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How Oil Shocks Propagate: Evidence on the Monetary Policy Channel*

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Abstract

Using high-frequency responses of oil futures prices to prominent oil market news, we estimate the effects of oil supply news shocks when systematic monetary policy is switched off by the zero lower bound (ZLB) and when it is not (normal periods) in Japan, the United Kingdom, and the United States. We find that negative oil supply news shocks are less contractionary (and even expansionary) at the ZLB compared to normal periods. Inflation expectations increase during both periods, while the short nominal interest rates remain constant at the ZLB, pointing to the importance of monetary policy for oil shock propagation.

Keywords: oil price shocks, high-frequency identification, zero lower bound, systematic monetary policy

JEL Classification: E5, E7, G4

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1 Introduction

Oil price changes are among the most prominent macroeconomic disturbances. They are large and sudden, especially when caused by wars involving oil producers. Oil price changes affect all countries at the same time because crude oil is an internationally traded commodity. Moreover, according to the conventional supply-side view, it is difficult for monetary policy to stabilize both output and inflation in the face of oil price changes because inflation and output move in opposite directions. Recent events, such as the oil price increase in 2021-22, the 2022 Russian invasion of Ukraine, and the burst of high inflation worldwide in 2021-22, reignited interest in exploring the impact and mechanisms of the effects of oil price changes.

The fact that most post-WWII recessions in the United States followed oil price spikes has inspired a vast empirical literature on oil shock effects (Hamilton, 1983; Kilian, 2009; Kilian and Murphy, 2012; Baumeister and Hamilton, 2019; Kanzig, 2021). This literature has documented substantial effects of oil shocks on macroeconomic variables, relying on a variety of econometric approaches to identify oil shocks. At the same time, how exactly oil shocks propagate is still unclear. Theoretical literature proposes aggregate supply channels, such as variations in the cost of production, which can be amplified by markup endogeneity (Rotemberg and Woodford, 1996) and production network nonlinearities (Baqaee and Farhi, 2019), as well as aggregate demand channels, such as the interaction between nominal price stickiness and real wage rigidity (Blanchard and Gali, 2009) and the presence of financially constrained households (Chan *et al.*, 2022; Auclert *et al.*, 2023).

One aggregate demand channel stands out. It stems from the observation made in Bernanke, Gertler and Watson (1997) that most post-WWII recessions in the United States have followed monetary policy hikes. This raises the possibility that the estimated effects of oil shocks on real activity are also driven by systematic monetary policy reaction to these shocks. Such logic is central in standard New Keynesian models (Woodford, 2003). In particular, the central bank may tighten monetary policy following an oil price spike because the inflation rate increases, or inflation expectations go up, or because the central bank fears a future inflation rate increase. Such systematic monetary policy reactions can exacerbate the direct adverse effect of oil price spikes on output. Moreover, New Keynesian models can imply that oil price spikes are expansionary (Eggertsson, 2008; Bodenstein, Guerrieri and Gust, 2013), for example, if the central bank does not change the short-term nominal interest rate following an oil price spike. Intuitively, with higher inflation expectations and a constant nominal interest rate, the real interest rate drops, stimulating aggregate economic activity by increasing consumption and investment demand.

In this paper, we empirically evaluate the role of monetary policy in oil shock propagation. We leverage the fact that several countries have recently experienced zero lower bound (ZLB) episodes, when the short-term nominal interest rate—a standard conventional monetary policy tool—stayed close to zero.[1] This allows us to compare the effects of oil shocks during the ZLB periods, when monetary policy did not respond actively, and the normal periods, when the short-term nominal interest rate was not constrained by the ZLB. Specifically, we examine how Japan, the United Kingdom, and the United States responded to oil shocks between 1975:1 and 2019:12, differentiating between the ZLB and the normal periods.

Our primary focus is on Japan because it has the longest ZLB experience, which started at the end of 1995. Moreover, Japan experienced several business cycles during the ZLB periods, making it possible to average out the effects of potentially differential impacts of shocks during booms and busts. In addition, Japan was a net oil importer throughout our sample. Looking at the data in other countries, such as the United Kingdom and the United States, is helpful because the ZLB periods do not coincide, which alleviates the concern that some global factor could be responsible for results in all countries.

To extract exogenous and unexpected variation in oil prices, we build on the recent literature that uses high-frequency data to identify macroeconomic shocks (Kuttner, 2001; Gürkaynak, Sack and Swanson, 2004; Gertler and Karadi, 2015; Nakamura and Steinsson, 2018a). In particular, we follow Kanzig (2021), who uses the changes in oil futures prices in a tight window around the Organization of the Petroleum Exporting Countries (OPEC) production announcements. The series of oil futures price changes then becomes an external instrument in an oil market vector-autoregression (VAR), allowing us to estimate structural oil shocks, which we refer to as oil supply news shocks. These shocks change oil prices on impact and oil production gradually.

Using the state-dependent local projections method (Jorda, 2005), we estimate that oil supply news shocks are less contractionary, and at times even expansionary, in the ZLB compared to the normal periods. In particular, in our baseline specification using monthly data from Japan, industrial production increases by 1.3 percent after one year during the ZLB periods following a shock that increases the oil price by 10 percent. Outside of the ZLB periods, industrial production falls by up to 1 percent 12 months after the shock. The differences in the two responses are statistically significant at conventional levels. It is particularly remarkable that the increase in industrial production in response to the negative oil shock during the ZLB periods is significantly different from zero at some

[1] In practice, the short-term nominal interest can go slightly below zero. This has led researchers to refer to the bound as the effective lower bound. In this paper, we will use the term ZLB to refer to the effective lower bound.

horizons. The unemployment rate in Japan exhibits similar patterns with the opposite sign: it decreases significantly in the ZLB periods, while it stays near zero or increases outside of the ZLB. Since Japan has long periods with a zero nominal interest rate, we also estimate the effects of oil supply shocks using quarterly macro variables. Consistent with the results from industrial production and the unemployment rate, oil supply news shocks cause real per capita gross domestic product (GDP), consumption, and investment to increase in the ZLB periods and decrease (or do not change) outside of the ZLB.

To explore the relevance of the monetary policy channel, we estimate the responses of the Bank of Japan target interest rate (the Call rate) and 5-year nominal interest rates, realized inflation, and inflation expectations. In the ZLB periods, the short-term interest rate does not react to oil supply news shocks, and the 5-year rate increases slightly. In contrast, during the normal periods, both the short and 5-year rates increase considerably after an oil price increase, and these changes are statistically different from those during the ZLB periods. Moreover, inflation expectations react more during the ZLB. These results are consistent with the channel that works through ex ante real interest rate movements. At the same time, we do not observe a stronger reaction of realized inflation during the ZLB periods. In fact, realized inflation increases more in the normal periods.

Our estimated effects of oil supply news shocks in the United Kingdom and the United States are consistent with those in Japan. For example, in the United States, industrial production increases by 0.9 percent after 15 months following a shock that increases the oil price by 10 percent. Outside of the ZLB period, industrial production falls up to 0.8 percent. The difference in the two responses is statistically significant at conventional levels, and the ZLB response is significantly above zero at the 5 percent level for multiple horizons. The unemployment rate repeats this pattern with the opposite sign. The nominal interest rate responds more in the normal than in the ZLB periods, and both expected and realized inflation rates respond positively in the two periods. The results in the United Kingdom are qualitatively similar. These findings imply that the patterns obtained in Japan are not a particular feature of the Japanese economy.

We extend our main analysis in a number of ways. First, we consider alternative oil supply shocks obtained in two prominent recent studies: Kilian (2009) and Baumeister and Hamilton (2019). These papers rely on structural VAR identification of oil supply shocks. We find that the differences between the responses of the Japanese economy in the ZLB and the normal periods following oil supply shocks identified as in Kilian (2009) are not statistically significant. This is consistent with Wieland (2019), who also used these oil supply shocks to estimate the response of the Japanese economy during and outside of the ZLB. At the same time, the oil supply shocks identified as in Baumeister and Hamilton (2019) produce qualitatively similar results to our baseline findings. These differences can potentially be attributed to the fact that the oil supply shocks in Kilian (2009) do not move the oil price significantly, while they do in Baumeister and Hamilton (2019). Second, we estimate the responses of several other variables, such as output without oil production, exchange rates, and stock market prices, all of which exhibit differential responses across the ZLB and the normal periods. We estimate the reaction of economic activity in countries that experienced near-constant nominal interest rate periods, such as Canada and the euro area, and countries that did not experience their own ZLB episodes, and we do not find evidence of differential responses. Third, we investigate subsample properties of our results and re-estimate results following a refined version of the oil supply news shock that corrects for a potential information revelation effect (Jarociński and Karadi, 2020; Degasperi, 2021). We find largely similar results. Fourth, we measure the effects of positive and negative oil supply news shocks, and the effects of the shocks in booms and recessions and find no difference. Finally, we discuss the sensitivity of our results to variations in the number of control variables used in our regressions.

The last part of the paper presents a stylized model based on Galí and Monacelli (2002) and consisting of a small open economy and the rest of the world, where oil supply shocks propagate through aggregate supply, aggregate demand (via monetary policy), and international spillovers that affect both aggregate supply and demand. We formally illustrate the mechanisms of oil shock propagation and compare our empirical results to numerical responses to oil supply shocks in this model.

Related literature. Our paper contributes to the literature that studies the role of monetary policy in shock propagation and, in particular, a differential impact of shocks during the ZLB (or constant nominal interest rate periods) and the normal periods. On the aggregate demand side, Miyamoto, Nguyen and Sergeyev (2018) and Ramey and Zubairy (2018) estimated higher government spending multipliers in the ZLB periods than outside of it in Japan and the United States, respectively. Different from these papers, we focus on the shocks traditionally attributed to aggregate supply disturbances.

We contribute to the supply-side literature by analyzing state-dependent responses of several countries to oil shocks identified using high-frequency techniques. Bernanke, Gertler and Watson (1997) showed that most of the U.S. macroeconomic response to oil shocks is due to a systematic monetary policy.[2] Their analysis relies on the triangular VAR identification of oil shocks and a counterfactual simulation with a fixed policy interest rate in the spirit of Sims and Zha (2006). This method aims to remove systematic

[2] Hamilton and Herrera (2004) show that the quantitative importance of monetary policy in propagating oil shocks in Bernanke, Gertler and Watson (1997) depends on the lag order of the estimated VAR. Kilian and Lewis (2011) argue that Volcker's 1979 tightening is crucial for estimating a strong response by the Fed to oil price shock in Bernanke, Gertler and Watson (1997).

monetary policy effects but can be subject to the Lucas critique.[3] Unlike Bernanke, Gertler and Watson (1997), we directly estimate our results from the episodes with fixed interest rates. Garin, Lester and Sims (2019) find that *positive* total factor productivity shocks, measured by Fernald (2014), are more expansionary in the ZLB than in the normal periods in the United States. As mentioned above, Wieland (2019) finds no difference in Japanese output reactions to oil supply shocks identified as in Kilian (2009). Our results rely on the oil shocks identified using a high-frequency approach. Moreover, we use data from several countries, which is essential when studying relatively short ZLB periods. Unlike Garin, Lester and Sims (2019) and Wieland (2019), we find that shocks that depress aggregate supply increase output during the ZLB periods relative to the normal periods. Finally, our findings are consistent with the indirect evidence in Datta, Johannsen, Kwon and Vigfusson (2021). The authors estimate that oil prices and equity returns are more positively correlated in the recent past than before and that both variables become more responsive to macroeconomic news, consistent with the prediction of a model where the ZLB alters the economic environment.

Our paper also speaks to the literature on unconventional monetary policies. A constant short-term nominal interest rate, which we use to define ZLB episodes, does not logically imply that the medium- or long-term rates are constant. Swanson and Williams (2014b,a) estimate that, while the monetary policy rate is nearly constant during ZLB episodes, the medium- and long-term rates continue to respond to macroeconomic news. Such behavior can be caused by an active use of unconventional monetary policies, such as quantitative easing and forward guidance. Consistent with these findings we also find that longer term nominal interest rates respond to oil supply news shocks. However, these responses tend to be smaller during the ZLB episodes than during the normal periods. Debortoli *et al.* (2020) estimate a time-varying-coefficient structural VAR driven by shocks identified through the long-run and sign restrictions. They found no evidence for differential responses of macro variables to identified macroeconomic shocks. In contrast, we use a high-frequency identification of oil supply news shocks and find significantly different responses of macro variables during the ZLB and normal periods.

The rest of the paper is organized as follows. Section 2 starts by explaining how we measure the effects of oil supply news shocks on the aggregate economy and summarizes the data. We then present the main results for Japan in Section 3 and the United Kingdom and United States in Section 4. We describe our sensitivity analysis in Section 5. Section 6 presents a stylized model and compares its numerical predictions to our empirical results. Section 7 concludes. Online Appendix A lists the data sources, Appendix B and C collect

[3] Wolf and McKay (2022) extend the Sims and Zha (2006) methodology by incorporating the expectations effects via additional information about the responses of economic variables to *news* shocks. Importantly, this method assumes that people are aware of counterfactual policy changes.

figures omitted from the main text, and Appendix D presents model details.

2 Measurement, Data, and ZLB Definition

This section describes our empirical specification, details the data used for estimation, and concludes by defining ZLB periods.

2.1 Measurement of Oil Shock Effects

We measure the effects of oil supply news shocks in three steps: we define a daily oil supply surprise series, use it as an external instrument in an oil market VAR to identify oil supply news shocks, and employ these shocks to estimate state-dependent local projections.

High-frequency identification. To isolate exogenous and unanticipated changes in oil prices, we follow Kanzig (2021) and use a high-frequency approach, which has been used widely in monetary economics (Kuttner, 2001; Gürkaynak, Sack and Swanson, 2004; Gertler and Karadi, 2015; Nakamura and Steinsson, 2018a). The main idea relies on the observation that the oil market is dominated by a few large players, one of which—the Organization of Petroleum Exporting Countries (OPEC)—is responsible for about half of the world's oil production. OPEC is an intergovernmental organization that coordinates its oil production during conference meetings that end with public announcements about oil supply changes. Asset prices, particularly oil futures prices, react immediately to these announcements. Hence, looking at oil futures price changes in a narrow window of one day around OPEC announcements reveals unanticipated and arguably exogenous news about the future oil supply.[4]

Our implementation closely follows Kanzig (2021). First, we extract the first principle component of the daily price changes of the West Texas Intermediate (WTI) crude oil futures with maturities from one to twelve months in a narrow window around the OPEC announcements. These futures contracts are the most liquid contracts with the longest sample size, starting in 1983. We aggregate the daily first principle component up to a monthly frequency by summing the values if there were more than one OPEC meeting during a particular month. As Kanzig (2021), we will refer to this monthly series as *the oil supply surprise series*.

[4]The size of the window is motivated by the fact that the exact time of the announcement is unavailable in the initial part of our sample.

External instruments approach. The information contained in the OPEC announcements summarized by the oil supply surprise series is only a subset of all news about oil supply changes affecting the oil market. In addition, this series can be subject to a measurement error.[5] As a result, we do not use the series as a shock. Instead, following Stock and Watson (2012) and Mertens and Ravn (2013), we employ the oil supply surprise series as an external instrument—a variable that is correlated with the shock of interest but not the other shocks—in an oil market VAR.

Formally, consider a VAR(p) process of the form

$$Y_t = C + \sum_{l=1}^{p} B_l Y_{t-l} + \mathcal{U}_t,$$

where p is the number of lags, Y_t is an $n \times 1$ vector of endogenous variables, $\mathcal{U}_t$ is an $n \times 1$ vector of reduced-form errors with the variance-covariance matrix Σ, C is an $n \times 1$ vector of constants, $\{B_l\}$ are $n \times n$ matrices of coefficients. The reduced-form errors are related to an $n \times 1$ vector of uncorrelated structural shocks $\mathcal{E}_t$ linearly as $\mathcal{U}_t = S\mathcal{E}_t$, where S is an unknown $n \times n$ matrix. If we order the oil supply news shocks to be the first element of $\mathcal{E}_t$, the goal is to identify the first column of matrix S, which we denote as $S_{1:n,1}$.

Let z_t be an external instrument, that is, the oil supply surprise series in our application. If it is a valid instrument, it must be correlated with the shock of interest, i.e., $\mathbb{E}\left[z_t \mathcal{E}_{1,t}\right] \neq 0$, where $\mathbb{E}\left[\cdot\right]$ is the expectations operator, and uncorrelated with the other shocks, i.e., $\mathbb{E}\left[z_t \mathcal{E}_{k,t}\right] = 0$ for $k = 2, 3, \ldots, n$. Under this assumption, we can write

$$S_{2:n,1} = \mathbb{E}\left[z_t \mathcal{U}_{2:n,t}\right] / \mathbb{E}\left[z_t \mathcal{U}_{1,t}\right],$$

and the whole vector $S_{1:n,1}$ equals $(x, xS_{2:n,1})'$, where x is a normalization constant which can take any value. We will normalize x so that the shock $\mathcal{E}_{1,t}$ increases oil price on impact by 10 percent. Having computed $S_{1:n,1}$, we obtain the oil supply news shock $\widehat{\mathcal{E}}_{1,t}$ from the estimated reduced-form error terms $\widehat{\mathcal{U}}_t$. We will refer to these shocks as *oil supply news shocks* or *oil shocks*, for brevity. Importantly, to be able to estimate $\widehat{\mathcal{E}}_{1,t}$, the structural shock $\mathcal{E}_{1,t}$ has to be invertible, which would allow us to recover these shocks from current and past values of the variables entering the oil market VAR (Nakamura and Steinsson, 2018b; Stock and Watson, 2018). In Section 3.1, we present evidence supporting the assumption that the shock $\mathcal{E}_{1,t}$ is invertible.

[5] A high-frequency identification of monetary policy shocks faces a similar challenge (Nakamura and Steinsson, 2018a). Nevertheless, in Section 5.4, we present the results based on the specification where the oil supply surprise series is used directly as a shock.

The state-dependent effects of oil supply news shocks. Finally, to evaluate the impact of oil supply news shocks on a variable of interest y_t during and outside of the ZLB periods, we estimate a series of regressions at each horizon h from $h = 0$ to H of the form[6]

$$
\begin{aligned}
y_{t+h} - y_{t-1} = & \mathbb{I}_{t-1} \cdot \left[\alpha^{y}_{A,h} + \beta^{y}_{A,h} P^{oil}_{t} + \psi^{y}_{A,h}(L) x_{t-1}\right] \\
& + (1 - \mathbb{I}_{t-1}) \cdot \left[\alpha^{y}_{B,h} + \beta^{y}_{B,h} P^{oil}_{t} + \psi^{y}_{B,h}(L) x_{t-1}\right] + \epsilon^{y}_{t+h}, \qquad (1)
\end{aligned}
$$

where $\mathbb{I}_{t-1}$ is the indicator variable that takes the value of one if the economy is in the ZLB in period $t-1$, and zero otherwise, and the subscripts A and B indicate the ZLB and the normal periods, respectively, P^{oil}_{t} is the log real oil price, and the controls $\psi^{y}_{A,h}(L)x_{t-1}$ are *lags* of the variable of interest, the unemployment rate, and the measure of oil supply news shock $\widehat{\mathcal{E}}_{1,t}$. We use 12 lags with monthly data and 4 lags with quarterly data. In principle, the estimation does not require the addition of these controls when the data are infinite. In practice, however, our sample is far from infinite. In this case, it is helpful to increase the signal-to-noise ratio of the oil supply news shocks by removing the variation predicted by past values of the variable of interest, the variation attributed to business cycles but unrelated to the oil supply news shocks (we achieve this by adding the unemployment rate), and the variation attributed to past oil supply news shocks. In some cases, there are additional controls, which we specify separately.

Moreover, instead of directly regressing the variables of interest on the oil supply news shock $\widehat{\mathcal{E}}_{1,t}$, we instrument the log of oil price P^{oil}_{t} with the shock $\widehat{\mathcal{E}}_{1,t}$. There are two main reasons to do this. First, the shock $\widehat{\mathcal{E}}_{1,t}$ is a generated regressor whose measurement error needs to be accounted for when computing standard errors. Instead, generated instruments do not require standard error correction under very general conditions, as explained in Section 6.1 of Wooldridge (2010).[7] Second, an instrumental variable approach naturally normalizes the impulse responses. Specifically, the estimated coefficients $\beta^{y}_{A,h}$ and $\beta^{y}_{B,h}$ can be interpreted as the impulse response of variable y at horizon h to a 1 percent change in real oil price upon impact of oil supply news shocks. We use heteroskedasticity and autocorrelation consistent (HAC) standard errors and statistics that are robust to both arbitrary heteroskedasticity and autocorrelation. We choose automatic bandwidth

[6]The variables of interest, such as industrial production, real GDP, real aggregate consumption, and real aggregate investment are in logs, while nominal interest rates, the unemployment rate, the inflation rate, inflation expectations, and trade balance to GDP ratio are in levels, that is, we do not apply logs.

[7]We compare our baselines results to those where we use the shocks $\widehat{\mathcal{E}}_{1,t}$ directly in equation (1) in place of the log oil price P^{oil}_{t}. The results reported in Appendix Figure B.1 are virtually identical. Coibion and Gorodnichenko (2012) present similar findings in a different context and argue that the generated regressor problem is likely to be minimal when the generated regressor is a residual rather than a fitted variable from a regression.

selection in the estimation.

Finally, the state-dependent specification allows us to test the null hypothesis that the responses are identical in the two periods, that is, $\beta^{y}_{A,h} = \beta^{y}_{B,h}$.

2.2 Data

We estimate the four-variable oil market VAR using the log levels of the following monthly variables. The real oil price is the WTI spot oil price deflated by the U.S. consumer price index. For world industrial production, we use the Baumeister and Hamilton (2019) index, which covers the OECD countries plus six major economies (Brazil, China, India, Indonesia, the Russian Federation, and South Africa). Together, these countries jointly represent 75 percent of world's GDP. World oil inventories are OECD petroleum stocks from Kilian and Murphy (2014). World oil production comes from the U.S. Energy Information Administration.[8] The detailed description of the data sources is in Appendix A. Because of the availability of oil futures data, the oil supply surprise series covers a shorter period between 1983:4 and 2019:12, but the external instrument VAR uses data between 1974:1 and 2019:12 to estimate oil supply news shocks. The VAR includes a constant term and has 12 lags.

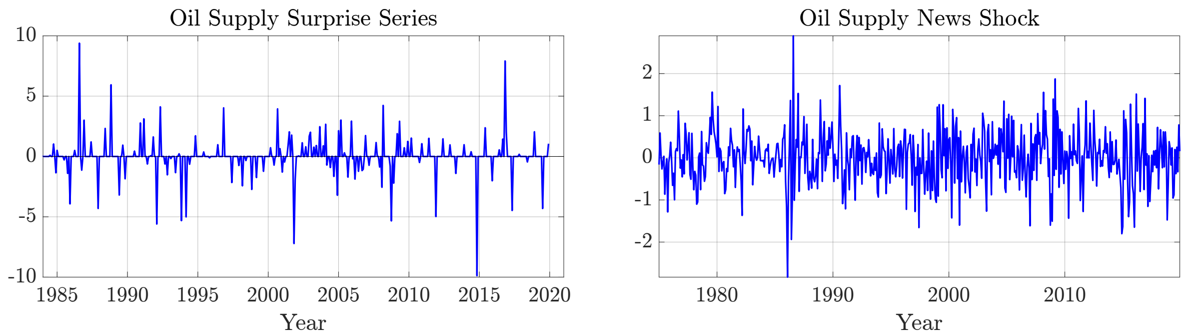
We use monthly and quarterly macro data for three advanced countries—Japan, the United Kingdom, and the United States—that have experienced a sufficiently long period where the nominal interest rate is at the ZLB. The sample period is between 1975:1 and 2019:12, which corresponds to the sample of the extracted oil supply news shock $\widehat{\mathcal{E}}_{1,t}$. We exclude the COVID pandemic by dropping data after 2019:12.

2.3 ZLB Definition

We define the ZLB periods as those when the short-term nominal interest rate—the standard instrument of conventional monetary policy—was close enough to zero that it did not respond substantially to macroeconomic developments. In particular, the ZLB periods for Japan are October 1995 to June 2006 and January 2009 to December 2019. This definition is similar to Wieland (2019), who follows the previous literature on the timing of the ZLB spell in Japan. This period of the ZLB in Japan coincides with a 0.5 percent cutoff before 1998 and 0.25 percent after that. We define the ZLB periods in the United Kingdom and the United States as the months during which each country's short-term nominal interest rate is at or below 0.5 percent. The ZLB period in the United States was

[8]Unlike Kanzig (2021), we do not directly use the U.S. industrial production and inflation rate in the oil market VAR. This is because the United States can exhibit a state-dependent response of these two variables.

Figure 1: The oil supply surprise series and oil supply news shock.



Notes: The left panel plots the oil supply surprise series from 1983:4 to 2019:12. The right panel presents the oil supply news shock $\widehat{\mathcal{E}}_{1,t}$ from 1975:1 to 2019:12, estimated using a four-variable oil VAR instrumented by the oil supply surprise series. The details are in Section 2.1.

between 2008:11 and 2016:11.[9] According to our definition, the United Kingdom experienced two ZLB episodes: 2009:4–2010:9 and 2012:2–2018:7.[10] Finally, as mentioned in the introduction, we rely only on the stance of the conventional monetary policy to define ZLB episodes. We estimate the responses of longer-term rates to analyze if these alternative policies managed to overcome the ZLB constraint.

3 Oil Supply News Shocks and their Effects in Japan

This section presents the estimates of the oil supply news shocks and their effects on the Japanese economy when the nominal interest rate is at the zero lower bound and outside of it. Specifically, we present the results for real monthly variables, including industrial production and the unemployment rate, then quarterly variables, such as real GDP, real private consumption, and real investment per capita. Finally, we examine the New Keynesian mechanism by estimating the responses of the nominal interest rate and inflation expectations.

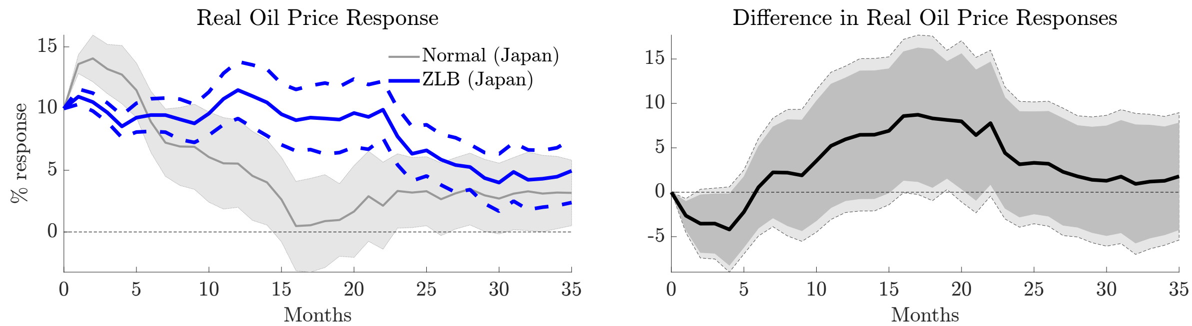
3.1 Oil Supply News Shocks

We start by describing properties the oil supply surprise series. First, the left panel of Figure 1, which plots the series, illustrates that OPEC announcements had a sizable impact on the oil futures prices in both directions throughout our sample. Second, the external

[9]This 0.5 percent cutoff results in a ZLB period similar to Ramey and Zubairy (2018). Lowering the cutoff to 0.25 percent shrinks the ZLB period to 2008:12-2015:12, however, the estimation results remain qualitatively similar.

[10]The U.K. interest rate rose just above 0.5 percent during 2009:5-2010:8. Our results do not materially change if we define this period as ZLB as well.

Figure 2: Impulse responses of real oil prices.



Notes: The left panel plots the impulse response functions and one-standard-deviation (HAC) confidence bands for the real oil price in the ZLB (thick blue line) and in the normal (thin gray line) periods. The Japanese ZLB definition is used to break the full sample into the ZLB and normal periods. The right panel plots the differences in the responses in the ZLB and normal periods along with the 90 percent (dark gray area) and 95 percent (light gray area) confidence bands.

instrument approach to estimating the oil supply news shocks requires the oil supply surprise series to be a relevant instrument. The first-stage regression of the oil price residuals from the VAR on the instrument, which is the oil news surprise series, yields an F-statistic of 19.9, well over a recommended threshold of 10 (Montiel Olea *et al.*, 2021). The instrument explains 3.6 percent of the variation in the oil price residual. Third, we find evidence supporting the assumption of the invertibility of the oil supply news shocks. Specifically, we test whether the oil supply surprise series jointly Granger causes the four variables in the oil VAR (Giannone and Reichlin, 2006; Forni and Gambetti, 2014; Plagborg-Møller and Wolf, 2022). The p-value for the Wald test of no Granger causality null hypothesis is 0.90.[11]

The right panel of Figure 1 plots the estimate of the oil supply news shocks $\widehat{\mathcal{E}}_{1,t}$ from 1975:1 to 2019:12. There is no noticeable difference between the different parts of the sample in terms of the size and frequency of the shocks. There was a relatively large negative shock in February 1986, followed by a positive shock of similar size in August 1986.

Since the estimated oil supply news shocks are used as an instrument for the real oil price in equation (1), we also report the F-statistics from the first-stage regression of the log real oil price on this instrument. The smallest value of the F-statistics using the data from 1975:1 to 2019:12 for horizons up to 36 months is 16.3, well above 10. As mentioned earlier, we estimate the responses to an oil supply news shock by directly regressing variables of interest on this shock using the local projections method. These additional results, reported in Appendix B, are consistent with the baseline results presented next.

[11]This result also holds when we test for Granger causality using first-differences, a shorter sample from 1983:4 to 2017:12, and a 6-variable VAR as in Kanzig (2021) instead of the 4-variable VAR as in our baseline specification.

Before turning to the analysis of the Japanese economy, in Figure 2, we plot the responses of the real oil prices to the oil supply news shock that increases the real oil price on impact by 10 percent. We use the ZLB definition for Japan and generate these responses by estimating equation (1) with the real oil price as the variable of interest. There is some evidence that the shock in the ZLB periods leads to a more persistent increase in the real oil price than that in the normal periods. The differences between the real oil price responses in the ZLB periods and those in the normal periods are statistically different at the 5 percent level at horizons of 1 and 19 months after the shock, as seen from the right panel of Figure 2. However, the responses in the two subperiods are qualitatively similar.[12]

3.2 Real Economy

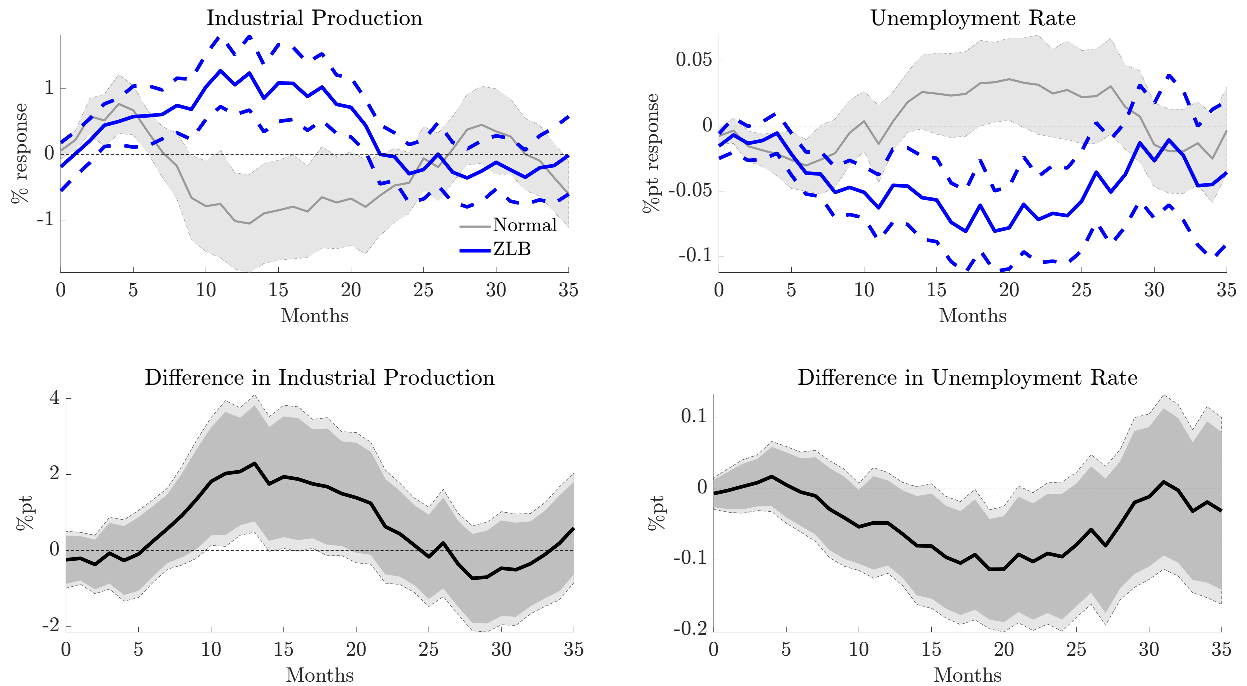
The top left panel of Figure 3 shows the responses of industrial production (a volume measure) to a 10 percent increase in real oil prices driven by the oil supply news shocks at horizons up to 36 months after the shock. The error bands show a one-standard-deviation confidence interval. In normal times, industrial production slowly falls, declining almost 1 percent one year after the shock. This decline reverts around two years after the shock. In the ZLB periods, the response of industrial production increases to just above 1 percent one year after the shock, and it is statistically different from zero at the 5 percent level at horizons between 11 and 14 months. The difference between the two responses of industrial production in the ZLB and the normal periods are in the bottom left panel of Figure 3. This difference is statistically significant at the 10 percent level at horizons between 9 and 18 months and at the 5 percent level at horizons between 10 and 14 months after the shock.[13] Appendix Figure B.1 shows that using the oil supply news shock directly as a regressor in specification (1) in place of the oil price keeps the results virtually unchanged.

Oil supply news shocks also lead to different labor market dynamics in the ZLB and the normal periods. The top right panel of Figure 3 plots the responses of the unemployment rate to a 10 percent increase in real oil prices driven by the oil supply news shocks in both the normal and the ZLB periods. Initially, the responses of the unemployment rate to an increase in real oil prices are negative and not significantly different from zero. Then, the unemployment rate increases up to 0.03 percentage points during normal times at the one-year horizon, while it decreases 0.08 percentage points in the ZLB periods a

[12]A higher persistence of the oil supply news shocks in the ZLB period implies a larger disruption in aggregate economic activity under the conventional supply-side view. Our estimation below presents evidence to the contrary.

[13]The Anderson and Rubin (1949) p-values, which take into account the fact the instrument in specification (1) can be weak, are higher, but the difference in the industrial production response is significant at the 10 percent level at horizons of 12 and 13 months.

Figure 3: Japanese industrial production and unemployment rate impulse responses.



Notes: Each figure in the top panel plots the impulse response functions and one-standard-deviation confidence bands of industrial production and the unemployment rate in Japan in the ZLB (thick blue line) and in the normal (thin gray line) periods. The lower panels present the differences in the responses of industrial production and the unemployment rate in the normal and ZLB periods, with the 95 percent (dark gray area) and 90 percent (ligh gray area) confidence bands.

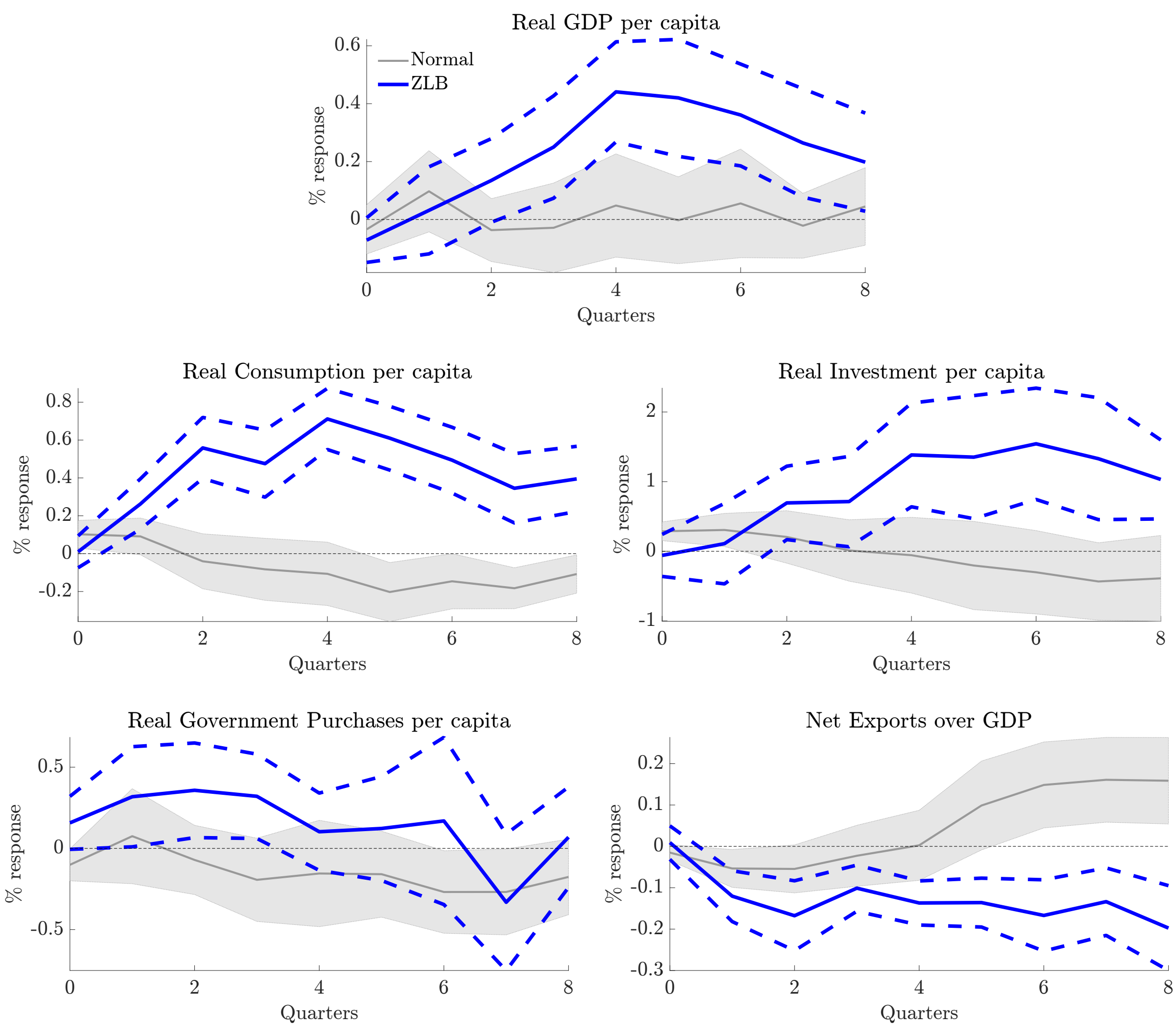
year after the shock. This decrease is statistically significant at the 5 percent level at horizons between 5 and 24 months after the shock. We test the differences in the responses in the normal and ZLB periods, and the p-values are below 5 percent for horizons between 16 and 22 months, after the shock, except in month 21, as depicted in the bottom-right panel of Figure 3.[14]

Long ZLB spells in Japan allow us to perform estimation using quarterly data. Figure 4 plots the impulse responses of real per capita GDP, private consumption, and private investment, government purchases, and net exports in Japan to a 10 percent increase in the real oil price driven by the oil supply news shock. We sum all monthly shocks within a quarter to obtain quarterly oil supply news shocks that we use as the instrument in equation (1). The controls are identical to those described in Section 2.1, and the number of lags at quarterly frequency is four. Consistent with the monthly real variables, we observe that real GDP, consumption, and investment per capita responses are larger in the ZLB periods than in the normal periods. In particular, GDP virtually does not change on impact but increases above 0.4 percent after one year at the ZLB. It remains roughly

[14]The Anderson and Rubin (1949) p-values are below 10 percent for horizons 16 to 22 months, again excluding month 21.

constant in the normal periods. Appendix Figure C.1 plots the difference in responses in the ZLB and normal periods.[15]

Figure 4: Japanese quarterly variable impulse responses.



Notes: Each figure plots the impulse response functions and one-standard-deviation confidence bands of real GDP, real consumption, real investment, real government purchases (all per capita), and nominal trade balance over nominal GDP in Japan to an oil supply news shock that increases the oil price by 10 percent on impact in the ZLB (thick blue line) and the normal (thin gray line) periods. The estimated specification is in equation (1). The sample is from 1975:Q1 to 2019:Q4. The differences in the impulse responses across the ZLB and the normal periods are plotted in Figure C.1.

Interestingly, the point estimate for government purchase responses are also higher in the ZLB periods than outside of it. However, the standard errors are substantial, so the differences in government purchase responses in the two periods are not statistically different at conventional significance levels. Finally, the trade balance exhibits a differential response across the two states as well.[16] The difference between the responses in the ZLB

[15]From now on, we present the difference in responses in the ZLB and normal periods in Appendix C.

[16]To estimate the response of the real trade balance at various horizons, we normalize the change in the

and normal periods is negative a year after the shock. This is consistent with the fact that aggregate consumption and investment increase at the ZLB relative to the normal period, which can drive an increase in demand for imports.[17]

3.3 Interest Rates, Inflation, and Inflation Expectations

One possible explanation for the observed differences in the responses of real macro variables in Japan to oil supply news shocks is that the monetary policy stance during these periods differs. To inspect this mechanism, we estimate the responses of nominal interest rates, inflation, and inflation expectations to an oil supply news shock. In the specification for the nominal interest rates, we add 12 lags of the CPI inflation rate and the real oil price in addition to the controls described in Section 2.1.[18]

The top left panel of Figure 5 plots the impulse response of the short-term nominal interest rate to a 10 percent increase in real oil prices driven by the oil supply news shocks. In the ZLB periods, the response of the nominal interest rate stays near zero and is precisely estimated, as one would expect from the ZLB periods. In normal times, the short-term nominal interest rate does not change on impact, but then it gradually increases by 0.4 percentage points one year after the shock. Appendix Figure C.2 plots the difference in the responses of the short-term nominal interest rate, as well as of the other variables discussed in this subsection. The short-term nominal interest rate difference is significant at conventional levels for the first 15 months. In addition, the response in the normal periods is significantly different from zero at the 5 percent significance level from horizons of 1 to 15 months.

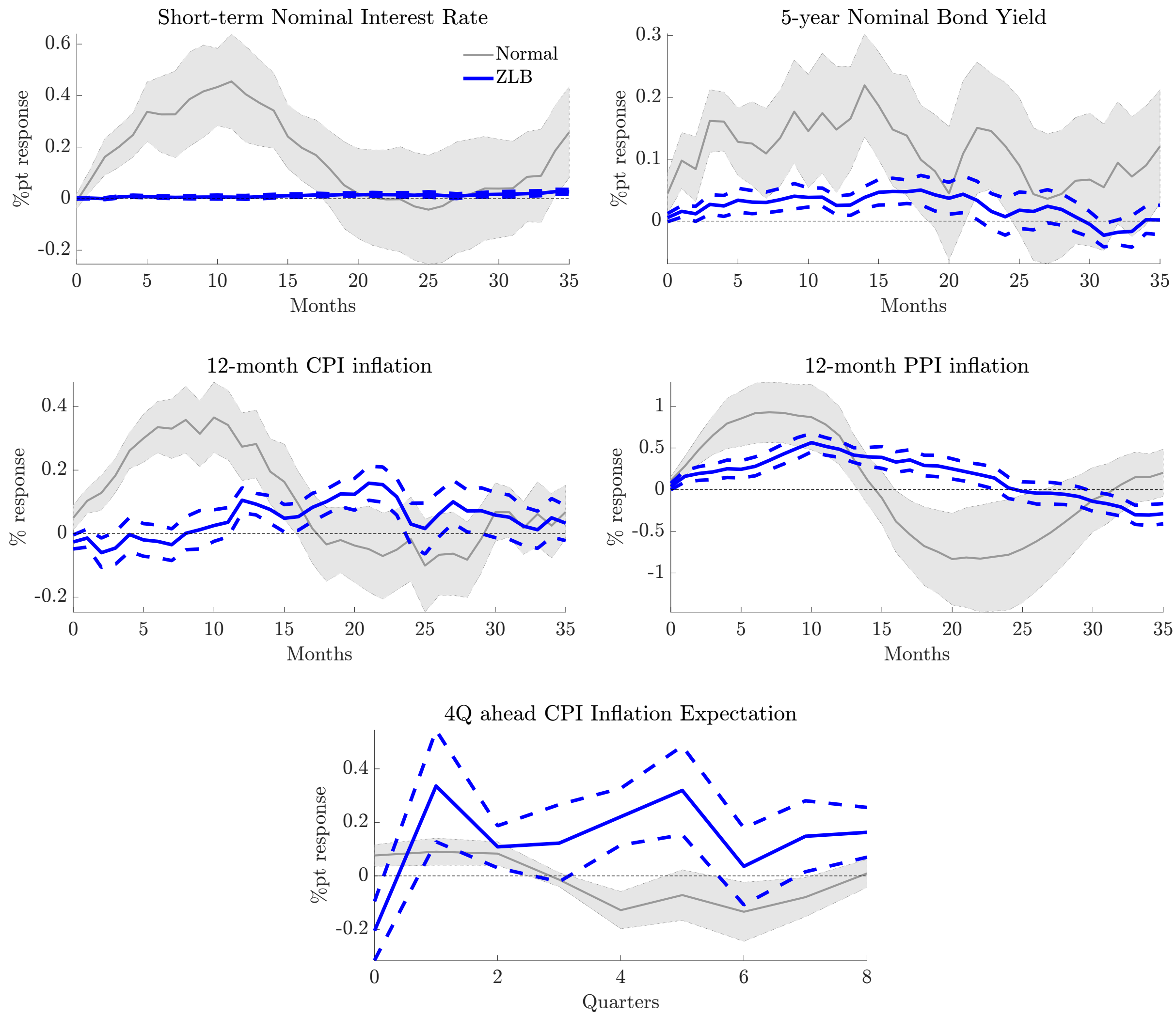
As mentioned earlier, the longer-term rates can vary even when the short-term nominal rate is constant at zero. The top right panel of Figure 5 presents the response of the 5-year nominal government bonds yield. This longer rate behaves qualitatively similar to the short rate, with the important difference being that it increases significantly but mildly during the ZLB periods. In addition, in the normal periods, this longer rate rises less than the short rate. Appendix Figure C.2 presents the differences in responses of these interest rates, together with the conventional confidence intervals. The differences are significant up to 6 months and at horizon of 14 months.

nominal trade balance by the nominal GDP just before the shock. That is, we use $(NX_{t+h} - NX_{t-1})/Y_{t-1}$ as the left-hand variable.

[17] Section 5.2 presents the responses of the real exchange rate.

[18] We add the inflation rate because it is typically an important factor that central bankers take into account when choosing monetary policy. We add the real oil price because it can have information above that contained in the lags of the oil supply shocks. The two variables can help predict the nominal interest rates and increase the signal-to-noise ratio in our regressions. In practice, however, the results are virtually unchanged without these two controls.

Figure 5: Interest rates, inflation, and inflation expectations responses in Japan.



Notes: The figures plot the impulse response functions and one-standard-deviation confidence bands of the short-term nominal interest rate, 5-year nominal interest rate, CPI inflation rate, PPI inflation rate, and CPI inflation expectations in Japan in the ZLB (thick blue line) and in the normal (thin gray line) periods. The responses are to an oil supply shock that increases the real oil price by 10 percent on impact. The differences in responses across ZLB and the normal periods are in Figure C.2.

The reaction of the CPI inflation rate to the oil shock is plotted in the middle left panel of Figure 5. The inflation rate, measured as the 12-month percentage change in the consumer price index, increases in in the normal periods about 0.3 percentage points after 8 months from the shock and reverts after 17 months. In the ZLB periods, the inflation rate does not react at the beginning and shows a moderate increase of 0.15 percentage points 20 months after the shock. The middle right panel of the same figure presents the reaction of the producer price index (PPI) inflation rate. The behavior qualitatively repeats the behavior of the CPI inflation rate. However, the response of the PPI inflation rate during the

ZLB periods is more pronounced than the response of the CPI inflation rate.[19]

The bottom panel of Figure 5 demonstrates the impulse responses of the one-year-ahead inflation expectations at quarterly frequency. We use the Japanese Center for Economic Research (JCER) CPI inflation forecast to proxy for inflation expectations.[20] Inflation expectations increase 0.3 percentage points in the ZLB one quarter after the shock, which is statistically significant at the 5 percent level. The inflation expectations response in the normal periods is close to zero on impact and becomes negative after one year. The differences of the responses between the ZLB and normal periods are significant at the 4, 5, and 7 quarter horizons. These results imply that the ex ante real ante interest rate falls during the ZLB periods but increases in the normal periods.

4 Oil Supply News Shocks Effects in the U.K. and U.S.

Since Japan is not the only country that has experienced ZLB episodes, we now investigate the effects of oil supply news shocks in the United Kingdom and the United States, both of which faced a near-zero interest rate in the recent past. The results provide additional evidence that oil supply news shocks have differential impacts during and outside of the ZLB periods, suggesting that the results using Japanese data are not specific only to Japan or to the period when Japan was in a liquidity trap.

4.1 Real Economy

Panel (a) of Figure 6 plots the impulse responses of industrial production to a 10 percent increase in real oil prices driven by an oil supply news shock, and the difference is in Appendix Figure C.3. Both in the United States and United Kingdom, the difference between the industrial production responses in the ZLB and normal periods becomes positive and statistically significant a few months after the shock. Moreover, in these countries, the industrial production responses become positive and statistically different from zero in the ZLB period several months after the shock. For example, in the United States, industrial production increases nearly 1 percent one year after the shock. In contrast, industrial production falls in the normal period: the responses are negative and statistically different from zero. These results are qualitatively consistent but numerically and statistically

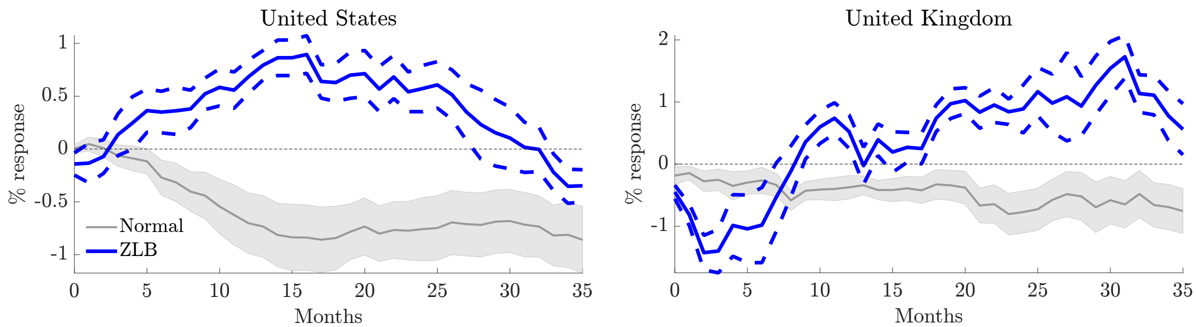
[19]The diminished reaction of price indices to oil shocks during the ZLB, relative to the normal period, aligns with Watanabe and Watanabe (2017) finding that goods level price inflation in Japan has shown reduced variability since the mid-1990s.

[20]Alternative inflation expectations datasets are too short to estimate the effects of oil shocks on inflation expectations in the normal periods in Japan. For example, the Consensus Economics Forecast data for Japan started in 1989.

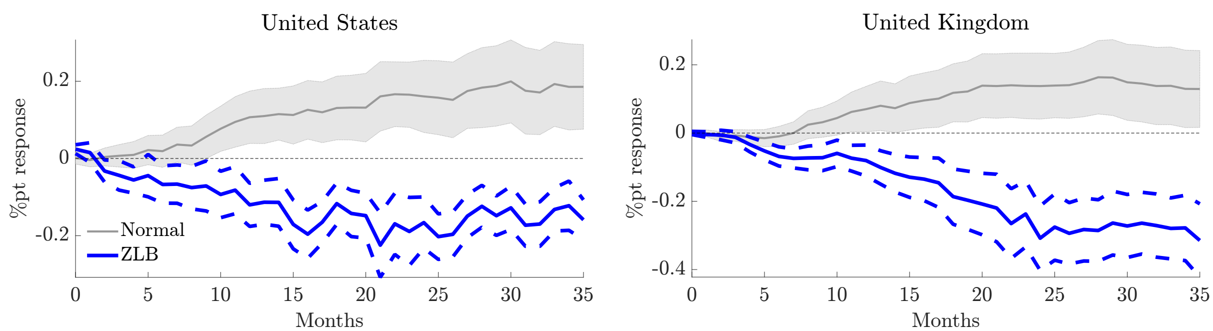
more pronounced compared to the results in Japan.

Figure 6: Industrial production and unemployment rate responses in the U.S. and U.K.

(a) Industrial production



(b) Unemployment rate



Notes: Each figure plots the impulse response functions and one-standard-deviation confidence bands. Panel (a) shows industrial production responses in the United Kingdom and the United States in the ZLB (thick blue line) and in the normal (thin gray line) periods. Panel (b) plots unemployment rate responses in the United Kingdom and the Unites States in the ZLB (thick blue line) and in the normal (thin gray line) periods. Figure C.3 plots the differences in the responses of these variables in the ZLB and outside of it.

The labor market responses, which we plot in Panel (b) of Figures 6 and C.3, mirror the responses of industrial production. Specifically, the unemployment rate in the United States and the United Kingdom falls after a negative oil supply news shock that increases the oil price in the ZLB period a few months after the shock. These ZLB responses are statistically different from zero and from those in the normal periods.

4.2 Interest Rates, Inflation, and Inflation Expectations

To investigate whether the monetary policy channel can be responsible for the above differences in the United States and the United Kingdom, we estimate the effects of oil supply news shocks on the nominal variables. Panel (a) of Figure 7 shows that responses of the short-term nominal interest rate in the ZLB periods remain zero. In contrast, the responses in the normal periods are positive and statistically different from those in the ZLB periods for a few initial months, as shown in Figure C.4. Although the results are noisier compared to Japan. The 5-year nominal yields respond to oil shocks in the two countries both during and outside the ZLB. Nevertheless, the responses during the ZLB are statistically different from those in the normal periods at several horizons. The inflation rate behavior in the ZLB and normal periods are indistinguishable in the United States. In the United Kingdom, the initial inflation responses in both periods are close to each other. However, inflation in the ZLB periods starts falling substantially after one year.

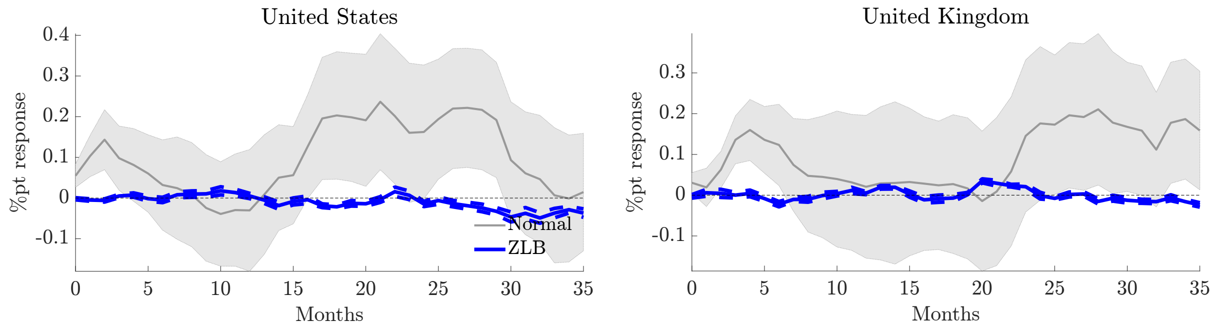
We close this section by describing the behavior of inflation expectations. We focus on households' inflation expectations because this group of economic agents is responsible for consumption choices. In addition, recent literature, such as Weber *et al.* (2022) and Candia *et al.* (2023), documents that inflation expectations of firm managers are closer to those of households than of professional forecasters. In the bottom panel of Figure 7, we plot the responses of the median one-year-ahead inflation expectations in the United States using the Michigan Survey of Consumers data and in the United Kingdom using the Bank of England/Ipsos Inflation Attitudes Survey data. The former survey collects data monthly, while the latter is a quarterly survey. As in the case of regressions with quarterly Japanese data, we construct quarterly oil supply news shocks by adding up the shocks within the same quarter. In both countries, we observe an increase in inflation expectations on impact in both the ZLB and normal periods. In the United States, the response in the normal periods is larger than that in the ZLB periods: the reaction in the normal periods reaches 0.24 percentage points compared to 0.1 percentage point in the ZLB periods.[21] In contrast, the response in the ZLB periods in the United Kingdom is larger than the response in the normal periods: the ZLB largest response is almost 0.5 percentage points compared to less than 0.1 percentage point in the normal periods.

Overall these results parallel those in Japan in Section 3.3. Specifically, the reaction of the nominal interest rates and expected inflation indicate the differential reaction of the ex ante real interest rate in the ZLB and normal period.

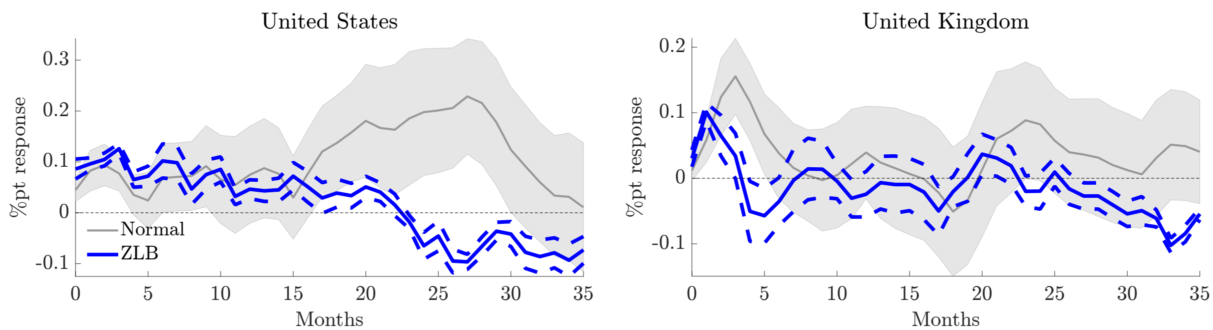
[21] Figure B.2 shows the response of one-year ahead inflation expectations by professional forecasters using the data from Consensus Economics. There is also evidence of a stronger reaction of the expectations in the normal period, however, the difference is smaller than in the case of the U.S. consumers.

Figure 7: Interest rates and inflation in the the United States and United Kingdom

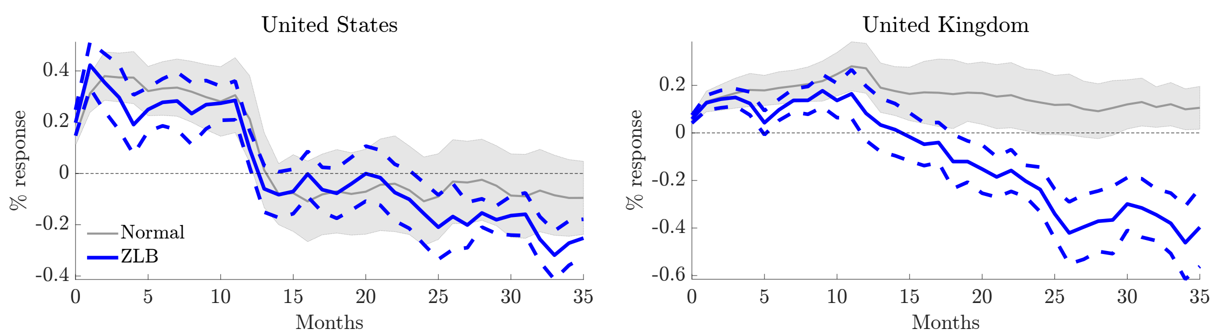
(a) Short-term nominal interest rate



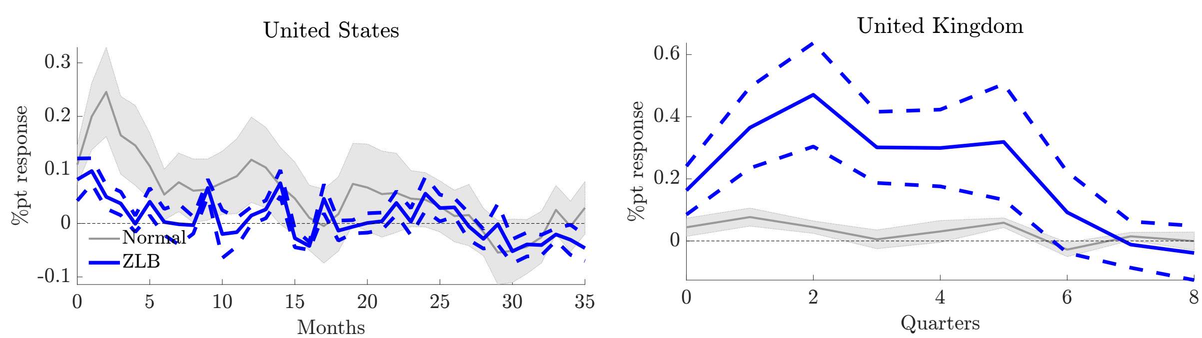
(b) 5-year nominal government bond yield



(c) CPI inflation rate



(d) Median household inflation expectations



Notes: Each figure plots the impluse response functions and one-standard-deviation confidence bands the central bank interest rate, the 5-year nominal yield on government bonds, the CPI inflation rate, and household inflation expectations responses to the oil supply shock that increases the real oil price by 10 percent in the United States and United Kingdom in the ZLB (thick blue line) and in the normal (thin gray line) periods. The U.S. inflation expectations are 12-month-ahead inflation expectations from the Michigan Survey of Consumers measured at monthly frequency. The U.K. inflation expectations are 4-quarter-ahead inflation expectations from the Bank of England/Ipsos Inflation Attitudes Survey measured at quarterly frequency. Figure C.4 plots the differences in the responses of these variables in the ZLB and outside of it.

5 Other Shocks, Variables, Countries, and Specifications

Our estimates for Japan are consistent with those in the United States and the United Kingdom, suggesting that the less contractionary (and even expansionary) effects of an oil supply news shock at the ZLB are not particular to Japan. We now extend our analysis in several dimensions to provide new insights and explore the sensitivity of the results just presented. First, we estimate the effects of alternative oil shocks using two prominent identifications of these shocks in the literature. Second, we look into the responses of additional variables to oil supply news shocks. Third, we estimate the effects in the economies areas that experienced near-ZLB episodes, such as Canada and the euro area, and countries that were far away from the ZLB constraint, such as Mexico, Sweden, and the Republic of Korea. Finally, we check how our main results in Sections 3 and 4 change when we vary our benchmark specification in a number of ways, such as the sample size, exclusion of influential historical episodes, and modification of control variables.

5.1 Other Oil Shocks

We begin by analyzing the effects of oil supply shocks, identified through alternative methods, to demonstrate the importance of oil shocks identification for our findings. This analysis clarifies why our baseline results differ from those of Wieland (2019), which found no difference in Japanese industrial production and unemployment rate responses to oil supply shocks.

We focus on two prominent identification strategies for oil supply shocks. The first one is the oil supply shocks identified in Kilian (2009), also used in Wieland (2019). We will refer to these oil supply shocks as the Kilian shocks. The second set of oil supply shocks is from Baumeister and Hamilton (2019), which we will call the BH shocks. To identify oil supply shocks in a structural VAR, Kilian (2009) assumes that there is no short-run reaction of the oil supply to changes in the real oil price.[22] In contrast, Baumeister and Hamilton (2019) do not restrict the oil supply elasticity to be small and estimate it to be a much larger number of 0.15. Baumeister and Hamilton (2019)'s identification strategy of oil supply shocks recovers a series of oil supply shocks that is substantially more important in accounting for the historical oil price movements than Kilian (2009)'s identified oil supply shocks. We use these two oil supply shocks in the same way as in our baseline specification described in Section 2.1.

Appendix Figure B.3 plots the impulse responses of the Japanese industrial production and unemployment rate to the Kilian (first row) and BH (second row) shocks. The results

[22] Kilian and Murphy (2012) later modified this assumption with an alternative that the short-run price elasticity of the oil supply is small or, more specifically, less than 0.0258.

based on the Kilian shocks are consistent with Wieland (2019).[23] In contrast, the responses of industrial production and the unemployment rate to the BH shocks are qualitatively similar to our baseline estimation using oil supply news shocks. In particular, industrial production increases in the ZLB periods but decreases in the normal periods, and the unemployment rate decreases in the ZLB periods but increases in the normal periods. The differences between the responses of the unemployment rate to BH shocks in the ZLB period and those in the normal period are not statistically significant, while the industrial production responses are significantly different at 10 percent at the horizon of 20 and 21 months after the shock, as can be seen in Figure C.5.

What can be behind the different effects of these shocks? As displayed in the top right panel of Figure B.5, the Kilian (2009) oil supply shocks generate a response of the real oil price that is not statistically different from zero, consistent with one of his main results that his oil supply shocks are not an important driver of the historical oil price variations. This observation might explain why these shocks do not generate statistically significant responses of the real variables in the United States, as reported in Kilian (2009). Additionally, this could account for the lack of different responses of real variables at the ZLB and outside of it in Japan, as shown in our Figures B.3 and B.4. At the same time, the response of the real oil price to the BH shock (the bottom right panel of Figure B.5) is more significant than that to the Kilian shock, but still not as precise as the response of the real oil price to the oil supply news shocks identified in Kanzig (2021) (the top left panel of Figure B.5). Since we use all these shocks to instrument the oil price in specification (1), it is not surprising that the standard errors following the BH shocks are somewhat larger compared to those following the Kanzig shocks. Notably, when we regress the real variables on the shocks directly (i.e., without instrumenting), the effects of the BH oil supply shocks become much more precise, as seen in panel (b) of Figure B.4.

We close this section by presenting the responses of the Japanese economy to another oil shock estimated in Kilian (2009). This shock is ordered last in a triangular identification of his three-variable VAR, making it effectively a residual to the oil supply and global aggregate demand shocks, which come first and second. Kilian (2009) refers to this shock as the oil-market specific demand shock. When normalized to increase the oil price, this shock reduces U.S. GDP and, at the same time, increases the U.S. CPI, unlike his oil supply shock that does not change U.S. GDP and CPI. As shown in Figure B.6, where we use

[23] Wieland (2019) does not use the Kilian shocks to instrument for the real oil price. He uses the shocks directly in equation (1) instead of the oil price. We present the estimation based on this direct approach in Appendix Figure B.4 Panel (a). These responses between the ZLB and normal periods are not statistically significant. Note that our responses are not identical to the responses in Figure 6 (panels B and C) in Wieland (2019) because of at least two differences. First, we compute the Kilian shocks using the 1973:2-2019:12 sample instead of the 1973:2-2015:9 sample in Wieland (2019). Second, following our analysis in Section 3, we start our sample in 1975:1 instead of starting in 1986:1 as in Wieland (2019).

this shock to estimate the responses in Japan, the results are qualitatively similar to our baseline.

5.2 Other Variables

Production without oil. Our analysis used a measure of industrial production that includes mining, such as oil extraction. This raises a concern that oil price variations can mechanically affect the mining sector's output. This problem can be potentially more pronounced in the United States, where the importance of oil production has grown in the last two decades following the shale oil extraction boom. To address this concern, we present the responses of industrial production in the United States, the United Kingdom, and Japan only for the manufacturing sector in Panel (a) of Figure B.7 and for industrial production when we remove oil production for the United States in Panel (b). Our main findings remain qualitatively unchanged.

Stock market response. If oil supply news shocks lead to different responses in aggregate economic activity during and outside of the ZLB, it is logical to expect this pattern to be reflected in financial markets. To investigate this possibility, we estimate the response of the U.S. stock market represented by the S&P 500 index.[24] The left panel of Figure B.8 plots the reaction of the S&P 500 index estimated using specification (1) on monthly data. The results confirm our previous finding that the U.S. economy responds positively to the shocks at the ZLB and negatively in the normal periods. The positive response is significant at almost all horizons at the 5 percent significance level, while the negative response is only significant at this level at the 13-month horizon.

In addition to looking at the stock market data at a monthly frequency, we leverage the fact that stock market data are also available at a higher daily frequency. This allows us to use the oil supply surprise series directly to measure the importance of monetary policy for oil shock propagation. Specifically, we estimate a state-dependent specification where a log-change of the S&P500 index is projected on the oil supply surprise series. We use ten lags of the S&P 500 index as only controls. The sample covers all OPEC press-conference announcements between January 1, 1983 and December 31, 2019. The right panel of Figure B.8 presents the results, which are consistent with the monthly data results and our earlier findings on the responses of aggregate economic activity. The different stock market responses based on the direct use of the oil supply surprise series reinforces

[24] We primarily look at the US stock market for two reasons. First, it is the biggest stock market in the world. Second, the recent literature, documenting the presence of the global financial cycle (Rey, 2015; Miranda-Agrippino and Rey, 2022), notes that financial market developments are correlated across countries.

our main results based on the oil supply news shock.

Exchange rates. Another financial variable that can exhibit different responses during the ZLB and the normal periods is exchange rates. This can be due to the differential response of trade balance and monetary policy we estimated earlier.[25] Figure B.9 shows that the real exchange rate depreciates in the ZLB period and appreciates in the normal periods during the first two years after the shock, although the responses in the normal periods are not statistically significant at conventional levels. This reaction is in line with a relative tightening of monetary policy during normal times.

Unconventional monetary policy. During the ZLB periods, central banks often use unconventional monetary policies, such as forward guidance (announcements about future conventional monetary policy) or quantitative easing (purchases of long-term private and public financial assets). As a result, central banks can potentially respond to oil price shocks even during the ZLB periods. One way to assess this possibility is to look at longer-term nominal interest rates, which we did in Sections 3.3 and 4.2. We found evidence of the reaction of long-term rates to oil supply news shocks. An alternative assessment tool is a measure of monetary policy stance summarized by the shadow rate (Wu and Xia, 2016). The shadow rate is a synthetic short-term nominal interest rate that is estimated to be consistent with the part of the yield curve that is above zero. By construction, the shadow rate closely follows the actual short-term nominal interest rate when the ZLB constraint does not bind, but it may go negative when the ZLB constraint is active. Figure B.10 presents the impulse responses of the shadow rate to an oil supply news shock in the ZLB periods in the United States. To be consistent with the estimation of the interest rate responses in Sections 3.3 and 4.2, we add the lags of inflation rate and real oil price as controls. Using updated data from Wu and Xia (2016), we estimate projections on the sample from 1990:1 to 2019:12 and report the ZLB shadow rate responses only.[26] The shadow rate falls significantly after an oil price hike. This pattern is consistent with the expansion of real economic activity during the ZLB we obtained earlier. One reason why the shadow rate might fall after the oil price spike is that the inflation rate is typically below target during ZLB episodes, potentially making central bankers less concerned about inflation and more concerned about a possible output decline following oil price spikes.

[25]Most macro models emphasize the role of trade balance and monetary policy in determining exchange rates. However, the exchange rate disconnect puzzle (Obstfeld and Rogoff, 2000)—the absence of correlation between exchange rate and macro fundamentals—is still standing. See Engel and Wu (2023)for recent progress in identifying the effects of macro variables on the exchange rate.

[26]The normal period response of the shadow rate is qualitatively similar to the response of the short-term nominal rate in Figure 7 but nosier at longer horizons.

5.3 Other Countries

Canada and the euro area. Japan, the United States, and the United Kingdom experienced clear episodes when the interest rate was constant. Two more economies experienced close-to-zero interest rate episodes: Canada and the euro area. Figure B.11 plots their short-term nominal interest rates. We define the ZLB period to be between 2009:4 and 2017:4 in Canada and 2012:7–2019:12 in the euro area. It is important to note that the interest rates continued changing even when they were close to zero in both economies, which is why it is reasonable to expect noisier results.

Figure B.12 demonstrates a general lack of difference in responses of industrial production and unemployment in the ZLB and normal periods in Canada and the euro area. In contrast to the results in Japan, the United Kingdom, and the United States, there is some evidence that the initial response of industrial production goes up by more, and the unemployment rate goes down by more in the normal period than in the ZLB period in the euro area. In Canada, while not significant, the point estimates are consistent with our results for Japan, the United States, and the United Kingdom.

Placebo countries. We next present the responses of three other countries—Mexico, Sweden, and the Republic of Korea—that did not experience ZLB during the Global Financial Crisis. We chose them among the OECD countries and ensured they are located in different parts of the world. Figure B.13 shows that neither industrial production nor unemployment rates respond differently in the normal and the ZLB periods based on the U.S. definition of the ZLB period. Only Mexico shows some signs of the expansionary effect of oil shocks during the U.S. ZLB period, which can be driven by international spillovers from an expansion in the United States following an increase in oil prices.

5.4 Other Specifications

Subperiod analysis. The advantage of showing results from several countries that experienced ZLB episodes is that these episodes may not coincide, providing evidence from different subperiods. In practice, however, the ZLB episodes in the United States and the United Kingdom occur almost simultaneously and partly overlap with the ZLB periods in Japan. To avoid the potential spillover of the United States during its ZLB period to other countries, we estimate the reaction of the Japanese industrial production and the unemployment rate in the normal and the ZLB periods when we shorten the sample to before the start of the ZLB period in the United States. Specifically, we choose the ending date for the sample to be June 2006, corresponding to the end of the first ZLB sub-period in Japan. The results, plotted in Figure B.14, qualitatively repeat the patterns for the full sample estimation in Figure 3.

The first half of 2008 was marked by the world plunging into the Global Financial Crisis, with recession in Japan starting in March 2008. At the same time, the oil price continued rising until July 2008, partly due to rising oil demand in China and a stable oil supply in oil-producing countries (Hamilton, 2009). Such a coincidence may have an important effect on the estimation of the oil supply news shocks on economic activity. To investigate this possibility, we re-estimate the responses of Japanese industrial production and the unemployment rate using specification (1), where we not only have dummies for the ZLB and the normal periods as defined above, but also a third dummy that equals one during 2008:3-2008:7, which effectively dummies out this period. Figure B.15 presents the responses of Japanese variables during the ZLB and normal periods, and we do not add the responses during 2008:3-2008:7 to the figure. The responses of industrial production and the unemployment rate in the ZLB period remain virtually unchanged compared with our baseline results. However, the point estimate responses in the normal periods become less contractionary than before, consistent with the hypothesis that the developments in 2008 may be important for generating a negative relationship between economic activity and the oil supply news shocks. This point can also be seen in Figure B.14 where the point estimates in the normal periods are less contractionary than in our baseline results. Finally, if we were to assign the 2008:3-2008:7 period to the ZLB episode, we would estimate a weaker expansionary effect of the shocks in Japan during the ZLB episode.[27]

The United States responded to the large oil price swings of the 1970s by introducing rationing of gasoline. To check if these episodes affect our U.S. results, we estimate the responses of U.S. industrial production and the unemployment rate with the sample starting in 1986:1, following Ramey and Vine (2011). Figure B.16 shows that our results are robust to shortening of the sample. The only notable difference is that the normal period responses become more persistent than our baseline results in Figure 6.

Controls. In Section 2.1, we argued in favor of using controls, such as a lagged dependent variable, lagged oil shocks, and the lagged unemployment rate, to enhance the signal-to-noise ratio of the oil supply news shocks. To verify this claim, we present results obtained without these controls. Appendix Figure B.17 displays the impulse response functions of Japanese industrial production and the unemployment rate, where we have omitted controls $\psi^y_{A,h}(L)x_{t-1}$ and $\psi^y_{B,h}(L)x_{t-1}$ from equation (1). While the point estimates are similar to those with controls, the standard errors increase significantly. This motivates the inclusion of the controls. Furthermore, Appendix Figure B.18 demonstrates

[27]We thank Christiane Baumeister for pointing out the importance of the developments in the first half of 2008.

that our findings hold up even when we add additional controls, such as the lagged inflation rate and the lagged 1-year nominal government bond yield.[28]

Information effect. In monetary economics, a high-frequency approach to identifying monetary policy shocks is subject to a criticism that it cannot distinguish between a true shock and the information revelation about the state of the economy by a central bank (Nakamura and Steinsson, 2018a). The same criticism may apply to the announcements by OPEC. For example, OPEC may have better information about the state of the global oil market than market participants themselves. To check whether our results are driven by the information revelation about the state of the economy, we use recent insights from the monetary shocks identification literature (Cieslak and Schrimpf, 2019; Cieslak and Pang, 2021; Jarociński and Karadi, 2020) that were applied to OPEC announcements in Degasperi (2021). The idea is to separate the movements in the oil supply surprise series on the days of the OPEC announcements into those that positively co-move with the stock market, and are likely to represent the information revelation effect, and those that co-move negatively, and are likely to proxy true oil supply surprises. We use the S&P 500 stock price index to measure the stock market reaction. The advantage of using the U.S. stock market index is that it may not react strongly to monetary policy in Japan, which, as we illustrated above, can influence the sign of the stock market reaction. Figure B.19 presents the results for Japan based on the oil supply news shocks that were estimated using a part of the oil supply surprise series that generates a negative comovement between the U.S. stock market and the real oil price. The results are qualitatively similar to the baseline results: the difference in industrial production and unemployment responses in the normal and the ZLB periods remain significant.[29]

Sign of oil shocks. The strength of the effects of macroeconomic shocks can also depend on the sign of these shocks (Barnichon *et al.*, 2022). This can be rationalized by, for example, nonlinearities in macro models, such as downward nominal wage rigidity, which binds when a shock reduces nominal wages but is slack when it increases them. In

[28] The results are not sensitive to using two or five-year nominal interest rates.

[29] One can object to even using the U.S. stock market by arguing that the U.S. economy entered the ZLB in 2008 and the S&P 500 changed the sign of its co-movement with the oil supply series after that, as we previously illustrated. To address this concern, we re-estimate the results in Figure B.19 by removing a period when the U.S. short-term nominal interest rate was constant. Specifically, we choose a sample between 1975:1 and 2006:6. The main finding that the shock that increases the real oil price is less contractionary remains true even in this case.

Figure C.8, we present the results from estimating the following specification

$$y_{t+h} - y_{t-1} = \mathbb{I}\left(\widehat{\mathcal{E}}_{1,t} \geq 0\right) \cdot \left[\alpha^{y}_{A,h} + \beta^{y}_{A,h}\widehat{\mathcal{E}}_{1,t} + \psi^{y}_{A,h}(L)x_{t-1}\right] + \mathbb{I}\left(\widehat{\mathcal{E}}_{1,t} < 0\right) \cdot \left[\alpha^{y}_{B,h} + \beta^{y}_{B,h}\widehat{\mathcal{E}}_{1,t} + \psi^{y}_{B,h}(L)x_{t-1}\right] + \epsilon^{y}_{t+h},$$

where $\mathbb{I}(\widehat{\mathcal{E}}_{1,t} < 0)$ is an indicator variable that equals one when the shock $\widehat{\mathcal{E}}_{1,t}$ is negative and zero otherwise, and $\mathbb{I}(\widehat{\mathcal{E}}_{1,t} \geq 0) = 1 - \mathbb{I}(\widehat{\mathcal{E}}_{1,t} < 0)$. We do not find evidence for sign-dependent effects. Figure C.8 shows that the difference $\widehat{\beta}^{y}_{A,h} - \widehat{\beta}^{y}_{B,h}$ is not statistically significant from zero at conventional levels.

Expansions and recessions. Aggregate economic variables may also react differently to shocks depending on the state of the economy. For example, there is evidence that government purchases have different impacts on GDP in expansions and recessions (Auerbach and Gorodnichenko, 2013). Since ZLB episodes often coincide with recessions, the results that we obtain can potentially be driven by the different impacts of the oil supply news shocks in recessions and booms. To explore this possibility, we estimate state-dependent responses of Japanese macro variables to the oil supply news shocks using a specification in equation (1) with the same controls but where the dummy variable takes a value of one if Japan is in a recession and zero if it is in an expansion. We use the Cabinet Office of Japan dating of business cycles.[30] If our results in Section 3.2 were driven only by the different responses in recessions and expansions, we should be able to detect the difference across recessions and booms. Figure B.20 plots the responses of industrial production and the unemployment rate in recessions and booms, and Figure C.9 plots the difference. The difference in industrial production responses is not statistically different from zero at conventional levels at all horizons, the difference in the unemployment rate is barely different from zero at 10 percent level at horizons of 18 and 22 months. We interpret this result as not being strong evidence for different responses across recessions and booms.[31]

Doing without external instruments. In our baseline analysis, we assumed that the four-variable oil market VAR spans all the relevant information to estimate the series of the oil supply news shocks $\widehat{\mathcal{E}}_{1,t}$, an assumption for which we could find supporting evi-

[30] The data can be found at `https://www.esri.cao.go.jp/en/stat/di/rdates.html`. For example, according to this dating, Japan experienced a recession from March 2008 to March 2009 during the Global Financial Crisis.

[31] It is sometimes argued that the entire ZLB experience in Japan coincides with a long recession. We cannot separate the effects of this long recession from a constant interest rate period. However, following Section 6.2 in Miyamoto *et al.* (2018), we note that in modern macro models that generate different responses in booms and busts, a crucial indicator is labor market tightness, which did not exhibit drastically different behavior before and during the ZLB.

dence. Nevertheless, for completeness, we now show the responses using an alternative strategy, where we directly use the oil supply surprise series z_t as a shock. More precisely, we first regress the series on twelve lags four variables from the oil VAR and extract the residual.[32] We then estimate specification (1), replacing the log oil price with the residual just obtained. The advantage of this approach is that it does not require the invertability assumption. However, we can only use the sample where the oil supply surprise series is available, namely 1983:4-2019:12. The left panel of Figure B.21 shows that industrial production in Japan responds differentially in the ZLB and normal periods, consistent with our baseline results. The estimates are, however, much noisier, which is expected from a shorter sample and less-frequent independent variable. The magnitudes of the point estimates are larger than those in our baseline estimation, which can be related to the fact that, unlike in our baseline estimation, the response of the real oil price peaks at higher than 10 percent level after several months following the shock. The right panel of Figure B.21 presents the responses of the unemployment rate. The standard errors are large, preventing us from making any statistical statements. However, the point estimates paint a similar picture to our baseline results.

6 Oil Shocks in an Open Economy New Keynesian Model

This section examines a standard Open Economy New Keynesian model with oil shocks with the following two goals. First, we formalize the intuition that oil market shocks that increase oil price can sometimes be expansionary. Second, we present a stylized quantitative exercise aimed at quantifying the model-implied effects of oil price spikes on macro variables under constant nominal interest rate monetary policy.

6.1 The Model

Our model is a version of Galí and Monacelli (2002) extended with oil intermediate inputs used by firms in the spirit of Bodenstein, Guerrieri and Gust (2013).[33] It consists of a small open home country and a foreign country representing the world economy. The small open economy is intended to approximate Japan. Each country has a representative household that consumes home and foreign goods with home-biased preferences, saves

[32] This step is not necessary and does not change the results. However, by doing so, we make sure that the oil supply surprise series is not predicted by other oil variables. This step also relates the exercise to the internal instrument approach (Ramey, 2011; Plagborg-Møller and Wolf, 2021), where the instrument is added to the VAR and ordered first.

[33] Galí and Monacelli (2002) is a working paper of the classical Gali and Monacelli (2005) paper. The key difference between the two is that the working paper features two countries, a small open economy and the world economy, while the published version has a continuum of small open-economies.

via complete domestic and international asset markets, and supplies labor to domestic firms. Monopolistically competitive firms produce differentiated goods from domestic labor and oil using a constant-returns-to-scale production function. These firms set their nominal prices in domestic currencies and change them infrequently a la Calvo. Only foreign country receives a stochastic endowment of oil in each period. All goods, including oil, cannot be stored. In addition, all goods are traded globally, implying the law of one price.[34]

The foreign country—the world economy—can be described by the following log-linearized equilibrium conditions, which we derive in Appendix D. Aggregate consumption follows the Euler equation:

$$\mathbb{E}_t\left[\widehat{c}^*_{t+1}\right] - \widehat{c}^*_t = \frac{1}{\sigma}\left(i^*_t - \mathbb{E}_t\left[\pi^*_{F,t+1}\right] - \iota\right), \tag{2}$$

where asterisks denote foreign country variables, $\widehat{c}^*_t$ is the log deviation of world consumption from its steady state, i^*_t is the nominal interest rate, $\pi^*_{F,t+1}$ is both a consumer price index (CPI) and producer price index (PPI) inflation rate because the world economy is closed, ι is the steady-state nominal interest rate, and σ is the inverse of the intertemporal elasticity of substitution. The world variables satisfy the Phillips curve

$$\pi^*_{F,t} = \kappa\zeta^*\widehat{c}^*_t + \beta\mathbb{E}_t\left[\pi^*_{F,t+1}\right] + \kappa\psi^*_o\widehat{r}_t, \tag{3}$$

where ζ^* and ψ^*_o are composite parameters that represent the sensitivities of firm's marginal costs to consumption $\widehat{c}^*_t$ and the real oil price $\widehat{r}_t$, which is expressed in units of foreign goods, κ is the sensitivity of inflation to changes in the marginal cost.[35] The main difference between a standard closed economy model and this world economy is the last term in the Phillips curve equation, $\kappa\psi^*_o\widehat{r}_t$, which relates changes in the real oil price and inflation. The foreign central bank sets the interest rate according to the Taylor rule

$$i^*_t = \iota + \varphi^*_\pi\pi^*_{F,t}, \tag{4}$$

where the inflation target is zero, the effective lower bound is not explicitly introduced, and φ^*_π is non-negative. Finally, the equilibrium real oil price is determined in the world's

[34]This stylized model can be extended in many realistic dimensions that we do not pursue here. It can feature non-rational expectations (Milani, 2009), incomplete markets within (Chan *et al.*, 2022; Auclert *et al.*, 2023) and across (Itskhoki and Mukhin, 2021) countries, local or dollar currency pricing (Gopinath and Itskhoki, 2022), production and storage of oil (Pindyck, 2001), and many more.

[35]The composite coefficients $\kappa, \zeta^*, \psi^*_o$, are expressed in equations (D.24), (D.27), and (D.28) in Appendix D through preferences and production function parameters.

oil market:

$$\widehat{r}_t = \phi_c \widehat{c}_t^* - \phi_o \widehat{o}_t^*, \tag{5}$$

where $\widehat{o}_t^*$ is the oil endowment and ϕ_c, ϕ_o are positive.[36] This equation implies that an increase in the world economic activity or a decrease in the oil supply lead to an increase in the real oil price.

Home country consumption dynamics follows a similar Euler equation

$$\mathbb{E}_t [\widehat{c}_{t+1}] - \widehat{c}_t = \frac{1}{\sigma} (i_t - \mathbb{E}_t [\pi_{t+1}] - \iota), \tag{6}$$

while the home-produced goods inflation rate obeys the following Phillips curve

$$\pi_{H,t} = \kappa \zeta_H \widehat{c}_t + \beta \mathbb{E}_t [\pi_{H,t+1}] + \kappa \psi_o \widehat{r}_t + \kappa \zeta_F \widehat{c}_t^* + \kappa \psi_q \widehat{q}_t, \tag{7}$$

where, in addition, to the channels present in the Phillips curve of the foreign country in equation (3), the term $\kappa \zeta_F \widehat{c}_t^*$ incorporates the effect of global aggregate demand on domestic inflation, and the last term shows the influence of the real exchange rate $\widehat{q}_t$ on the marginal costs of home firms and, hence, the inflation rate. The real exchange rate is defined as the price of foreign goods in home goods unit, so an increase in $\widehat{q}_t$ is a real depreciation. When the global aggregate demand drops, foreign residents purchase fewer home goods, reducing marginal costs of home firms and, hence, the inflation rate. In addition, when the home currency depreciates in real terms ($\widehat{q}_t$ increases), oil becomes more expensive to buy at home for any level of the real oil price $\widehat{r}_t$ denominated in foreign goods. As a result, home firms' marginal costs and the inflation rate go up.

The central bank at home follows the Taylor rule

$$i_t = \iota + \varphi_\pi \pi_{H,t}, \tag{8}$$

where $\phi_\pi \geq 0$. The link between the CPI and PPI inflation rates is

$$\pi_t = \pi_{H,t} + \frac{1 - \Omega}{\Omega} \Delta \widehat{q}_t, \tag{9}$$

where $\Omega \in [0, 1]$ is the share of home goods in home household spending, which represents the degree of home bias. The second term expresses the effect of real depreciation of domestic currency on the consumption basket inflation.

With complete financial markets, there is perfect risk sharing across countries, so the

[36] The composite coefficients ϕ_c and ϕ_o are in equations (D.25) and (D.26).

real exchange rate is proportional to the difference in consumption in the two countries:

$$\widehat{q}_t = \sigma \left(\widehat{c}_t - \widehat{c}_t^* \right). \tag{10}$$

The production of home goods is a sum of home and foreign demand for home goods

$$\widehat{y}_{H,t} = \frac{\Omega}{\Omega + 1 - \Omega^*} \widehat{c}_t + \frac{1 - \Omega^*}{\Omega + 1 - \Omega^*} \widehat{c}_t^* + \frac{\Omega \left(1 - \Omega\right) + \left(1 - \Omega^*\right)}{\Omega + 1 - \Omega^*} \cdot \frac{\gamma_n}{\Omega} \widehat{q}_t. \tag{11}$$

where $1 - \Omega^*$ is the proportion of consumption that foreign households allocate to home goods.[37] The last two terms express two forces that take into account that oil shocks affect home country through the world economy. The presence of $\widehat{c}_t^*$ captures the spillover of world consumption demand to domestic goods. The term featuring the real exchange rate $\widehat{q}_t$ measures the strength of the expenditure switching effect between foreign to home goods, which crucially depends on the elasticity of substitution between home and foreign goods γ_n.

We look into the consequences of a one-time unanticipated negative global oil supply shock $\widehat{o}_0^* = \widehat{o}^* < 0$ that reverts back to its steady-state value of zero with probability $\alpha \in [0, 1]$.[38] There are no other sources of uncertainty.

Finally, instead of explicitly representing the zero lower bound on the nominal interest rate, we assume that the nominal interest rate stays constant following an oil price shock. This assumption retains the key property of the ZLB that the interest rate does not respond to developments in the economy. At the same time, it avoids the need to introduce additional notation. See Nakamura and Steinsson (2014) for a similar treatment in the context of fiscal multipliers.

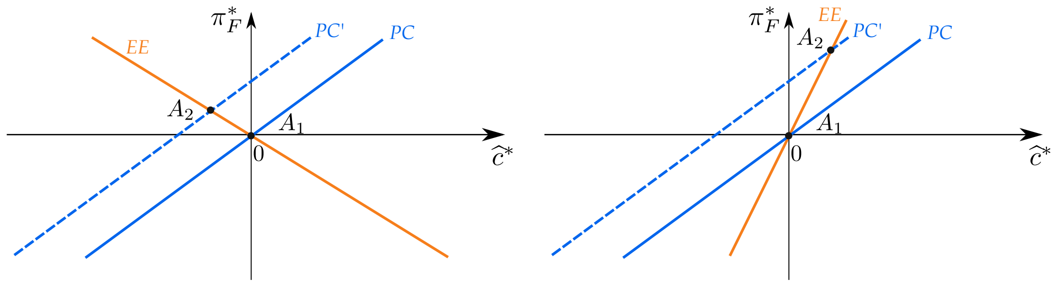
6.2 The Effects of Oil Shocks

We start by presenting the responses of foreign goods production, which equals consumption, and the inflation rate. We look for a unique bounded Markov equilibrium where

[37] As Appendix D formally shows, we assume that foreign demand for domestic goods is proportional to the size of the home country. Because we consider a limit when the size of the home country tends to zero (the small open economy assumption), both home production and foreign demand for home goods shrink to zero. However, their ratio stays positive in the limit.

[38] In our empirics, we use the oil supply news shocks that affect the oil price immediately but the actual oil supply gradually with a lag of several months. In this stylized model, where oil cannot be stored and the price of oil is determined in the spot market, oil supply news can only affect current oil price through other forward-looking macro variables, such as consumption. Because of this, we choose to focus on the realized oil supply shocks in this model. From a perspective of a small open economy, the distinction between oil supply news and oil supply shocks may not be crucial when the key manifestation of the shock is an increase in the oil price. Finally, in our numerical exercise, we employ quarterly calibration, which brings the supply news shocks closer to the realized supply shocks.

Figure 8: The World Economy.



Notes: Both panels show the response of the world economy to a decline in oil supply when monetary policy is active (the left panel) and when it does not respond following an oil shock (the right panel).

endogenous variables $\{\widehat{c}_t^*, \pi_{F,t+1}^*, i_t^*, \widehat{r}_t\}$ are linear functions of oil supply $\widehat{o}_t^*$. The Phillips curve and the Euler equation (together with the Taylor rule) are

$$\pi_F^* = \frac{\kappa\left(\zeta^* + \psi_o^* \phi_c\right)}{1-\beta\alpha}\widehat{c}^* + \frac{\kappa \psi_o^* \phi_o}{1-\beta\alpha}\left(-\widehat{o}^*\right), \tag{12}$$

$$\widehat{c}^* = -\frac{\varphi_\pi^* - \alpha}{\left(1-\alpha\right)\sigma}\pi_F^*. \tag{13}$$

Figure 8 plots the two equations under the assumption that monetary policy is active, that is, $\varphi_\pi^* > 1$ (the left panel), and when monetary policy does not change following an oil shock, that is, $\varphi_\pi^* = 0$ (the right panel). The upward-sloping Euler equation on the right panel is due to the fact that higher inflation in the absence of the response of the nominal interest rate reduces the real interest rate and increases aggregate consumption. For the unique bounded solution to exist, the Euler equation should be steeper than the Phillips curve when they have slopes with the same sign.[39]

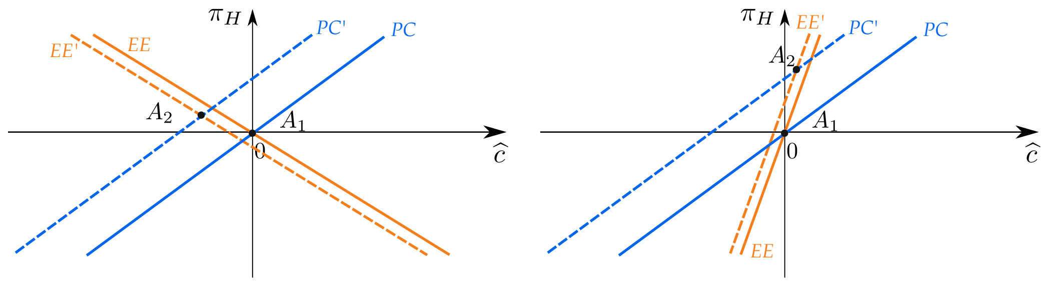
A decline in the oil supply results in an upward shift in the Phillips curve in both panels of Figure 8. Intuitively, a decline in the oil supply increases the real oil price, which, in turn, pushes up the inflation rate.

Under active monetary policy, the central bank increases the nominal interest rate such that the real interest rate rises, reducing aggregate output. At the same time, under the constant nominal interest rate policy, the real rate falls, stimulating the world economy.

Similarly to the world economy, we can represent the Phillips curve and Euler equa-

[39] Formally, we focus on the parameters that satisfy $(1-\beta\alpha)(1-\alpha)\sigma + \kappa\left(\varphi_\pi^* - \alpha\right)\left(\zeta^* + \psi_o^*\phi_c\right) > 0$. This inequality always holds when the monetary policy is active. When $\varphi_\pi^* = 0$, however, the inequality restricts the parameter space.

Figure 9: The Small-Open Economy.



Notes: Both panels show the response of the small-open economy to a decline in oil supply when the monetary policy is active (the left panel) and when it does not respond following the oil shock (the right panel). Both panels assume that the world economy's output declines following an increase in the oil price.

tion of the home economy as

$$\pi_H = \frac{\kappa\left(\zeta_H + \psi_q \sigma\right)}{1-\alpha\beta}\widehat{c} + \frac{\kappa}{1-\alpha\beta}\psi_o \widehat{r} + \frac{\kappa\left(\zeta_F - \psi_q \sigma\right)}{1-\alpha\beta}\widehat{c}^*, \tag{14}$$

$$\widehat{c} = -\Omega \frac{\varphi_\pi - \alpha}{\sigma(1-\alpha)}\pi_H + (1-\Omega)\,\widehat{c}^*. \tag{15}$$

The two equations closely resemble the world equations (12) and (13), with the exception of additional terms. When the oil supply decreases, real oil prices increase, pushing down global demand $\widehat{c}^*$ when the world monetary policy tightens, the case we will focus on. The last term in the Phillips curve encapsulates two global forces. First, a slowdown in global demand leads domestic marginal costs and inflation to fall. Second, to keep the marginal utility of consumption equalized in both countries as required by equation (10), the home currency depreciates in real terms. This makes the purchase of oil from the global market costlier and consequently increases the marginal costs of production at home. The net effect of these two forces is ambiguous, and it depends on the degree of openness of the home country. In Figure 9, we plot equation (14) when monetary policy in the rest of the world actively responds to the oil shock. In the graph, we assume that the second force is stronger, so a decrease in the oil supply pushes the PC to shift to the left.

In addition, when there is a decline in global economic activities, the last term in the Euler equation implies that the EE line shifts to the left, as plotted in Figure 9. The reason is that a decline in global economic activity, which is expected to revert in the future, appreciates home currency in real terms over time. This, in turn, reduces home inflation and, hence, increases the real interest rate for any level of the nominal interest rate. As a result, domestic aggregate demand falls.

In equilibrium, as demonstrated in Figure 9, a decline in oil supply that increases the oil price can also be expansionary when the leftward shift in the Euler equation is small enough, which is the case with a sufficiently high degree of home bias, i.e., when the coefficient Ω is close to one.[40]

Table 1: Calibration Parameters

$\beta = 0.99$	Discount factor
$\sigma^{-1} = 1$	Intertemporal elasticity of substitution
$\varphi^{-1} = 0.5$	Frisch elasticity of labor supply
$\gamma_n = 0.5$	Elasticity of substitution between home and foreign goods
$\Omega = 0.8$	Share of home goods in home consumption basket
$\alpha = 0.58$	Persistence of oil shocks
$\varphi_\pi = \varphi_\pi^* = 1.5$	Taylor rule coefficient on inflation
$\gamma_y = 0.1$	Elasticity between labor and oil in production
$\omega_{oy} = 0.057$	share of oil in production
$1 - \theta = 0.25$	Probability of price adjustment
Composite parameters	
$\kappa = (1-\theta)(1-\beta\theta)/\theta = 0.086$	$\zeta^* = (1-\omega_{oy})(\sigma+\varphi)/(1+\omega_{oy}\gamma_y\varphi) = 2.769$
$\psi_o^* = \psi_o = \frac{(1+\gamma_y\varphi)\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}} = 0.077$	$\phi_c = \left\{1+\gamma_y\left[\varphi+(1-\omega_{oy})\sigma\right]\right\}/\left[(1-\omega_{oy})\gamma_y\right] = 5.167$
$\phi_o = \frac{1+\omega_{oy}\gamma_y\varphi}{(1-\omega_{oy})\gamma_y} = 2.167$	$\psi_q = \frac{1-\Omega}{\Omega}\cdot\frac{(1+\varphi\gamma_n)(1-\omega_{oy})}{1+\gamma_y\varphi\omega_{oy}} = 1.27$
$\zeta_H = \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left(\sigma+\varphi\frac{\Omega}{\Omega+1-\Omega^*}\right) = 2.05$	$\zeta_F = \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\varphi\frac{1-\Omega^*}{\Omega+1-\Omega^*} = 0.808$
$\Omega^* = \Omega - \omega_{oy}/(1-\omega_{oy}) = 0.558$	$\iota = -\log\beta = 0.01$

Numerical exercise. To illustrate the quantitative importance of the highlighted channels, we calibrate the model by choosing the values used in the recent macro literature. The quarterly calibration parameters are listed in Table 1.[41] Most of the parameters are taken from Gali and Monacelli (2005). The exceptions are the share of oil in production, which come from Bodenstein *et al.* (2013). The persistence of oil shocks is set to 0.58 (half life of five months) to match the maximum point estimate decline in Japanese aggregate consumption by 0.2 percent in the normal period following a shock that increases oil prices by 10 percent. The elasticity of substitution between home and foreign goods consumption is set to 0.5 and between labor and oil in production to 0.3. We chose these somewhat lower than usual values to highlight the short-run effects of oil supply shocks. The weight of home goods in the home consumption basket is $\Omega = 0.8$, which is set to approximate this value in Japan.[42]

Under these parameters and a shock to oil supply that increases oil prices by 10 percent, world consumption falls 0.18 percent under active monetary policy and increases 1

[40] Note that for the bounded solution to be unique, the following condition must hold $\sigma(1-\alpha\beta)(1-\alpha)+\kappa(\varphi_\pi-\alpha)(\zeta_H+\psi_q\sigma)\Omega > 0$. This condition always holds when the central bank actively responds to the oil shock, i.e., $\varphi_\pi > 1$. However, when $\varphi_\pi = 0$, the above condition restricts the set of model parameters.

[41] We use quarterly calibration because we estimate the response of consumption and GDP in Japan using quarterly variables.

[42] This value is lower than the one typically used for the U.S., but higher that that for euro area countries.

percent when the nominal interest rate is constant. The home economy's consumption falls 0.2 percent when monetary policy is active both at home and abroad, while home consumption increases 0.9 percent when monetary policy does not respond at home but responds in the rest of the world. The ZLB consumption response is larger than the maximum estimated response of Japanese consumption at the ZLB of 0.7 percent, presented in Figure 4. Nevertheless, this stylized numerical exercise captures our empirical result that the ZLB response is larger than the absolute value of the response in the normal period.

The model allows us to look at other variables. For example, home production increases by 1.1 percent when home monetary policy does not respond (but foreign monetary policy does), and it falls by 0.2 percent when home monetary policy is active. Furthermore, using equation (11), we can decompose the 1.1 percent change in output into 0.67 percent due to a higher aggregate demand at home, -0.04 percent due to a lower foreign demand, and 0.42 percent due to expenditure switching, suggesting the importance of the international spillovers of the oil price variations.

7 Conclusion

This paper presents new evidence on the effects of oil supply news shocks when the nominal interest rate is not responsive due to the ZLB constraint. We focus on Japan, which has the longest spell at the ZLB, and supplement with evidence from the United Kingdom and the United States, which have also experienced considerable periods with fixed interest rates. We find that oil supply shocks are less contractionary in the ZLB period than in the periods outside of the ZLB in Japan, the United Kingdom, and the United States. Notably, we document that these economies expand following an oil price increase during the ZLB periods. In addition, we find that inflation expectations rise after an oil supply news shock, while the nominal interest rate remains at zero during the ZLB and increases in normal periods. This suggests that the monetary policy channel can play a significant role in the transmission of oil shocks.

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Online Appendix

A Data Details

This section details data sources.

1. **Industrial production**. Monthly industrial production data for all countries (i.e., Japan, UK, US, as well as Canada, Eurozone, Mexico, the Republic of Korea, and Sweden) are taken from Haver Analytics. The series name is "Industrial production excluding construction, seasonally adjusted 2015=100." For all countries, the data are from 1974:1 to 2019:12.

2. **Unemployment rate**. The unemployment rate data for all countries are taken from the OECD database. The series name is Unemployment rate (monthly), Total, All persons.

3. **Inflation rate**. CPI data for all countries are taken from the OECD database. The subject of the series is "CPI: 01-12 All items, not seasonally adjusted, 2015=100." Core CPI data (2015=100, not seasonally adjusted) for all countries are downloaded from Haver Analytics, which populates data from the OECD Major Economic Indicator (MEI) database. The codename for the series is PCXG_N023. We seasonally adjusted core CPI using X-12-ARIMA in Stata.

4. **Inflation expectations.** The inflation expectations for Japan come from the Japan Center for Economic Research (JCER). The US inflation expectations are 12-month ahead median inflation expectations from the Michigan Survey of Consumers measured at monthly frequency. The UK inflation expectations are 4-quarter ahead median inflation expectations from the Bank of England/Ipsos Inflation Attitudes Survey measured at quarterly frequency.

5. **Short-term nominal interest rate.**

    (a) For Japan, the short-term nominal interest rate is the uncollateralized overnight call rate from July 1985. Prior to that date, we use the collateralized overnight call rate. Both data series are taken from the Bank of Japan website.

    (b) For the United States and the United Kingdom, the short-term interest rate is the overnight interbank rate downloaded from Haver Analytics, with a ticker FRUO, which populates data from OECD MEI.

    (c) For the Euro Area, the short-term nominal interest rate is the 3-month treasury yield, downloaded from the Global Financial Database (GFD).

6. **Long-term nominal interest rate.** For all countries, the long-term nominal interest rate is the 5-year government bond yield. Data are taken from the GFD.

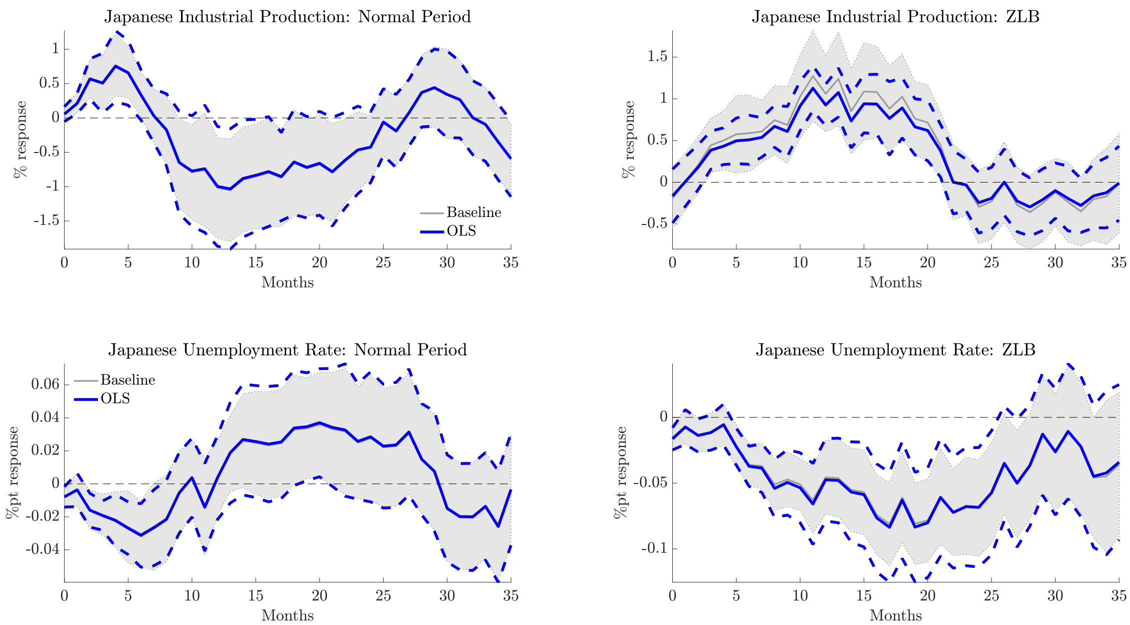
7. **Shadow rate.** The U.S. shadow rate series is from Wu and Xia (2016). The updated version of the series was downloaded from Jing Cynthia Wu personal website: `https://sites.google.com/view/jingcynthiawu/`.

8. **Nominal and real exchange rates.** The nominal and real exchange rates for all countries are taken from the BIS, end of period.

9. **Oil VAR data.** All four variables for the VAR are taken from Kanzig (2021).

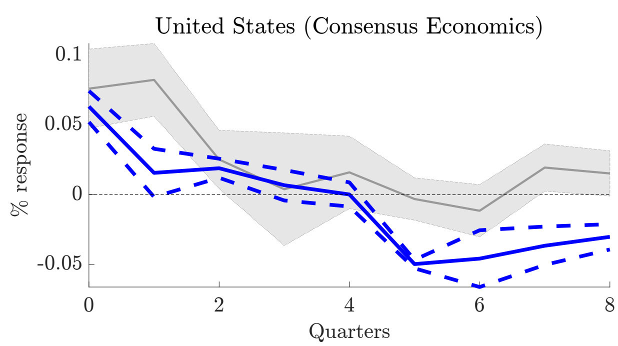
B Additional Figures and Tables

Figure B.1: Japan impulse responses using OLS vs. baseline specification with instruments.



Notes: Each row displays the impulse response functions and one-standard-deviation confidence bands for the responses of Japanese industrial production and the unemployment rate to oil price shocks, as identified in Kanzig (2021). The OLS estimates (thick blue line) represent the results obtained by regressing the variables of interest directly on the oil supply news shocks. Meanwhile, the Baseline estimates illustrate the responses of these same variables to the real oil price, which is instrumented by the oil supply news shock

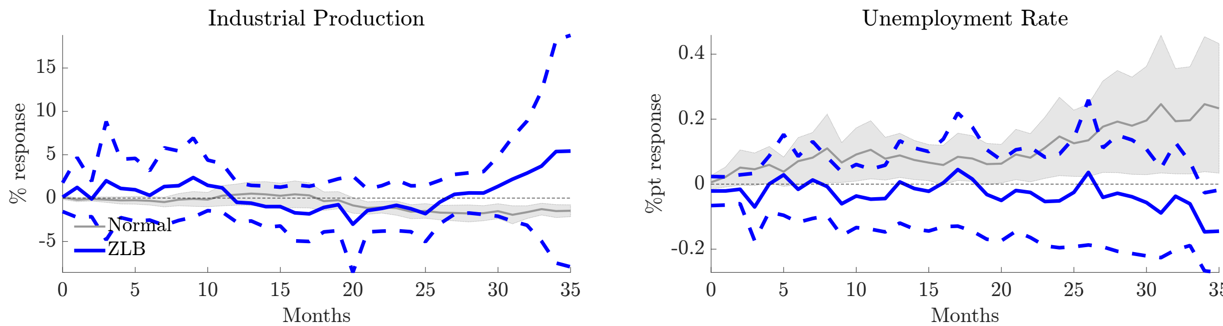
Figure B.2: Responses of inflation expectations in the United States in the ZLB and normal periods.



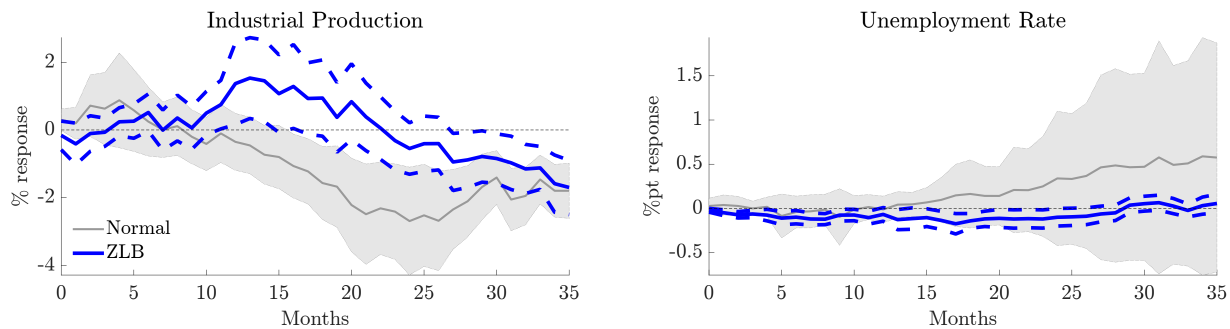
Notes: The figure plots the impulse response function and one-standard-deviation confidence bands for the professional forecasters' (Consensus Economics) inflation expectations in the Unites States in the ZLB (thick blue line) and normal (thin gray line) periods.

Figure B.3: Japan industrial production and unemployment rate: alternative oil supply shocks.

(a) Kilian (2009) oil supply shocks



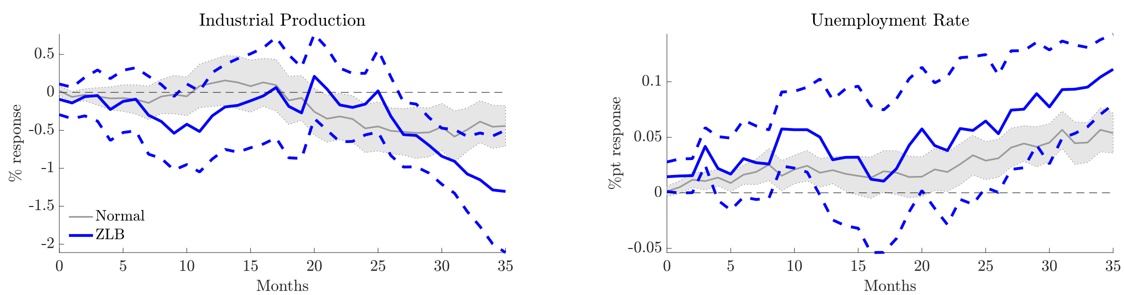
(b) Baumeister and Hamilton (2019) oil supply shocks



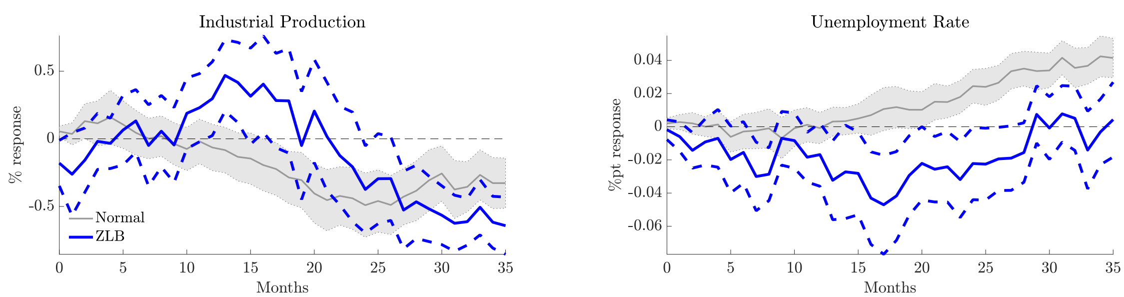
Notes: Each figure plots the impulse response functions and 68 percent confidence bands of industrial production and the unemployment rate in Japan to oil shocks identified in Kilian (2009) and Baumeister and Hamilton (2019). Figure C.5 plots the differences in the responses of these variables in the ZLB and outside of it.

Figure B.4: Japanese real variables responses to different oil supply shocks: OLS estimation

(a) The Kilian oil supply shocks

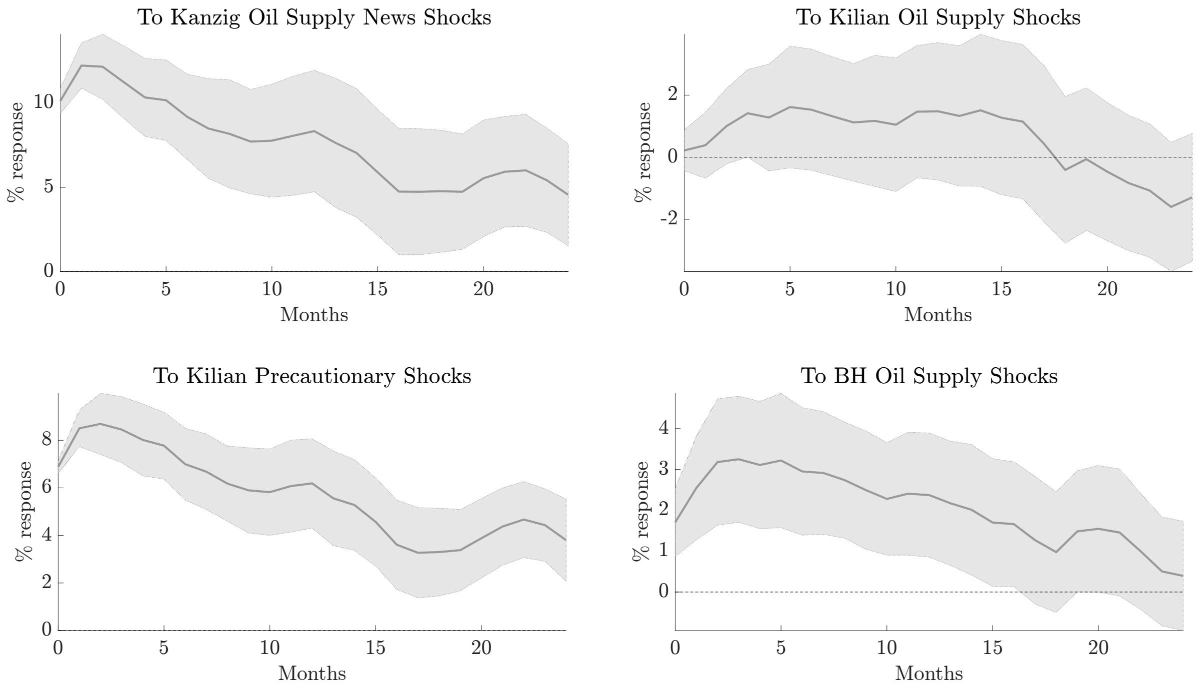


(b) The Baumeister-Hamilton oil supply shocks



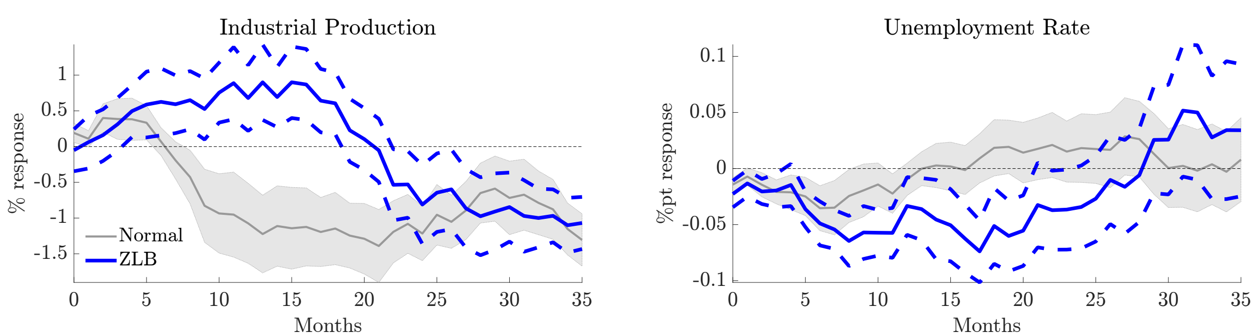
Notes: Each row plots the impulse response functions and 68 percent confidence bands for the responses of Japanese industrial production and the unemployment rate to oil supply shocks, as identified in Kilian (2009) and Baumeister and Hamilton (2019), using OLS estimation instead of IV. The estimation period is from 1975:1 to 2019:12. Note that the oil supply shocks lead to an increase in real oil prices.

Figure B.5: Real oil price response to different identified oil shocks.



Notes: Each figure plots the impulse response functions and 90 percent confidence bands for the responses of oil price to oil price shocks identified in Kanzig (2021) (oil supply news shock), Kilian (2009) (oil supply and oil-market specific demand shocks), and Baumeister and Hamilton (2019) (oil supply shocks). In the case of the Kanzig (2021) shocks, we use the whole sample of 1974:1 to 2019:12 to estimate the response of oil prices.

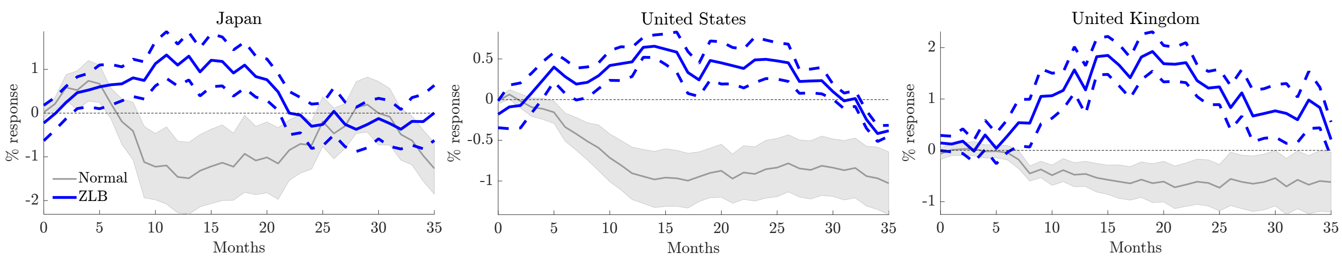
Figure B.6: Japan impulse responses to Kilian (2009) oil-market specific demand shocks.



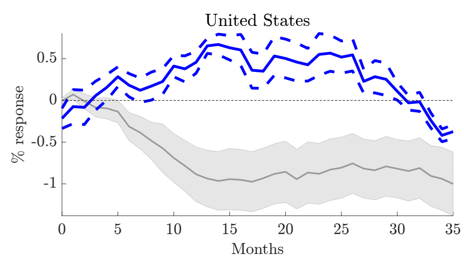
Notes: Each figure plots the impulse response functions and 68 percent confidence bands for the responses of Japanese industrial proudction and unemployment rate to oil price shocks identified in Kilian (2009) oil-market specific demand shocks. Figure C.6 plots the differences in the responses of these variables in the ZLB and outside of it.

Figure B.7: Non-oil output responses in Japan, the United Kingdom, and the United States.

(a) Manufacturing Industrial Production

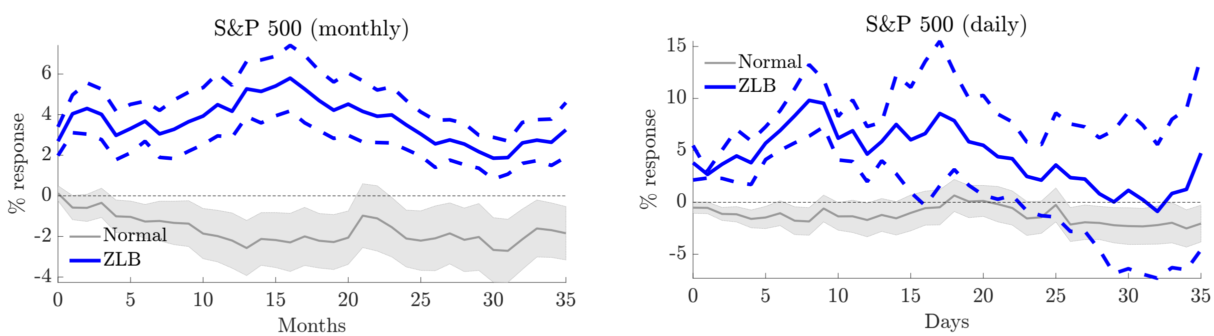


(b) Non-Oil Industrial Production



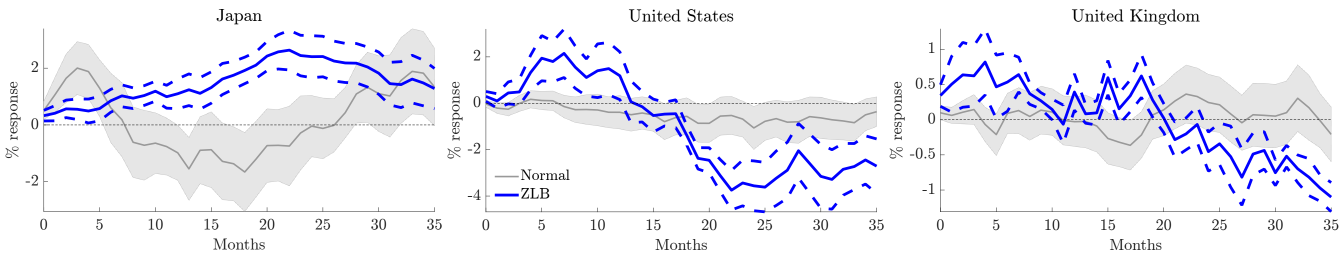
Notes: This figure plots the impulse response functions and one-standard-deviation confidence bands of the responses of manufacturing industrial production in Japan, the Unites States, and the United Kingdom, and non-oil industrial production in the United States (lower panel) in the ZLB (thick blue line) and outside of it (think gray line) to an oil supply news shock that increases the real oil price by 10 percent on impact. Formally, we estimate equation (1) using data from 1975:1 to 2019:12.

Figure B.8: The S&P 500 responses to oil supply news shocks and surprises.



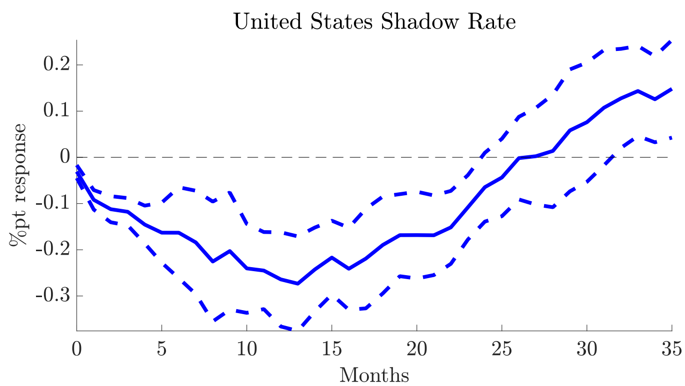
Notes: This figure plots the impulse response functions and one-standard-deviation confidence bands of the responses of the S&P 500 index to the oil supply news shock (left) and to the oil supply surprise series (right) both of which increase the oil price by 10 percent on impact. The left panel presents the results based on the specification (1) using monthly data, while the right panel uses daily data. The daily estimation uses 10 lags of the daily stock price as the only controls.

Figure B.9: Real exchange rate responses.



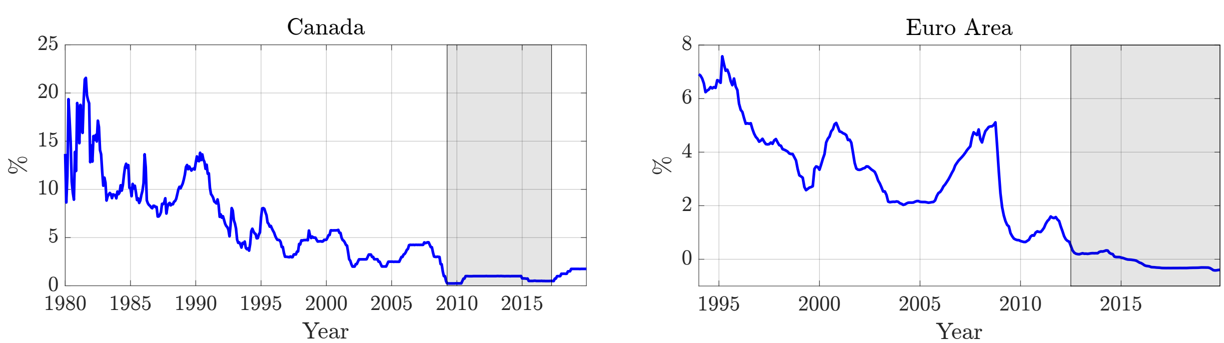
Notes: Notes: Each figure plots the impulse response functions and one-standard-deviation confidence bands of the responses of the real effective exchange rates in Japan, the United States, and the United Kingdom. An increase in the real exchange rate is a depreciation. The differences are plotted in Figure C.7.

Figure B.10: Shadow rate impulse responses in the United States in the ZLB period.



Notes: This figure plots the impulse response function and one-standard-deviation confidence bands of the responses of the shadow interest rate in the Unites States in the ZLB.

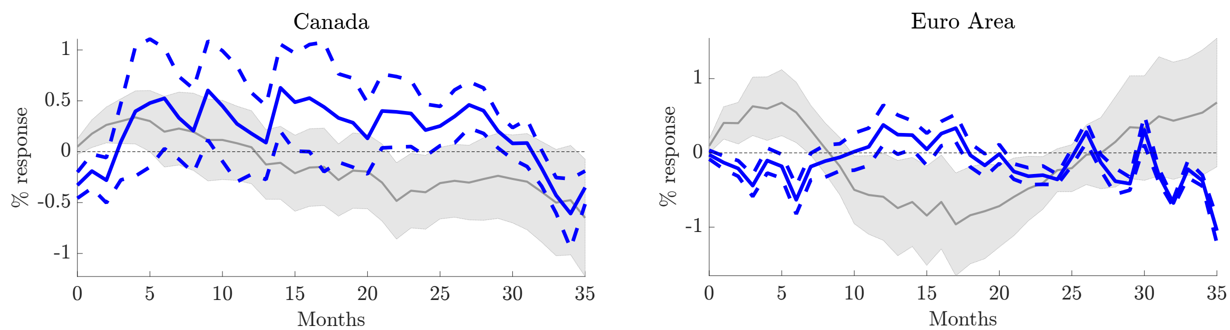
Figure B.11: The short-term nominal interest rates.



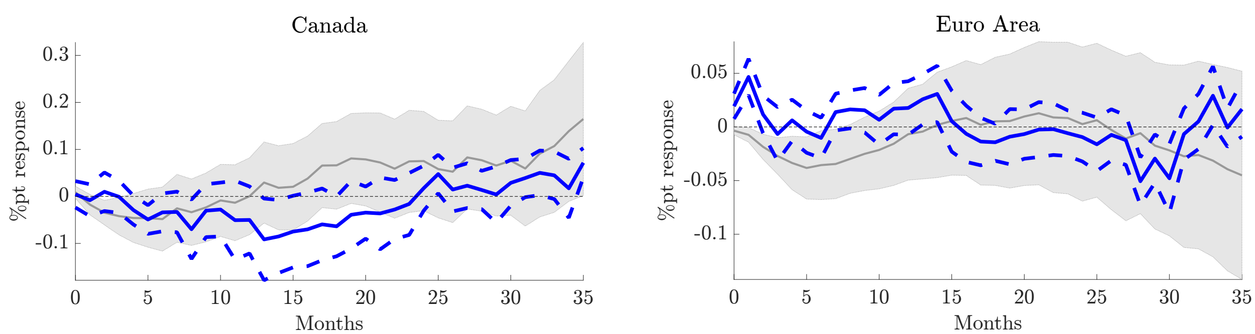
Notes: Each panel presents central bank's policy rate for each country. The shaded areas are the zero lower bound periods defined in Section 5.3.

Figure B.12: Industrial production and unemployment rate responses in Canada and the euro area.

(a) Industrial production



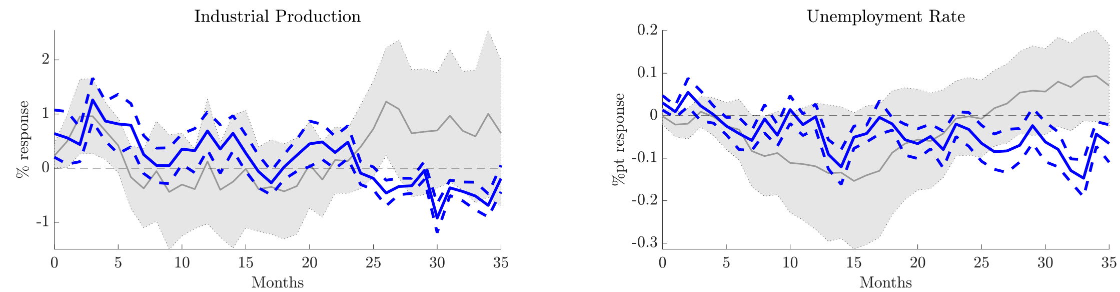
(b) Unemployment rate



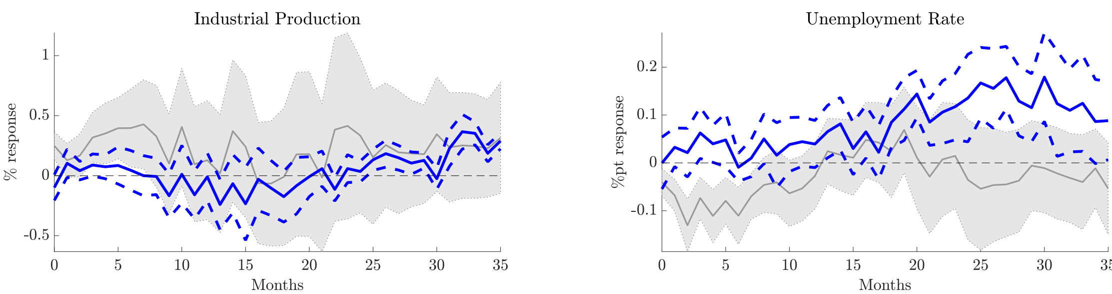
Notes: Each figure plots the impulse response functions and one-standard-deviation confidence bands of the responses. Panel (a) shows industrial production responses in Canada and the Euro Area in the ZLB (thick blue line) and in the normal (thin gray line) periods. Panel (b) plots unemployment rate responses in Canada and the Euro Area in the ZLB (thick blue line) and in the normal (thin gray line) periods.

Figure B.13: Responses in other countries: Korea, Mexico and Sweden.

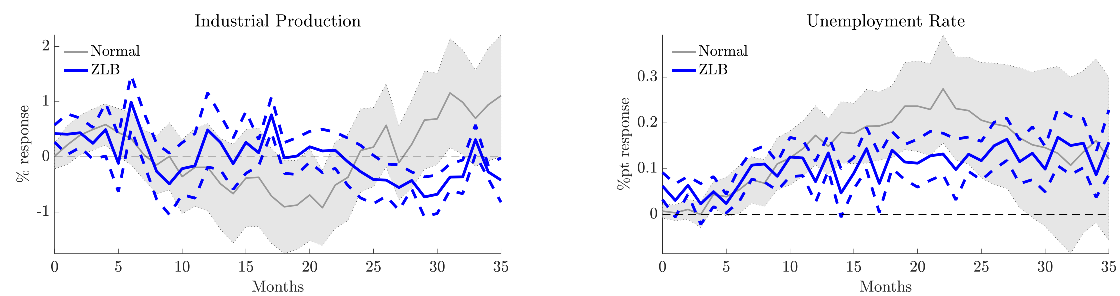
(a) Republic of Korea



(b) Mexico

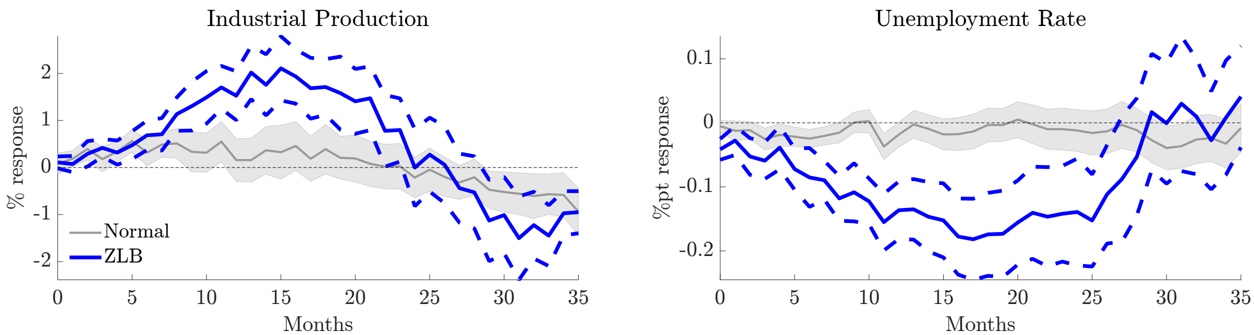


(c) Sweden



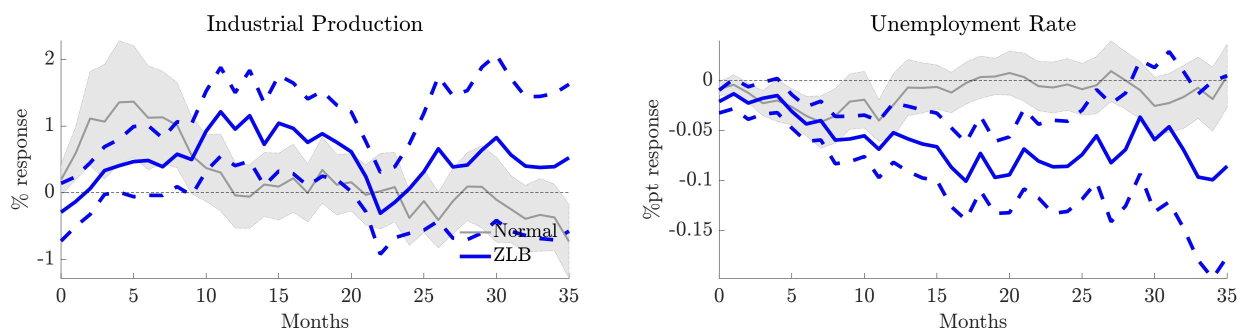
Notes: Each figure plots the impulse response function and one-standard-deviation confidence bands of the responses of industrial production (left panels) and the unemployment rate (right panels) in Republic of Korea, Mexico and Sweden in the ZLB (thick blue line) and in the normal (thin gray line) periods, where the ZLB period is defined based on the behavior of the short term nominal interest rate in the US.

Figure B.14: Industrial production and unemployment rate responses in Japan when ZLB is 1995:10-2006:6.



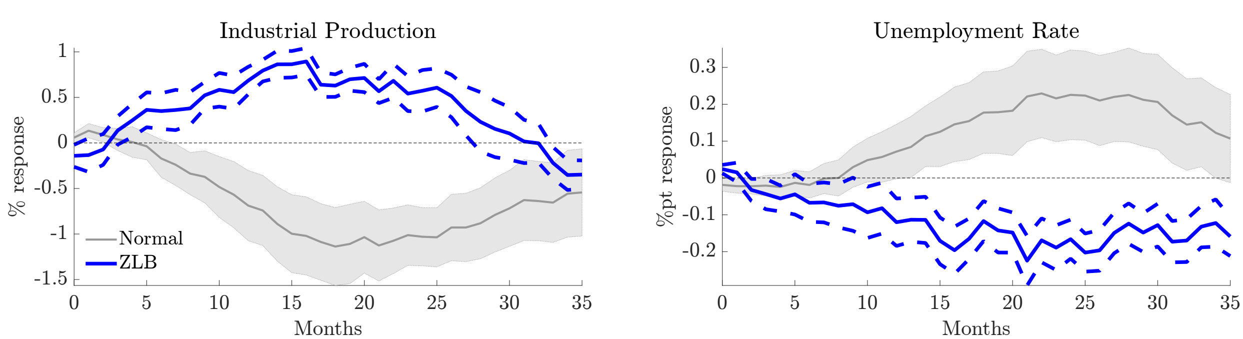
Notes: The impulse responses and one-standard-deviation confidence bands of industrial production and unployment rate to an oil supply news shock that increases oil price by 10 percent in the ZLB (think blue line) and normal (thin gray line) periods. The estimation sample is from 1975:1 to 2006:6.

Figure B.15: Industrial production and unemployment rate responses in Japan without 2008:3-2008:7.



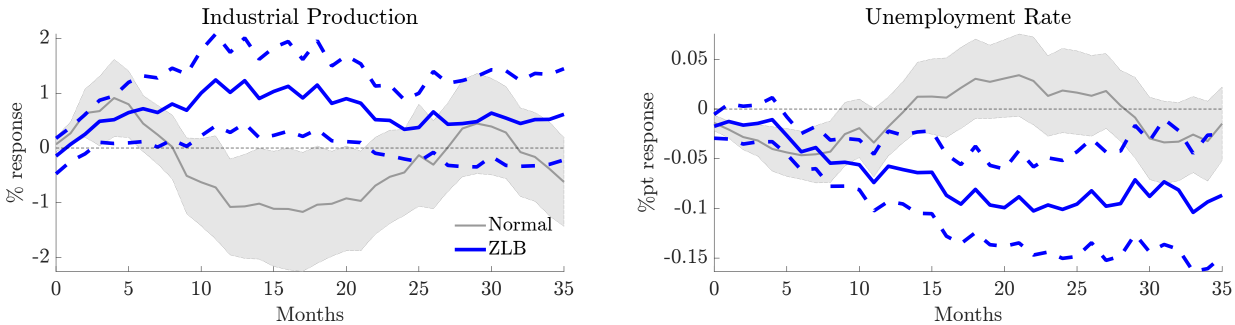
Notes: The figures plot impulse response functions and one-standard-deviation confidence bands of industrial production (left) and unployment rate (right) to an oil supply news shock that increases oil price by 10 percent in the ZLB (thick blue line) and normal (thin gray line) periods. The results based on specification when the 2008:3-2008:7 period is dummied out.

Figure B.16: Industrial production and unemployment rate responses in the US: the 1986:1-2019:12 sample.



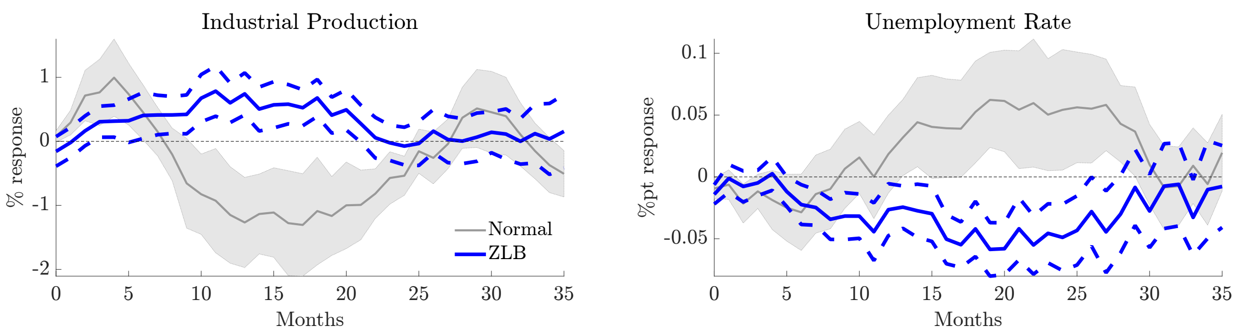
Notes: The figures plot the impulse response functions and one-standard-deviation confidence bands of industrial production (left) and unployment rate (right) to an oil supply news shock that increases oil price by 10 percent in the ZLB (thick blue line) and normal (thin gray line) periods in the United States. The sample is from 1986:1 to 2019:12.

Figure B.17: Japanese industrial production and unemployment rate responses: no controls



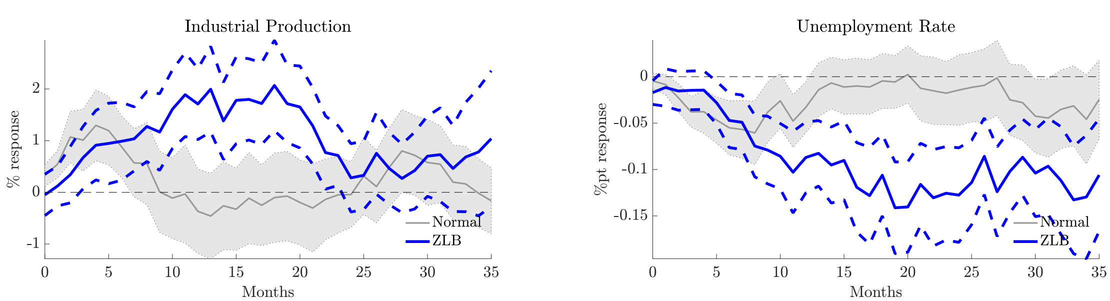
Notes: Each figure plots the impulse response functions and one-standard-deviation confidence bands of the responses of Japanese industrial proudction and unemployment rate to oil supply news shocks that increase the real oil price by 10 percent on impact in the ZLB (thick blue line) and the normal (thin gray line) periods. The controls only include a constant and lags of dependent variable, unlike the baseline specification that features the unemployment rate and the lags of the oil supply news shocks.

Figure B.18: Japanese industrial production and unemployment rate responses: additional controls



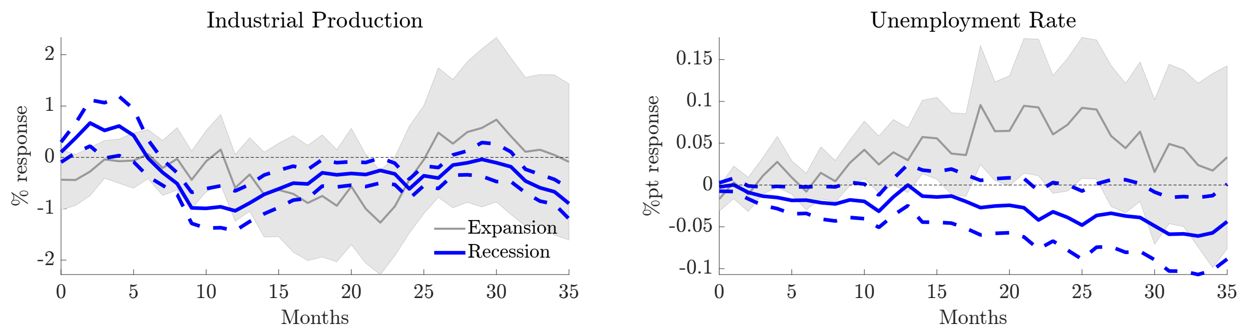
Notes: Each figure plots the impulse response functions and one-standard-deviation confidence bands of the responses of Japanese industrial proudction and unemployment rate to oil supply news shocks that increase the real oil price by 10 percent on impact in the ZLB (thick blue line) and the normal (thin gray line) periods. In addition to the baseline specification control variables (the lagged left-hand variable, unemployment rate, and the oil supply news shocks), we add two nominal variables to the list of controls: the lags of inflation rate and lags of 1-year nominal interest rate.

Figure B.19: Industrial production and unemployment rate responses in Japan (without info revelation).



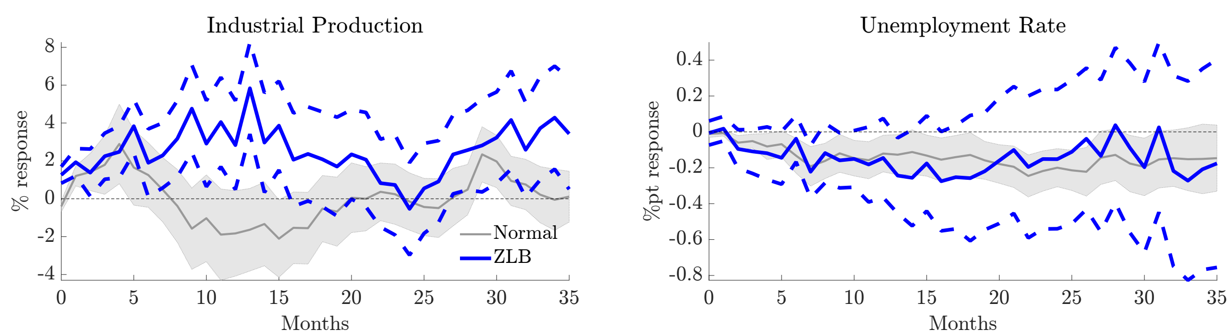
Notes: Each figure plots the impulse response functions and one-standard-deviation confidence bands of industrial production (left) and unployment rate (right) to a modified oil supply news shock that increases oil price by 10 percent in the ZLB (thick blue line) and the normal (thin gray line) periods. A modified oil supply news shock was obtained by only using only a part of the oil supply surprise series that generates a negative comovement between oil prices and the U.S. stock market index (Degasperi, 2021).

Figure B.20: Japanese industrial production and unemployment rate responses in booms and busts.



Notes: Notes: Each figure plots the impulse response functions and one-standard-deviation confidence bands of the responses of Japanese industrial production and the unemployment rate in recessions and booms following a 10-percent increase in real oil price generated by the oil supply news shock. The difference is plotted in Figure C.9.

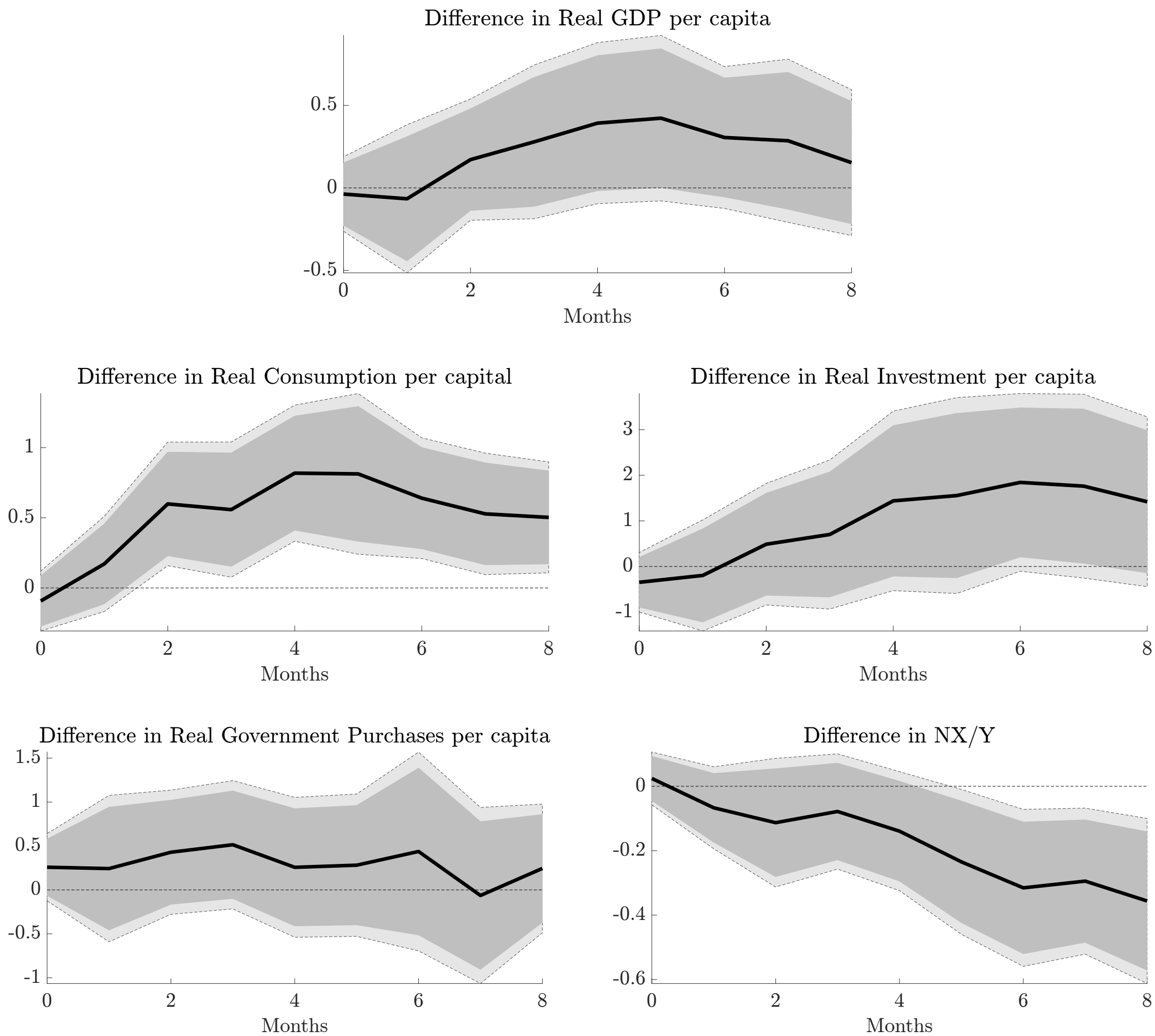
Figure B.21: Japanese Industrial Production and Unemployment Rate Responses: the oil supply surprise series



Notes: Each figure plots the IRFs and 68% confidence bands of the responses of Japanese industrial proudction and unemployment rate to an oil supply suprise that increase the real oil price by 10 percent on impact.

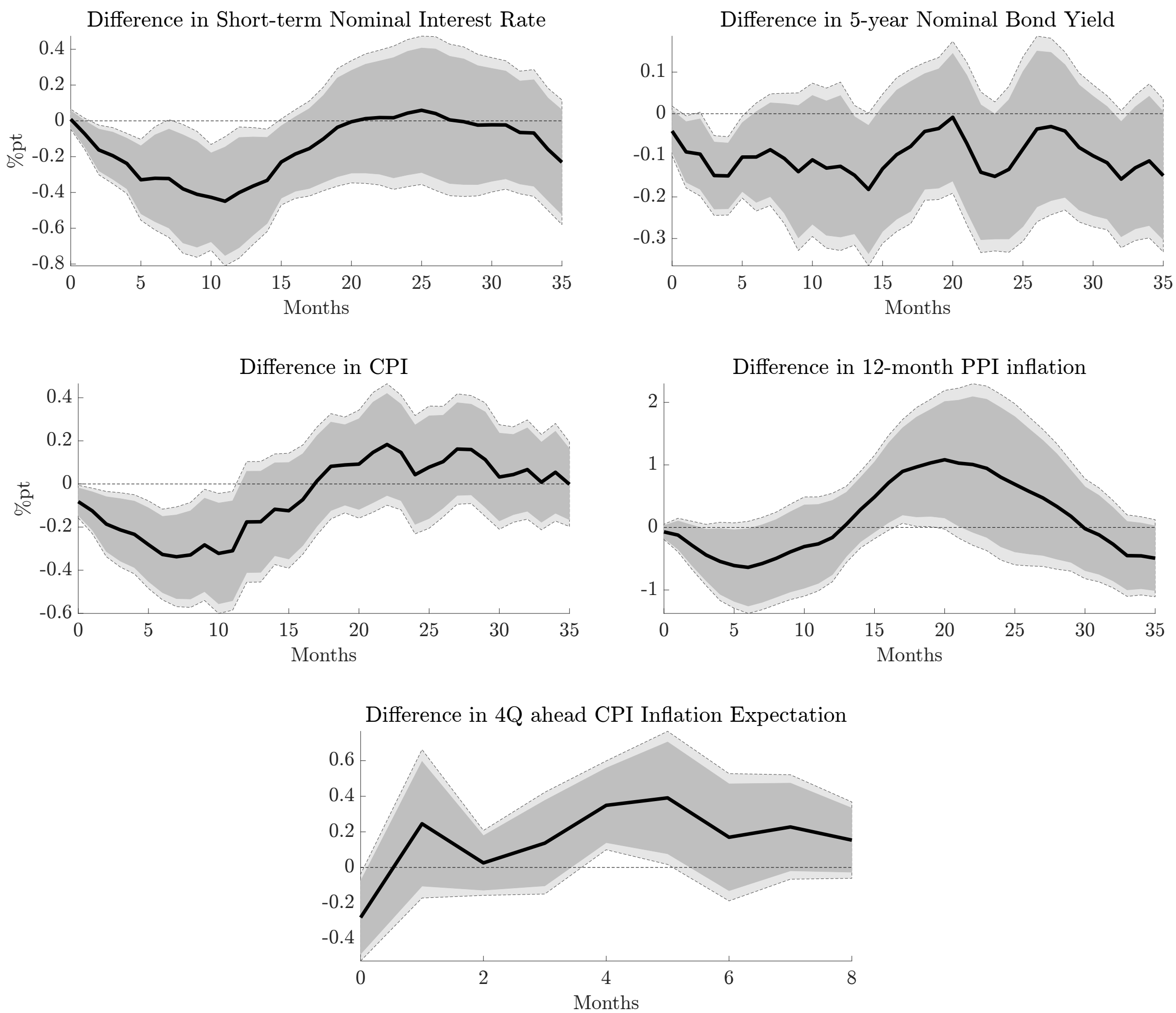
C Additional Figures: Differences in Estimates

Figure C.1: Difference in quarterly impulse responses in Japan.



Notes: Each figure plots the difference between the responses of real GDP, real consumption, real investment, real government purchases (all per capita), and nominal trade balance over nominal GDP in Japan to the oil supply news shock that increase the oil price by 10 percent on impact across the ZLB and in the normal periods. The estimated specification is in equation (1). The sample is from 1975Q1 to 2019Q4. Each plot also presents the 90-percent (dark grey) and 95-percent (light grey) confidence bands. The levels of the impulse responses are plotted in Figure 4.

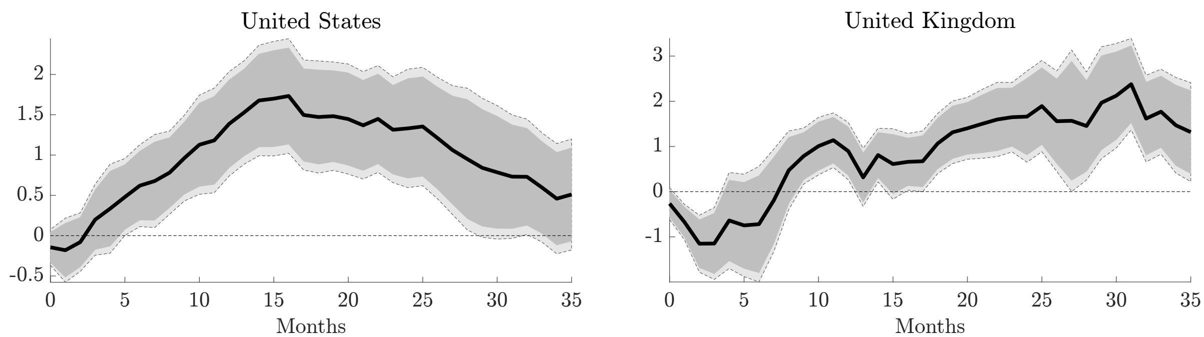
Figure C.2: Difference in interest rates, inflation, and inflation expectations responses in Japan.



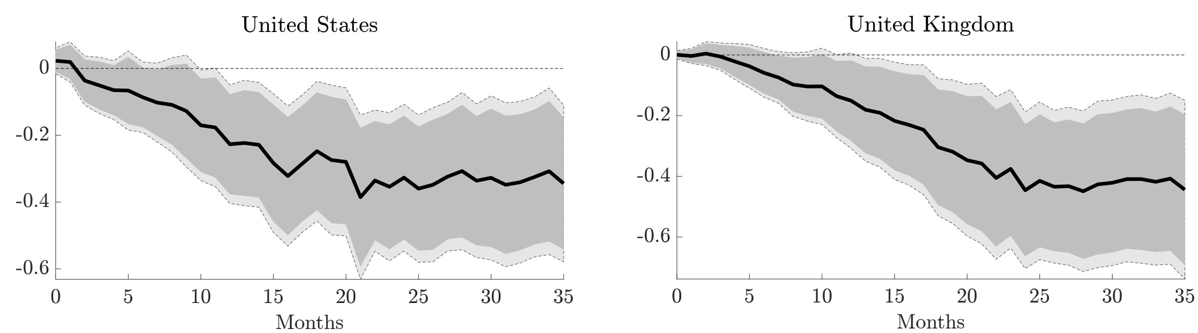
Notes: The figures plot the difference in impulse responses of the short-term nominal interest rate, 5-year nominal interest rate, CPI inflation rate, PPI inflation rate, and CPI inflation expectations across ZLB and normal periods in Japan. The errors bands are 90- and 95-percent confidence intervals. The corresponding impulse responses in the ZLB and normal periods are in Figure 5.

Figure C.3: Difference in industrial production and unemployment rate responses in the US and UK.

(a) Difference in the industrial production responses



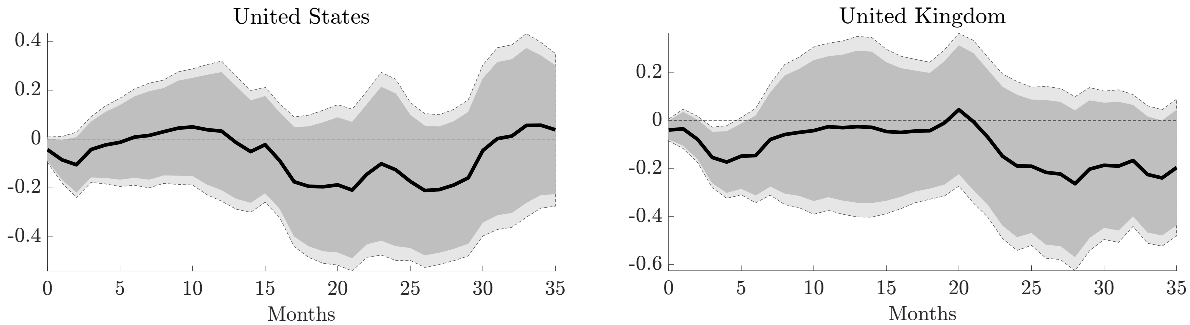
(b) Difference in the unemployment rate responses



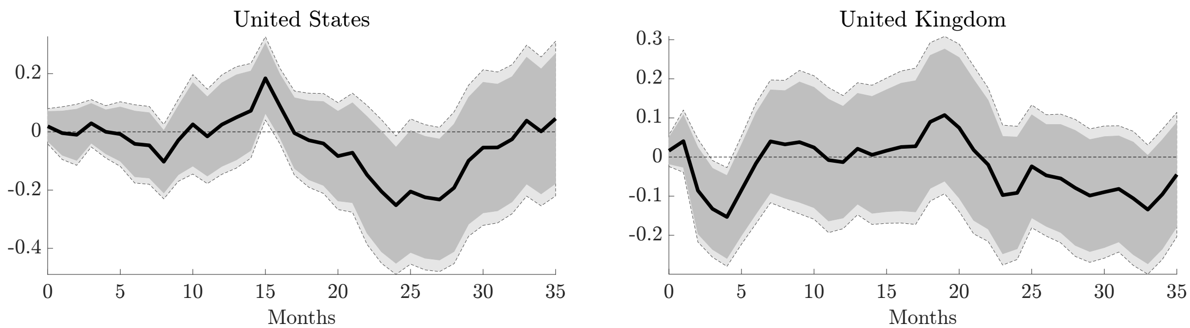
Notes: Each figure plots the mean and one standard-deviation confidence bands of the responses. Panel (a) shows industrial production responses in the United Kingdom and the Unites States in the ZLB (blue) and in the normal (grey) periods. Panel (b) plots unemployment rate responses in the United Kingdom and the Unites States in the ZLB (blue) and in the normal (grey) periods. The levels of the impulse responses are plotted in Figure 6.

Figure C.4: Difference in interest rates and inflation in the the United States and United Kingdom.

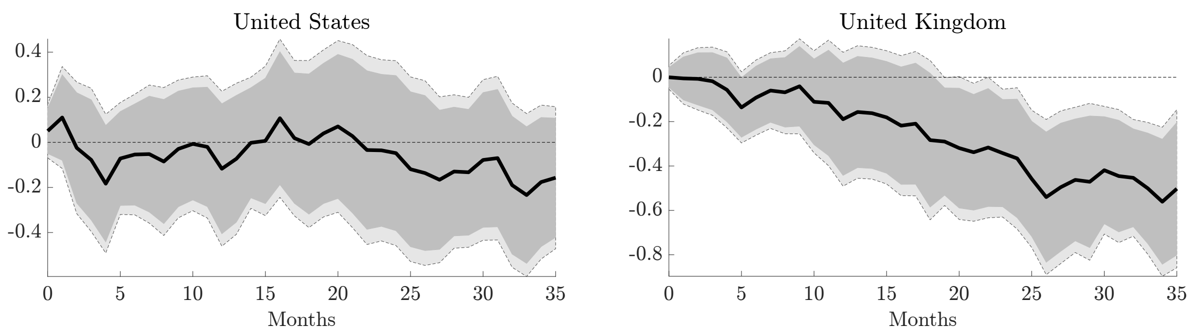
(a) Short-term nominal interest rate difference



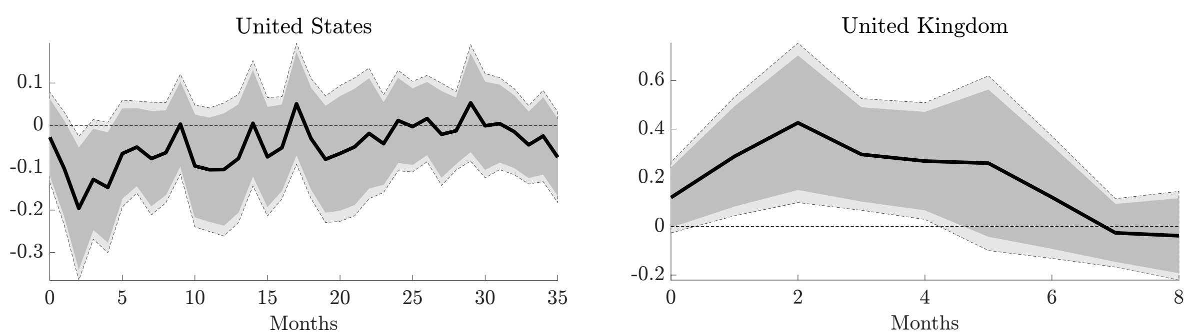
(b) 5-year nominal government bond yield difference



(c) CPI inflation rate difference



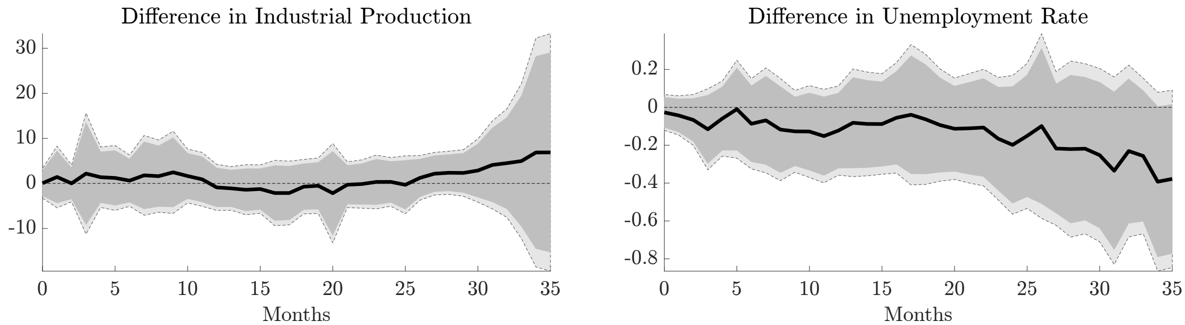
(d) Household inflation expectations difference



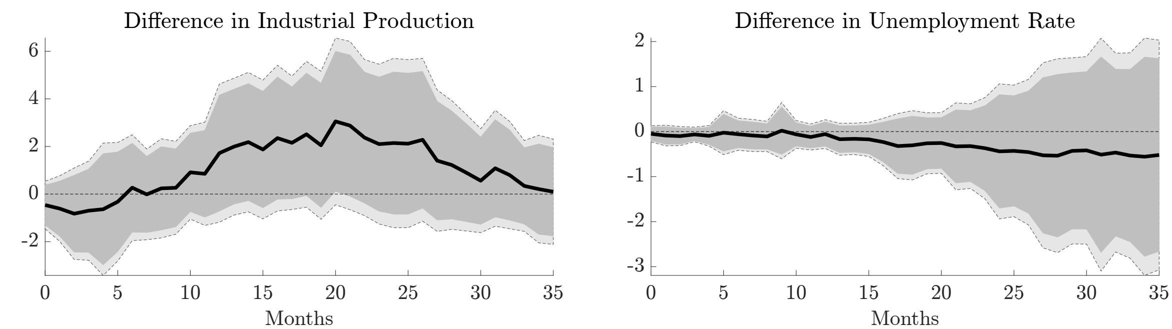
Notes: Each figure plots the mean, 90- and 95-percent confidence intervals of the difference in the responses of the central bank nominal interest rate, the 5-year nominal yield on government bonds, and the CPI inflation, and the household inflation expectations in the Unites States and the United Kingdom in the ZLB (thick blue line) and in the normal (thin grey line) periods. All the units on thy Y axes are percentage points. The levels of the impulse responses are plotted in Figure 7.

Figure C.5: Difference in responses of Japan industrial production and unemployment rate: different oil supply shocks.

(a) Kilian (2009) oil supply shocks

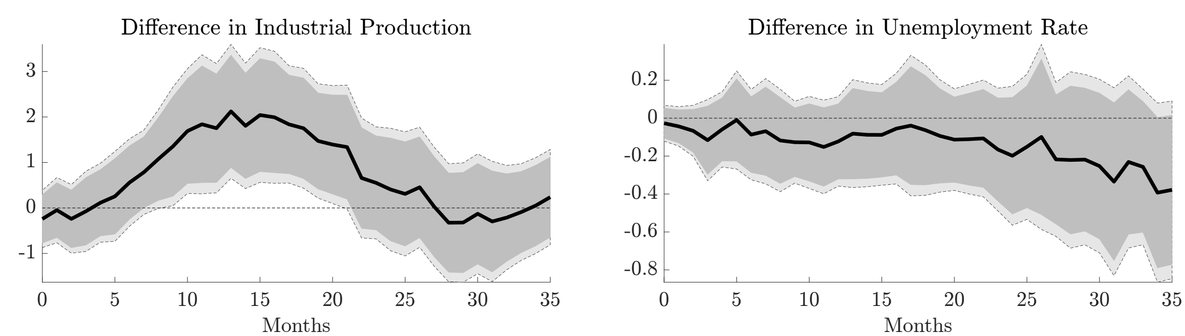


(b) Baumeister and Hamilton (2019) oil supply shocks



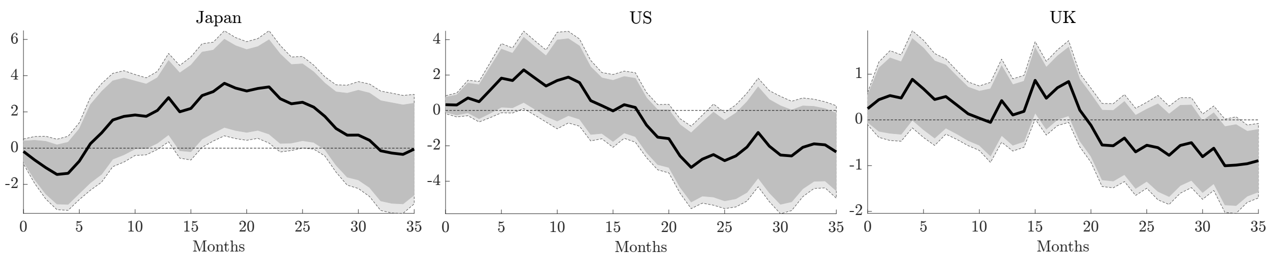
Notes: Each figure plots the mean, 90- and 95-percent confidence intervals of the difference in the responses of industrial production and the unemployment rate in Japan to oil shocks identified as in Kilian (2009) and Baumeister and Hamilton (2019). The levels of the impulse responses are plotted in Figure B.3.

Figure C.6: Difference in responses of Japan industrial production and unemployment rate: oil-market specific demand shock.



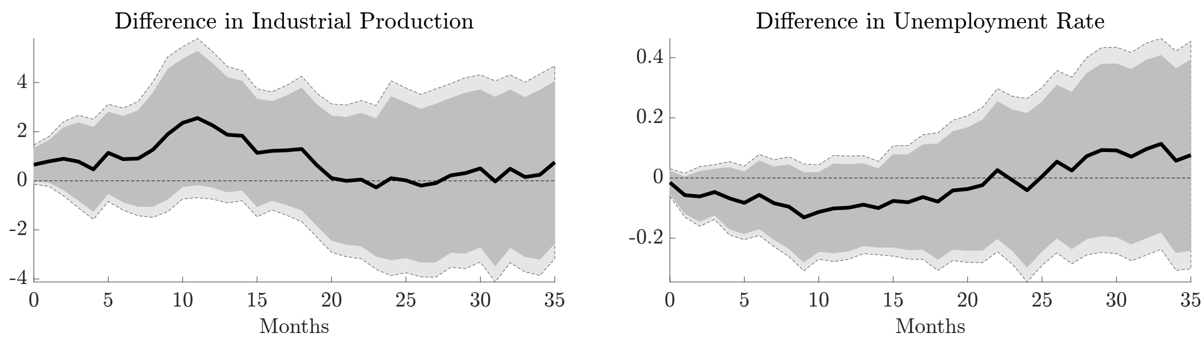
Notes: Each figure plots the mean, 90 and 95 percent confidence intervals of the difference in the responses of industrial production and the unemployment rate in Japan to oil-market specific demand shocks identified as in Kilian (2009). The levels of the impulse responses are plotted in Figure B.6.

Figure C.7: Difference in the real effective exchange rate responses.



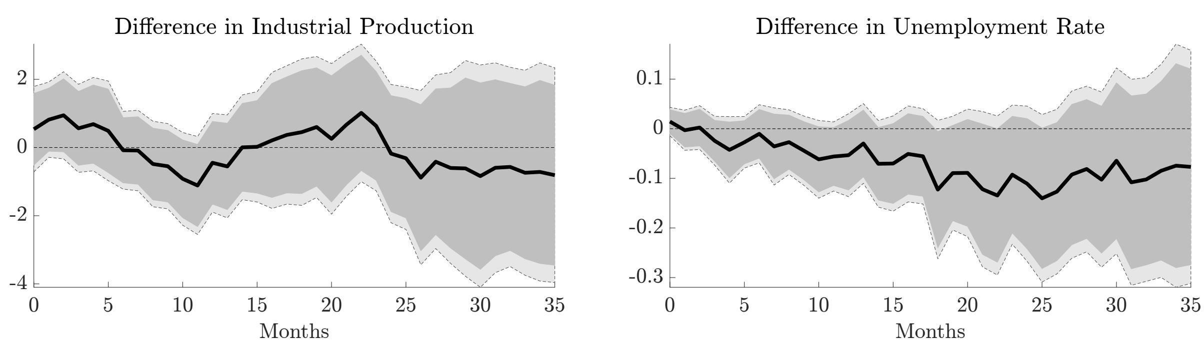
Notes: Notes: Each figure plots the mean, 90 and 95 percent confidence intervals of the difference in the responses of the real effective exchange rate in Japan, the United States, and the United Kingdom in the ZLB and normal periods. The levels of the impulse responses are plotted in Figure B.9.

Figure C.8: Positive and negative oil supply shocks.



Notes: Notes: Each figure plots the mean, 90- (dark gray) and 95-percent (light gray) confidence intervals of the difference in the responses of the Japanese industrial production and the unemployment rate to positive and negative oil supply news shocks normalized to change the real oil price by 10 percent.

Figure C.9: Difference in Industrial Production and Unemployment Rate: booms and busts.



Notes: Notes: Each figure plots the mean, 90- and 95-percent confidence intervals of the difference in the responses of the Japanese industrial production and the unemployment rate in recessions and booms following a 10-percent increase in real oil price generated by the oil supply news shock. The levels of the impulse responses are plotted in Figure B.20.

D Model Details

This section presents a standard open-economy New Keynesian model extended with demand for oil by firms. The goal of this section is to illustrate the direct effects of oil shocks on aggregate supply, systematic monetary policy, and the world economy.

There are two countries home (H) and foreign (F). We denote variables of foreign country with an asterisk. Each country is populated by measure n and $n^* \equiv 1-n$ residents. All quantity variables are expressed in per capita terms. Monopolistically competitive producers in home country produce varieties with indexes i on the interval $[0,n)$, while foreign firms on the interval $[n,1]$.

We describe the agent in home country. Foreign country agents description is symmetric.

D.1 Households

At home, there is a representative household with preferences represented by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \tag{D.1}$$

where

$$U(C,N) \equiv \frac{C^{1-\sigma}}{1-\sigma} - \Theta \frac{N^{1+\varphi}}{1+\varphi},$$

and N_t is labor supply, and C_t is consumption index[43]

$$C_t \equiv \left[\omega^{\frac{1}{\gamma_n}} C_{H,t}^{\frac{\gamma_n - 1}{\gamma_n}} + (1-\omega)^{\frac{1}{\gamma_n}} C_{F,t}^{\frac{\gamma_n - 1}{\gamma_n}} \right]^{\frac{\gamma_n}{\gamma_n - 1}}.$$

The indices of home and foreign goods consumption $C_{H,t}$ and $C_{F,t}$ by home residents are

$$C_{H,t} \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\epsilon}} \int_0^n C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} \, di \right]^{\frac{\epsilon}{\epsilon-1}},$$

and

$$C_{F,t} \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\epsilon}} \int_n^1 C_{F,t}(i)^{\frac{\epsilon-1}{\epsilon}} \, di \right]^{\frac{\epsilon}{\epsilon-1}},$$

where $i \in [0,1]$ is the index of individual good variety, ϵ is the elasticity of substitution across varieties, γ_n is the elasticity of substitution between home and foreign goods, $1-\omega \equiv (1-n)(1-\Omega)$ is a share of foreign-good expenditures in total expenditures, and $\Omega \in [0,1)$ is an index of home bias. The value of $\Omega = 0$ corresponds to the case of no home bias, while $\Omega > 0$ is required for non-zero degree of home bias. For the foreign economy, $1-\omega^* \equiv n(1-\Omega^*)$ is the share of home-good expenditures in total expenditures.

The household maximizes its preferences by choosing a plan $\{\{C_{F,t}(i)\}, \{C_{H,t}(i)\}, C_{F,t}, C_{H,t}, C_t, N_t,$

[43] The consumption index in foreign country is symmetric to home consumption index and equals: $C_t^* \equiv [(\omega^*)^{1/\gamma_n} (C_{F,t}^*)^{(\gamma_n-1)/\gamma_n} + (1-\omega^*)^{1/\gamma_n} (C_{H,t}^*)^{(\gamma_n-1)/\gamma_n}]^{\gamma_n/(\gamma_n-1)}$, where ω^* is a share of foreign goods in foreign consumption basket.

$B_{H,t+1}, B_{F,t+1}\}$ subject to the flow budget constraint

$$\int_0^n P_{H,t}(i)C_{H,t}(i)di + \mathcal{E}_t \int_n^1 P_{F,t}^*(j)C_{F,t}(j)dj + \mathbb{E}_t\left[M_{t,t+1}B_{H,t+1}\right]$$
$$+\mathbb{E}_t\left[\mathcal{E}_t M_{t,t+1}^* B_{F,t+1}\right] \leq B_{H,t} + \mathcal{E}_t B_{F,t} + W_t N_t + P_{O,t}O_t + \Pi_t - T_t, \tag{D.2}$$

where $P_{H,t}(i)$ and $P_{F,t}^*(j)$ are the prices of home and foreign varieties denoted in home and foreign currencies, respectively, $\mathcal{E}_t$ is the nominal exchange rate in units of domestic currency per foreign currency (an increase in $\mathcal{E}_t$ is a depreciation of home currency), $P_{O,t}$ is the oil price in units of home currency, $M_{t,t+1}$ and $M_{t,t+1}^*$ are prices of domestic and foreign state-contingent securities in home and foreign currencies, W_t is nominal wage at home, Π_t represent nominal profits, and T_t is nominal lump-sum taxes. The holdings of home and foreign state-contingent securities are $B_{H,t+1}$ and $B_{F,t+1}$. O_t represents the oil endowment at home.

Price indexes, terms of trade, and real exchange rate. We define the price index of home and foreign produced goods as $P_{H,t} \equiv [n^{-1}\int_0^n P_{H,t}(i)^{1-\epsilon}di]^{1/(1-\epsilon)}$ and $P_{F,t}^* \equiv [(1-n)^{-1}\int_n^1 P_{F,t}^*(i)^{1-\epsilon}di]^{1/(1-\epsilon)}$, and the home CPI is

$$P_t \equiv \left[\omega P_{H,t}^{1-\gamma_n} + (1-\omega)\left(\mathcal{E}_t P_{F,t}^*\right)^{1-\gamma_n}\right]^{1/(1-\gamma_n)}, \tag{D.3}$$

In addition, we assume that the law of one price holds for all goods. We define the terms of trade as the relative price of imported goods $S_t \equiv P_{F,t}/P_{H,t}$ and the real exchange rate as

$$Q_t \equiv \mathcal{E}_t P_t^*/P_t \tag{D.4}$$

The real oil price as the nominal oil price expressed in home currency divided by the foreign producers price index $P_{F,t}$ expressed in home currency:

$$R_t \equiv \frac{P_{O,t}}{P_{F,t}}.$$

Optimality conditions. Consumption

$$C_{H,t}(i) = \frac{1}{n}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} C_{H,t}, \tag{D.5}$$

$$C_{F,t}(i) = \frac{1}{1-n}\left(\frac{P_{F,t}^*(i)}{P_{F,t}^*}\right)^{-\epsilon} C_{F,t}, \tag{D.6}$$

where

$$C_{H,t} = \omega\left(\frac{P_{H,t}}{P_t}\right)^{-\gamma_n} C_t, \tag{D.7}$$

$$C_{F,t} = (1-\omega)\left(\frac{\mathcal{E}_t P_{F,t}^*}{P_t}\right)^{-\gamma_n} C_t, \tag{D.8}$$

Labor supply

$$-\frac{U_2(C_t, N_t)}{U_1(C_t, N_t)} = \frac{W_t}{P_t}. \tag{D.9}$$

where $U_1(C_t, N_t)$ and $U_2(C_t, N_t)$ are partial derivatives with respect to the first and the second arguments.
Assets

$$M_{t,t+1} = \beta \frac{U_1(C_{t+1}, N_{t+1})}{U_1(C_t, N_t)} \cdot \frac{P_t}{P_{t+1}}, \tag{D.10}$$

$$M^*_{t,t+1} = \beta \frac{U_1(C_{t+1}, N_{t+1})}{U_1(C_t, N_t)} \cdot \frac{P_t}{P_{t+1}} \cdot \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}. \tag{D.11}$$

Foreign households. Conditions symmetric to (D.5)-(D.11) hold for foreign country. In particular, the first-order condition with respect to $B^*_{H,t}$ imply

$$M_{t,t+1} = \beta \frac{U_1(C^*_{t+1}, N^*_{t+1})}{U_1(C^*_t, N^*_t)} \cdot \frac{P^*_t}{P^*_{t+1}} \cdot \frac{1/\mathcal{E}_{t+1}}{1/\mathcal{E}_t}. \tag{D.12}$$

Combining equations (D.10) and (D.12), we get

$$\frac{U_1(C_{t+1}, N_{t+1})}{U_1(C^*_{t+1}, N^*_{t+1})} Q_{t+1} = \frac{U_1(C_t, N_t)}{U_1(C^*_t, N^*_t)} Q_t \equiv \phi_q, \tag{D.13}$$

which implies that the ratio of marginal utility of consumption in the two countries multiplied by the real exchange rate is constant over time, which we denote as ϕ_q.

Log-linarization. We next log-linearize around the steady state where all relative goods prices are 1 and all prices and quantities are constant. Section D.8 will provide more details about steady state. For now, we obtain

$$\begin{aligned}
\widehat{c}_{H,t} &= -\gamma_n \left(\widehat{p}_{H,t} - \widehat{p}_t\right) + \widehat{c}_t, \\
\widehat{c}_{F,t} &= -\gamma_n \left(\widehat{e}_t + \widehat{p}^*_{F,t} - \widehat{p}_t\right) + \widehat{c}_t,
\end{aligned}$$

where hats denote log-deviations from steady state values.

The labor supply

$$\sigma \widehat{c}_t + \varphi \widehat{n}_t = \widehat{w}_t - \widehat{p}_t.$$

The Euler equation

$$\mathbb{E}_t\left[\widehat{c}_{t+1}\right] - \widehat{c}_t = \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \iota\right),$$

where $i_t \equiv -\log \mathbb{E}_t[M_{t,t+1}]$ is the safe short-term nominal interest rate and $\iota \equiv -\log \beta$.

The international risk sharing condition—a log-linear version of the second equality in equation (D.13)—is

$$\widehat{c}^*_t + \frac{1}{\sigma} q_t = \widehat{c}_t.$$

Total consumption is

$$\widehat{c}_t = \omega \widehat{c}_{H,t} + (1-\omega)\widehat{c}_{F,t}.$$

D.2 Price indices

The log-linearization of price indexes in home and foreign countries are

$$\widehat{p}_t = \omega \widehat{p}_{H,t} + (1-\omega)\widehat{p}_{F,t} = \widehat{p}_{H,t} + (1-\omega)\left(\widehat{p}_{F,t} - \widehat{p}_{H,t}\right),$$
$$\widehat{p}^*_t = \omega^* \widehat{p}^*_{F,t} + (1-\omega^*)\widehat{p}^*_{H,t} = \widehat{p}^*_{F,t} + (1-\omega^*)\left(\widehat{p}^*_{H,t} - \widehat{p}^*_{F,t}\right).$$

The real exchange rate is

$$\widehat{q}_t = \widehat{e}_t + \widehat{p}^*_t - \widehat{p}_t = (\omega + \omega^* - 1)\left(\widehat{p}_{F,t} - \widehat{p}_{H,t}\right).$$

Home relative prices are

$$\begin{aligned}
\widehat{p}_{H,t} - \widehat{p}_t &= -\frac{1-\omega}{\omega+\omega^*-1}\widehat{q}_t, \\
\widehat{p}_{F,t} - \widehat{p}_t &= \frac{\omega}{\omega+\omega^*-1}\widehat{q}_t, \\
\widehat{p}_{O,t} - \widehat{p}_t &= \widehat{p}_{O,t} - \widehat{p}_{F,t} + \frac{\omega}{\omega+\omega^*-1}\widehat{q}_t.
\end{aligned}$$

Foreign relative prices are

$$\begin{aligned}
\widehat{p}^*_{F,t} - \widehat{p}^*_t &= \frac{1-\omega^*}{\omega+\omega^*-1}\widehat{q}_t, \\
\widehat{p}^*_{H,t} - \widehat{p}^*_t &= -\frac{\omega^*}{\omega+\omega^*-1}\widehat{q}_t, \\
\widehat{p}^*_{O,t} - \widehat{p}^*_t &= \widehat{p}^*_{O,t} - \widehat{p}^*_{F,t} + \frac{1-\omega^*}{\omega+\omega^*-1}\widehat{q}_t.
\end{aligned}$$

D.3 Firms

A home producer of variety i combines labor $N_t(i)$ and oil inputs $O_{Y,t}(i)$ to produce good i according to the CES production function of the form

$$Y_t(i) = \left[(1-\omega_{oy})^{\frac{1}{\gamma_y}}\left(AN_t(i)\right)^{\frac{\gamma_y-1}{\gamma_y}} + \omega_{oy}^{\frac{1}{\gamma_y}} O_{Y,t}(i)^{\frac{\gamma_y-1}{\gamma_y}}\right]^{\frac{\gamma_y}{\gamma_y-1}},$$

where A_t is labor productivity, the intra-temporal elasticity of substitution γ_y is positive, and $\omega_{oy} \in [0,1]$ is the share of oil in production. The firm is free to optimize its inputs every period. Because of the constant-elasticity-of-substitution assumption about the form of the production function, the *nominal* marginal cost of production does not depend on output and equals

$$MC_t = \left[(1-\omega_{oy})\left(\frac{W_t}{A}\right)^{1-\gamma_y} + \omega_{oy} P_{O,t}^{1-\gamma_y}\right]^{\frac{1}{1-\gamma_y}}, \tag{D.14}$$

and the optimal labor and oil choices are

$$O_{Y,t}(i) = \omega_{oy} \left(\frac{P_{O,t}}{MC_t} \right)^{-\gamma_y} Y_t(i), \tag{D.15}$$

$$N_t(i) = (1 - \omega_{oy}) \left(\frac{W_t / A}{MC_t} \right)^{-\gamma_y} \frac{Y_t(i)}{A}. \tag{D.16}$$

The firm resets its price infrequently in the spirit of Calvo. When allowed, a firm chooses the reset price $P^r_{H,t}(i)$ to maximizes an expected discounted sum of future profits

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left(P^r_{H,t}(i) - (1+\tau) MC_{t+k} \right) Y_{H,t+k|t}(i) \right]$$

where $1 - \theta \in [0, 1]$ is the probability of price reset, $Y_{H,t+k|t}(i)$ is the output in period $t+k$ conditional on the price set in period t, and τ is the government's proportional tax. The price is set in producer's currency and the firm does not differentiate between domestic and foreign consumers. In other words, the law of one price holds for individual varieties.

The demand for variety i is

$$Y_{H,t+k|t}(i) = nC_{H,t+k|t}(i) + (1-n) C^*_{H,t+k|t}(i) = \left(\frac{P^r_{H,t}(i)}{P_{H,t+k}} \right)^{-\epsilon} \left(C_{H,t+k} + \frac{1-n}{n} C^*_{H,t+k} \right),$$

where $C_{H,t+k}$ and $C^*_{H,t+k}$ are the home and foreign demand for home goods.

Firm's optimal choice of its reset price requires

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \theta^k M_{t,t+k} Y_{H,t+k|t}(i) \left(\frac{P^r_{H,t}(i)}{P_{H,t-1}} - \frac{\epsilon}{\epsilon - 1} (1+\tau) \frac{MC_{t+k}}{P_{H,t+k}} \cdot \frac{P_{H,t+k}}{P_{H,t-1}} \right) \right] = 0, \tag{D.17}$$

where we deflated the expression by $P_{H,t-1}$.

Log-linearization. We log-linearize around a steady state with constant prices and quantities and where all relative prices equal one. The marginal costs are

$$\widehat{mc}_t = (1 - \omega_{oy}) \widehat{w}_t + \omega_{oy} \widehat{p}_{O,t}.$$

The demand for labor and oil are

$$\widehat{n}_t = -\gamma_y \left(\widehat{w}_t - \widehat{mc}_t \right) + \frac{nC_H}{nC_H + (1-n) C^*_H} \widehat{c}_{H,t} + \frac{(1-n) C^*_H}{nC_H + (1-n) C^*_H} \widehat{c}^*_{H,t}$$

$$\widehat{o}_{Y,t} = -\gamma_y \left(\widehat{p}_{O,t} - \widehat{mc}_t \right) + \frac{nC_H}{nC_H + (1-n) C^*_H} \widehat{c}_{H,t} + \frac{(1-n) C^*_H}{nC_H + (1-n) C^*_H} \widehat{c}^*_{H,t},$$

The optimal reset price satisfies

$$p^r_{H,t} - p_{H,t-1} = (1 - \beta\theta) \left(\widehat{mc}_t - \widehat{p}_{H,t} \right) + \pi_{H,t} + \beta\theta \mathbb{E}_t \left[p^r_{H,t+1} - p_{H,t} \right]. \tag{D.18}$$

D.4 Government

The home government consists of fiscal and monetary authorities. The fiscal authority undoes monopolistic competition distortion by subsidizing production. Specifically, it sets the production tax $\tau = -1/\epsilon$ and runs a balanced budget each period.

The monetary authority sets the short-term safe nominal interest rate $i_t \equiv -\log \mathbb{E}_t[M_{t,t+1}]$ according to the Taylor rule that reacts to the producers price inflation rate $\pi_{H,t} \equiv \log(P_{H,t}/P_{H,t-1})$. Formally,

$$i_t = \iota + \varphi_\pi \pi_t, \tag{D.19}$$

where $\iota \equiv -\log \beta$ and $\varphi_\pi \geq 0$. We do not explicitly introduce the zero lower bound on the interest rate here.

The foreign government acts analogous to home government. In particular, the central bank set the interest rate according to

$$i_t^* = \iota + \varphi_\pi^* \pi_{H,t}^*, \tag{D.20}$$

where $\varphi_\pi^* \geq 0$.

D.5 Oil

Oil is supplied as endowment that equals O_t and O_t^* in per capita terms, and these endowments change randomly every period. Households own oil and it cannot be stored. The oil prices $P_{O,t}$ and $P_{O,t}^*$ are perfectly flexible and the law of one price applies to oil.

D.6 Market Clearing

Local markets. The home labor supply equals home labor demand

$$N_t = \int_0^n \left(1 - \omega_{oy}\right) \left(\frac{W_t/A}{MC_t}\right)^{-\gamma_y} Y_t(i) di.$$

The market for every home variety $i \in [0, 1]$ clears

$$Y_{H,t}(i) = nC_{H,t}(i) + (1-n)C_{H,t}^*(i). \tag{D.21}$$

The analogoes conditions hold for foreign country.

Global markets. In equilibrium, all markets clear. Specifically, all state-contingent asset markets clear

$$n\left(B_{H,t} + \mathcal{E}_t B_{F,t}\right) + (1-n)\left(B_{H,t}^* + \mathcal{E}_t B_{F,t}^*\right) = 0.$$

The oil market clears

$$nO_{Y,t} + (1-n)\, O_{Y,t}^* = nO_t + (1-n)O_t^*.$$

and its log-linear approximation is

$$nO_Y \widehat{o}_{Y,t} + (1-n) O_Y^* \widehat{o}_{Y,t}^* = nO\widehat{o}_t + (1-n)O^* \widehat{o}_t^*.$$

D.7 Aggregation

The law of motion of the price index

$$\Pi_{H,t}^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_{H,t}^{r}}{P_{H,t-1}}\right)^{1-\epsilon},$$
$$\left(\Pi_{F,t}^{*}\right)^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_{F,t}^{*,r}}{P_{F,t-1}^{*}}\right)^{1-\epsilon}.$$

The price dispersion in home country is

$$\Delta_{H,t} \equiv \frac{1}{n}\int_0^n \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} di = \theta\left(\frac{P_{H,t-1}}{P_{H,t}}\right)^{-\epsilon}\Delta_{H,t-1} + (1-\theta)\left(\frac{P_{H,t}^{r}}{P_{H,t}}\right)^{-\epsilon}.$$

Aggregate goods market clearing in home and foreign countries

$$\left[(1-\omega_{oy})^{\frac{1}{\gamma_y}}\left(AN_t\right)^{\frac{\gamma_y-1}{\gamma_y}} + \omega_{oy}^{\frac{1}{\gamma_y}} O_{Y,t}^{\frac{\gamma_y-1}{\gamma_y}}\right]^{\frac{\gamma_y}{\gamma_y-1}} = \left(C_{H,t+k} + \frac{1-n}{n}C_{H,t+k}^{*}\right)\Delta_{H,t},$$
$$\left[(1-\omega_{oy})^{\frac{1}{\gamma_y}}\left(A^{*}N_t^{*}\right)^{\frac{\gamma_y-1}{\gamma_y}} + \omega_{oy}^{\frac{1}{\gamma_y}}\left(O_{Y,t}^{*}\right)^{\frac{\gamma_y-1}{\gamma_y}}\right]^{\frac{\gamma_y}{\gamma_y-1}} = \left(C_{F,t+k}^{*} + \frac{n}{1-n}C_{F,t+k}\right)\Delta_{F,t}^{*}.$$

Value added in terms of produced goods

$$Z_{H,t} = Y_{H,t} - \frac{P_{O,t}}{P_{H,t}}O_{Y,t} = Y_{H,t} - R_t\frac{P_{F,t}}{P_{H,t}}O_{Y,t},$$
$$Z_{F,t}^{*} = Y_{F,t}^{*} - \frac{P_{O,t}^{*}}{P_{F,t}^{*}}O_{Y,t}^{*} = Y_{F,t}^{*} - R_t O_{Y,t}^{*}.$$

where $Y_{H,t}$ and $Y_{F,t}^{*}$ is the demand for home and foreign-produced goods.

Trade balance in units of domestically produced goods over steady state output

$$\frac{NX_t}{Y_H} = \frac{1}{Y_H}\left(Y_{H,t} + \frac{P_{O,t}}{P_{H,t}}O_t - \frac{P_t}{P_{H,t}}C_t - \frac{P_{O,t}}{P_{H,t}}O_{Y,t}\right),$$
$$\frac{NX_t^{*}}{Y_F^{*}} = \frac{1}{Y_F^{*}}\left(Y_{F,t}^{*} + \frac{P_{O,t}^{*}}{P_{F,t}^{*}}O_t^{*} - \frac{P_t^{*}}{P_{F,t}^{*}}C_t^{*} - \frac{P_{O,t}^{*}}{P_{F,t}^{*}}O_{Y,t}^{*}\right).$$

This formulas takes into account that the countries trade in goods and oil.

Log-linearization. Home and foreign inflation rates are

$$\pi_{H,t} = (1-\theta)\left(\widehat{p}_{H,t}^{r} - \widehat{p}_{H,t-1}\right),$$
$$\pi_{F,t}^{*} = (1-\theta)\left(\widehat{p}_{F,t}^{*,r} - \widehat{p}_{F,t-1}^{*}\right).$$

Combining the last two equations with the optimal reset price equation at home D.18 and a similar equation abroad, we get two standard Phillips curves

$$\pi_{H,t} = \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\widehat{mc}_t - \widehat{p}_{H,t}\right) + \beta\mathbb{E}_t\left[\pi_{H,t+1}\right],$$
$$\pi^*_{F,t} = \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\widehat{mc}^*_t - \widehat{p}^*_{F,t}\right) + \beta\mathbb{E}_t\left[\pi^*_{F,t+1}\right].$$

Goods market clearing

$$\begin{aligned}
&(1-\omega_{oy})^{\frac{1}{\gamma_y}}\left(\frac{AN}{C_H+\frac{1-n}{n}C^*_H}\right)^{\frac{\gamma_y-1}{\gamma_y}}\widehat{n}_t + \omega_{oy}^{\frac{1}{\gamma_y}}\left(\frac{O_Y}{C_H+\frac{1-n}{n}C^*_H}\right)^{\frac{\gamma_y-1}{\gamma_y}}\widehat{o}_{Y,t}\\
=&\frac{nC_H}{nC_H+(1-n)\,C^*_H}\widehat{c}_{H,t} + \frac{(1-n)\,C^*_H}{nC_H+(1-n)\,C^*_H}\widehat{c}^*_{H,t},\\
&(1-\omega_{oy})^{\frac{1}{\gamma_y}}\left(\frac{A^*N^*}{C^*_F+\frac{n}{1-n}C_F}\right)^{\frac{\gamma_y-1}{\gamma_y}}\widehat{n}^*_t + \omega_{oy}^{\frac{1}{\gamma_y}}\left(\frac{O^*_Y}{C^*_F+\frac{n}{1-n}C_F}\right)^{\frac{\gamma_y-1}{\gamma_y}}\widehat{o}^*_{Y,t}\\
=&\frac{(1-n)\,C^*_F}{(1-n)\,C^*_F+nC_F}\widehat{c}^*_{F,t} + \frac{nC_F}{(1-n)\,C^*_F+nC_F}\widehat{c}_{F,t},
\end{aligned}$$

Value added

$$\widehat{z}_{H,t} = \frac{1}{1-\omega_{oy}}\widehat{y}_{H,t} - \frac{\omega_{oy}}{1-\omega_{oy}}\left(\widehat{r}_t + \widehat{p}_{F,t} - \widehat{p}_{H,t} + \widehat{o}_{y,t}\right),$$
$$\widehat{z}^*_{F,t} = \frac{1}{1-\omega_{oy}}\widehat{y}^*_{F,t} - \frac{\omega_{oy}}{1-\omega_{oy}}\left(\widehat{r}_t + \widehat{o}^*_{y,t}\right).$$

The trade balance is

$$\widehat{nx}_t = \widehat{y}_{H,t} + \frac{O-O_Y}{Y_H}\left(\widehat{r}_t + \frac{1}{\omega+\omega^*-1}\widehat{q}_t\right) + \frac{O\widehat{o}_t - O_Y\widehat{o}_{Y,t}}{Y_H} - \frac{C}{Y_H}\left(\frac{1-\omega}{\omega+\omega^*-1}\widehat{q}_t + \widehat{c}_t\right), \tag{D.22}$$
$$\widehat{nx}^*_t = \widehat{y}^*_{F,t} + \frac{O^*-O^*_Y}{Y^*_F}\widehat{r}_t + \frac{O^*\widehat{o}^*_t - O^*_Y\widehat{o}^*_{Y,t}}{Y^*_F} - \frac{C^*}{Y^*_F}\left(-\frac{1-\omega^*}{\omega+\omega^*-1}\widehat{q}_t + \widehat{c}^*_t\right). \tag{D.23}$$

D.8 Steady State

We assume that in steady state all prices are constant and the relative prices equal one:

$$\frac{P_O}{P} = \frac{P_F}{P} = \frac{P_H}{P} = 1.$$

This implies that the real exchange rate is

$$Q = \frac{\mathcal{E}P^*}{P} = \mathcal{E}\frac{\left[(1-\omega^*)\left(P_H/\mathcal{E}\right)^{1-\gamma_n} + \omega^*\left(P^*_F\right)^{1-\gamma_n}\right]^{1/(1-\gamma_n)}}{\left[\omega P_H^{1-\gamma_n} + (1-\omega)\left(\mathcal{E}P^*_F\right)^{1-\gamma_n}\right]^{1/(1-\gamma_n)}} = 1$$

We get

$$-\frac{U_2\left(C,N\right)}{U_1\left(C,N\right)} = A$$

$$
\begin{aligned}
e^{-\iota} &= \beta, \\
\frac{W}{P} &= A, \\
\frac{MC}{P} &= \frac{W}{AP} = 1, \\
C_H &= \omega C, \\
C_F &= (1-\omega) C, \\
Y_H &= C_H + \frac{1-n}{n} C_H^*, \\
Y_H &= \left[(1-\omega_{oy})^{\frac{1}{\gamma_y}} (AN)^{\frac{\gamma_y - 1}{\gamma_y}} + \omega_{oy}^{\frac{1}{\gamma_y}} O_Y^{\frac{\gamma_y - 1}{\gamma_y}} \right]^{\frac{\gamma_y}{\gamma_y - 1}}, \\
AN &= \left(1-\omega_{oy}\right) Y_H, \\
O_Y &= \omega_{oy} Y_H, \\
C &= Y_H + O - O_Y.
\end{aligned}
$$

Observe that that the last equation is the flow budget constraint of the country. Also note that if there were no oil, then we would have $C = Y_H$ and $C^* = C_F^* + \frac{n}{1-n} C_F$ and $C_H = \omega C$ and $C_F = (1-\omega) C$ and $C_F^* = \omega^* C^*$ and $C_H^* = (1-\omega^*) C^*$. Combining these together, we get $(1-\omega) nC = (1-n)(1-\omega^*) C^*$ and $(1-n)(1-\omega^*) C^* = n(1-\omega) C$. The last two equations are identical. They determine the ratio: $C/C^* = (1-n)/n \cdot (1-\omega^*)/(1-\omega) = (1-\Omega^*)/(1-\Omega)$. How can we determine steady state consumption level? The Backus-Smith condition $C = \phi_q^{-\frac{1}{\sigma}} C^* Q^{\frac{1}{\sigma}}$ implies $C/C^* = \phi_q^{-\frac{1}{\sigma}} = (1-\Omega^*)/(1-\Omega)$.

Simplifying we get 3 equations and 2 unknowns C, N as functions of parameters and foreign consumption C^*. The first two equations below unambiguously determine C and AN, while the third equation is a constraint on the parameter Θ.

$$
\begin{aligned}
nC &= \frac{(1-n)(1-\omega^*)\left(1-\omega_{oy}\right) C^* + nO}{1-\left(1-\omega_{oy}\right)\omega}, \\
AN &= \left(1-\omega_{oy}\right)\left[\omega C + \frac{1-n}{n}(1-\omega^*) C^*\right], \\
-\frac{U_2(C,N)}{U_1(C,N)} &= \frac{\Theta N^{\varphi}}{C^{-\sigma}} = A,
\end{aligned}
$$

We can write symmetric equations for the foreign economy. Specifically,

$$
(1-n) C^* = \frac{n(1-\omega)\left(1-\omega_{oy}\right) C + (1-n) O^*}{1-\left(1-\omega_{oy}\right)\omega^*},
$$

$$
N^* = \frac{\left(1-\omega_{oy}\right)\left(\omega^* C^* + \frac{n}{1-n}(1-\omega) C\right)}{A}.
$$

For the Backus-Smith condition to be satisfied, the constant ϕ_q has to take a certain value.

$$
\frac{C}{C^*} = \phi_q^{-\frac{1}{\sigma}}.
$$

Solving jointly for C and C^*, we get:

$$nC = \frac{\left[1 - \left(1 - \omega_{oy}\right)\omega^*\right] nO + \left(1 - \omega_{oy}\right)\left(1 - \omega^*\right)\left(1 - n\right) O^*}{\omega_{oy}\left\{1 + \left(1 - \omega_{oy}\right)\left[1 - \left(\omega^* + \omega\right)\right]\right\}}$$

and

$$\left(1 - n\right) C^* = \frac{\left(1 - \omega\right)\left(1 - \omega_{oy}\right) nO + \left[1 - \omega\left(1 - \omega_{oy}\right)\right]\left(1 - n\right) O^*}{\omega_{oy}\left\{1 + \left(1 - \omega_{oy}\right)\left[1 - \left(\omega^* + \omega\right)\right]\right\}}.$$

The steady state share of home goods domestic demand in home good demand

$$\frac{nC_H}{nC_H + \left(1 - n\right) C_H^*} = \frac{n\omega C}{n\omega C + \left(1 - n\right)\left(1 - \omega^*\right) C^*}$$

And

$$\frac{nC_F}{nC_F + \left(1 - n\right) C_F^*} = \frac{n\left(1 - \omega\right) C}{n\left(1 - \omega\right) C + \left(1 - n\right)\omega^* C^*}$$

Consumption normalization. We further normalize steady state to have unit consumption in home and foreign countries: $C = C^* = 1$.This normalization requires the following restriction on the parameters

$$1 = \left(1 - \omega_{oy}\right)\left[1 + \left(1 - n\right)\left(\Omega - \Omega^*\right)\right] + O.$$
$$1 = \left(1 - \omega_{oy}\right)\left[1 + n\left(\Omega^* - \Omega\right)\right] + O^*,$$

Small open economy limit. In the case when n goes to zero, we get

$$\left(1 - \Omega^*\right) = \left(1 - \Omega\right) + \frac{\omega_{oy} - O}{1 - \omega_{oy}}.$$
$$O^* = \omega_{oy}.$$

In this case, we have

$$\frac{nC_H}{nC_H + \left(1 - n\right) C_H^*} = \frac{\Omega}{1 - O}\left(1 - \omega_{oy}\right) = \Omega\frac{1 - O^*}{1 - O} = \frac{\Omega}{1 + \Omega - \Omega^*}.$$

As a benchmark case, we assume that home country does not produce oil, that is, $O = 0$**.** In this case,

$$\left(1 - \Omega^*\right) = \left(1 - \Omega\right) + \frac{\omega_{oy}}{1 - \omega_{oy}}.$$
$$O^* = \omega_{oy},$$

and the share of domestic and foreign consumption are

$$\frac{nC_H}{nC_H + \left(1 - n\right) C_H^*} = \Omega\left(1 - \omega_{oy}\right),$$
$$\frac{nC_F}{nC_F + \left(1 - n\right) C_F^*} = 0.$$

D.9 Equilibrium

Unknowns (27).

$$
\begin{aligned}
\textbf{Home quantities (6): } & \widehat{c}_t, \widehat{c}_{H,t}, \widehat{c}_{F,t}, \widehat{o}_{C,t}, \widehat{o}_{Y,t}, \widehat{n}_t, \\
\textbf{Home prices (6):} & \widehat{p}_t, \widehat{p}_{H,t}, \widehat{w}_t, i_t, \widehat{mc}_t, \pi_{H,t} \\
\textbf{Foreign quantities (6): } & \widehat{c}^*_t, \widehat{c}^*_{H,t}, \widehat{c}^*_{F,t}, \widehat{o}^*_{C,t}, \widehat{o}^*_{Y,t}, \widehat{n}^*_t, \\
\textbf{Foreign prices (6):} & \widehat{p}^*_t, \widehat{p}^*_{F,t}, \widehat{w}^*_t, i^*_t, \widehat{mc}^*_t, \pi^*_{F,t} \\
\textbf{International prices (3): } & \widehat{e}_t, \widehat{q}_t, \widehat{p}_{O,t}
\end{aligned}
$$

Home conditions (11 equations). Households

$$
\begin{aligned}
\widehat{c}_{H,t} &= -\gamma_n \left(\widehat{p}_{H,t} - \widehat{p}_t\right) + \widehat{c}_t, \\
\widehat{c}_{F,t} &= -\gamma_n \left(\widehat{e}_t + \widehat{p}^*_{F,t} - \widehat{p}_t\right) + \widehat{c}_t, \\
\sigma \widehat{c}_t + \varphi \widehat{n}_t &= \widehat{w}_t - \widehat{p}_t, \\
\mathbb{E}_t\left[\widehat{c}_{t+1}\right] - \widehat{c}_t &= \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \iota\right).
\end{aligned}
$$

The Price index

$$
\widehat{p}_t = \omega \widehat{p}_{H,t} + (1-\omega)\left(\widehat{e}_t + \widehat{p}^*_{F,t}\right).
$$

Firms

$$
\begin{aligned}
\widehat{mc}_t &= (1-\omega_{oy})\widehat{w}_t + \omega_o \widehat{p}_{O,t}, \\
\widehat{n}_t &= -\gamma_y \left(\widehat{w}_t - \widehat{mc}_t\right) + \frac{nC_H}{nC_H + (1-n)\,C^*_H}\widehat{c}_{H,t} + \frac{(1-n)\,C^*_H}{nC_H + (1-n)\,C^*_H}\widehat{c}^*_{H,t}, \\
\widehat{o}_{Y,t} &= -\gamma_y \left(\widehat{p}_{O,t} - \widehat{mc}_t\right) + \frac{nC_H}{nC_H + (1-n)\,C^*_H}\widehat{c}_{H,t} + \frac{(1-n)\,C^*_H}{nC_H + (1-n)\,C^*_H}\widehat{c}^*_{H,t}, \\
\pi_{H,t} &= \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\widehat{mc}_t - \widehat{p}_{H,t}\right) + \beta \mathbb{E}_t\left[\pi_{H,t+1}\right].
\end{aligned}
$$

Goods market clearing

$$
\left(1-\omega_{oy}\right)\widehat{n}_t + \omega_{oy}\widehat{o}_{Y,t} = \frac{nC_H}{nC_H + (1-n)\,C^*_H}\widehat{c}_{H,t} + \frac{(1-n)\,C^*_H}{nC_H + (1-n)\,C^*_H}\widehat{c}^*_{H,t}.
$$

Government

$$
i_t = \iota + \varphi_\pi \pi_{H,t}.
$$

Foreign condition (11 equations). Households

$$
\begin{aligned}
\widehat{c}_{H,t}^{*} &= -\gamma_n\left(\widehat{p}_{H,t}-\widehat{e}_t-\widehat{p}_t^{*}\right)+\widehat{c}_t^{*},\\
\widehat{c}_{F,t}^{*} &= -\gamma_n\left(\widehat{p}_{F,t}^{*}-\widehat{p}_t^{*}\right)+\widehat{c}_{N,t}^{*},\\
\sigma\widehat{c}_t^{*}+\varphi\widehat{n}_t^{*} &= \widehat{w}_t^{*}-\widehat{p}_t^{*},\\
\mathbb{E}_t\left[\widehat{c}_{t+1}^{*}\right]-\widehat{c}_t^{*} &= \frac{1}{\sigma}\left(i_t^{*}-\mathbb{E}_t\left[\pi_{t+1}^{*}\right]-\iota\right).
\end{aligned}
$$

The price index

$$
\widehat{p}_t^{*}=\omega^{*}\widehat{p}_{F,t}^{*}+(1-\omega^{*})\widehat{p}_{H,t}^{*}.
$$

Firms

$$
\begin{aligned}
\widehat{mc}_t^{*} &= (1-\omega_{oy})\widehat{w}_t^{*}+\omega_o\left(\widehat{p}_{O,t}-\widehat{e}_t\right),\\
\widehat{n}_t^{*} &= -\gamma_y\left(\widehat{w}_t^{*}-\widehat{mc}_t^{*}\right)-\widehat{a}_t+\frac{nC_F}{nC_F+(1-n)\,C_F^{*}}\widehat{c}_{F,t}+\frac{(1-n)\,C_F^{*}}{nC_F+(1-n)\,C_F^{*}}\widehat{c}_{F,t}^{*}\\
\widehat{o}_{Y,t}^{*} &= -\gamma_y\left(\widehat{p}_{O,t}^{*}-\widehat{mc}_t^{*}\right)+\frac{nC_F}{nC_F+(1-n)\,C_F^{*}}\widehat{c}_{F,t}+\frac{(1-n)\,C_F^{*}}{nC_F+(1-n)\,C_F^{*}}\widehat{c}_{F,t}^{*}\\
\pi_{F,t}^{*} &= \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\widehat{mc}_t^{*}-\widehat{p}_{F,t}^{*}\right)+\beta\mathbb{E}_t\left[\pi_{F,t+1}^{*}\right]
\end{aligned}
$$

Goods market clearing

$$
(1-\omega_{oy})\widehat{n}_t^{*}+\omega_{oy}\widehat{o}_{Y,t}^{*}=\frac{(1-n)\,C_F^{*}}{(1-n)\,C_F^{*}+nC_F}\widehat{c}_{F,t}^{*}+\frac{(1-n)\,C_H^{*}}{(1-n)\,C_F^{*}+nC_F}\widehat{c}_{F,t}.
$$

Government

$$
i_t^{*}=\iota+\varphi_\pi^{*}\pi_{F,t}^{*}.
$$

International conditions (3 equations).

$$
\begin{aligned}
\widehat{c}_t-\widehat{c}_t^{*} &= \frac{\widehat{q}_t}{\sigma},\\
\widehat{q}_t &= \widehat{e}_t+\widehat{p}_t^{*}-\widehat{p}_t,\\
nO_Y\widehat{o}_{Y,t}+(1-n)\,O_Y^{*}\widehat{o}_{Y,t}^{*} &= nO\widehat{o}_t+(1-n)O^{*}\widehat{o}_t^{*}.
\end{aligned}
$$

D.10 Euler Equations, Phillips Curves, Oil Price, and Risk Sharing

This section reduces all the equilibrium equations to only six: two Euler equations, two Phillips curves, the Backus-Smith condition and the equilibrium on a global oil market. The unknowns are consumption $(\widehat{c}_t, \widehat{c}_t^{*})$, inflation $(\pi_{H,t}, \pi_{F,t}^{*})$, the real exchange rate q_t and the real price of oil expressed in units of *foreign* goods $\widehat{r}\equiv\widehat{p}_{O,t}-\widehat{p}_{F,t}=\widehat{p}_{O,t}^{*}-\widehat{p}_{F,t}^{*}$.

Euler equations. Rewrite the Euler equation as

$$
\mathbb{E}_t\left[\widehat{c}_{t+1}\right]-\widehat{c}_t=\frac{1}{\sigma}\left(i_t-\mathbb{E}_t\left[\pi_{H,t+1}\right]-\iota\right)-\frac{1}{\sigma}\cdot\frac{1-\omega}{\omega+\omega^{*}-1}\mathbb{E}_t\left[\Delta\widehat{q}_{t+1}\right],
$$

where $\pi_{r,t} \equiv \pi_{O,t} - \pi_{F,t} = (\widehat{p}_{O,t} - \widehat{p}_{O,t-1}) - (\widehat{p}_{F,t} - \widehat{p}_{F,t-1})$.

Foreign Euler equation

$$\mathbb{E}_t\left[\widehat{c}^*_{t+1}\right] - \widehat{c}^*_t = \frac{1}{\sigma}\left(i^*_t - \mathbb{E}_t\left[\pi^*_{F,t+1}\right] - \iota\right) + \frac{1}{\sigma}\cdot\frac{1-\omega^*}{\omega+\omega^*-1}\mathbb{E}_t\left[\Delta\widehat{q}_{t+1}\right].$$

Consumption. Rewrite consumption of domestic goods through overall consumption and prices at home

$$\widehat{c}_{H,t} = \frac{1-\omega}{\omega+\omega^*-1}\gamma_n\widehat{q}_t + \widehat{c}_t,$$

and abroad

$$\widehat{c}^*_{H,t} = \frac{\omega^*}{\omega+\omega^*-1}\gamma_n\widehat{q}_t + \widehat{c}^*_t.$$

And consumption of foreign goods

$$\widehat{c}_{F,t} = -\frac{\omega}{\omega+\omega^*-1}\gamma_n\widehat{q}_t + \widehat{c}_t,$$

and

$$\widehat{c}^*_{F,t} = -\frac{1-\omega^*}{\omega+\omega^*-1}\gamma_n\widehat{q}_t + \widehat{c}^*_t.$$

Production. Domestic production of home goods

$$\begin{aligned}\widehat{y}_{H,t} =& \frac{n\omega C}{n\omega C + (1-n)(1-\omega^*)C^*}\widehat{c}_{H,t} + \frac{(1-n)(1-\omega^*)C^*}{n\omega C + (1-n)(1-\omega^*)C^*}\widehat{c}^*_{H,t}\\ =& \widehat{c}^M_t + \frac{n\omega C(1-\omega) + (1-n)(1-\omega^*)C^*\omega^*}{n\omega C + (1-n)(1-\omega^*)C^*}\cdot\frac{\gamma_n}{\omega+\omega^*-1}\widehat{q}_t.\end{aligned}$$

where

$$\widehat{c}^M_t \equiv \frac{n\omega C}{n\omega C + (1-n)(1-\omega^*)C^*}\widehat{c}_t + \frac{(1-n)(1-\omega^*)C^*}{n\omega C + (1-n)(1-\omega^*)C^*}\widehat{c}^*_t.$$

Foreign production of foreign goods

$$\begin{aligned}\widehat{y}^*_{F,t} =& \frac{nC_F}{nC_F + (1-n)C^*_F}\widehat{c}_{F,t} + \frac{(1-n)C^*_F}{nC_F + (1-n)C^*_F}\widehat{c}^*_{F,t}\\ =& \widehat{c}^{M*}_t - \frac{n\omega(1-\omega)C + (1-n)(1-\omega^*)\omega^*C^*}{n(1-\omega)C + (1-n)\omega^*C^*}\cdot\frac{\gamma_n}{\omega+\omega^*-1}\widehat{q}_t\end{aligned}$$

where

$$\widehat{c}^{M,*}_t \equiv \frac{n(1-\omega)C}{n(1-\omega)C + (1-n)\omega^*C^*}\widehat{c}_t + \frac{(1-n)\omega^*C^*}{n(1-\omega)C + (1-n)\omega^*C^*}\widehat{c}^*_t.$$

Home marginal costs. Replace real wage (using the labor supply equation) from the labor demand and the equation for the marginal costs

$$\widehat{mc}_t - \widehat{p}_t = (1-\omega_{oy})(\sigma\widehat{c}_t + \varphi\widehat{n}_t) + \omega_{oy}(\widehat{p}_{O,t} - \widehat{p}_t),$$

$$\begin{aligned}\widehat{n}_t =& -\frac{\gamma_y}{1+\gamma_y\varphi}\sigma\widehat{c}_t + \frac{\gamma_y}{1+\gamma_y\varphi}\left(\widehat{mc}_t - \widehat{p}_t\right)\\ &+ \frac{1}{1+\gamma_y\varphi}\left(\frac{nC_H}{nC_H+(1-n)\,C_H^*}\widehat{c}_{H,t} + \frac{(1-n)\,C_H^*}{nC_H+(1-n)\,C_H^*}\widehat{c}^*_{H,t}\right).\end{aligned}$$

Solve for equilibrium labor and the marginal costs

$$\begin{aligned}\widehat{mc}_t - \widehat{p}_t =& \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left[\sigma\widehat{c}_t + \varphi\left(\frac{nC_H}{nC_H+(1-n)\,C_H^*}\widehat{c}_{H,t} + \frac{(1-n)\,C_H^*}{nC_H+(1-n)\,C_H^*}\widehat{c}^*_{H,t}\right)\right]\\ &+ \frac{1+\gamma_y\varphi}{1+\gamma_y\varphi\omega_{oy}}\omega_{oy}\left(\widehat{p}_{O,t} - \widehat{p}_t\right).\end{aligned}$$

and

$$\begin{aligned}\widehat{n}_t =& -\frac{\sigma\gamma_y\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\widehat{c}_t + \frac{\gamma_y\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left(\widehat{p}_{O,t} - \widehat{p}_t\right)\\ &+ \frac{1}{1+\gamma_y\varphi\omega_{oy}}\left(\frac{nC_H}{nC_H+(1-n)\,C_H^*}\widehat{c}_{H,t} + \frac{(1-n)\,C_H^*}{nC_H+(1-n)\,C_H^*}\widehat{c}^*_{H,t}\right).\end{aligned}$$

Replace the CPI $\widehat{p}_t$ with $\widehat{p}_{H,t}$ in the marginal cost equation above at home

$$\begin{aligned}\widehat{mc}_t - \widehat{p}_{H,t} =& \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left[\sigma\widehat{c}_t + \varphi\left(\frac{nC_H}{nC_H+(1-n)\,C_H^*}\widehat{c}_{H,t} + \frac{(1-n)\,C_H^*}{nC_H+(1-n)\,C_H^*}\widehat{c}^*_{H,t}\right)\right]\\ &+ \frac{1+\gamma_y\varphi}{1+\gamma_y\varphi\omega_{oy}}\omega_{oy}\left(\widehat{p}_{O,t} - \widehat{p}_{F,t}\right) + \frac{\left(1+\gamma_y\varphi\right)\omega_{oy} + (1-\omega)\left(1-\omega_{oy}\right)}{\left(1+\gamma_y\varphi\omega_{oy}\right)(\omega+\omega^*-1)}\widehat{q}_t.\end{aligned}$$

Rewrite the marginal costs

$$\widehat{mc}_t - \widehat{p}_{H,t} = \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left(\sigma\widehat{c}_t + \varphi\widehat{c}_t^M\right) + \psi_o\left(\widehat{p}_{O,t} - \widehat{p}_{F,t}\right) + \psi_q\widehat{q}_t,$$

where

$$\begin{aligned}\psi_o \equiv& \frac{\left(1+\gamma_y\varphi\right)\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}},\\ \psi_q \equiv& \frac{\left(1+\gamma_y\varphi\right)\omega_{oy} + (1-\omega)\left(1-\omega_{oy}\right)}{\left(1+\gamma_y\varphi\omega_{oy}\right)(\omega+\omega^*-1)}\\ &+ \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\cdot\frac{1}{\omega+\omega^*-1}\cdot\frac{n\omega C\,(1-\omega) + (1-n)\,(1-\omega^*)C^*\omega^*}{n\omega C + (1-n)\,(1-\omega)C^*}\varphi\gamma_n\end{aligned}$$

Foreign marginal costs. A similar expression for foreign country marginal costs is

$$\begin{aligned}\widehat{mc}^*_t - \widehat{p}^*_t =& \frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\left[\sigma\widehat{c}^*_t + \varphi\left(\frac{nC_F}{nC_F+(1-n)\,C_F^*}\widehat{c}_{F,t} + \frac{(1-n)\,C_F^*}{nC_F+(1-n)\,C_F^*}\widehat{c}^*_{F,t}\right)\right]\\ &+ \omega_{oy}\frac{1+\varphi\gamma_y}{1+\omega_{oy}\gamma_y\varphi}\left(\widehat{p}^*_{O,t} - \widehat{p}^*_t\right)\end{aligned}$$

and replace $\widehat{p}_t^*$

$$
\begin{aligned}
\widehat{mc}_t^* - \widehat{p}_{F,t}^* = & \frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\left[\sigma\widehat{c}_t^* + \varphi\left(\frac{nC_F}{nC_F+(1-n)\,C_F^*}\widehat{c}_{F,t} + \frac{(1-n)\,C_F^*}{nC_F+(1-n)\,C_F^*}\widehat{c}_{F,t}^*\right)\right] \\
& + \left[\omega_{oc} + \omega_{oy}\frac{1+\varphi\gamma_y}{1+\omega_{oy}\gamma_y\varphi}\left(1-\omega_{o,c}\right)\right]\left(\widehat{p}_{O,t}^* - \widehat{p}_{F,t}^*\right) - \frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\cdot\frac{1-\omega^*}{(\omega+\omega^*-1)}\widehat{q}_t
\end{aligned}
$$

Foreign country marginal costs are

$$
\widehat{mc}_t^* - \widehat{p}_{F,t}^* = \frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\left[\sigma\widehat{c}_t^* + \varphi\widehat{c}_t^{M,*}\right] + \psi_o^*\left(\widehat{p}_{O,t}^* - \widehat{p}_{F,t}^*\right) + \psi_q^*\widehat{q}_t
$$

where

$$
\begin{aligned}
\psi_o^* \equiv & \frac{\omega_{oy}\left(1+\varphi\gamma_y\right)}{1+\omega_{oy}\gamma_y\varphi} = \psi_o, \\
\psi_q^* \equiv & -\frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\cdot\frac{1-\omega^*}{\omega+\omega^*-1} \\
& -\frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\cdot\frac{1}{\omega+\omega^*-1}\cdot\frac{n\omega C_F + (1-n)\left(1-\omega^*\right)C_F^*}{nC_F+(1-n)\,C_F^*}\varphi\gamma_n.
\end{aligned}
$$

Note that ψ_q^* and ψ_q are not symmetric. This is because in both equations for the marginal costs the real oil price is expressed in terms of foreign goods.

Phillips curves. The home Phillips curve is

$$
\pi_{H,t} = \kappa\left\{\frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left[\sigma\widehat{c}_t + \varphi\widehat{c}_t^M\right] + \psi_o\widehat{r}_t + \psi_q\widehat{q}_t\right\} + \beta\mathbb{E}_t\left[\pi_{H,t+1}\right],
$$

and the foreign Phillips curve is

$$
\pi_{F,t}^* = \kappa\left\{\frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\left[\sigma\widehat{c}_t^* + \varphi\widehat{c}_t^{M,*}\right] + \psi_o^*\widehat{r}_t + \psi_q^*\widehat{q}_t\right\} + \beta\mathbb{E}_t\left[\pi_{F,t+1}^*\right],
$$

where

$$
\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}. \tag{D.24}
$$

Oil demand. The home firms demand for oil

$$
\begin{aligned}
\widehat{o}_{Y,t} = & \gamma_y\frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\sigma\widehat{c}_t - \frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\gamma_y\widehat{r}_t + \frac{1+\varphi\gamma_y}{1+\gamma_y\varphi\omega_{oy}}\widehat{c}_t^M \\
& + \left\{\left[(\omega+\omega^*-1)\,\psi_q - 1\right]\gamma_y + \frac{n\omega C\,(1-\omega) + (1-n)\,(1-\omega^*)C^*\omega^*}{n\omega C + (1-n)\,(1-\omega)C^*}\gamma_n\right\}\frac{\widehat{q}_t}{\omega+\omega^*-1}.
\end{aligned}
$$

The foreign firm demand for oil

$$\widehat{o}^*_{Y,t} = \gamma_y \frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\sigma\widehat{c}^*_t - \frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\gamma_y\widehat{r}_t + \frac{1+\gamma_y\varphi}{1+\omega_{oy}\gamma_y\varphi}\widehat{c}^{M*}_t$$
$$- \frac{1}{1+\omega_{oy}\gamma_y\varphi}\left(\gamma_n \frac{n\omega C_F + (1-n)(1-\omega^*)C^*_F}{nC_F + (1-n)C^*_F} + \gamma_y\left(1-\omega_{oy}\right)(1-\omega^*)\right)\frac{\widehat{q}_t}{\omega+\omega^*-1}$$

Oil market. Oil market equilibrium condition

$$nO_Y\widehat{o}_{Y,t} + (1-n)O^*_Y\widehat{o}^*_{Y,t} = nO\widehat{o}_t + (1-n)O^*\widehat{o}^*_t,$$

which determines $\widehat{r}_t = \widehat{p}^*_{O,t} - \widehat{p}^*_{F,t}$.

D.11 A Small-Open Economy Limit

We now take the limit as the size of home economy approaches zero and the size of the foreign economy approaches one, that is, $n \to 0$ and $n^* = 1-n \to 1$. Taking into account the following definitions

$$1-\omega = (1-n)(1-\Omega),$$
$$1-\omega^* = n(1-\Omega^*),$$

we have that in the limit the fraction of domestic goods expenditure

$$\omega = \Omega,$$
$$\omega^* = 1.$$

This implies that

$$\frac{1-\omega^*}{\omega+\omega^*-1} = \frac{n(1-\Omega^*)}{1-(1-n)(1-\Omega)-n(1-\Omega^*)} = 0,$$
$$\frac{nC_H}{nC_H+(1-n)C^*_H} = \frac{\Omega C}{\Omega C + (1-\Omega^*)C^*},$$
$$\frac{nC_F}{nC_F+(1-n)C^*_F} = 0,$$
$$\frac{n\omega C_F + (1-n)(1-\omega^*)C^*_F}{nC_F+(1-n)C^*_F} = 0.$$

The relative prices of home goods in units of domestic and foreign consumption baskets are

$$\widehat{p}_{H,t} - \widehat{p}_t = -\frac{1-\omega}{\omega+\omega^*-1}\widehat{q}_t = -\frac{1-\Omega}{\Omega}\widehat{q}_t,$$
$$\widehat{p}^*_{H,t} - \widehat{p}^*_t = -\frac{1}{\Omega}\widehat{q}_t.$$

The world economy. The world equilibrium consists of six unknowns $\left(\widehat{y}_F^*, \widehat{c}_t^*, \pi_t^*, \pi_{F,t}^*, \widehat{r}_t, i_t^*\right)$ and six equations are

$$\begin{aligned}
\widehat{r}_t &= \phi_c \widehat{c}_t^* - \phi_o \widehat{o}_t^*, \\
\pi_{F,t}^* &= \kappa \zeta^* \widehat{c}_t^* + \beta \mathbb{E}_t \left[\pi_{F,t+1}^*\right] + \kappa \psi_o^* \widehat{r}_t, \\
\mathbb{E}_t \left[\widehat{c}_{t+1}^*\right] - \widehat{c}_t^* &= \frac{1}{\sigma} \left(i_t^* - \mathbb{E}_t \left[\pi_{t+1}^*\right] - \iota\right), \\
i_t^* &= \iota + \varphi_\pi^* \pi_{F,t}^*, \\
\pi_t^* &= \pi_{F,t}^*, \\
\widehat{y}_{F,t}^* &= \widehat{c}_t^*.
\end{aligned}$$

where $\widehat{r}_t \equiv \widehat{p}_{O,t}^* - \widehat{p}_{F,t}^*$ and

$$\phi_o = \frac{1 + \omega_{oy} \gamma_y \varphi}{\left(1 - \omega_{oy}\right) \gamma_y}, \tag{D.25}$$

$$\phi_c = \frac{1 + \gamma_y \left[\varphi + \left(1 - \omega_{oy}\right) \sigma\right]}{\left(1 - \omega_{oy}\right) \gamma_y}, \tag{D.26}$$

$$\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta},$$

$$\zeta^* \equiv \frac{1 - \omega_{oy}}{1 + \omega_{oy} \gamma_y \varphi} (\sigma + \varphi), \tag{D.27}$$

$$\psi_o^* = \frac{\left(1 + \gamma_y \varphi\right) \omega_{oy}}{1 + \gamma_y \varphi \omega_{oy}}. \tag{D.28}$$

We took into account that $\psi_q^* = 0, \widehat{c}_t^{M*} = \widehat{c}_t^*$.

Substituting oil price into the Phillips curve, we get

$$\pi_{F,t}^* = \kappa \left(\zeta^* + \psi_o^* \phi_c\right) \widehat{c}_t^* + \beta \mathbb{E}_t \left[\pi_{F,t+1}^*\right] - \kappa \psi_o^* \phi_o \widehat{o}_t^*,$$

The value added is

$$\widehat{z}_{F,t}^* = \frac{\left(1 - \omega_{oy} \phi_c\right) \widehat{c}_t^* - \omega_{oy} \left(1 - \phi_o\right) \widehat{o}_t^*}{1 - \omega_{oy}}.$$

Trade balance is

$$\widehat{nx}_t^* = 0.$$

Small-open economy. The SOE block consists of six unknowns $\left(\widehat{c}_t, \widehat{y}_{H,t}, \pi_t, \pi_{H,t}, i_t, q_t\right)$ and six equations

$$\begin{aligned}
\mathbb{E}_t\left[\widehat{c}_{t+1}\right]-\widehat{c}_t &= \frac{1}{\sigma}\left(i_t-\mathbb{E}_t\left[\pi_{t+1}\right]-\iota\right), \\
\pi_{H,t} &= \kappa\zeta_H\widehat{c}_t+\beta\mathbb{E}_t\left[\pi_{H,t+1}\right]+\kappa\psi_o\widehat{r}_t+\kappa\zeta_F\widehat{c}_t^*+\kappa\psi_q\widehat{q}_t, \\
\pi_t &= \pi_{H,t}+\frac{1-\Omega}{\Omega}\Delta\widehat{q}_t, \\
i_t &= \iota+\phi_\pi\pi_{H,t}, \\
\widehat{q}_t &= \sigma\left(\widehat{c}_t-\widehat{c}_t^*\right), \\
\widehat{y}_{H,t} &= \frac{\Omega}{\Omega+1-\Omega^*}\widehat{c}_t+\frac{1-\Omega^*}{\Omega+1-\Omega^*}\widehat{c}_t^*+\frac{\Omega\left(1-\Omega\right)+\left(1-\Omega^*\right)}{\Omega+1-\Omega^*}\cdot\frac{\gamma_n}{\Omega}\widehat{q}_t.
\end{aligned}$$

where

$$\begin{aligned}
\zeta_H &= \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left(\sigma+\varphi\frac{\Omega}{\Omega+1-\Omega^*}\right), \\
\zeta_F &= \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\varphi\frac{1-\Omega^*}{\Omega+1-\Omega^*}, \\
\psi_q &= \frac{1-\Omega}{\Omega}\cdot\frac{\left(1+\varphi\gamma_n\right)\left(1-\omega_{oy}\right)}{1+\gamma_y\varphi\omega_{oy}}, \\
\psi_o &= \frac{\left(1+\gamma_y\varphi\right)\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}},
\end{aligned}$$

and we took into account the fact that $\widehat{c}_t^M = \widehat{c}_t\Omega/\left(\Omega+1-\Omega^*\right)+\widehat{c}_t^*\left(1-\Omega^*\right)/\left(\Omega+1-\Omega^*\right)$. Note that the Phillips curve has three additional terms compared to the standard closed-economy formulation. The term $\kappa\psi_o\widehat{r}_t$ shows that higher real oil price increase the marginal cost and, hence, inflation. The term $\kappa\zeta_F\widehat{c}_t^*$ is due to the fact that the world aggregate demand affects the demand for home products and increases the cost of production. The term $\kappa\psi_q\widehat{q}_t$ reflects the fact that oil prices are quoted in units of foreign goods. This implies that absence any chance in the real oil price (in units of foreign goods), a real depreciation of domestic currency (an increase in $\widehat{q}_t$) acts to increase the oil price in units of home goods, which, in turn, increases the marginal cost and inflation. Finally, note that $\zeta_H+\zeta_F=\zeta^*$.

Substituting away the real exchange rate from the Euler equation and the Phillips curve, we obtain

$$\begin{aligned}
\mathbb{E}_t\left[\widehat{c}_{t+1}\right]-\widehat{c}_t &= \frac{\Omega}{\sigma}\left(i_t-\mathbb{E}_t\left[\pi_{H,t+1}\right]-\iota\right)+\left(1-\Omega\right)\left\{\mathbb{E}_t\left[\widehat{c}_{t+1}^*\right]-\widehat{c}_t^*\right\}, \\
\pi_{H,t} &= \kappa\left(\zeta_H+\psi_q\sigma\right)\widehat{c}_t+\beta\mathbb{E}_t\left[\pi_{H,t+1}\right]+\kappa\psi_o\widehat{r}_t+\kappa\left(\zeta_F-\psi_q\sigma\right)\widehat{c}_t^*,
\end{aligned}$$

This form of the Euler equation implies that the home aggregate demand is affected by the rest of the world because it changes the relative price of imported goods. A booming world economy has two opposing effects on the Phillips curve. On the one hand, this boom increases demand for home goods and pushes up inflation. On the other hand, a booming world economy appreciates the home currency in real terms and makes oil cheaper at home. The net effect of these two forces is captured by the coefficient

$$\zeta_F-\psi_q\sigma=\frac{1-\Omega^*}{\Omega+1-\Omega^*}\varphi-\frac{1-\Omega}{\Omega}\left(1+\varphi\gamma_n\right)\sigma.$$

In general, the sign of this coefficient is ambiguos. However, under the empirical relevant parameters $\Omega=\Omega^*=2/3, \sigma=1, \varphi=2$, this coefficient is positive.

Oil demand is

$$\begin{aligned}\widehat{o}_{Y,t} =&\gamma_y\frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left(\sigma\widehat{c}_t-\widehat{r}_t\right)+\frac{1+\varphi\gamma_y}{1+\gamma_y\varphi\omega_{oy}}\left(\frac{\Omega}{\Omega+1-\Omega^*}\widehat{c}_t+\frac{1-\Omega^*}{\Omega+1-\Omega^*}\widehat{c}_t^*\right)\\&+\left\{\left(\Omega\psi_q-1\right)\gamma_y+\frac{\Omega\left(1-\Omega\right)+\left(1-\Omega^*\right)}{\Omega+1-\Omega^*}\gamma_n\right\}\frac{\widehat{q}_t}{\Omega}.\end{aligned}$$

Trade balance is

$$\widehat{nx}_t =\widehat{y}_{H,t}-\omega_{oy}\widehat{r}_t-\left(\omega_{oy}+\frac{1-\Omega}{\Omega+1-\Omega^*}\right)\frac{\widehat{q}_t}{\Omega}-\omega_{oy}\widehat{o}_{Y,t}-\frac{1}{\Omega+1-\Omega^*}\widehat{c}_t.$$

D.12 Neo-classical Effects

When prices are completely flexible, the Phillips curve in foreign country implies

$$\widehat{c}_t^* = \Gamma_O^{c,*}\phi_o\widehat{o}_t^*.$$

where

$$\Gamma_O^{c,*}\equiv\frac{\psi_o^*}{\zeta^*+\psi_o^*\phi_c}=\frac{\frac{\omega_{oy}}{1-\omega_{oy}}\left(1+\varphi\gamma_y\right)}{\sigma+\varphi+\frac{\omega_{oy}}{1-\omega_{oy}}\left(1+\varphi\gamma_y\right)\frac{1-\gamma_y\omega_{oy}\sigma+\gamma_y(\varphi+\sigma)}{\gamma_y\left(1-\omega_{oy}\right)}}.$$

The consumption effect of oil supply change

$$\begin{aligned}\Gamma_O^{c,*}\phi_o &=\frac{\frac{\omega_{oy}}{1-\omega_{oy}}\left(1+\varphi\gamma_y\right)}{\frac{\left(1-\omega_{oy}\right)\gamma_y}{\left(1+\omega_{oy}\gamma_y\varphi\right)}\left[\left(\sigma+\varphi\right)+\frac{\omega_{oy}}{1-\omega_{oy}}\left(1+\varphi\gamma_y\right)\frac{\left(1-\gamma_y\omega_{oy}\sigma\right)+\gamma_y(\varphi+\sigma)}{\left(1-\omega_{oy}\right)\gamma_y}\right]}\\&=\frac{\omega_{oy}\left(1+\varphi\gamma_y\right)}{\omega_{oy}+\gamma_y\left(\sigma+\varphi-\omega_{oy}\sigma\right)}.\end{aligned}$$

The oil price

$$\begin{aligned}\widehat{r}_t &=-\frac{\left(\sigma+\varphi\right)}{\sigma+\varphi+\frac{\omega_{oy}}{1-\omega_{oy}}\left(1+\varphi\gamma_y\right)\frac{\left(1-\gamma_y\omega_{oy}\sigma\right)+\gamma_y(\varphi+\sigma)}{\left(1-\omega_{oy}\right)\gamma_y}}\phi_o\widehat{o}_t^*\\&=-\frac{\left(\sigma+\varphi\right)\left(1-\omega_{oy}\right)}{\omega_{oy}+\gamma_y\left(\sigma+\varphi-\omega_{oy}\sigma\right)}\widehat{o}_t^*.\end{aligned}$$

A change in output normalized by a change in oil price

$$\frac{\widehat{c}_t^*}{\widehat{r}_t}=\frac{\frac{\omega_{oy}\left(1+\varphi\gamma_y\right)}{\omega_{oy}+\gamma_y\left(\sigma+\varphi-\omega_{oy}\sigma\right)}}{-\frac{\left(\sigma+\varphi\right)\left(1-\omega_{oy}\right)}{\omega_{oy}+\gamma_y\left(\sigma+\varphi-\omega_{oy}\sigma\right)}}=-\frac{\omega_{oy}}{1-\omega_{oy}}\cdot\frac{1+\varphi\gamma_y}{\sigma+\varphi}.$$

D.13 A Two-state Markov Shock

With probability α, the shock persists, with the remaining probability it goes away. Oil shock $\widehat{o}_t^*\in\{\widehat{o}^*,0\}$,where $\widehat{o}^*<0$.

D.13.1 The World Economy

We start from the world economy. Using the equation for the oil price in equilibrium, we obtain the following Phillips curve

$$\pi_F^* = \frac{\kappa\left(\zeta^* + \psi_o^*\phi_c\right)}{1-\beta\alpha}\widehat{c}^* + \frac{\kappa\psi_o^*\phi_o}{1-\beta\alpha}\left(-\widehat{o}^*\right), \tag{D.29}$$

The Euler equation is

$$\widehat{c}^* = -\frac{\left(\varphi_\pi^* - \alpha\right)\pi_F^*}{\left(1-\alpha\right)\sigma}. \tag{D.30}$$

The response of the inflation rate that solves the above two equations is

$$\frac{d\pi_F^*}{d\left(-\widehat{o}^*\right)} = \phi_o \frac{\frac{\kappa\psi_o^*}{(1-\beta\alpha)}}{1 + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\left(\zeta^* + \psi_o^*\phi_c\right)} > 0. \tag{D.31}$$

There are two notable features here. First, the effect on inflation is non-negative (when the denominator is positive). Second, when the shock is permanent, inflation response is zero. The response of inflation rate when oil shock is such that the oil price increases by one percent is

$$\pi_F^* = \frac{\frac{\kappa\psi_o^*}{1-\beta\alpha}}{1 + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\zeta^*}.$$

The response of consumption (and output) that solve the Euler equation (combined with the monetary policy rule) and the Phillips curve is

$$\frac{d\widehat{c}^*}{d\left(-\widehat{o}^*\right)} = -\phi_o \frac{\frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)}\psi_o^*}{\sigma + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)}\left(\zeta^* + \psi_o^*\phi_c\right)} = \frac{\left(\zeta^* + \psi_o^*\phi_c\right)\frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)}}{\sigma + \left(\zeta^* + \psi_o^*\phi_c\right)\frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)}}\Gamma_y^{nc,*} \lesseqgtr 0. \tag{D.32}$$

where $\Gamma_y^{nc,*} = -\phi_o\psi_o^* / \left(\zeta^* + \psi_o^*\phi_c\right)$ is the neo-classical response. There are several notable observations here. When the Taylor rule response to the inflation rate is strong enough, that is, $\varphi_\pi^* - \alpha > 0$, the aggregate consumption unambiguously falls following a hike in oil prices. This is because of the increase in the real interest rate and the fall in demand for oil consumption. Second, when the shock is permanent: $d\widehat{c}^*/d\left(-\underline{o}^*\right) = -\phi_o\psi_o^*/(\zeta^* + \psi_o^*\phi_c) < 0$. Third, when goods prices are completely sticky, i.e., $\kappa = 0$, or the central bank targets a fixed real rate, i.e., $\varphi_\pi^* = \alpha$, the response of consumption is zero: $d\widehat{c}^*/d\left(-\underline{o}^*\right) = 0$. Fourth, the response is smaller compared to the case of completely flexible prices when $\varphi_\pi^* - \alpha > 0$.

The reaction of the oil price is

$$\widehat{r} = \frac{1 + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\zeta^*}{1 + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\left(\zeta^* + \psi_o^*\phi_c\right)}\phi_o\left(-\underline{o}^*\right). \tag{D.33}$$

Note that an oil supply decline unambiguously raises oil price when $\varphi_\pi^* - \alpha > 0$. The size of the oil shock that increase the oil price by one percent equals

$$\phi_o\left(-\widehat{o}^*\right) = \frac{1 + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\left(\zeta^* + \psi_o^*\phi_c\right)}{1 + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\zeta^*}.$$

The log-deviation of the consumption level from its steady state c^* following the oil supply shock that increases oil price by one percent, $\widehat{r}=1$, is

$$\widehat{c}^* = -\frac{\frac{\kappa(\varphi_\pi^*-\alpha)}{\sigma(1-\beta\alpha)(1-\alpha)}\zeta^*}{1+\frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\zeta^*}\cdot\frac{\psi_o^*}{\zeta^*}. \tag{D.34}$$

When prices are flexible, i.e., $\kappa\to\infty$,we have $\widehat{c}^* = -\psi_o^*/\zeta^*$, while when they are completely rigid, consumption does not respond, i.e., $\widehat{c}^*=0$.

ZLB. All the above formulas can be applied to the case of the liquidity trap or inelastic interest rates by setting $\varphi_\pi^*=0$. We have

$$\begin{aligned}
\frac{d\pi_F^*}{d\left(-\widehat{o}^*\right)} &= \phi_o\frac{\frac{\kappa\psi_o^*}{1-\beta\alpha}}{1-\frac{\kappa\alpha}{(1-\beta\alpha)(1-\alpha)\sigma}\left(\zeta^*+\psi_o^*\phi_c\right)}>0,\\
\frac{d\widehat{c}^*}{d\left(-\widehat{o}^*\right)} &= -\phi_o\frac{-\frac{\alpha\kappa\psi_o^*}{(1-\beta\alpha)(1-\alpha)\sigma}}{1-\frac{\alpha\kappa}{(1-\beta\alpha)(1-\alpha)\sigma}\left(\zeta^*+\psi_o^*\phi_c\right)}>0,\\
\frac{d\widehat{r}}{d\left(-\widehat{o}^*\right)} &= \phi_o\frac{1-\frac{\alpha\kappa\zeta^*}{(1-\beta\alpha)(1-\alpha)\sigma}}{1-\frac{\alpha\kappa}{(1-\beta\alpha)(1-\alpha)\sigma}\left(\zeta^*+\psi_o^*\phi_c\right)}>0.
\end{aligned}$$

For the equilibrium to be unique, we assume that

$$1-\frac{\alpha\kappa}{\sigma\left(1-\beta\alpha\right)\left(1-\alpha\right)}\left(\zeta^*+\psi_o^*\phi_c\right)>0.$$

The inflation rate response is

$$\begin{aligned}
&\left.\frac{d\pi_F^*}{d\left(-\widehat{o}^*\right)}\right|_{\text{ZLB}}-\left.\frac{d\pi_F^*}{d\left(-\widehat{o}^*\right)}\right|_{\text{normal}}\\
=&\frac{\kappa\left(\zeta^*+\psi_o^*\phi_c\right)\varphi_\pi^*\phi_o\left(1-\alpha\right)\kappa\psi_o^*\sigma}{\left[\left(1-\beta\alpha\right)\left(1-\alpha\right)\sigma-\alpha\kappa\left(\zeta^*+\psi_o^*\phi_c\right)\right]\left[\left(1-\beta\alpha\right)\left(1-\alpha\right)\sigma+\kappa\left(\zeta^*+\psi_o^*\phi_c\right)\left(\varphi_\pi^*-\alpha\right)\right]}>0.
\end{aligned}$$

The consumption response is

$$\left.\frac{d\widehat{c}^*}{d\left(-\widehat{o}^*\right)}\right|_{\text{ZLB}}-\left.\frac{d\widehat{c}^*}{d\left(-\widehat{o}^*\right)}\right|_{\text{normal}}=\phi_o\frac{\frac{\kappa\varphi_\pi^*}{(1-\beta\alpha)(1-\alpha)}\psi_o^*\sigma}{\left[\sigma+\frac{-\alpha\kappa}{(1-\beta\alpha)(1-\alpha)}\left(\zeta^*+\psi_o^*\phi_c\right)\right]\left[\sigma+\frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)}\left(\zeta^*+\psi_o^*\phi_c\right)\right]}.$$

The absolute response

$$\left.\frac{d\widehat{c}^*}{d\left(-\widehat{o}^*\right)}\right|_{\text{ZLB}}=\phi_o\frac{\frac{\alpha\kappa}{(1-\beta\alpha)(1-\alpha)}\cdot\frac{\left(1+\gamma_y\varphi\right)\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}}{\sigma+\frac{\kappa(0-\alpha)}{(1-\beta\alpha)(1-\alpha)}\left(\zeta^*+\psi_o^*\phi_c\right)}$$

D.13.2 The SOE Economy

The Euler equation

$$\widehat{c}=-\Omega\frac{(\varphi_{\pi}-\alpha)}{\sigma(1-\alpha)}\pi_{H}+(1-\Omega)\widehat{c}^{*}.$$

The Phillips curve

$$\pi_{H}=\frac{\kappa\left(\zeta_{H}+\psi_{q}\sigma\right)}{1-\alpha\beta}\widehat{c}+\frac{\kappa}{1-\alpha\beta}\psi_{o}\widehat{r}+\frac{\kappa}{1-\alpha\beta}\left(\zeta_{F}-\psi_{q}\sigma\right)\widehat{c}^{*}.$$

The inflation rate response is

$$\pi_{H}=\frac{\psi_{o}\frac{\kappa}{1-\alpha\beta}}{1+\frac{\kappa(\varphi_{\pi}-\alpha)}{(1-\alpha)(1-\alpha\beta)\sigma}\left(\zeta_{H}+\sigma\psi_{q}\right)\Omega}\widehat{r}+\frac{\frac{\kappa}{1-\alpha\beta}\left[\zeta_{H}(1-\Omega)+\zeta_{F}-\psi_{q}\sigma\Omega\right]}{1+\frac{\kappa(\varphi_{\pi}-\alpha)}{\sigma(1-\alpha\beta)(1-\alpha)}\left(\zeta_{H}+\psi_{q}\sigma\right)\Omega}\widehat{c}^{*},$$

The first term represents the reaction of the economy to the world oil price, while the second term represents the reaction to a change in the global economic activity. Specifically, a higher demand in the foreign economy increases the demand for home products and, at the same time, appreciates domestic real exchange rate making it less costly to produce conditional on an unchanged real oil price in the units of foreign goods.

The response of aggregate consumption is

$$\widehat{c}=\frac{1-\Omega+\frac{\kappa(\varphi_{\pi}-\alpha)}{\sigma(1-\alpha\beta)(1-\alpha)}\left(\zeta_{H}+\psi_{q}\sigma\right)\Omega\frac{\psi_{q}\sigma-\zeta_{F}}{\zeta_{H}+\psi_{q}\sigma}}{1+\frac{\kappa(\varphi_{\pi}-\alpha)}{\sigma(1-\alpha\beta)(1-\alpha)}\left(\zeta_{H}+\psi_{q}\sigma\right)\Omega}\widehat{c}^{*}-\frac{\frac{(\varphi_{\pi}-\alpha)\kappa}{\sigma(1-\alpha)(1-\alpha\beta)}\left(\zeta_{H}+\sigma\psi_{q}\right)\Omega}{1+\frac{\kappa(\varphi_{\pi}-\alpha)}{(1-\alpha)(1-\alpha\beta)\sigma}\left(\zeta_{H}+\sigma\psi_{q}\right)\Omega}\cdot\frac{\psi_{o}}{\zeta_{H}+\sigma\psi_{q}}\widehat{r}.$$

The response of aggregate consumption conditional on the oil supply shock that increases oil price by one percent

$$\widehat{c}=\frac{1-\Omega+\Omega\frac{\kappa(\varphi_{\pi}-\alpha)}{\sigma(1-\alpha\beta)(1-\alpha)}\left(\psi_{q}\sigma-\zeta_{F}\right)}{1+\frac{\kappa(\varphi_{\pi}-\alpha)}{\sigma(1-\alpha\beta)(1-\alpha)}\left(\zeta_{H}+\psi_{q}\sigma\right)\Omega}\widehat{c}^{*}-\frac{\frac{(\varphi_{\pi}-\alpha)\kappa}{\sigma(1-\alpha)(1-\alpha\beta)}\left(\zeta_{H}+\sigma\psi_{q}\right)\Omega}{1+\frac{\kappa(\varphi_{\pi}-\alpha)}{(1-\alpha)(1-\alpha\beta)\sigma}\left(\zeta_{H}+\sigma\psi_{q}\right)\Omega}\cdot\frac{\psi_{o}}{\zeta_{H}+\sigma\psi_{q}},$$

where $\widehat{c}^{*}$ is from equation (D.34).

We note that when $\varphi_{\pi}=\varphi_{\pi}^{*}$, we get $\widehat{c}=\widehat{c}^{*}$.

Home production is

$$\widehat{y}_{H,t}=\frac{\Omega}{\Omega+1-\Omega^{*}}\widehat{c}_{t}+\frac{1-\Omega^{*}}{\Omega+1-\Omega^{*}}\widehat{c}_{t}^{*}+\frac{\Omega(1-\Omega)+(1-\Omega^{*})}{\Omega+1-\Omega^{*}}\cdot\frac{\gamma_{n}}{\Omega}\widehat{q}_{t}.$$

This formula clearly illustrates that the oil shock affects the non-oil production through three distinct channels corresponding to three terms in the formula: (i) a change in domestic aggregate demand; (ii) a change in foreign aggregate demand; (iii) an expenditure switching effect from foreign to domestic goods. We replace the real exchange rate and domestic consumption

$$\begin{aligned}\widehat{y}_{H,t}&=\frac{\Omega}{\Omega+1-\Omega^{*}}\widehat{c}_{t}+\frac{1-\Omega^{*}}{\Omega+1-\Omega^{*}}\widehat{c}_{t}^{*}+\frac{\Omega(1-\Omega)+(1-\Omega^{*})}{\Omega+1-\Omega^{*}}\cdot\frac{\gamma_{n}}{\Omega}\sigma\left(\widehat{c}_{t}-\widehat{c}_{t}^{*}\right)\\&=\frac{\Omega}{\Omega+1-\Omega^{*}}\left(1+\frac{\Omega(1-\Omega)+(1-\Omega^{*})}{\Omega}\cdot\frac{\gamma_{n}\sigma}{\Omega}\right)\widehat{c}_{t}+\frac{1-\Omega^{*}}{\Omega+1-\Omega^{*}}\left(1-\frac{\Omega(1-\Omega)+(1-\Omega^{*})}{1-\Omega^{*}}\cdot\frac{\gamma_{n}\sigma}{\Omega}\right)\widehat{c}_{t}^{*}.\end{aligned}$$

The response of the real exchange rate is

$$\widehat{q}_t = \sigma\left(\widehat{c}_t - \widehat{c}_t^*\right) = -\sigma \frac{\frac{\kappa(\varphi_\pi - \varphi_\pi^*)}{\sigma(1-\beta\alpha)(1-\alpha)}\psi_o\Omega}{1 + \frac{\kappa(\varphi_\pi - \alpha)}{\sigma(1-\alpha\beta)(1-\alpha)}\left(\zeta_H + \psi_q\sigma\right)\Omega} \cdot \frac{1}{1 + \frac{\kappa(\varphi_\pi^* - \alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\zeta^*}.$$

The last expression implies that the real exchange rate does not respond when the home and foreign central banks respond to domestic inflation in the same way, that is, $\varphi_\pi = \varphi_\pi^*$. When, however, home country is at the ZLB while foreign country actively responds to oil shock, i.e., $\varphi_\pi - \varphi_\pi^* < 0$, the real exchange rate depreciates $\widehat{q}_t > 0$.

For uniqueness of bounded ZLB solution, the following condition must hold

$$1 - \frac{\alpha\kappa}{\sigma\left(1-\alpha\beta\right)\left(1-\alpha\right)}\left(\zeta_H + \psi_q\sigma\right)\Omega > 0.$$