 70 bps.

Using swaps is the more direct measure since OIS contracts are written on the effective federal funds rate, the policy rate in our model. The time series is, however, shorter as swap trading became liquid about a decade after TIPS markets had been introduced.

The discrepancy between the two estimates shares a wider pattern in the literature. The exact level of the natural rate of interest is hard to pin down.[2] However, estimates generally share the feature that the natural rate of interest has been declining over the sample period.

The advantage of an r^*-estimate based on financial markets is that new data is available whenever markets are open. Our methodology of using a cross-section of data takes maximum advantage of this feature. At the same time, the variation in the estimates is limited, suggesting that our estimation produces stable results. Furthermore, our estimate of the natural rate is consistent with the inflation risk premium and inflation expectations within the context of the New Keynesian model.

4.3 Demand and Supply Shocks

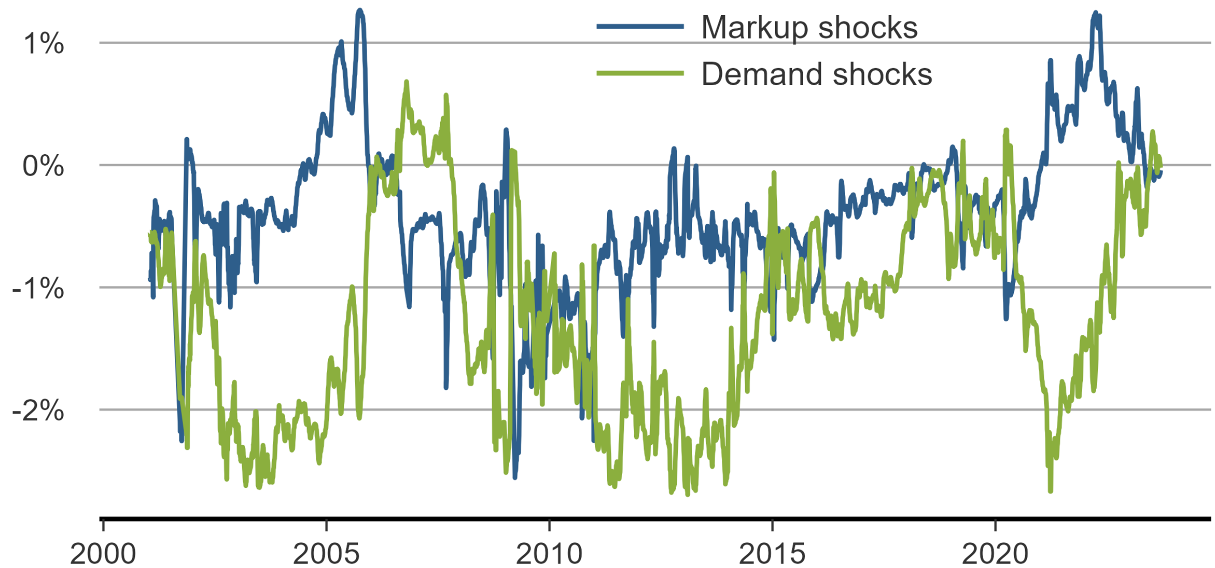
Among the variables we recover from the estimation are the current level of the demand shock and the level of the mark-up shock on the supply side. Given that we estimate these shocks from data five years out, we interpret them as longer-term shocks rather than representing transition dynamics or other shocks that might drive the short-run.

Figure 6 shows the time series of daily estimates for demand and mark-up shocks, normalized by the standard deviation of the mark-up shock. For the interpretation, it is important to keep in mind that the mark-up shocks are extracted from the inflation curve only. The demand shocks then soak up variation in nominal yields to match the Treasury yield curve. Also note that the estimation does not impose a restriction that the estimated shocks have to be zero on average.

Despite the lack of such a constraint, estimated mark-up shocks are close to zero on average. There are two periods where they are notably positive: The mid-2000s and during the post-COVID

[2]Our literature review lists a set of papers deriving measures of the natural rate.

Figure 5
Estimates of Supply and Demand Shocks



Notes: This figure presents estimates of demand and mark-up shocks, standardized by the standard deviation of mark-up shocks, based on the nominal Treasury yield curve and break-even inflation rates. Estimates are winsorized at the 1 percent level and smoothed over a 10-day window. Grey shading indicates NBER recessions.

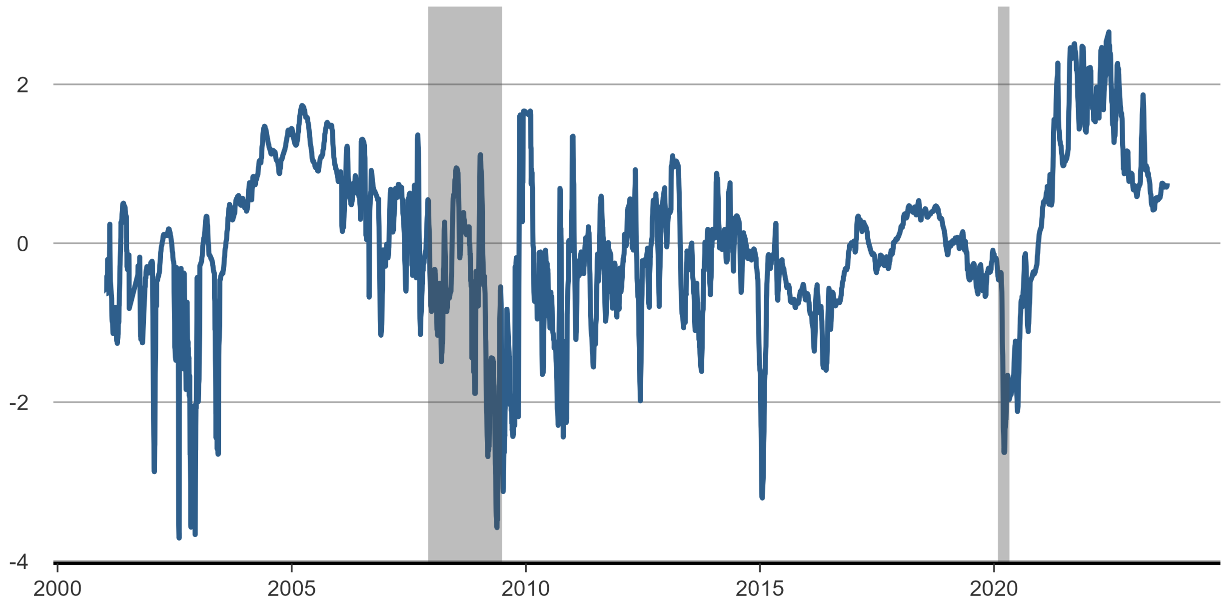
spike in inflation. During most of the 2010s, when inflation ran close to, but slightly below, the Federal Reserve's target level of two percent, mark-up shocks were below their unconditional mean.

The estimated series series for demand shocks is substantially more volatile. Demand shocks are mostly negative, reflecting an upward sloping yield curve conditional on the estimates for mark-up shocks. Particularly in the aftermath of the Great Financial Crisis and the COVID pandemic, estimated demand is below average. The only periods of positive demand shocks appear during post-COVID inflationary episode and before the Great Financial Crisis.

4.4 The Stance of Policy

The model allows us to dig deeper into the workings of monetary policy to offset these demand- and supply-side disturbances. The main measure for monetary policy in the United States is the federal funds rate. Not only does its current level matter but also the path. A summary statistic for this path is a longer-term interest rate, which is affected by the sequence of short-term interest rates through the Expectations Hypothesis.

Figure 6
Medium- to Long-run Policy Stance



Notes: This figure presents estimates of the difference between the 5-year-5-year nominal interest rate and the corresponding neutral rate of interest. Estimates are standardized by the standard deviation, winsorized at the 1 percent level and smoothed over a 10-day window. Grey shading indicates NBER recessions.

We provide a measure of the current stance of monetary policy by looking at longer-term forward interest rates in relation to a neutral rate implied by the model. The stance of policy is restrictive if interest rates are above the neutral rate, and accommodative when interest rates are below. In our model, we derive the term structure for the neutral policy rate by computing the interest rate that sets the expected change in the output gap to zero at each horizon. Appendix A.6 shows that the n-year neutral rate, $\tilde{i}_{t,t+n}$, can be expressed as:

$$\tilde{i}_{t,t+n} = r^* + \frac{1}{n}\sum_{s=1}^{n}\hat{\mathbb{E}}_t[\pi_{t+1}] + \frac{\gamma}{n}\left(\sum_{s=0}^{n-1}\rho_g^s\right)g_t, \tag{18}$$

where $\frac{1}{n}\sum_{s=1}^{n}\hat{\mathbb{E}}_t[\pi_{t+1}]$ is just the n-year break-even inflation rate or inflation swap rate. Intuitively, equation (18) provides a neutral stance through the following consideration. The *nominal* neutral interest rate over the next n years is equal to the long-run real rate, r^*, plus the risk-neutral expectation for inflation over the next n years, plus an additional factor that exactly cancels out the effects of the current demand shock.

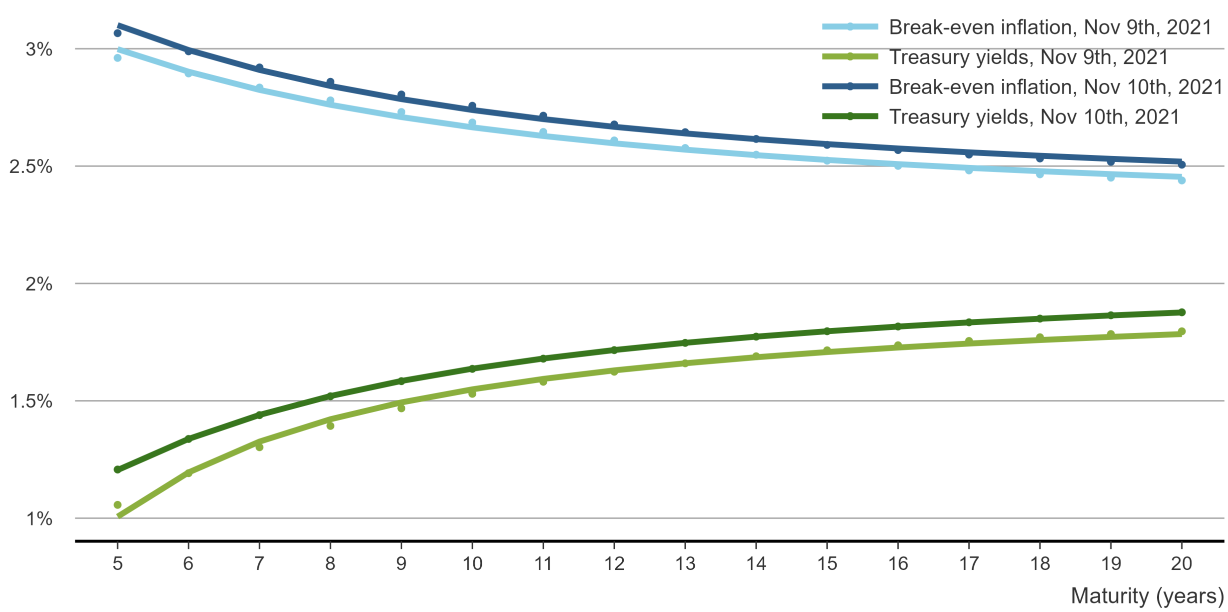
Figure 6 presents the difference between the 5-year-5-year nominal interest rate and the corresponding neutral interest rate over our sample period, which we interpret as an indicator for the medium- to long-run policy stance. Throughout the Global Financial Crisis (GFC) of 2007-09 and the post-GFC period, the longer term policy stance remained largely accommodative. As the Federal Reserve raised interest rates above the zero lower bound, our policy stance indicator approached neutrality. In response to the Covid-19 pandemic, policy turned accommodative again.

There were only two periods in our sample when the stance of policy was persistently tight according to our measure. The first is during the mid-2000s with declining restrictiveness before the financial crisis. And, more recently, during the post-COVID episode of high inflation.

4.5 Event Study: CPI Release on November 10, 2021

In this subsection, we show how the estimation in this paper lends itself to event study analysis. Therefore, we investigate the impact of a specific news release on inflation and interest rate expectations in the context of our framework.

FIGURE 7
IMPACT OF OCTOBER 2021 CPI RELEASE ON INFLATION EXPECTATIONS AND NOMINAL YIELDS



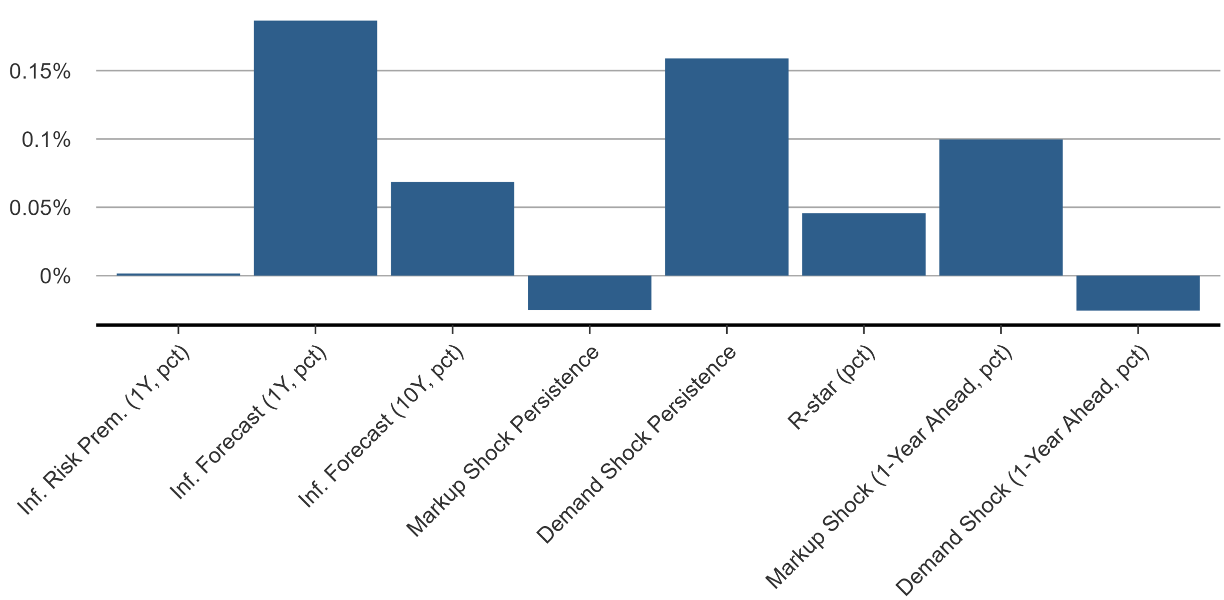
Notes: This figure presents the change in the risk-neutral inflation expectations and the nominal yield curve in response to the October 2021 CPI release, which occurred on November 10, 2021.

On November 10, 2021, the Bureau of Labor Statistics released the October 2021 CPI report, which showed annual inflation had hit 6.2 percent. At the time, the U.S. had not seen an inflation reading this high since 1990. This surprisingly high October CPI print shifted up the break-even inflation curve and Treasury yields.

Figure 7 shows this shift in the inflation and interest rate curves. It adds the model fit on the day of the October 2021 CPI release to the Figure 1, which already contained the model fit on the day before the data release. Analogous to Figure 1, the points represent data and the lines represent the model implied break-even rates and nominal yields.

Applying the estimation to the day before and after the news release allows us to parse out how the perceived state of the economy changed following the news release.

FIGURE 8
IMPACT OF OCTOBER 2021 CPI RELEASE ON OTHER VARIABLES



Notes: This figure presents the impact of the October 2021 CPI release on the one-year inflation risk premium, 1-year inflation expectations, 10-year inflation expectations, the persistence of the mark-up shock and the long-run real interest rate. Each bar represents the change in the estimate between November 9, 2021 and November 10, 2021.

Figure 8 presents the changes different variables of interest as a result of the October 2021 CPI release. Our estimates show that the change in the break-even inflation curve is mainly explained by expected inflation, rather than a change in inflation risk premiums. The first bar in Figure 8 shows the one-year inflation risk premium barely moved, while the physical expectation of inflation over the 10 years increased substantially.

While long-term inflation expectations rose, inflation expectations at short horizons (second bar) rose substantially more. As a result, the implied inflation expectations curve started out at a higher level and thus had to revert more quickly to the mean. The fourth bar in Figure 8 shows the estimated persistence of the mark-up shock decreased. Since inflation rose, the level of the estimated mark-up shock had to increase as well. The shift in the Treasury curve implied a higher demand shock with an increased persistence.

With the rise in expected inflation, the stance of policy was perceived to be more accommodative. Consequently, the perceived long-run neutral real interest rate declined following the announcement (sixth bar).

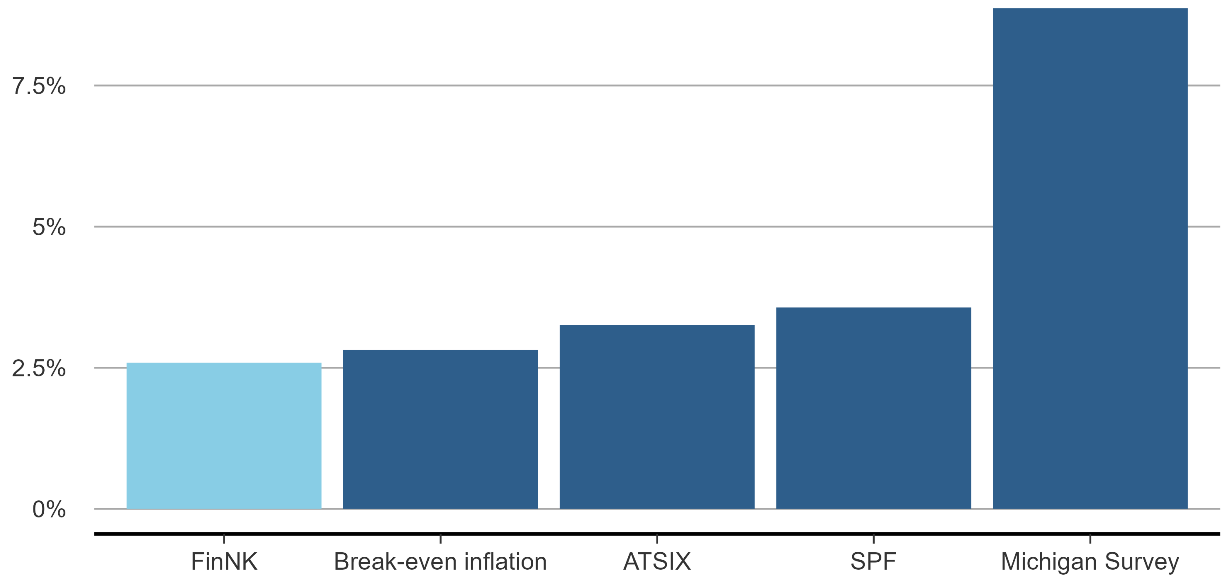
4.6 Inflation Forecasting

The ability to switch between physical and risk-neutral expectations in the model allows us to provide an inflation forecast. This forecast applies to longer horizons since we match the long end of the break-even inflation curve.

In this section, we analyze the accuracy of the forecast. We use our model to compute physical expectations of average inflation over a 10-year horizon by subtracting inflation risk premiums from break-even inflation rates. We compare the accuracy of our model's long-run inflation forecasts to various well known inflation forecasts. These other forecasts include the Michigan Survey, the Survey of Professional Forecasters, and the Aruoba Term Structure of Inflation Expectations (ATSIX). We also include the break-even inflation rates and inflation swap rates as additional forecasts for inflation, even though these two series both include risk-premium components.

Due to the different frequencies at which surveys are conducted, we first aggregate all of our forecasts to the quarter level by averaging forecasts within the quarter. We then compute the square root of the the sum of squared distance between each forecast and the average realized inflation over a horizon of 10 years. Our sample period therefore stops in 2013, 10 years before the full sample period to be able to compute the ex-post realized rate of inflation.

Figure 9
Accuracy of Inflation Forecasts



Notes: This figure compares the accuracy of long-run inflation forecasts from our model (FinNK) against the accuracy other inflation forecasts. Each bar represents the square root of the sum of squared distance between 10-year inflation forecasts and realized average inflation over a 10-year horizon. The data are aggregated to a quarterly level to account for the fact that surveys are conducted at different frequencies.

Figure 9 presents the accuracy of the various forecast methods, as measured by the sum of squared forecast errors.

Long-run inflation forecasts generated by the financial New Keynesian are the most accurate since the start of our sample. Their accuracy exceed those of the raw break-even inflation rates, unadjusted for risk premia. This result is by no means guaranteed: There is no condition in the estimation strategy that ensures that the absence of risk premia leads to better forecasts.

Survey evidence from ATSIX and the Survey of Professional Forecasters perform relatively well. They outperform (risk-neutral) forecasts from inflation swaps that struggle particularly in the early part of the sample when liquidity was low. The Michigan Survey provides the least accurate forecast. This is mainly due to the fact that the level of the forecasts exceeds the true rate of inflation.

Figure 9 also suggests that risk-neutral expectations for inflation provide reasonable forecasts for inflation even without adjusting for risk premiums. The financial New Keynesian model improves on this forecast and thus tends to forecast long-run inflation better than professional forecasters and households.

5 Conclusion

This paper shows that financial market data can partially identify shocks and parameters in the standard New Keynesian model. The estimation can be performed based on a single cross section of asset prices. It gives rise to measures of the inflation risk premia, short- and long-term inflation expectations, as well as the long-run neutral real interest rate. The methodology further lends itself to analyzing the effects of events such as news releases.

While we take a strong position by relying on financial market data at a snapshot in time, future research can augment the use of financial data with macroeconomic data to jointly estimate the entire set of parameters.

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Appendix

A The New Keynesian model

A.1 Households

Time is discrete and there exists a unit mass of households that live forever. In each period, households consume, they supply labor, and they save. The household objective is to maximize lifetime utility given by equation (1).

A.2 IS Curve: Details

In the following section, we derive the log-linear IS relationship. We deviate from the canonical derivation by substituting in the definition of the inflation swap rate prior to log-linearizing the equilibrium.

For any real rate of return R_{t+1} realized at time $t+1$, the household's Euler equation reads:

$$\mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} \right] = 1 \tag{19}$$

Under a flexible price economy the Euler equation reads:

$$\mathbb{E}_t \left[\beta \left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\gamma} R^*_t \right] = 1 \tag{20}$$

Putting together equations (19) and (20) together shows:

$$\mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{I_t}{\Pi_{t+1}} \right] = \mathbb{E}_t \left[\beta \left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\gamma} R^*_t \right] \tag{21}$$

where $R_{t+1} = I_t/\Pi_{t+1}$, and I_t is a nominal rate of return known at time t. We substitute equation (6) on the left-hand side with $X_{t+1} = \Pi_{t+1}$ to show:

$$\frac{I_t}{S_{t,t+1}}\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right] = \mathbb{E}_t\left[\beta\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma} R^*_t\right], \tag{22}$$

where $\hat{\mathbb{E}}_t[\Pi_{t+1}] = S_{t,t+1}$ is the risk-neutral expectation of inflation (alternatively the inflation swap rate). The first-order approximation of the above expression simplifies to equation (7).

A.3 Firms

Assume there exists a unit mass of monopolisticly competitive firms with the production function $Y_t(i) = A_t N_t(i)$. Furthermore, assume firms pay a nominal adjustment cost in order to update their prices following Rotemberg (1982). This adjustment cost is:

$$\frac{\eta}{2}\left(\frac{P_t(i) - P_{t-1}(i)}{P_{t-1}}\right)^2 P_{t-1}Y_{t-1}.$$

Substitute the production function and household demand into the firm's objective to write the firm's real value function as:

$$V(P_{t-1}(i)) = \max_{P_t(i)}\left(\frac{P_t(i)}{P_t}\right)^{1-\varepsilon} Y_t - \Psi_t\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t - \frac{\eta}{2}\left(\frac{P_t(i) - P_{t-1}(i)}{P_{t-1}}\right)^2 \frac{P_{t-1}}{P_t} Y_{t-1} + \mathbb{E}\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} V(P_t(i))\right]$$

where

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$$

and

$$\Psi_t = \frac{W_t}{P_t A_t}$$

is the firm's real marginal cost. We assume the firm is owned by the households, and therefore discount the future using the household's stochastic discount factor. The first order condition with respect to $P_t(i)$ yields

$$(1-\epsilon)\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_t}+\epsilon\Psi_t\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_t(i)}-\eta\left(\frac{P_t(i)-P_{t-1}(i)}{P_{t-1}}\right)\frac{Y_{t-1}}{P_t}=-\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}V'(P_t(i))\right] \quad (23)$$

Iterating forward the envelope condition yields:

$$V'(P_t(i))=\eta\left(\frac{P_{t+1}(i)-P_t(i)}{P_t}\right)\frac{Y_t}{P_{t+1}} \quad (24)$$

Combining equations (23) and (24), and dividing through by Y_t shows:

$$\begin{aligned}&(1-\epsilon)\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}\frac{1}{P_t}+\epsilon\Psi_t\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}\frac{1}{P_t(i)}-\eta\left(\frac{P_t(i)-P_{t-1}(i)}{P_{t-1}}\right)\frac{1}{P_t}\frac{Y_{t-1}}{Y_t}=\\&-\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\eta\left(\frac{P_{t+1}(i)-P_t(i)}{P_t}\right)\frac{1}{P_{t+1}}\right]\end{aligned}$$

Imposing symmetry across price-setters, multiplying through by P_t, and plugging in the equilibrium consumption of households yields:

$$\begin{aligned}(1-\epsilon)+\epsilon\Psi_t+\eta\left(1-\Pi_t\right)\frac{Y_{t-1}}{Y_t}&=\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\eta\left(1-\Pi_{t+1}\right)\frac{1}{\Pi_{t+1}}\right]\\&=\eta\mathbb{E}_t\left[\beta\left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma}\frac{1}{\Pi_{t+1}}\right]-\eta\mathbb{E}_t\left[\beta\left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma}\right]\\&=\eta\mathbb{E}_t\left[M_{t+1}\right]-\eta\mathbb{E}_t\left[M_{t+1}\Pi_{t+1}\right]\\&=\eta\mathbb{E}_t\left[M_{t+1}\right]\left(1-S_{t,t+1}\right).\end{aligned}$$

The log-linear approximation of the equation above is:

$$\varepsilon(\psi_t-\overline{\psi}_t)-\eta(y_t-y_{t-1})-\eta\pi_t+\eta(y_t-y_{t-1})=-\eta\beta s_{t,t+1} \quad (25)$$

In the textbook New Keynesian model, the real marginal cost ψ_t is a function of the output gap x_t. We also add in an ad-hoc cost-push shock u_t, and we re-write the previous equation as:

$$\pi_t = \lambda x_t + \beta \widehat{\mathbb{E}}_t \left[\pi_{t+1} \right] + u_t,$$

where $u_t = \rho_u u_{t-1} + \varepsilon_{u,t}$ and $\varepsilon_u \sim N(0, \sigma_u)$.

A.4 Central Bank

We assume that the central bank sets the policy rate i_t to minimize the expected quadratic loss:

$$\mathcal{L} = (1-\beta)\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \alpha x_t^2 \right) \right], \tag{26}$$

where expectations are taken under the physical distribution. Following Clarida, Gali and Gertler (1999), we assume future inflation and output are not affected by today's actions and the central bank cannot directly manipulate expectations. In other words, we assume policy is discretionary, and we reformulate the central bank's objective as:

$$\min_{x_t} \pi_t^2 + \alpha x_t^2 \quad s.t. \quad \pi_t = \lambda x_t + \beta \mathbb{E}_t \left[\pi_{t+1} \right] + u_t, \tag{27}$$

which obtains a maximum at

$$x_t = -\frac{\lambda}{\alpha} \pi_t \tag{28}$$

A.5 Equilibrium

In this section, we derive the equilibrium inflation process, optimal monetary policy and risk premiums assuming that the central bank sets interest rates optimally. The equilibrium is described by the following set of equations:

$$x_t = g_t - \frac{1}{\gamma}(i_t - \hat{\mathbb{E}}_t[\pi_{t+1}] - r_t^*) + \mathbb{E}_t[x_{t+1}] \tag{IS Curve}$$

$$\pi_t = u_t + \lambda x_t + \beta\hat{\mathbb{E}}_t[\pi_{t+1}] \tag{Phillips Curve}$$

$$x_t = -\frac{\lambda}{\alpha}\pi_t \tag{Optimality Condition}$$

$$g_{t+1} = \rho_g g_t + \varepsilon_{g,t+1} \tag{Demand shock}$$

$$u_{t+1} = \rho_u u_t + \varepsilon_{u,t+1} \tag{Cost-push Shock}$$

with $\mathbb{E}[\varepsilon_g] = \mathbb{E}[\varepsilon_u] = 0$. In risk-neutral expectations, $\hat{\mathbb{E}}[\varepsilon_g] = 0$ and $\hat{\mathbb{E}}[\varepsilon_u] = \mu_u$.

Lemma 1 (Inflation) *The inflation process is given by*

$$\pi_t = \frac{\alpha}{\lambda^2 + \alpha}(\beta\hat{\mathbb{E}}_t[\pi_{t+1}] + u_t) = \frac{\alpha^2\beta}{(\lambda^2 + \alpha(1-\beta))(\lambda^2 + \alpha(1-\beta\rho_u))}\mu_u + \frac{\alpha}{\lambda^2 + \alpha(1-\beta\rho_u)}u_t$$

where inflation expectations are

$$\hat{\mathbb{E}}_t[\pi_{t+1}] = \frac{\alpha(\lambda^2 + \alpha)}{(\lambda^2 + \alpha(1-\beta))(\lambda^2 + \alpha(1-\beta\rho_u))}\mu_u + \frac{\alpha\rho_u}{\lambda^2 + \alpha(1-\beta\rho_u)}u_t$$

Proof: We will use a guess and verify strategy to prove the lemma. Conjecture:

$$\pi_t = \kappa_0^\pi + \kappa_E^\pi\hat{\mathbb{E}}_t[\pi_{t+1}] + \kappa_u^\pi u_t$$

Plug conjecture into the Phillips curve, use the optimality condition for monetary policy, and match coefficients:

$$\kappa_0^\pi + \kappa_E^\pi\hat{\mathbb{E}}_t[\pi_{t+1}] + \kappa_u^\pi u_t = -\frac{\lambda^2}{\alpha}(\kappa_0^\pi + \kappa_E^\pi\hat{\mathbb{E}}_t[\pi_{t+1}] + \kappa_u^\pi u_t) + u_t + \beta\hat{\mathbb{E}}_t[\pi_{t+1}]$$

$$\kappa_0^\pi = 0 \qquad \kappa_E^\pi = \frac{\alpha\beta}{\lambda^2 + \alpha} \qquad \kappa_0^\pi = \frac{\alpha}{\lambda^2 + \alpha}$$

As a result:

$$\pi_t = \frac{\alpha}{\lambda^2 + \alpha}(\beta\hat{\mathbb{E}}_t[\pi_{t+1}] + u_t) \tag{29}$$

Conjecture:

$$\hat{\mathbb{E}}_t[\pi_{t+1}] = \kappa_0^E + \kappa_u^E u_t$$

Use this guess and match coefficients via

$$\begin{aligned}
\hat{\mathbb{E}}_t[\pi_{t+1}] &= \kappa_0^E + \kappa_u^E u_t \\
&= \hat{\mathbb{E}}_t\left[\frac{\alpha\beta}{\lambda^2 + \alpha}\hat{\mathbb{E}}_{t+1}[\pi_{t+2}] + \frac{\alpha}{\lambda^2 + \alpha}u_{t+1}\right] \\
&= \hat{\mathbb{E}}_t\left[\frac{\alpha\beta}{\lambda^2 + \alpha}(\kappa_0^E + \kappa_u^E u_{t+1})) + \frac{\alpha}{\lambda^2 + \alpha}u_{t+1}\right] \\
&= \frac{\alpha\beta}{\lambda^2 + \alpha}\kappa_0^E + \frac{\alpha}{\lambda^2 + \alpha}(1 + \beta\kappa_u^E)\hat{\mathbb{E}}_t\left[u_{t+1}\right] \\
&= \frac{\alpha\beta}{\lambda^2 + \alpha}\kappa_0^E + \frac{\alpha}{\lambda^2 + \alpha}(1 + \beta\kappa_u^E)(\rho_u u_t + \mu_u) \\
&= \frac{\alpha\beta}{\lambda^2 + \alpha}\kappa_0^E + \frac{\alpha}{\lambda^2 + \alpha}(1 + \beta\kappa_u^E)\mu_u + \frac{\alpha}{\lambda^2 + \alpha}(1 + \beta\kappa_u^E)\rho_u u_t
\end{aligned}$$

Matching coefficients results in:

$$\begin{aligned}
\kappa_u^E &= \frac{\alpha}{\lambda^2 + \alpha}(1 + \beta\kappa_u^E)\rho_u \\
\Leftrightarrow (\lambda^2 + \alpha)\kappa_u^E &= \alpha\beta\rho\kappa_u^E + \alpha\rho_u
\end{aligned}$$

And therefore

$$\kappa_u^E = \frac{\alpha\rho_u}{\lambda^2 + \alpha(1 - \beta\rho_u)} \tag{30}$$

Now for the intercept:

$$
\begin{aligned}
\kappa_0^E &= \frac{\alpha\beta}{\lambda^2+\alpha}\kappa_0^E + \frac{\alpha}{\lambda^2+\alpha}(1+\beta\kappa_u^E)\mu_u \\
\Leftrightarrow (\lambda^2+\alpha(1-\beta))\kappa_0^E &= \alpha\left(1+\beta\frac{\alpha\rho_u}{\lambda^2+\alpha(1-\beta\rho_u)}\right)\mu_u \\
\Leftrightarrow (\lambda^2+\alpha(1-\beta))\kappa_0^E &= \frac{\alpha(\lambda^2+\alpha)}{\lambda^2+\alpha(1-\beta\rho_u)}\mu_u
\end{aligned}
$$

And therefore

$$
\kappa_0^E = \frac{\alpha(\lambda^2+\alpha)}{(\lambda^2+\alpha(1-\beta))(\lambda^2+\alpha(1-\beta\rho_u))}\mu_u \tag{31}
$$

Now write inflation as

$$
\begin{aligned}
\pi_t &= \kappa_0^* + \kappa_u^* u_t = \frac{\alpha}{\lambda^2+\alpha}(\beta\hat{\mathbb{E}}_t[\pi_{t+1}] + u_t) \\
&= \frac{\alpha}{\lambda^2+\alpha}\left(\beta\left(\frac{\alpha(\lambda^2+\alpha)}{(\lambda^2+\alpha(1-\beta))(\lambda^2+\alpha(1-\beta\rho_u))}\mu_u + \frac{\alpha\rho_u}{\lambda^2+\alpha(1-\beta\rho_u)}u_t\right) + u_t\right) \\
&= \frac{\alpha}{\lambda^2+\alpha}\left(\frac{\alpha\beta(\lambda^2+\alpha)}{(\lambda^2+\alpha(1-\beta))(\lambda^2+\alpha(1-\beta\rho_u))}\mu_u + \frac{\alpha\beta\rho_u}{\lambda^2+\alpha(1-\beta\rho_u)}u_t +\right) u_t \\
&= \frac{\alpha^2\beta}{(\lambda^2+\alpha(1-\beta))(\lambda^2+\alpha(1-\beta\rho_u))}\mu_u + \frac{\alpha}{\lambda^2+\alpha}\left(1+\frac{\alpha\beta\rho_u}{\lambda^2+\alpha(1-\beta\rho_u)}\right)u_t \\
&= \frac{\alpha^2\beta}{(\lambda^2+\alpha(1-\beta))(\lambda^2+\alpha(1-\beta\rho_u))}\mu_u + \frac{\alpha}{(\lambda^2+\alpha(1-\beta\rho_u))(\lambda^2+\alpha)}\left(\lambda^2+\alpha\right)u_t \\
&= \frac{\alpha^2\beta}{(\lambda^2+\alpha(1-\beta))(\lambda^2+\alpha(1-\beta\rho_u))}\mu_u + \frac{\alpha}{\lambda^2+\alpha(1-\beta\rho_u)}u_t
\end{aligned}
$$

□

The physical expectation of inflation at the one year horizon is:

$$
\mathbb{E}_t[\pi_{t+1}] = \frac{\alpha^2\beta}{(\lambda^2+\alpha(1-\beta))(\lambda^2+\alpha(1-\beta\rho_u))}\mu_u + \frac{\alpha\rho_u}{\lambda^2+\alpha(1-\beta\rho_u)}u_t \tag{32}
$$

Analogously, we get physical expectations of the output gap

$$
\mathbb{E}_t[x_{t+1}] = -\frac{\alpha\lambda\beta}{(\lambda^2+\alpha(1-\beta))(\lambda^2+\alpha(1-\beta\rho_u))}\mu_u - \frac{\lambda\rho_u}{\lambda^2+\alpha(1-\beta\rho_u)}u_t \tag{33}
$$

Optimal interest rate policy under discretion takes the form:

$$i_t = r_t^* + \gamma \mathbb{E}[x_{t+1}] + \left(1 + \frac{\beta\lambda\gamma}{\lambda^2 + \alpha}\right)\hat{\mathbb{E}}[\pi_{t+1}] + \frac{\lambda\gamma}{\lambda^2 + \alpha} u_t + \gamma g_t$$

Using the relationship that

$$\mathbb{E}[x_{t+1}] = \frac{\lambda}{\lambda^2 + \alpha(1 - \beta\rho_u)}\mu_u - \frac{\lambda}{\alpha}\hat{\mathbb{E}}[\pi_{t+1}] \tag{34}$$

we get the interest rate rule

$$i_t = r_t^* + \frac{\gamma\lambda}{\lambda^2 + \alpha(1 - \beta\rho_u)}\mu_u + \left(1 + \frac{\beta\lambda\gamma}{\lambda^2 + \alpha} - \frac{\gamma\lambda}{\alpha}\right)\hat{\mathbb{E}}[\pi_{t+1}] + \frac{\lambda\gamma}{\lambda^2 + \alpha} u_t + \gamma g_t \tag{35}$$

After plugging in for expectations as a function of shocks, we get the interest rate as

$$i_t = r_t^* + \frac{\alpha\left(\alpha + \lambda^2\right)}{(\alpha(1-\beta) + \lambda^2)(\alpha(1 - \beta\rho_u) + \lambda^2)}\mu_u + \frac{\alpha\rho_u + \gamma\lambda(1-\rho_u)}{\lambda^2 + \alpha(1 - \beta\rho_u)} u_t + \gamma g_t \tag{36}$$

We therefore get a term premium of

$$\hat{\mathbb{E}}_t[i_{t+1}] - \mathbb{E}_t[i_{t+1}] = \frac{\alpha\rho_u + \gamma\lambda(1-\rho_u)}{\lambda^2 + \alpha(1 - \beta\rho_u)}\mu_u$$

Using the optimality condition for x_t, equation (28), we can show that the risk-neutral expectation of x_{t+1} and the physical expectation of x_{t+1} are related by

$$\hat{\mathbb{E}}_t[x_{t+1}] - \mathbb{E}_t[x_{t+1}] = -\frac{\lambda}{\lambda^2 + \alpha(1 - \beta\rho_u)}\mu_u$$

A.6 The Neutral Interest Rate

We define the neutral as the interest rate that sets the expected change in the output gap to zero. In period t, the one-year neutral interest rate is derived directly from the IS Curve by setting

$\mathbb{E}[x_{t+1}] = x_t$:

$$\tilde{i}_t = r^*_t + \hat{\mathbb{E}}_t[\pi_{t+1}] + \gamma g_t.$$

To solve for the two-year neutral interest rate, we replace $\mathbb{E}_t[x_{t+1}]$ with its risk-neutral counterpart given by equation (A.5), and then we iterate forward the IS curve:

$$\begin{aligned}
x_t &= -\frac{1}{\gamma}\left(i_t - \hat{\mathbb{E}}_t[\pi_{t+1}] + r^*\right) + \hat{\mathbb{E}}_t[x_{t+1}] + \frac{\lambda}{\lambda^2 + \alpha(1-\beta\rho_u)}\mu_u + g_t \\
&= -\frac{1}{\gamma}\left(i_t - \hat{\mathbb{E}}_t[\pi_{t+1}] + r^*\right) + \hat{\mathbb{E}}_t\left[-\frac{1}{\gamma}\left(i_{t+1} - \hat{\mathbb{E}}_{t+1}[\pi_{t+2}] + r^*\right) + \mathbb{E}_{t+1}[x_{t+2}] + g_{t+1}\right] + g_t \\
&= -\frac{1}{\gamma}\left(i_t - \hat{\mathbb{E}}_t[\pi_{t+1}] + r^*\right) + \hat{\mathbb{E}}_t\left[-\frac{1}{\gamma}\left(i_{t+1} - \hat{\mathbb{E}}_{t+1}[\pi_{t+2}] + r^*\right) + \hat{\mathbb{E}}_{t+1}[x_{t+2}] + \frac{\lambda}{\lambda^2 + \alpha(1-\beta\rho_u)}\mu_u + g_{t+1}\right] + g_t \\
&= -\frac{1}{\gamma}\left(i_t - \hat{\mathbb{E}}_t[\pi_{t+1}] + r^*\right) - \frac{1}{\gamma}\left(\hat{\mathbb{E}}_t[i_{t+1}] - \hat{\mathbb{E}}_t[\pi_{t+2}] + r^*\right) + \hat{\mathbb{E}}_t[x_{t+2}] + \frac{\lambda}{\lambda^2 + \alpha(1-\beta\rho_u)}\mu_u + (1+\rho_g)g_t \\
&= -\frac{1}{\gamma}\left(i_t - \hat{\mathbb{E}}_t[\pi_{t+1}] + r^*\right) - \frac{1}{\gamma}\left(\hat{\mathbb{E}}_t[i_{t+1}] - \hat{\mathbb{E}}_t[\pi_{t+2}] + r^*\right) + \mathbb{E}_t[x_{t+2}] + (1+\rho_g)g_t
\end{aligned}$$

Setting $x_t = \mathbb{E}_t[x_{t+1}]$ and dividing by 2 shows the 2-year neutral rate is:

$$\tilde{i}_{t,t+2} = \frac{1}{2}\left(\tilde{i}_t + \hat{\mathbb{E}}_t[\tilde{i}_{t+1}]\right) = r^* + \underbrace{\frac{1}{2}\left(\hat{\mathbb{E}}_t[\pi_{t+1} + \pi_{t+2}]\right)}_{\text{2-year inflation swap}} + \frac{\gamma(1+\rho_g)g_t}{2}$$

Analogously, the n-year neutral rate is:

$$\tilde{i}_{t,t+n} = \frac{1}{n}\sum_{s=0}^{n-1}\hat{\mathbb{E}}_t[\tilde{i}_{t+s}] = r^* + \underbrace{\frac{1}{n}\sum_{s=1}^{n}\hat{\mathbb{E}}_t[\pi_{t+1}]}_{\text{n-year inflation swap}} + \frac{\gamma\left(\sum_{s=0}^{n-1}\rho_g^s\right)g_t}{n}$$

B Estimation

B.1 Data Sources

Break-even Inflation. U.S. break-even inflation rates are found in Bloomberg under the ticker USGGBEX, where X is the maturity of the break-even rate. We use break-even rates of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 30 year maturities in our paper.

Treasury Yields. Treasury yields are found in FRED under the mnemonic DSX, where X is the maturity. We use yields of 1, 2, 3, 5, 7, 10, 20 and 30 year maturities in our paper.

Inflation Swap Rates. Inflation swap rates are found in Bloomberg under the ticker USSWITX Curncy, where X is the maturity of the swap rate. We use swap rates of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 25 and 30 year maturities in our paper.

OIS Rates written on the Federal Funds Rate. OIS rates written on the Federal Funds Rate are found in Bloomberg under the ticker USSOX Curncy, where X is the maturity of the OIS rate. We use OIS rates of 1, 2, 3, 4, 5, 7, 10 and 20 year maturities in our paper.

B.2 Estimating the Parameter Vector

In the following appendix, we provide additional details about the estimation of the parameters. As discussed in section 3, we estimate parameters using a four step procedure.

Step 1: Estimate ρ_u. We estimate ρ_u by minimizing the sum-of-squared errors between the model implied forward break-even rates and the data. At each date t, we can write the one-period forward break-even inflation rate maturing in period $t+s$ as a function of ρ_u and two other one-period forward break-even inflation rates.

We take one-period forward break-even rate maturing in 6-years (i.e., the risk-neutral expectation of inflation from year 5 to year 6) and in 20-years with the data, and we use the model to compute one-period forward break-even rates maturing 7-years to 19-years in the future. Given the one-period forward break-even rates maturing 6-years and 20-years in the future, the model implied one-period forward break-even rates are functions of ρ_u alone:

$$\hat{\mathbb{E}}_t\left[\pi_{t+6+s}\right] = \frac{\left(\sum_{i=s}^{13} \rho_u^i\right) \hat{\mathbb{E}}_t[\pi_{t+6}] + \left(\sum_{i=0}^{s-1} \rho_u^i\right) \hat{\mathbb{E}}_t[\pi_{t+20}]}{\sum_{i=0}^{13} \rho_u^i} \tag{37}$$

We compute the ρ_u that minimizes the sum of squared errors between the model-implied one-period forward break-even rates given by equation (37) and their data counterparts at 7-year to 19-year maturities (s in equation (37) ranges from 1 to 13).

If the resulting $\rho_u > 1$, then we redo the process but instead take the one-period forward break-even rate maturing in 11-years and in 20-years from the data, and we minimize the sum-of-squared errors between model implied forward break-even rates and data at the 12-year to 19-year maturities. When we use the 11-year and 20-year forward break-even rate data, the expression for one-period break-even rates maturing between 12-years and 19-years in the future is:

$$\hat{\mathbb{E}}_t\left[\pi_{t+11+s}\right] = \frac{\left(\sum_{i=s}^{8} \rho_u^i\right)\hat{\mathbb{E}}_t[\pi_{t+11}] + \left(\sum_{i=0}^{s-1} \rho_u^i\right)\hat{\mathbb{E}}_t[\pi_{t+20}]}{\sum_{i=0}^{8} \rho_u^i}, \tag{38}$$

where s ranges from 1 to 9.

Step 2: Estimate $\mu_{u,t}$ and u_t. After estimating ρ_u, we compute model-implied break-even rates maturing at 5-years to 20-years. The model-implied break-even rate maturing in S years equals $\frac{1}{S}\sum_{s=1}^{S}\hat{\mathbb{E}}_t[\pi_{t+s}]$. We derive an expression for $\hat{\mathbb{E}}_t[\pi_{t+s}]$ as a function of ρ_u, $\mu_{u,t}$ and u_t by iterating forward our expression for the inflation swap rate presented in Lemma 1. The one-period ahead inflation swap rate is:

$$\hat{\mathbb{E}}_t[\pi_{t+1}] = \frac{1}{\lambda^2 + \alpha(1-\beta\rho_u)}\left(\frac{\alpha(\lambda^2+\alpha)}{(\lambda^2+\alpha(1-\beta))}\mu_{u,t} + \alpha\rho_u u_t\right) \tag{39}$$

To derive the two-period ahead inflation swap rate, we iterate the inflation process forward one period, take risk-neutral expectations of the resulting expression, and plug in the solution for the one-period ahead inflation swap rate:

$$\begin{aligned}
\hat{\mathbb{E}}_t[\pi_{t+1}] &= \frac{\alpha}{\lambda^2+\alpha}\left(\beta\hat{\mathbb{E}}_t[\pi_{t+2}] + \hat{\mathbb{E}}_t[u_{t+1}]\right)\\
\Leftrightarrow \quad \hat{\mathbb{E}}_t[\pi_{t+2}] &= \frac{1}{\beta}\left(\left(\frac{\lambda^2+\alpha}{\alpha}\right)\hat{\mathbb{E}}_t[\pi_{t+1}] - \hat{\mathbb{E}}_t[u_{t+1}]\right),
\end{aligned}$$

which implies the two-period ahead inflation swap rate is:

$$\hat{\mathbb{E}}_t[\pi_{t+2}] = \frac{1}{\beta}\left(\left(\frac{\lambda^2+\alpha}{\alpha}\right)\hat{\mathbb{E}}_t[\pi_{t+1}] - \rho_u u_t - \mu_{u,t}\right) \tag{40}$$

Continuing to iterate inflation expectations forward, we show the three-period ahead inflation swap rate is:

$$\begin{aligned}\hat{\mathbb{E}}_t[\pi_{t+3}] &= \frac{1}{\beta}\left(\left(\frac{\lambda^2+\alpha}{\alpha}\right)\hat{\mathbb{E}}_t[\pi_{t+2}] - \hat{\mathbb{E}}_t[u_{t+2}]\right)\\ &= \frac{1}{\beta}\left(\left(\frac{\lambda^2+\alpha}{\alpha}\right)\hat{\mathbb{E}}_t[\pi_{t+2}] - (\rho_u^2 u_t + (1+\rho_u)\mu_u)\right)\end{aligned} \tag{41}$$

More generally, the $n+1$-period ahead inflation swap rate can be written recursively as:

$$\hat{\mathbb{E}}_t[\pi_{t+n+1}] = \frac{1}{\beta}\left(\left(\frac{\lambda^2+\alpha}{\alpha}\right)\hat{\mathbb{E}}_t[\pi_{t+n}] - \rho_u^n u_t - \left(\sum_{j=0}^{n-1}\rho_u^j\right)\mu_{u,t}\right) \tag{42}$$

Plugging in the expressions for $\hat{\mathbb{E}}_t[\pi_{t+1}]$ and iterating forward shows:

$$\hat{\mathbb{E}}_t[\pi_{t+n}] = \frac{\alpha(\alpha\beta + (\alpha(1-\beta)+\lambda^2)(\sum_{j=0}^{n-1}\rho_u^j)))}{(\lambda^2+\alpha(1-\beta))(\lambda^2+\alpha(1-\beta\rho_u))}\mu_{u,t} + \frac{\alpha\rho_u^n}{\lambda^2+\alpha(1-\beta\rho_u)}u_t, \tag{43}$$

which is a function of ρ_u, $\mu_{u,t}$, u_t and the calibrated parameters of the model.

We estimate $\mu_{u,t}$ and u_t by minimizing the sum-of-squared errors between the model-implied break-even rates and the data:

$$\{\mu_{u,t}, u_t\} = \arg\min_{\mu_u, u} \mathbf{G}(\rho_u, \mu_u, u)'\mathbf{G}(\rho_u, \mu_u, u),$$

where $\mathbf{G}(\rho_u, \mu_u, u)$ is a column vector in which element s is the squared difference between the model-implied break-even rate maturing in s-periods and its data counterpart. The expressions for the break-even inflation rate above show that $\mathbf{G}(\rho_u, \mu_u, u)$ depend ρ_u, $\mu_{u,t}$, u_t, and the slow-moving parameters of the model. Importantly, $\mathbf{G}(\rho_u, \mu_u, u)$ does not depend on r_t^*, ρ_g and g_t.

Step 3: Estimate ρ_g. Similar to ρ_u, we can write the one-period forward interest rate maturing in period $t + s + 1$ as a function of ρ_g and two other one-period forward interest rates.

We take one-period forward interest rates maturing in year 6 and in year 20 from the data, and we use the model to compute one-period interest rates maturing 7-years to 19-years in the future. These 1-period forward interest rates are a function of ρ_g, $\hat{\mathbb{E}}_t[i_{t+5}]$, $\hat{\mathbb{E}}_t[i_{t+19}]$, and other parameters of the model that are either calibrated or previously estimated:

$$\hat{\mathbb{E}}_t\left[i_{t+5+s}\right] = \frac{\left(\sum_{j=s}^{13} \rho_g^j\right)\left(\hat{\mathbb{E}}_t[i_{t+5}] + \kappa_g \sum_{j=1}^{s} \rho_u^{j+5}\right) + \left(\sum_{j=0}^{s-1} \rho_g^j\right)\left(\hat{\mathbb{E}}_t[i_{t+19}] - \kappa_g \sum_{j=s+1}^{14} \rho_u^{j+6}\right)}{\sum_{j=0}^{13} \rho_g^j}, \tag{44}$$

where

$$\kappa_g = \frac{\left(\alpha\rho_u + \gamma\lambda(1-\rho_u)\right)\left(\mu_{u,t} - u_t(1-\rho_u)\right)}{\alpha(1-\beta\rho_u) + \lambda^2}.$$

We compute the ρ_g that minimizes the sum of squared errors between the model-implied one-period forward interest rates given by equation (44) and their data counterparts maturing between 7-years and 19-years (s in equation (37) ranges from 1 to 13).

If the resulting $\rho_g > 1$, then we redo the process but instead take the one-period forward interest rate maturing in 11-years and in 20-years from the data, and we minimize the sum-of-squared errors between model implied forward interest rates and data from 12-year to 19-year maturities. When we use the 11-year and 20-year forward interest rate data, the expression for one-period interest rates maturing between 12-years and 19-years in the future is:

$$\hat{\mathbb{E}}_t\left[i_{t+10+s}\right] = \frac{\left(\sum_{j=s}^{8} \rho_g^j\right)\left(\hat{\mathbb{E}}_t[i_{t+10}] + \kappa_g \sum_{j=1}^{s} \rho_u^{j+8}\right) + \left(\sum_{j=0}^{s-1} \rho_g^j\right)\left(\hat{\mathbb{E}}_t[i_{t+19}] - \kappa_g \sum_{j=s+1}^{14} \rho_u^{j+11}\right)}{\sum_{j=0}^{8} \rho_g^j}, \tag{45}$$

Step 4: Estimate r_t^* and g_t. We compute model-implied interest rates as function of ρ_u, $\mu_{u,t}$, g_t, and ρ_g and the slow-moving parameters. The model-implied interest rate for the N-year horizon equals $\frac{1}{N}\sum_{n=1}^{N} \hat{\mathbb{E}}_t[i_{t+n-1}]$. We derive an expression for $\hat{\mathbb{E}}_t[i_{t+n}]$ by iterating forward our expression for the 1-period interest rate given by equation (36):

$$\begin{aligned}\hat{\mathbb{E}}_t[i_{t+1}] &= r^* + \frac{\alpha\left(\alpha+\lambda^2\right)}{(\alpha(1-\beta)+\lambda^2)(\alpha(1-\beta\rho_u)+\lambda^2)}\mu_u + \frac{\alpha\rho_u+\gamma\lambda(1-\rho_u)}{\lambda^2+\alpha(1-\beta\rho_u)}(\rho_u u_t+\mu_u)+\gamma\rho_g g_t \\ &= i_t + \frac{\alpha\rho_u+\gamma\lambda(1-\rho_u)}{\lambda^2+\alpha(1-\beta\rho_u)}((\rho_u-1)u_t+\mu_u)+\gamma(\rho_g-1)g_t\end{aligned} \tag{46}$$

We derive a general expression for the risk-neutral expectation of the 1-period interest rate from period $t+n$ to $t+n+1$ by iterating equation (36) forward and taking risk-neutral expectations:

$$\hat{\mathbb{E}}_t[i_{t+n}] = i_t + \frac{\alpha\rho_u+\gamma\lambda(1-\rho_u)}{\lambda^2+\alpha(1-\beta\rho_u)}\left((\rho_u^n-1)u_t+\left(\sum_{j=0}^{n-1}\rho_u^j\right)\mu_u\right)+\gamma(\rho_g^n-1)g_t. \tag{47}$$

We estimate r_t^* and g_t by minimizing the sum-of-squared errors between the model-implied interest rates and the data:

$$\left\{r_t^*, g_t\right\} = \arg\min_{r^*,g} \mathbf{H}(\rho_u,\mu_{u,t},u_t,\rho_g,r^*,g)'\mathbf{H}(\rho_u,\mu_{u,t},u_t,\rho_g,r^*,g),$$

where $\mathbf{H}(\rho_u,\mu_{u,t},u_t,\rho_g,r^*,g)$ is a column vector in which element n is the squared difference between the model-implied interest rate from t to $t+5+n$ and its data counterpart. n ranges from 1 to 15.