FEDERAL RESERVE BANK OF SAN FRANCISCO

WORKING PAPER SERIES

A Simple Measure of Anchoring for Short-Run Expected Inflation in FIRE Models

Peter Lihn Jørgensen
Copenhagen Business School

Kevin J. Lansing
Federal Reserve Bank of San Francisco

October 2024

Working Paper 2024-34

https://doi.org/10.24148/wp2024-34

Suggested citation:

Jørgensen, Peter Lihn and Kevin J. Lansing. 2024. “A Simple Measure of Anchoring for Short-Run Expected Inflation in FIRE Models.” Federal Reserve Bank of San Francisco Working Paper 2024-34. https://doi.org/10.24148/wp2024-34

A Simple Measure of Anchoring for Short-Run Expected Inflation in FIRE Models*

Peter Lihn Jørgensen[†]
Copenhagen Business School

Kevin J. Lansing[‡]
Federal Reserve Bank of San Francisco

October 30, 2024

Abstract

We show that the fraction of non-reoptimizing firms that index prices to the inflation target, rather than lagged inflation, provides a simple measure of anchoring for short-run expected inflation in a New Keynesian model with full-information rational expectations. Higher values of the anchoring measure imply less sensitivity of rational inflation forecasts to movements in actual inflation. The approximate value of the model's anchoring measure can be inferred from observable data generated by the model itself, as given by 1 minus the autocorrelation statistic for quarterly inflation. We show that a shift in the collective indexing behavior of firms allows the model to account for numerous features of evolving U.S. inflation behavior since 1960.

Keywords: *Anchored inflation expectations, Phillips curve, Indexation, Inflation persistence.*
JEL Classification: E31, E32, E37

---

[*] Forthcoming in *Economics Letters*. The views in this paper are our own and not necessarily those of the Federal Reserve Bank of San Francisco or of the Federal Reserve System.

[†] Department of Economics, Copenhagen Business School, Porcelænshaven 16A, 1st floor, DK-2000 Frederiksberg, email: pljo.eco@cbs.dk, https://sites.google.com/view/peterlihnjorgensen.

[‡] Corresponding author. Research Department, Federal Reserve Bank of San Francisco, P.O. Box 7702, San Francisco, CA 94120-7702, email: kevin.j.lansing@sf.frb.org, www.frbsf.org/economics/economists/klansing.html

1 Introduction

U.S. inflation behavior has shifted dramatically over the past sixty-plus years. Figure 1 plots annualized quarterly inflation π_t based on the personal consumption expenditures (PCE) price index together with summary statistics computed from a 20-year rolling window of data, recording the value at the end of the window.[1] The rolling measures of volatility and persistence have declined substantially since the late 1970s, but both have increased in recent years. The reduced form Phillips curve, which links inflation to economic activity, has also undergone profound changes. Starting in the late 1990s, the rolling slope of the "accelerationist" Phillips curve, given by $Cov_{20}(\Delta\pi_t, y_t)/Var_{20}(y_t)$, has become flatter while the corresponding slope of the "original" Phillips curve, given by $Cov_{20}(\pi_t, y_t)/Var_{20}(y_t)$, has become steeper, where y_t is the output gap based on potential output from the Congressional Budget Office (CBO).[2] Jørgensen and Lansing (2024) show that these patterns are robust to different measures of inflation, including detrended inflation, or different measures of economic activity.

[Insert Figure 1 about here]

This paper proposes a simple measure of anchoring for short-run expected inflation in a standard New Keynesian model with a constant inflation target and full-information rational expectations (FIRE). The model's anchoring measure is the fraction of non-reoptimizing firms that index prices to the inflation target, rather than lagged inflation. We show that an increase in this anchoring measure can account for the shifting patterns of U.S. inflation behavior in Figure 1.[3]

The approximate value of the model's anchoring measure can be inferred from observable data generated by the model itself, as given by 1 minus the autocorrelation statistic for quarterly inflation. The bottom right panel of Figure 1 plots the model-implied anchoring measure using the 20-year rolling autocorrelation statistic for quarterly PCE inflation. The anchoring measure starts trending up in the late 1990s when the Great Inflation era drops out of the

---

[1] The basic patterns in Figure 1 are robust to different window lengths. A 20-year time frame allows us to roughly span the most recent era of consistent monetary policy together with stable *long-run* inflation expectations, which is the setting of our model.

[2] Following Jørgensen and Lansing (2024), the accelerationist Phillips curve regression takes the form $\pi_t - \pi_{t-1} \equiv \Delta\pi_t = c_0 + c_1 y_t$. The original Phillips curve regression takes the form $\pi_t = c_0 + c_1 y_t$.

[3] Prior to the late 1990s, the U.S. anchoring measure in Figure 1 is relatively stable while the value of $Cov_{20}(\pi_t, y_t)/Var_{20}(y_t)$ goes from negative to positive and then back to negative. From the perspective of our model, this pattern can be explained by shifts in the relative importance of demand versus cost-push shocks (and the resulting policy responses) during the early decades of the data sample.

rolling window and is replaced by an era of low and stable inflation. Near the end of the data sample, the anchoring measure declines somewhat due to a rebound in inflation persistence.

Bernanke (2007) defines the term "anchored" to mean that *long-run* inflation expectations are "relatively insensitive to incoming data." In FIRE models with a constant inflation target, long-run expected inflation remains well anchored by construction. But if the structural slope of the New Keynesian Phillips curve (NKPC) is relatively flat as suggested by many empirical studies, then short-run expected inflation becomes very important for determining movements in inflation. In such an environment, improved anchoring of short-run expected inflation can help the central bank achieve its goals.

The autocorrelation-based anchoring measure in Figure 1 comoves strongly with a survey-based anchoring measure constructed by Lansing and Nucera (2023) that gauges how much professional economists adjust their one-year ahead inflation forecast in response to recent movements in actual inflation. We use the model's equilibrium solution to show that there is a direct theoretical link between our autocorrelation-based anchoring measure and an alternative measure that is based on a regression of one-quarter ahead expected inflation on actual inflation. The autocorrelation-based anchoring measure also comoves strongly with a measure that is based on the mean absolute gap between professional economists' inflation forecasts and an inflation target of 2%, along the lines of the anchoring measures constructed by Bems, et al. (2021) and Naggert, Rich, and Tracy (2023).[4]

Jørgensen and Lansing (2024) employ an imperfect information model to show that the transition to a policy regime with a transparent and constant inflation target serves to anchor long-run inflation expectations, allowing their model to account for the patterns observed in Figure 1. Our contribution here is to show that a FIRE model with a constant inflation target can account for the same set of stylized facts when there is an increase in the fraction of non-reoptimizing firms that index prices to the inflation target. A plausible driver of this increase would be a shift to a more vigilant monetary policy regime that keeps inflation close to target, thereby inducing more firms to view inflation shocks as transitory. Consistent with this idea, the contribution of permanent versus transitory shocks to U.S. inflation has declined over time in a manner that can account for the observed decline in U.S. inflation persistence (Stock and Watson 2007, Lansing 2009). A decline in inflation peristence translates directly to an increase in our simple anchoring measure.

---

[4] Appendix B summarizes various anchoring measures employed in the literature and compares our autocorrelation-based anchoring measure to two alternatives.

2 Model

The framework for our analysis is the following New Keynesian model

$$y_t = \mu_y E_t y_{t+1} + (1-\mu_y) y_{t-1} - \alpha[i_t - E_t \pi_{t+1} - r^*] + v_t, \qquad v_t \sim N\left[0,\, \sigma_v^2\right], \tag{1}$$

$$\pi_t - \pi^* = \underbrace{\frac{\beta}{1+\beta(1-\mu_\pi)}}_{\equiv \gamma_f}(E_t \pi_{t+1} - \pi^*) + \underbrace{\frac{(1-\mu_\pi)}{1+\beta(1-\mu_\pi)}}_{\equiv \gamma_b}(\pi_{t-1} - \pi^*)$$

$$+ \kappa y_t + u_t, \qquad u_t \sim N\left[0,\, \sigma_u^2\right], \tag{2}$$

$$i_t - \pi^* - r^* = g_\pi\left(E_t \pi_{av,t+1} - \pi^*\right) + g_y E_t y_{t+1} + \varepsilon_t, \qquad \varepsilon_t \sim N\left[0,\, \sigma_\varepsilon^2\right], \tag{3}$$

where (1) is the consumption Euler equation, (2) is the NKPC, (3) is the monetary policy rule. The variable y_t is the output gap, π_t is the annualized quarterly inflation rate, and i_t is the "proxy" or "shadow" policy interest rate.[5]

The variable $\pi_{av,t} \equiv \omega\pi_t + (1-\omega)\pi_{av,t-1}$ is "average inflation" where the value of ω is set so that $\pi_{av,t}$ approximates the compound average inflation rate over the past 4 quarters—a typical central bank target variable. Similar to Coibion and Gorodnichenko (2011), the interest rate responds to the rational forward-looking forecasts $E_t\pi_{av,t+1}$ and $E_t y_{t+1}$. The parameters π^* and r^* represent the inflation target and the neutral real rate of interest (r-star). The model allows for a demand shock v_t, a cost-push shock u_t, and a monetary policy shock ε_t.

The presence of y_{t-1} in (1) can be motivated by habit formation in consumption behavior (Fuhrer 2000). The presence of π_{t-1} in (2) can be motivated by price indexation, as in the NKPC specification derived by Cogley and Sbordonne (2008) which allows for drifting trend inflation. For our analysis here, we impose constant trend inflation equal to π^* as in the version described by Mavroeidis, Plagborg-Møller, and Stock (2014, p. 131).

Given the set of firms that do not optimally reset prices each period, a fraction $\mu_\pi \in [0,1]$ index prices to π^* while the remainder index prices to π_{t-1}. We will show that the value of μ_π serves as a simple measure of anchoring for short-run expected inflation.

---

[5] As in Wu and Zhang (2019), use of a shadow rate allows us to sidestep complications of solving the model subject to an occasionally-binding lower bound on the policy interest rate.

3 Parameter values

Table 1 shows the parameter values for our quantitative analysis.

Table 1. Baseline parameter values

Parameter	Value	Description/Target
μ_y	0.5	Weight on $E_t y_{t+1}$ in Euler equation.
α	0.1	Interest rate coefficient in Euler equation.
β	0.995	Discount factor in Phillips curve.
κ	0.03	Slope coefficient in Phillips curve.
σ_ν	1.1%	Std. dev. of demand shock.
σ_u	1.0%	Std. dev. of cost push shock.
σ_ε	0.5%	Std. dev. of monetary policy shock.
r^*	1.5%	Steady state real interest rate.
π^*	2.0%	Inflation target.
ω	0.464	$\pi_{av,t} \simeq$ 4-quarter PCE inflation rate.
g_π	1.20	Policy rule response to inflation.
g_y	0.75	Policy rule response to output gap.
μ_π	0.5	$1 - Corr(\pi_t, \pi_{t-1}) \simeq 0.5$ in U.S. data.

The values $\mu_y = 0.5$ and $\alpha = 0.1$ are close to those estimated by Fuhrer and Rudebusch (2004). The values $\beta = 0.995$ and $\kappa = 0.03$ imply a low rate time preference together with a relatively flat Phillips curve, consistent with recent empirical estimates (Hazell, et al. 2022, Inoue, Rossi, and Wang 2024).

The values of σ_ν, σ_u and σ_ε deliver persistence and volatility measures that approximate those in U.S. data since 1988.Q1—a sample period of consistent U.S. monetary policy. The value $r^* = 1.5\%$ is based on estimates from Lubik and Matthes (2023) for data since 1988.Q1. The value $\pi^* = 2\%$ is based on the Federal Reserve's stated goal for PCE inflation. Following Lansing (2021), we compute the value $\omega = 0.464$ so that $\pi_{av,t}$ approximates the 4-quarter PCE inflation rate from 1961.Q1 to 2024.Q2. The policy rule coefficients g_π and g_y are close to values obtained by regressing the proxy federal funds rate from Choi, et al. (2022) on 4-quarter PCE inflation and the CBO output gap using data from 1988.Q1 through 2019.Q4.[6]

Mavroeidis, Plagborg-Møller, and Stock (2014) report estimates of μ_π in the literature that range from 0.35 to 0.95. As a baseline, we set $\mu_\pi = 0.5$ which delivers an autocorrelation coefficient for quarterly inflation that is close to that observed in U.S. data since 1988.Q1.

---

[6] Including data from the pandemic and its aftermath reduces the value of both regression coefficients.

4 Effects of improved anchoring

Using U.S. data from 1970.Q1 to 2021.Q4, Inoue, Rossi, and Wang (2024) estimate time-varying values for both γ_f and γ_b in (2). Their study employs a time-varying instrumental variable approach that is robust to weak instruments. The estimated value of γ_f gradually increases from around 0.4 to 0.6 while the estimated value of γ_b gradually decreases from around 0.4 to 0.1. In our model, these results would imply a gradual increase in the value of μ_π over time.

Table 2 compares the moments of U.S. data variables for two different sample periods to those predicted by the model for two different values of μ_π.

Table 2. Unconditional moments: Data versus model

	U.S. Data		Model	
	1960.Q1-1987.Q4	1988.Q1-2024.Q2	$\mu_\pi = 0.1$	$\mu_\pi = 0.5$
$Std\ Dev\ (\pi_t)$	3.03%	1.76%	3.08%	1.71%
$Std\ Dev\ (\pi_t^d)$	1.60%	1.45%	–	–
$Corr(\pi_t, \pi_{t-1})$	0.89	0.55	0.81	0.50
$Corr(\pi_t^d, \pi_{t-1}^d)$	0.62	0.34	–	–
$Std\ Dev\ (y_t)$	2.72%	2.00%	2.36%	2.26%
$Corr(y_t, y_{t-1})$	0.93	0.85	0.73	0.71
$Cov(\Delta\pi_t, y_t)/Var(y_t)$	0.114	0.028	0.160	0.086
$Cov(\Delta\pi_t^d, y_t)/Var(y_t)$	0.078	0.022	–	–
$Cov(\pi_t, y_t)/Var(y_t)$	−0.077	0.283	0.074	0.162
$Cov(\pi_t^d, y_t)/Var(y_t)$	0.096	0.166	–	–

Notes: For U.S. data, y_t is the CBO output gap, π_t is quarterly PCE inflation (annualized), π_t^d is detrended quarterly inflation using the HP filter with $\lambda = 1600$, $\Delta\pi_t \equiv \pi_t - \pi_{t-1}$, and $\Delta\pi_t^d \equiv \pi_t^d - \pi_{t-1}^d$. Model moments are computed analytically, as described in Appendix A.

An increase in μ_π from 0.1 to 0.5 serves to reduce inflation persistence and volatility, flatten the slope of the accelerationist Phillips curve as given by $Cov(\Delta\pi_t, y_t)/Var(y_t)$, and steepen the slope of the original Phillips curve as given by $Cov(\pi_t, y_t)/Var(y_t)$. All of these patterns are consistent with U.S. data when moving from the first sample period to the second sample period. Notably, the patterns in U.S. data are robust to using a measure of detrended quarterly inflation π_t^d.

[Insert Figure 2 about here]

The top panels of Figure 2 show that higher values of μ_π reduce $Corr(\pi_t, \pi_{t-1})$ and $Std\ Dev(\pi_t)$. In contrast, higher values of μ_π have only small effects on the properties of the output gap y_t and the persistence of the policy interest rate i_t. The bottom left panel of Figure 2 shows that higher values of μ_π reduce values of the statistic $Cov(E_t\pi_{t+h}, \pi_t)/Var(\pi_t)$ for $h = 1,\ 4,\ 8,\ 12$. These are regression coefficients that measure the response of expected inflation at different forecast horizons to movements in π_t. The plot confirms that higher values of μ_π capture the flavor of the term "anchored" employed by Bernanke (2007) to mean "relatively insensitive to incoming data."

The bottom right panel of Figure 2 shows that, starting from low levels, higher values of μ_π serve to flatten the slope of the accelerationist Phillips curve as given by $Cov(\Delta\pi_t, y_t)/Var(y_t)$ but steepen the slope of the original Phillips curve as given by $Cov(\pi_t, y_t)/Var(y_t)$.

To understand the intuition for the shifting slope patterns, consider a simplified version of the model that sets $\mu_y = 1$, $E_t y_{t+1} = 0$, $\beta = 1$, $\gamma_f = \mu_\pi$, $\gamma_b = 1 - \mu_\pi$, $g_y = 0$, and $\omega = 1$. In this case, the model can be reduced to the following two equations

$$\begin{aligned} y_t \;&=\; -\,\alpha(g_\pi - 1)(E_t\,\pi_{t+1} - \pi^*) \,+\, v_t, & (4)\\ \pi_t - \pi^* \;&=\; \mu_\pi(E_t\,\pi_{t+1} - \pi^*) \,+\, (1-\mu_\pi)(\pi_{t-1} - \pi^*) \,+\, \kappa y_t \,+\, u_t, & (5)\end{aligned}$$

where the form of (4) resembles an optimal central bank targeting rule under discretion, except that here the targeted inflation variable would be $E_t\,\pi_{t+1}$ rather than π_t.[7]

First consider the accelerationist Phillips curve. When $\mu_\pi \simeq 0$ (weak anchoring), (5) implies $Cov(\Delta\pi_t, y_t)/Var(y_t) = \kappa$. When $\mu_\pi \simeq 1$ (strong anchoring), inflation is not persistent such that $E_t\pi_{t+1} \simeq \pi^*$. In this case, (5) implies $Cov(\Delta\pi_t, y_t)/Var(y_t) = \kappa[1 - Corr(y_{t-1}, y_{t-1})]$ which shows that the slope will become flatter when anchoring improves, provided that the output gap exhibits some persistence such that $Corr(y_{t-1}, y_{t-1}) \in (0,\ 1)$. The output gap in the full model does exhibit persistence due to the term involving y_{t-1} in (1).

Now consider the original Phillips curve. When $\mu_\pi \simeq 0$ (weak anchoring), inflation is highly persistent such that $E_t\pi_{t+1} \simeq \pi_t$ and (4) implies $Cov(\pi_t, y_t) < 0$ whenever $g_\pi > 1$, i.e., whenever the Taylor principle is satisfied. When $\mu_\pi \simeq 1$ (strong anchoring), inflation is not persistent such that $E_t\pi_{t+1} \simeq \pi^*$. In this case, (5) implies $Cov(\pi_t, y_t)/Var(y_t) = \kappa$ which shows that the slope will go from negative to positive as anchoring improves. This is exactly the pattern observed in U.S. data starting in the late 1990s (Figure 1).

In the full model, values of μ_π above 0.5 cause the rational forecast $E_t\pi_{t+1}$ to move substantially less in response to movements in y_t. This effect reduces the comovement between π_t

---

[7] For examples of such targeting rules, see McLeay and Tenreyro (2020) and Jørgensen and Lansing (2024). The case when $g_\pi \to \infty$ would correspond to "strict inflation targeting."

and y_t, accounting for the non-monotonic behavior of $Cov(\pi_t, y_t)/Var(y_t)$ in Figure 2.

The top left panel of Figure 2 shows that $Corr(\pi_t, \pi_{t-1}) \simeq 1 - \mu_\pi$. The inverse link between the model's anchoring measure and inflation persistence is consistent with the cross-country empirical findings of Bems, et al. (2021). They show that an improvement in a survey-based measure of anchoring is associated with less persistent responses of inflation to shocks.

In Appendix A, we show that that $Corr(\pi_t, \pi_{t-1})$ in the model is equal to the slope coefficient obtained by regressing the rational inflation forecast $E_t\pi_{t+1}$ on a constant and π_t. The slope coefficient is given by $Cov(E_t\pi_{t+1}, \pi_t)/Var(\pi_t)$. This result obtains because $E_t\pi_{t+1} = \pi_{t+1} + \eta_{t+1}$, where η_{t+1} is the rational forecast error. Hence, there is a direct theoretical link between our autocorrelation-based anchoring measure and an alternative measure that is based on a regression of expected inflation on actual inflation.

5 Conclusion

We show that there is a simple and observable anchoring measure for short-run expected inflation in a standard New Keynesian model with full-information rational expectations. The anchoring measure is the fraction of non-reoptimizing firms that index prices to the inflation target, rather than lagged inflation. The approximate value of the model's anchoring measure is given by 1 minus the autocorrelation statistic for quarterly inflation. Higher values of the anchoring measure imply less sensitivity of rational inflation forecasts to movements in actual inflation. We show that a shift in the collective indexing behavior of firms, driven plausibly by a shift to a more vigilant monetary policy regime that keeps inflation close to target, allows the model to account for numerous features of evolving U.S. inflation behavior since 1960.

References

Bems, R., F. Caselli, F. Grigoli, and B. Gruss (2021), "Expectations' Anchoring and Inflation Persistence," *Journal of International Economics*, 132, Article 103516.

Bernanke, B. (2007) "Inflation Expectations and Inflation Forecasting," Speech at Monetary Economics Workshop, NBER Summer Institute, Cambridge, Massachusetts (July 10).

Choi, J., T. Doh, A. Foerster, and Z. Martinez (2022) "Monetary Policy Stance Is Tighter than Federal Funds Rate." *FRBSF Economic Letter* 2022-30 (November 7).

Cogley, T. and A.M. Sbordone (2008) "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve," *American Economic Review* 98, 2101-2106.

Coibion, O. and Y. Gorodnichenko (2011) "Monetary Policy, Trend Inflation, and the Great Moderation: An Alternative Interpretation," *American Economic Review* 101, 341-370.

Fuhrer, J.C. (2000) "Habit Formation in Consumption and its Implications for Monetary Policy Models," *American Economic Review* 90, 367-390.

Fuhrer, J.C. and G.D. Rudebusch (2004) "Estimating the Euler Equation for Output," *Journal of Monetary Economics* 51, 1133-1153.

Hazell, J., J. Herreño, E. Nakamura, and J. Steinsson (2022) "The Slope of the Phillips Curve: Evidence from U.S. States," *Quarterly Journal of Economics*, 137, 1299-1344.

Inoue, A., B. Rossi and Y. Wang (2024) "Has the Phillips Curve Flattened?" CEPR Discussion Paper 18846.

Jørgensen, P.L. and K.J. Lansing (2024) "Anchored Inflation Expectations and the Slope of the Phillips Curve," Federal Reserve Bank of San Francisco, Working Paper 2019-27.

Lansing, K.J. (2009) "Time-Varying U.S. Inflation Dynamics and the New Keynesian Phillips Curve," *Review of Economic Dynamics* 12, 304-326.

Lansing, K.J. (2021) "Endogenous Forecast Switching Near the Zero Lower Bound," *Journal of Monetary Economics* 117, 153-169.

Lansing, K.J. and F.C. Nucera (2023) "Inflation Expectations, the Phillips Curve, and Stock Prices," *FRBSF Economic Letter* 2023-24 (September 25).

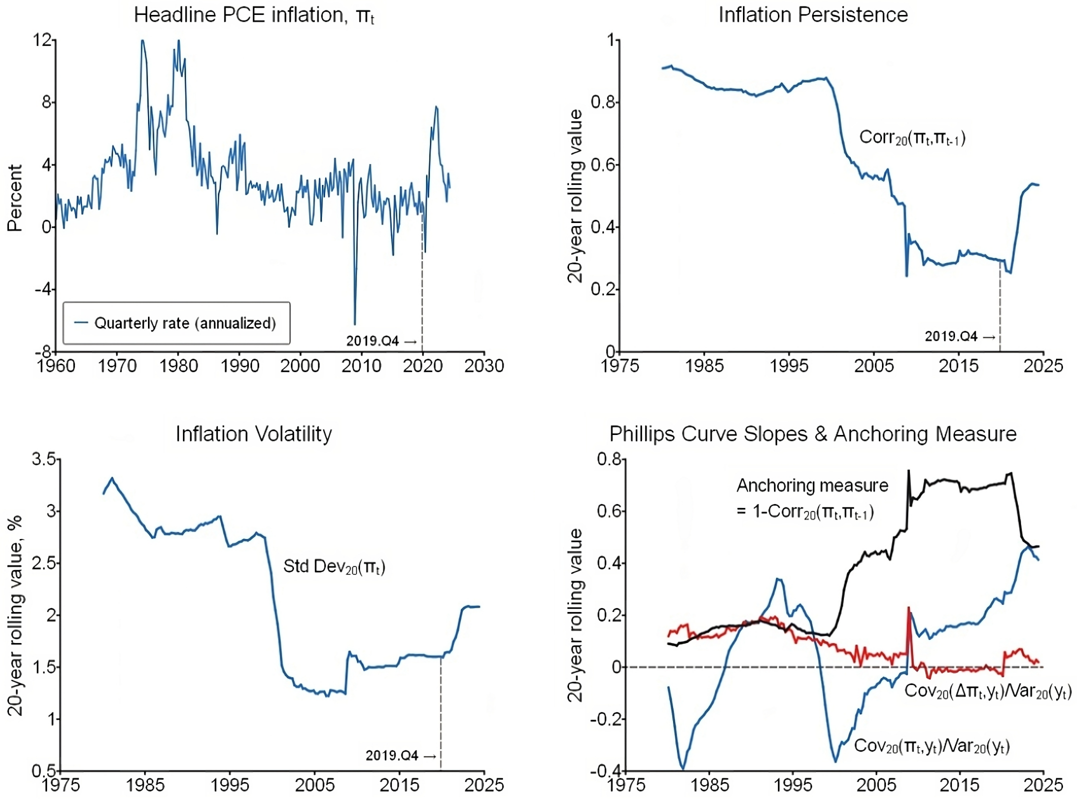
Lubik, T.A., and C. Matthes (2023) "The Stars Our Destination: An Update for Our R* Model." Federal Reserve Bank of Richmond, *Economic Brief* 23-32 (September).

Mavroeidis, S., M. Plagborg-Møller, and J.H. Stock (2014) "Empirical Evidence on Inflation Expectations in the New Keynesian Phillips curve,"*Journal of Economic Literature* 52(1), 124-188.

Stock, J.H. and M.W. Watson (2007) "Why Has Inflation Become Harder to Forecast?" *Journal of Money, Credit, and Banking*, 39, 3-34.

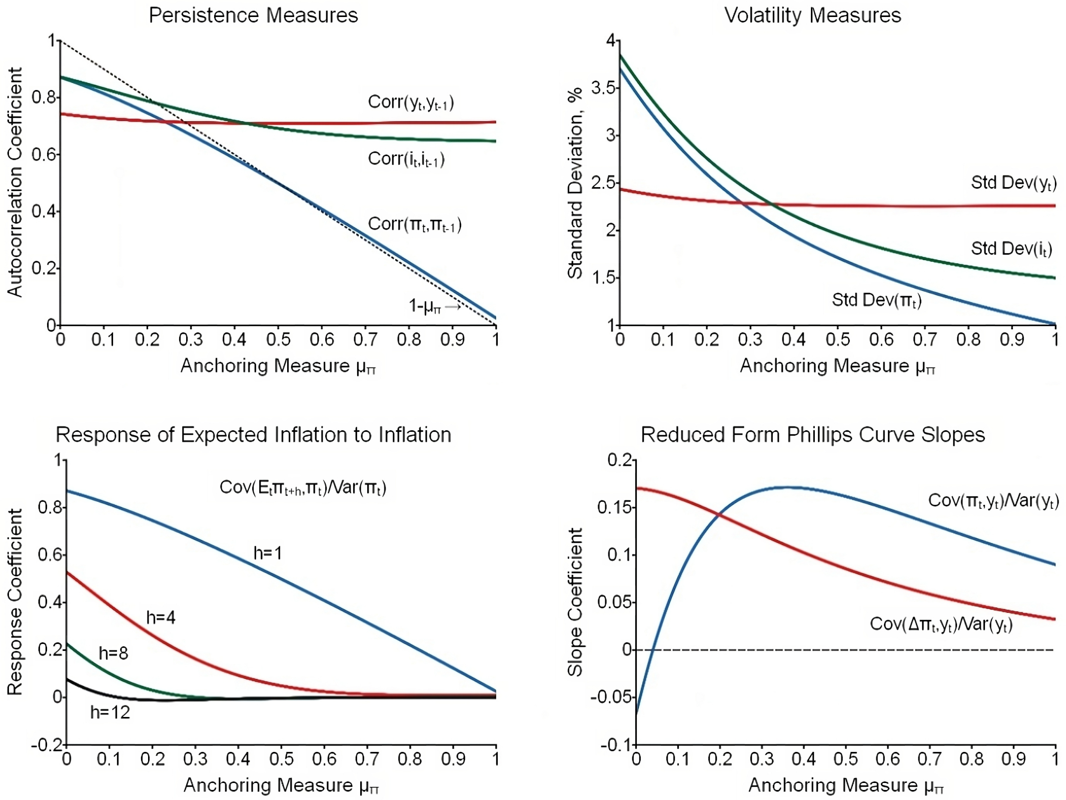
Wu, J.C. and J. Zhang (2019) "A Shadow Rate New Keynesian Model," *Journal of Economic Dynamics and Control* 107, Article 103728.

Figure 1: U.S. PCE Inflation and Statistics, 1960.Q1 to 2024.Q2



Notes: U.S. inflation behavior has shifted dramatically over the past sixty-plus years. A simple anchoring measure for short-run expected inflation is given by 1 minus the autocorrelation statistic for quarterly inflation.

Figure 2: Effects of Improved Anchoring



Notes: The model parameter μ_π is the fraction of non-reoptimizing firms that index prices to the inflation target rather than lagged inflation. Starting from low levels, higher values of μ_π serve to reduce inflation persistence and volatility, reduce the response of expected inflation to actual inflation, flatten the accelerationist Phillips curve, and steepen the original Phillips curve. The top left panel shows that $Corr(\pi_t, \pi_{t-1}) \simeq 1 - \mu_\pi$.

A Appendix: Model solution

Starting from the definition of average inflation $\pi_{av,t} \equiv \omega \pi_t + (1-\omega)\,\pi_{av,t-1}$ we have

$$E_t \pi_{av,t+1} - \pi^* = \omega\,(E_t \pi_{t+1} - \pi^*) + \omega\,(1-\omega)\,(\pi_t - \pi^*) + (1-\omega)^2\,(\pi_{av,t-1} - \pi^*)\,, \tag{A.1}$$

which can be substituted into the policy rule (3) to yield the following expression:

$$\begin{aligned} i_t - E_t\,\pi_{t+1} - r^* \;&=\; (g_\pi \omega - 1)\,(E_t \pi_{t+1} - \pi^*) + g_\pi \omega\,(1-\omega)\,(\pi_t - \pi^*) \\ &\quad + g_\pi\,(1-\omega)^2\,(\pi_{av,t-1} - \pi^*) + g_y E_t y_{t+1} + \varepsilon_t, \end{aligned} \tag{A.2}$$

which shows that i_t inherits persistence from four different endogenous variables. Equation (A.2) can be substituted into the Euler equation (1) to eliminate i_t. The resulting expression together with the NKPC (2) and the law of motion for $\pi_{av,t}$ form a linear system of three equations in the three unknown decision rules for y_t, π_t, and $\pi_{av,t}$. The state variables are y_{t-1}, π_{t-1}, $\pi_{av,t-1}$, v_t, u_t, and ε_t. Standard techniques yield a set of linear decision rules of the form

$$\begin{bmatrix} y_t \\ \pi_t - \pi^* \\ \pi_{av,t} - \pi^* \end{bmatrix} = \mathbf{A} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} - \pi^* \\ \pi_{av,t-1} - \pi^* \end{bmatrix} + \mathbf{B} \begin{bmatrix} v_t \\ u_t \\ \varepsilon_t \end{bmatrix}, \tag{A.3}$$

where $\mathbf{A}$ and $\mathbf{B}$ are 3×3 matrices of decision rule coefficients. The variance-covariance matrix $\mathbf{V}$ of the left-side variables in equation (A.3) can be computed analytically using the formula:

$$vec\,(\mathbf{V}) = [\mathbf{I} - \mathbf{A} \otimes \mathbf{A}]^{-1}\; vec\left(\mathbf{B}\boldsymbol{\Omega}\mathbf{B}'\right), \tag{A.4}$$

where $\boldsymbol{\Omega}$ is the variance-covariance matrix of the three fundamental shocks v_t, u_t, and ε_t. We use $\mathbf{V}$ and the other model equations to compute the analytical moments of model variables as the value of μ_π ranges from 0 to 1, as plotted in Figure 2.

For the parameter values shown in Table 1, the matrices $\mathbf{A}$ and $\mathbf{B}$ are

$$\mathbf{A} = \begin{bmatrix} 0.720 & -0.024 & -0.074 \\ 0.104 & 0.484 & -0.022 \\ 0.048 & 0.224 & 0.525 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 1.441 & -0.070 & -0.143 \\ 0.209 & 1.449 & -0.021 \\ 0.097 & 0.672 & -0.010 \end{bmatrix}. \tag{A.5}$$

Iterating the linear decision rules in equation (A.3) ahead one period and then taking the conditional expectation of both sides yields the following rational forecast rules:

$$E_t y_{t+1} = \mathbf{A}_{11} y_t + \mathbf{A}_{12}\,(\pi_t - \pi^*) + \mathbf{A}_{13}\,(\pi_{av,t} - \pi^*)\,, \tag{A.6}$$

$$E_t \pi_{t+1} - \pi^* = \mathbf{A}_{21} y_t + \mathbf{A}_{22}\,(\pi_t - \pi^*) + \mathbf{A}_{23}\,(\pi_{av,t} - \pi^*)\,, \tag{A.7}$$

$$E_t \pi_{av,t+1} - \pi^* = \mathbf{A}_{31} y_t + \mathbf{A}_{32}\,(\pi_t - \pi^*) + \mathbf{A}_{33}\,(\pi_{av,t} - \pi^*)\,, \tag{A.8}$$

where $\mathbf{A}_{i\,j}$ represents the corresponding element of the matrix $\mathbf{A}$.

From equation (A.3), we have

$$\pi_t - \pi^* = \mathbf{A}_{21} y_{t-1} + \mathbf{A}_{22} \left(\pi_{t-1} - \pi^*\right) + \mathbf{A}_{23} \left(\pi_{av,t-1} - \pi^*\right) + \mathbf{B}_{21} v_t + \mathbf{B}_{22} u_t + \mathbf{B}_{23} \varepsilon_t. \quad \text{(A.9)}$$

We can now demonstrate a direct theoretical link between our autocorrelation-based anchoring measure and an alternative anchoring measure that is based on a regression of expected inflation on actual inflation, along the lines of some empirical anchoring measures in the literature. Equation (A.9) implies

$$Cov(\pi_t, \pi_{t-1}) = \mathbf{A}_{21} Cov(\pi_t, y_t) + \mathbf{A}_{22} Var\left(\pi_t\right) + \mathbf{A}_{23} Cov\left(\pi_t, \pi_{av,t}\right). \quad \text{(A.10)}$$

Similarly, from equation (A.7) we have

$$Cov(E_t \pi_{t+1}, \pi_t) = \mathbf{A}_{21} Cov(\pi_t, y_t) + \mathbf{A}_{22} Var\left(\pi_t\right) + \mathbf{A}_{23} Cov\left(\pi_t, \pi_{av,t}\right). \quad \text{(A.11)}$$

Comparing equations (A.10) and (A.11) yields the result

$$Corr(\pi_t, \pi_{t-1}) \equiv \frac{Cov(\pi_t, \pi_{t-1})}{Var(\pi_t)} = \frac{Cov(E_t \pi_{t+1}, \pi_t)}{Var(\pi_t)}, \quad \text{(A.12)}$$

which shows that $Corr(\pi_t, \pi_{t-1})$ is equal to the slope coefficient obtained by regressing the rational inflation forecast on a constant and π_t.

B Appendix: Alternative anchoring measures

There is a vast literature on the topic of inflation expectations anchoring. Early examples include Roberts (2006), Williams (2006), Mishkin (2007), and Bernanke (2007). Anchoring measures for expected inflation can also be viewed as gauges of central bank credibility. Our autocorrelation-based anchoring measure complements other quantitative measures of anchoring for expected inflation at various forecast horizons. These include: (1) Measures based on the value of a gain parameter in imperfect information or learning models (Stock and Watson 2007, Lansing 2009, Milani 2014, Carvalho, et al. 2023, Gati 2023, Jørgensen and Lansing 2024, Jørgensen 2024), (2) Measures based on a regression of expected inflation on actual inflation (Demertzis, Massimiliano, and Nicola 2012, Ehrmann 2015, Ball and Mazumder 2019, Guerrieri, et al. 2023, Lansing and Nucera 2023), (3) Measures based on a regression of long-run expected inflation on short-run expected inflation (Strohsal, Melnick, and Nautz 2016, Buono and Formai 2018), (4) Measures based on high-frequency financial market data (Gürkaynak, Levin, and Swanson 2010, Beechey, Johannsen, and Levin 2011, Bauer 2015, Bundick and Smith 2024), and (5) Measures based on the deviation of actual inflation or agents' beliefs/forecasts from the central bank's inflation target (Meyer and Webster 1982, Huh and Lansing 2000, Andolfatto and Gomme 2003, Erceg and Levin 2003, Kozicki and Tinsley 2005, Wieland 2009, Gibbs and Kulish 2017, Bems, et al. 2021, Naggert, Rich, and Tracy 2023, and Diegel 2023).

Figure B.1 shows that an empirical version of our autocorrelation-based anchoring measure comoves strongly with two alternative anchoring measures for short-run expected inflation. The anchoring measure constructed by Lansing and Nucera (2023) gauges how much professional economists adjust their one-year ahead inflation forecast in response to recent movements in actual inflation. Using a 20-year rolling window of data, they regress the median inflation forecast from surveys on a constant and 4-quarter Consumer Price Index (CPI) inflation.[8] The resulting anchoring measure is given by $1 - Cov_{20}(F_t\pi_{4,t+4}, \pi_{4,t})/Var_{20}(\pi_{4,t})$, where $Cov_{20}(F_t\pi_{4,t+4}, \pi_{4,t})/Var_{20}(\pi_{4,t})$ is the estimated slope coefficient from the rolling regression. Bems, et al. (2021) and Naggert, Rich, and Tracy (2023) construct anchoring measures that are based on the deviation of expected inflation at various forecast horizons from the central bank's inflation target.[9] For comparison with our measure, we construct the time series $\exp\{-\text{abs}(F_t\pi_{4,t+4,} - \pi^*)_{20}\}$, where $\text{abs}(F_t\pi_{4,t+4,} - \pi^*)_{20}$ is the 20-year rolling mean absolute gap between professional economists' one-year ahead inflation forecasts and an inflation target of $\pi^* = 2\%$.[10] A larger mean absolute gap serves to lower the anchoring measure while the inverted exponential function delivers an index that ranges between 0 and 1. Figure B.1 plots the two alternative anchoring measures together with our autocorrelation-based anchoring measure constructed using quarterly CPI inflation, analogous to the anchoring measure plotted in Figure 1 for PCE inflation.

All three of the anchoring measures in Figure B.1 decline during the 1970s, rise gradually or hold steady during the 1990s, trend up substantially during subsequent decades, and then exhibit some modest end-of-sample declines. The correlation coefficient between the Lansing-Nucera measure and our measure is 0.81. The correlation coefficient between the gap-based measure and our measure is 0.88.

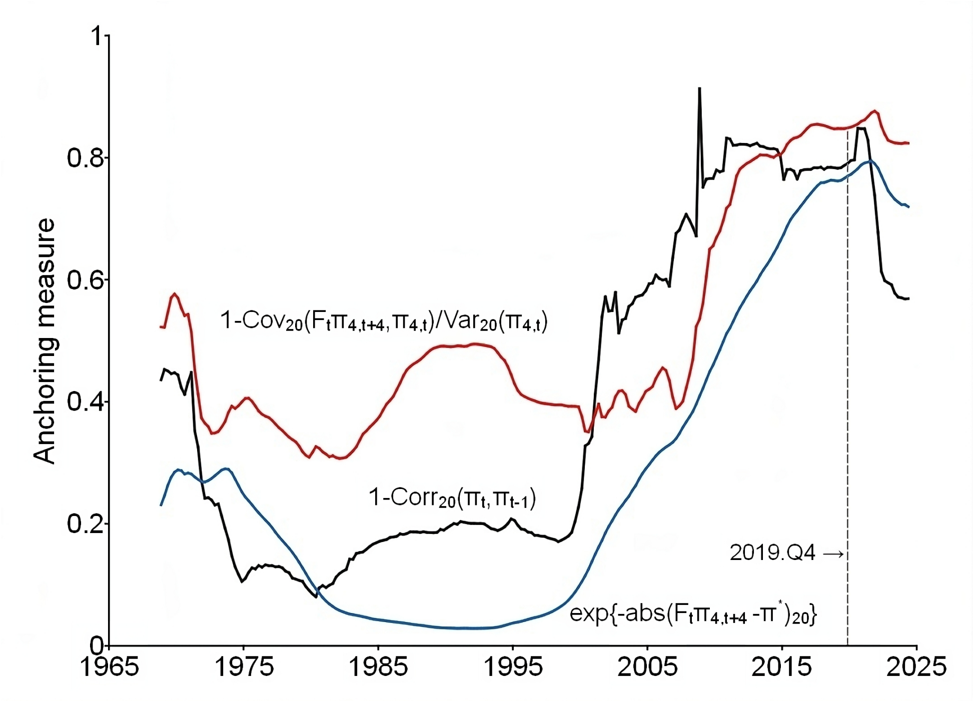
---

[8] Before 1981.Q3, expected inflation is the median response from the Philadelphia Fed's semiannual Livingston Survey, interpolated to obtain quarterly values. Starting in 1981.Q3, expected inflation is the median response from the Philadelphia Fed's quarterly Survey of Professional Forecasters.

[9] Wieland (2009) constructs an endogenous indexation measure that depends on the deviation of recent inflation from the central bank's target.

[10] Specifically, $F_t\pi_{4,t+4}$ is the same median one year ahead inflation forecast employed by Lansing and Nucera (2023).

Figure B.1: Anchoring Measures for Short-Run Expected Inflation



Notes: Our autocorrelation-based anchoring measure for short-run expected inflation comoves strongly with a survey-based anchoring measure constructed by Lansing and Nucera (2023) that gauges how much professional economists adjust their one-year ahead CPI inflation forecasts in response to movements in 4-quarter CPI inflation. Our anchoring measure also comoves strongly with a measure that is based on the mean absolute gap between professional economists' one-year ahead inflation forecasts and an inflation target of $\pi^* = 2\%$.

References for Appendix B

Andolfatto, D. and P. Gomme (2003) Monetary Policy Regimes and Beliefs," *International Economic Review* 44, 1-30.

Ball, L. and S. Mazumder (2019) "A Phillips Curve with Anchored Expectations and Short-Term Unemployment," *Journal of Money, Credit and Banking* 51, 111-137.

Bauer, M.D. (2015) "Inflation Expectations and the News," *International Journal of Central Banking* 11(2), 1-40.

Beechey, M., B. Johannsen, and A.T. Levin (2011) "Are Long-run Inflation Expectations Anchored more Firmly in the Euro Area than in the United States?" *American Economic Journal:Macroeconomics* 3, 104-129.

Bems, R., F. Caselli, F. Grigoli, and B. Gruss (2021), "Expectations' Anchoring and Inflation Persistence," *Journal of International Economics*, 132, Article 103516.

Bernanke, B. (2007) "Inflation Expectations and Inflation Forecasting," Speech at Monetary Economics Workshop, NBER Summer Institute, Cambridge, Massachusetts (July 10).

Buono, I. and S. Formai (2018) "New Evidence on the Evolution of the Anchoring of Inflation Expectations," *Journal of Macroeconomics* 57, 39-54.

Bundick, B. and A.L. Smith (2024) "Did the Federal Reserve Break the Phillips Curve? Theory and Evidence of Anchoring Inflation Expectations," *Review of Economics and Statistics*, forthcoming.

Carvalho, C., S. Eusepi, E. Moench, and B. Preston (2023) "Anchored Inflation Expectations," *American Economic Journal: Macroeconomics*, 15, 1-47.

Demertzis, M., M. Massimiliano, and N. Viegi (2012) "A Credibility Proxy: Tracking U.S. Monetary Developments," *B.E. Journal of Macroeconomics, Topics* 12(1), Article 12.

Diegel, M. (2023) "Time-varying Credibility, Anchoring and the Fed's Inflation Target," Working Paper.

Erceg, C.J. and A.T. Levin (2003) "Imperfect Credibility and Inflation Persistence," *Journal of Monetary Economics* 50, 915-944.

Ehrmann, M. (2015) "Targeting Inflation from Below: How Do Inflation Expectations Behave?" *International Journal of Central Banking* 11(S1), 213-249.

Gati, L.V. (2023) "Monetary Policy & Anchored Expectations–An Endogenous Gain Learning Model," *Journal of Monetary Economics* 140, S37-S47.

Gibbs, C.G. and M. Kulish (2017) "Disinflations in a Model of Imperfectly Anchored Expectations" *European Economic Review*, 100, 157-174.

Guerrieri, V., M. Marcussen, L. Reichlin and S. Tenreyro (2023) The Art and Science of Patience: Realtive Prices and Inflation," Geneva Reports on the World Economy 26.

Gürkaynak, R.S., A.T. Levin, and E.S. Swanson (2010) "Does Inflation Targeting Anchor Long-Run Inflation Expectations? Evidence from the U.S., U.K, and Sweden," *Journal of the European Economic Association* 8, 1208–1242.

Huh, C.G. and K.J. Lansing (2000) "Expectations, Credibility, and Disinflation in a Small Macroeconomic Model," *Journal of Economics and Business* 51, 51-86.

Jørgensen, P.L. (2024) "Anchored Inflation Expectations and the Output Cost of Disinflation," Working Paper.

Jørgensen, P.L. and K.J. Lansing (2024) "Anchored Inflation Expectations and the Slope of the Phillips Curve," Federal Reserve Bank of San Francisco, Working Paper 2019-27 (revised June 2024).

Kozicki, S. and P.A. Tinsley (2005) "Permanent and Transitory Policy Shocks in an Empirical Macro Model with Asymmetric Information," *Journal of Economic Dynamics and Control* 29, 1985-2015.

Lansing, K.J. (2009) "Time-Varying U.S. Inflation Dynamics and the New Keynesian Phillips Curve," *Review of Economic Dynamics* 12, 304-326.

Lansing, K.J. and F.C. Nucera (2023) "Inflation Expectations, the Phillips Curve, and Stock Prices," *FRBSF Economic Letter* 2023-24 (September 25).

Meyer, L.H. and C. Webster Jr. (1982) "Monetary Policy and Rational Expectations: A Comparison of Least Squares and Bayesian Learning," *Carnegie-Rochester Conference Series* *on Public Policy* 17, 67-98.

Milani, F. (2014) “Learning and Time-varying Macroeconomic Volatility,” *Journal of Economic Dynamics and Control*, 47, 94-114.

Mishkin, F.S. (2007) “Inflation Dynamics,” *International Finance* 10(3), 317-334.

Naggert, K., R. Rich, and J. Tracy (2023) “The Anchoring of U.S. Inflation Expectations Since 2012, Federal Reserve Bank of Cleveland, *Economic Commentary* 2023-11 (July 11).

Roberts, J. (2006) “Monetary Policy and Inflation Dynamics,” *International Journal of Central Banking* 2(3), 193-230.

Stock, J.H. and M.W. Watson (2007) “Why Has Inflation Become Harder to Forecast?” *Journal of Money, Credit, and Banking*, 39, 3-34.

Strohsal, T., R. Melnick, and D. Nautz (2016) “The Time-Varying Degree of Inflation Expectations Anchoring,” *Journal of Macroeconomics* 48, 62-71.

Wieland, V. (2009) “Learning, Endogenous Indexation and Disinflation in the New-Keynesian Model.” In: C. Walsh and K. Schmidt-Hebbel (eds.), *Monetary Policy under Uncertainty and Learning*, Volume XIII, Central Banking Series. Santiago, Chile: Central Bank of Chile, pp. 413-450.

Williams, J.C. (2006) “Inflation Persistence In An Era of Well-Anchored Inflation Expectations,” *FRBSF Economic Letter* 2006-27 (October 13).