Optimal Fiscal and Monetary Policy: Equivalence Results*

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Abstract

In this paper we analyze the way in which restrictions on price setting behavior shapes optimal fiscal and monetary policies in dynamic general equilibrium monetary models. We first show that the set of allocations that can be implemented as equilibria with taxes is independent of the price setting restrictions. We then derive two principles for optimal policy, independently of the price setting restrictions: Fiscal policy should be chosen as if all prices were flexible and monetary policy must replicate the flexible prices allocation, as if all prices were sticky.

1 Introduction

In this paper we analyze the implications of restrictions on price setting behavior for the conduct of fiscal and monetary policy, in dynamic general

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equilibrium monetary models. We compare flexible prices economies with economies in which prices are set one period in advance, but are identical in all other respects. We also consider mixed environments where there are both flexible an sticky prices firms. First, we analyze, in both environments, the sets of allocations that can be implemented as general equilibrium with taxes. We also analyze the sets of fiscal and monetary policies that implement those allocations.

The main contribution of this paper is to study fiscal and monetary policy in environments with sticky prices following the tradition of general equilibrium dynamic Ramsey problems. Thus, as in Ireland (1996), Carlstrom and Fuerst (1998), King and Wolman (1998), Khan, King and Wolman (2000) and Adao, Correia and Teles (2000), the relationship between policies and allocations is explicitly derived from a general equilibrium model. In contrast to that literature, however, our approach allows us to jointly study monetary and fiscal policy. In particular, we consider scenarios in which first best outcomes cannot be implemented.

The main finding of the paper is that the set of allocations that a government can implement is independent of the price setting rule. In particular, if as it is standard in Ramsey problems we assume a benevolent government, the second best allocation is the same, regardless of the price setting rules. It should be stressed that although we borrow from the Ramsey tradition the way to characterize the mapping between policies and allocations, our results refer to sets of implementable allocations.

On one hand, these results suggest that sticky prices are redundant, in terms of the implemenable allocations. On the other hand, they show that the characterization of the optimal distortions is independent of the restrictions on price setting. Therefore, the existing results in the optimal taxation literature - Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991), Zhu (1992) - extend in a straightforward way to the debate on stabilization policy.[1]

We also charaterize the policies that implement the optimal allocations. In the economy with flexible prices, where money is neutral - but not superneutral-, the fiscal instruments and the nominal interest rate determine the allocations. In these economies the optimal money supply is indeterminate. Under

[1] In a companion paper, Correia, Nicolini and Teles (2001a) we obtain that the Friedman rule is optimal in these environments, and extend results in Zhu (1992) on optimal smoothing of taxes.

sticky prices, where money is neither neutral nor superneutral, we obtain analogous results. In these environments, it is the state contingent fiscal policy that is indeterminate, suggesting short run neutrality of fiscal policy. A qualification should be made though, since as it is the case of the nominal interest rate under flexible prices, in this environment there are non-neutral fiscal instruments that play the same role as the money supply. Finally, in environments where both firms that set prices in advance and firms that set prices contemporaneously coexist, both fiscal and monetary instruments are non-neutral. In these economies it is always optimal, whatever are the government's preferences, to choose the optimal fiscal policy as if all prices were flexible, and monetary policy must replicate the flexible prices allocation, as if all prices were sticky. Interestingly enough, results derived in Adao, Correia and Teles (1999) in a somewhat different environment apply: the optimal allocation is independent of the shares of sticky or flexible firms, and so is the optimal policy[2].

The paper proceeds as follows. Section 2 presents the model and solves for equilibrium conditions on both the flexible and set-in-advance (SIA) prices economies. The main equivalence results are then derived. Section 3 presents a model that combines firms that set prices in advance with firms without price setting restrictions and discusses properties of optimal policies. Section 4 concludes.

2 The model

Our model economy follows closely the structures in Ireland (1996), Carlstrom and Fuerst (1998) and Adao, Correia and Teles (2000). The state of the economy will be represented by the realization of a random variable $\sigma_t \in \Sigma$ that follows a Markov process. The shocks to the economy will be time invariant functions of the state. That is, government expenditure shocks, $g_t = g(\sigma_t)$, and productivity shocks, $s_t = s(\sigma_t)$. In addition, we will let policy instruments to be functions of the state. That is, labor income taxes $\tau_t^n = \tau^n(\sigma_t)$, dividend taxes $\tau_t^d = \tau^d(\sigma_t)$, consumption taxes $\tau_t^c = \tau^c(\sigma_t)$ and money growth rates $\mu_t = \mu(\sigma_t)$. These are all the natural policy instruments to consider in this environment. We will analyze the optimal policy problem for alternative scenarios regarding restrictions on these functions.

[2] Adao, Correia and Teles (1999) also show that the polices that replicate flexible prices is independent of degree of not only price, but also portfolio stickiness.

The economy consists of a representative household, a continuum of producers of final goods indexed by $i \in [0,1]$, and a government. Each firm produces a distinct, perishable consumption good, indexed by i.

2.1 The households

Preferences are described by the expected utility function:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u \left(C_t, 1 - N_t \right) \right\} \tag{1}$$

where N_t is labor effort, $\beta \in (0,1)$ is a discount factor and the composite C_t is

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1.$$

The households start period t with outstanding nominal wealth, $\mathbb{W}_t$, and decide to buy money balances, M_t and B^h_{t+1} units of nominal bonds that pay $R_{t+1}B^h_{t+1}$ units of money one period later. They also buy Z^h_{t+1} units of state contingent nominal securities, that cost z_{t+1} in units of currency today, and each of them pays one unit of money at the beginning of period $t+1$ in a particular state. They can also buy $A(i)_{t+1}$ units of stocks of firm i, that cost $a(i)_t$ in units of currency. Households have to pay labor income, dividend and consumption taxes.

The purchases of consumption goods have to be made with cash, so,

$$\int_0^1 P_t(i) c_t(i)(1 + \tau^c_t) di \leq M_t \tag{2}$$

were $P_t(i)$ is the money price of final good i.

At the end of the period, households receive labor income, $W_t N_t$ where W_t is the nominal wage rate, and collect dividends, given by current period profits $d(i)_t$ that can be used to purchase consumption in the following period. Therefore, the households face the budget constraints

$$M_t + B^h_{t+1} + E_t Z^h_{t+1} z_{t+1} + \int_0^1 A(i)_{t+1} a(i)_t di \leq \mathbb{W}_t$$

$$\begin{aligned}\mathbb{W}_{t+1} &= M_t + R_{t+1}B^h_{t+1} + Z^h_{t+1} - (1+\tau^c_t)\int_0^1 P_t(i)c_t(i)di + \\ &\int_0^1 A(i)_{t+1}a(i)_{t+1}di + W_tN_t(1-\tau^n_t) + \int_0^1 A(i)_{t+1}d(i)_t di(1-\tau^d_t)\end{aligned} \quad (3)$$

The problem of the consumer can be stated as

$$V(\mathbb{W}_t,\sigma_t) = \text{Max } \{u(C_t, 1-N_t) + \beta E_t V(\mathbb{W}_{t+1},\sigma_{t+1})\}$$

subject to the cash in advance constraint, (2), and the budget constraints (3).

Let $P_t = \left[\int P_t(i)^{1-\theta}di\right]^{\frac{1}{1-\theta}}$. Then we have

$$\frac{c_t(i)}{C_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} \quad (4)$$

which defines the demand for each of the final consumption goods. The problem can be restated as

$$V(\mathbb{W}_t,\sigma_t) = \text{Max } \{u(C_t, 1-N_t) + \beta E_t V(\mathbb{W}_{t+1},\sigma_{t+1})\}$$

s.t.

$$P_tC_t(1+\tau^c_t) \leq M_t \quad (5)$$

$$M_t + B^h_{t+1} + E_tZ^h_{t+1}z_{t+1} + \int_0^1 A(i)_{t+1}a(i)_t di \leq \mathbb{W}_t$$

$$\begin{aligned}\mathbb{W}_{t+1} &\leq M_t + R_{t+1}B^h_{t+1} + Z^h_{t+1} - (1+\tau^c_t)P_tC_t + \\ &\int_0^1 A(i)_{t+1}a(i)_{t+1}di + (1-\tau^n_t)W_tN_t + \int_0^1 A(i)_{t+1}d(i)_t di(1-\tau^d_t)\end{aligned} \quad (6)$$

The following marginal conditions summarize households behavior

$$\frac{u_{1-Nt}}{u_{Ct}} = \frac{W_t}{P_t}\frac{(1-\tau^n_t)}{R_{t+1}(1+\tau^c_t)} \quad (7)$$

$$\frac{u_{Ct}}{P_t(1+\tau^c_t)} = R_{t+1}E_t\left[\frac{\beta u_{Ct+1}}{P_{t+1}(1+\tau^c_{t+1})}\right] \quad (8)$$

$$z_{t+1}\frac{P_{t+1}}{P_t} = \frac{\beta u_{Ct+1}}{u_{Ct}}\frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)} \tag{9}$$

$$E_t\left[z_{t+1}\right] = \frac{1}{R_{t+1}} \tag{10}$$

$$a(i)_t\frac{u_{Ct}}{P_t(1+\tau_t^c)} = E_t\left[\left[a(i)_{t+1} + d(i)_t(1-\tau_t^d)\right]\frac{\beta u_{Ct+1}}{(1+\tau_{t+1}^c)P_{t+1}}\right] \tag{11}$$

Condition (7) sets the intra-temporal marginal rate of substitution between consumption and leisure equal to the real wage times the corresponding taxes. Condition (8) is a requirement for the optimal savings decision. The last three conditions are arbitrage conditions for asset prices.

2.2 The government

The government must finance a given path of purchases $\{G_t\}_{t=0}^{\infty}$, such that

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}, \theta > 1$$

Given the prices on each good, $P_t(i)$, the government minimizes the expenditure on government purchases by deciding according to

$$\frac{g_t(i)}{G_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta}$$

A government policy consists of a sequence of a monetary and tax policy $\left(M_t, \tau_t^n, \tau_t^c, \tau_t^d\right)_{t=0}^{\infty}$.

2.3 Firms

There is a continuum of competitively monopolistic firms, each one produces a single differentiated consumption good. The technology is linear in labor, the only production input.

The pricing equation for stocks (11) implies that the problem of maximizing the value of a single monopolistic firm can be written as the following dynamic programing problem

$$a(i)_t\frac{u_{Ct}}{P_t(1+\tau_t^c)} = \max_{p(i)_t}\; d(i)_t(1-\tau_t^d)E_t\frac{\beta u_{Ct+1}}{(1+\tau_{t+1}^c)P_{t+1}} + E_t\left[\frac{a(i)_{t+1}\beta u_{Ct+1}}{(1+\tau_{t+1}^c)P_{t+1}}\right]$$

2.3.1 Flexible prices

Let $\Xi_t = (1-\tau_t^d)E_t\frac{\beta u_{Ct+1}}{(1+\tau_{t+1}^c)P_{t+1}}$. Note that Ξ_t does not depend on firm's actions. Thus, as long as $\Xi_t \geq 0$, the solution, as of time $t-1$, will be to set a state contingent rule for prices, satisfying

$$\max_{p(i)_t}\left[P_t(i)y_t(i) - W_t n_t(i)\right]$$

subject to the demand function

$$\frac{y_t(i)}{Y_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} \tag{12}$$

obtained from the households problem (4), were $Y_t = C_t + G_t$, and the technology

$$y_t(i) \leq s_t n_t(i) \tag{13}$$

where s_t is the level of technology. The optimal pricing rule is therefore

$$P_t(i)\left[1 + \frac{d\ln P_t(i)}{d\ln y_t(i)}\right] - MC_t = 0$$

where $\frac{d\ln P_t(i)}{d\ln y_t(i)} = -\frac{1}{\theta}$, so that θ is the demand elasticity. Thus,

$$P_t = P_t(i) = \frac{\theta}{\theta-1}MC_t$$

where

$$MC_t = \frac{W_t}{s_t}$$

The firms set a common price, a constant mark-up over their common marginal cost. Thus, given that technologies and demand functions are identical, an equilibrium will be characterized by equal prices and labor inputs across varieties. Thus, we will use the notation $c_t(i) = C_t$, $n_t(i) = N_t$. In equilibrium,

$$s_t = \frac{\theta}{\theta-1}\frac{W_t}{P_t}$$

Thus, as long as $\Xi_t \geq 0$, the rule $P_t(i) = \frac{\theta}{\theta-1}MC_t$ characterizes the behavior of the firms. If $\Xi_t < 0$, the solution for the firms is to set $y_t(i) = 0$.

2.3.2 When prices are set in advance

We consider now an environment where firms set the prices one period in advance and can only sell output in period t at the previously chosen price. As of time t, the firms are constrained in terms of the price at which they can sell, but are not constrained in terms of the quantity. Thus, at time t, and given a previously chosen price, they do choose quantities to maximize profits. That problem is given by

$$\max_{y(i)_t} \left[P_t(i) y_t(i) - W_t \frac{y_t(i)}{s_t} \right] \Xi_t$$

subject to

$$0 \leq y(i)_t \leq Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta}$$

or

$$0 \leq y(i)_t \leq Y_t$$

since all firms are symmetric. The problem can be restated as

$$\max_{y(i)_t} y_t(i) \left[P_t(i) - \frac{W_t}{s_t} \right] \Xi_t$$

The solution is to set $y(i)_t = Y_t$ as long as $\left[P_t(i) - \frac{W_t}{s_t} \right] \Xi_t \geq 0$, and $y(i)_t = 0$ otherwise. Thus, firms will satisfy demand as long as they do not make negative profits, and produce zero otherwise.

Firms at time $t-1$ must chose $p(i)_t$, to maximize the value of the firm next period

$$E_{t-1} a(i)_t \frac{u_{Ct}}{(1+\tau_t^c)P_t} = \max_{p(i)_{t+1}} E_{t-1}\ d(i)_t \frac{(1-\tau_t^d)\beta u_{Ct+1}}{P_{t+1}(1+\tau_{t+1}^c)} + E_{t-1} \left[\frac{a(i)_{t+1}\beta u_{Ct+1}}{P_{t+1}(1+\tau_{t+1}^c)} \right]$$

so, the optimal problem is to choose $P_t(i)$ to maximize,

$$E_{t-1} \left[\frac{u_{Ct+1}}{P_{t+1}(1+\tau_{t+1}^c)} (1-\tau_t^d) y_t(i) \left(P_t(i) - MC_t \right) \right] \tag{14}$$

subject to (12).

The solution is given by

$$P_t(i) = P_t = \frac{\theta}{(\theta - 1)} E_{t-1}\left[\upsilon_t MC_t\right]$$

where

$$\upsilon_t = \frac{\frac{(1-\tau_t^d)}{(1+\tau_{t+1}^c)} \frac{u_{Ct+1}}{P_{t+1}} y_t}{E_{t-1}\left[\frac{(1-\tau_t^d)}{(1+\tau_{t+1}^c)} \frac{u_{Ct+1}}{P_{t+1}} y_t\right]}.$$

As $MC_t = \frac{W_t}{s_t}$, we obtain

$$\frac{\theta - 1}{\theta} = E_{t-1}\left[\upsilon_t \frac{W_t}{s_t P_t}\right]$$

2.4 Market clearing

Market clearing requires

$$C_t + G_t = Y_t = s_t N_t \tag{15}$$

$$B_t^h = B_t^g$$

$$Z_{t,t+1}^h = Z_{t,t+1}^g, \text{ for all possible states at } t+1$$

where B_t^g and $Z_{t,t+1}^g$ represent government debt, and

$$A(i)_t = 1$$

2.5 Life-time budget constraint

The period by period budget constraints of the households can be written, once we take into account that $A(i)_t = 1, d(i)_t = D_t$ and $c(i) = C_t$ for all i, as

$$M_0 + B_1^h + E_0 Z_1^h z_1 \leq \mathbb{W}_0^-$$

where $\mathbb{W}_0^- = \mathbb{W}_0 - \int_0^1 a(i)_0 di$, and

$$\begin{aligned} M_t + B_{t+1}^h + E_t Z_{t+1}^h z_{t+1} \quad \leq \quad & M_{t-1} + R_t B_t^h + Z_t^h - P_{t-1} C_{t-1}(1 + \tau_{t-1}^c) + \\ & W_{t-1} N_{t-1}(1 - \tau_{t-1}^n) + D_{t-1}(1 - \tau_{t-1}^d) \end{aligned}$$

for $t \geq 1$. The present value budget constraint can be written if we multiply the equation by each state and period normalized contingent price $Q_t = \prod_{j=0}^{t} z_j$, take expectations and add for all t

$$E_0 \sum_{t=0}^{\infty} Q_{t+1} \left(P_t C_t (1+\tau_t^c) - W_t N_t (1-\tau_t^n) - D_t (1-\tau_t^d) + M_t \left(\frac{Q_t}{Q_{t+1}} - 1\right) \right) = \mathbb{W}_0^-$$

In order to avoid the well known time inconsistency problem of optimal monetary policy, we assume that $\mathbb{W}_0^- = 0$. Using asset price equations and the cash-in-advance constraint, we obtain

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{Ct}}{(1+\tau_t^c) P_t} \left(P_t C_t (1+\tau_t^c) - \frac{W_t N_t (1-\tau_t^n) + D_t (1-\tau_t^d)}{R_{t+1}} \right) = 0$$

Finally, using the definition of dividends and the production function, we obtain

$$E_0 \sum_{t=0}^{\infty} \beta^t U_{Ct} \left(C_t - \frac{w_t (1-\tau_t^n)}{R_{t+1}(1+\tau_t^c)} N_t + \frac{(1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)} s_t N_t - \frac{w_t (1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)} N_t \right) = 0 \tag{16}$$

2.6 Competitive equilibria

2.6.1 Flexible prices

With flexible prices, real wages must satisfy

$$s_t \frac{\theta - 1}{\theta} = w_t \tag{17}$$

every period and state. If we replace these conditions on the budget constraint (16) , we obtain

$$E_0 \sum_{t=0}^{\infty} \beta^t U_{Ct} \left(C_t - \frac{\theta - 1}{\theta} \frac{(1-\tau_t^n)}{R_{t+1}(1+\tau_t^c)} s_t N_t + \frac{1}{\theta} \frac{(1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)} s_t N_t \right) = 0 \tag{18}$$

The sequence $\{\tau_t^c, \tau_t^n, \tau_t^d, C_t, N_t, R_{t+1}\}_{t=0}^{\infty}$ is a competitive equilibria with taxes for the *flexible* prices economy, if it solves

$$\frac{u_{1-Nt}}{u_{Ct}} = s_t \frac{\theta-1}{\theta} \frac{(1-\tau_t^n)}{R_{t+1}(1+\tau_t^c)} \tag{19}$$

$$C_t + G_t = s_t N_t \tag{20}$$

for $t \geq 0$ and the budget constraint (18).

Given any value for M_t, the nominal interest rate R_{t+1} is pinned down by future expected monetary policy according to

$$\frac{u_{Ct}C_t}{M_t} = R_{t+1}E_t\left[\frac{\beta u_{Ct+1}C_{t+1}}{M_{t+1}}\right]$$

while the price level is determined by M_t according to the money demand equation

$$P_t C_t(1+\tau_t^c) = M_t \tag{21}$$

From a policy point of view, it is of particular interest to characterize the set of allocations $\{C_t, N_t\}_{t=0}^{\infty}$ that can be implemented as a competitive equilibria with taxes. In that respect, some of the policy instruments are equivalent, in the sense that many different combinations can implement the same allocation. Indeed, the relevant policy ratios to determine allocations are given by

$$\frac{(1-\tau_t^n)}{R_{t+1}(1+\tau_t^c)} \text{ and } \frac{(1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)}$$

Obviously R_{t+1} and $(1+\tau_t^c)$ are equivalent. Thus, let $R_{t+1} = 1$[3]. Obviously there is a continuum of values for the remaining three policy instruments that implement any given allocation.

Profit maximizing firms only produce output when $\Xi_t \geq 0$. Recall that

$$\Xi_t = (1-\tau_t^d)E_t\frac{\beta u_{Ct+1}}{(1+\tau_{t+1}^c)P_{t+1}} = (1-\tau_t^d)\frac{u_{Ct}}{P_t(1+\tau_t^c)R_{t+1}}$$

using intertemporal conditions. If we also use the intratemporal condition, we obtain

[3] It is convenient to consider this case, since this equivalence between R_{t+1} and $(1+\tau_t^c)$ is not robust to cash-credit or transactions technologies extensions of the model. See Correia, Nicolini and Teles (2001a).

$$\Xi_t = (1-\tau_t^d)\frac{u_{Ct}}{P_t(1+\tau_t^c)R_{t+1}} = \frac{(1-\tau_t^d)u_{1-Nt}}{(1-\tau_t^n)W_t}$$

Thus, we can write the constraint as

$$\frac{(1-\tau_t^d)u_{1-Nt}}{(1-\tau_t^n)W_t} \geq 0 \tag{22}$$

Note also that if $G_t > 0$ for all t, feasibility requires output to be positive. Thus, in the flexible prices economy, the restriction (22) must hold.

2.6.2 SIA prices

The pricing rule is

$$1 = E_t\left[\upsilon_{t+1}\frac{\theta}{\theta-1}\frac{w_{t+1}}{s_{t+1}}\right] \tag{23}$$

for $t \geq 0$. We can follow Adao, Correia and Teles (2000) and write (23)as

$$E_{t-1}\left[\frac{(1-\tau_t^d)u_{Ct+1}}{P_{t+1}(1+\tau_{t+1}^c)}y_t\frac{w_t}{s_t} - \frac{(1-\tau_t^d)u_{Ct+1}}{P_{t+1}(1+\tau_{t+1}^c)}y_t\frac{\theta-1}{\theta}\right] = 0.$$

Using the law of iterated expectations and the intertemporal conditions we obtain

$$E_{t-1}\left[\left(y_t\frac{w_t}{s_t} - y_t\frac{\theta-1}{\theta}\right)\frac{(1-\tau_t^d)u_{Ct}}{R_{t+1}(1+\tau_t^c)}\right] = 0$$

since P_t is in the information set at $t-1$. This can be written as

$$E_{t-1}\left(N_t w_t \frac{(1-\tau_t^d)u_{Ct}}{R_{t+1}(1+\tau_t^c)}\right) = \frac{\theta-1}{\theta}E_{t-1}\left(s_t N_t \frac{(1-\tau_t^d)u_{Ct}}{R_{t+1}(1+\tau_t^c)}\right)$$

Using the law of iterated expectations

$$E_0\left(N_t w_t \frac{(1-\tau_t^d)u_{Ct}}{R_{t+1}(1+\tau_t^c)}\right) = \frac{\theta-1}{\theta}E_0\left(s_t N_t \frac{(1-\tau_t^d)u_{Ct}}{R_{t+1}(1+\tau_t^c)}\right)$$

for $t \geq 1$.

On the other hand, the life-time budget constraint can be written

$$E_0 \sum_{t=0}^{\infty} \beta^t U_{Ct} \left(C_t - \frac{w_t(1-\tau_t^n)}{R_{t+1}(1+\tau_t^c)} N_t + \frac{(1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)} \left(s_t N_t - w_t N_t\right) \right) = 0$$

Using the equation above, it can be written as

$$U_{C0} \left(C_0 - \frac{w_0(1-\tau_0^n)}{R_1(1+\tau_0^c)} N_0 + \frac{(1-\tau_0^d)}{R_1(1+\tau_0^c)} (s_0 - w_0) N_0 \right) +$$

$$E_0 \sum_{t=1}^{\infty} \beta^t U_{Ct} \left(C_t - \frac{w_t(1-\tau_t^n)}{R_{t+1}(1+\tau_t^c)} N_t + \frac{(1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)} \frac{s_t N_t}{\theta} \right) = 0$$

Given P_0, the sequence $\{\tau_t^c, \tau_t^n, \tau_t^d, C_t, N_t, R_{t+1}, w_t\}_{t=0}^{\infty}$ is a competitive equilibria with taxes for the *SIA* prices economy, if it solves equations

$$\frac{u_{1-Nt}}{u_{Ct}} = w_t \frac{(1-\tau_t^n)}{R_{t+1}(1+\tau_t^c)} \tag{24}$$

$$C_t + G_t = s_t N_t \tag{25}$$

the budget constraint above and the condition

$$E_{t-1} \left(N_t w_t \frac{(1-\tau_t^d) u_{Ct}}{R_{t+1}(1+\tau_t^c)} \right) = \frac{\theta - 1}{\theta} E_{t-1} \left(s_t N_t \frac{(1-\tau_t^d) u_{Ct}}{R_{t+1}(1+\tau_t^c)} \right) \tag{26}$$

for $t \geq 1$.

Note that any competitive equilibria with taxes in the flexible prices economy is an equilibrium in this economy, since $w_t = \frac{\theta}{\theta-1} s_t$ obviously satisfies condition (26) .

As before, the nominal interest rate R_{t+1} is pinned down by future expected monetary policy according to

$$\frac{u_{Ct} C_t}{M_t} = R_{t+1} E_t \left[\frac{\beta u_{Ct+1} C_{t+1}}{M_{t+1}} \right] \tag{27}$$

while the quantity of money M_t that implements the allocation is the one that satisfies

$$P_t C_t (1 + \tau_t^c) = M_t \tag{28}$$

If $G_t > 0$, feasibility requires that firms produce positive amounts of output every period. As we argued before, this is the case when $\left[P_t(i) - \frac{W_t}{s_t}\right] \Xi_t \geq 0$. Thus, in the SIA prices economy, feasibility means that

$$\left[P_t(i) - \frac{W_t}{s_t}\right] \frac{(1-\tau_t^d)u_{1-Nt}}{(1-\tau_t^n)W_t} = \left[\frac{1}{w_t} - \frac{1}{s_t}\right] \frac{(1-\tau_t^d)u_{1-Nt}}{(1-\tau_t^n)} \geq 0 \qquad (29)$$

As before, if we are only interested in the set of allocations $\{C_t, N_t\}_{t=0}^{\infty}$ that can be implemented as competitive equilibria, there are redundant instruments. In this case, the relevant policy ratios to determine allocations are given by

$$\frac{w_t(1-\tau_t^n)}{R_{t+1}(1+\tau_t^c)}, \frac{(1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)} \text{ and } \frac{w_t(1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)}$$

Again, R_{t+1} and $(1+\tau_t^c)$ are equivalent, so we set $R_{t+1} = 1$. Clearly, w_t and any two of τ_t^c, τ_t^n and τ_t^d are sufficient instruments. The question, however, is if they are also necessary. The next proposition shows that this is not the case

Proposition 1 *Given P_0, if the sequence $\{\tau_t^c, \tau_t^n, \tau_t^d, C_t^*, N_t^*, R_{t+1} = 1, w_t\}_{t=0}^{\infty}$ is a competitive equilibrium with taxes for the* SIA *prices economy, then there is a different sequence $\{\widetilde{\tau}_t^c, \widetilde{\tau}_t^n, \widetilde{\tau}_t^d\}_{t=0}^{\infty}$ such that $\{\widetilde{\tau}_t^c, \widetilde{\tau}_t^n, \widetilde{\tau}_t^d, C_t^*, N_t^*, R_{t+1} = 1, \widetilde{w}_t\}_{t=0}^{\infty}$ is also a competitive equilibria with taxes for the* SIA *prices economy. One of these solutions has $\widetilde{w}_t = \frac{\theta-1}{\theta}s_t$ for all t.*

Pf: We first show it for the case in which $\widetilde{w}_t = \frac{\theta-1}{\theta}s_t$ for all t. Since quantities have not changed, (25)is satisfied. Let $\widetilde{\tau}_t^c = \tau_t^c$ for all t and let $\widetilde{\tau}_t^n$ satisfy

$$w_t(1-\tau_t^n) = \frac{\theta-1}{\theta}s_t(1-\widetilde{\tau}_t^n)$$

for all t and all states. By construction, it is clear that the sequence $\{\widetilde{\tau}_t^c, \widetilde{\tau}_t^n, \widetilde{\tau}_t^d, C_t^*, N_t^*, R_{t+1}, \frac{\theta-1}{\theta}s_t$ satisfies (24) for all t. As $w_t = \frac{\theta-1}{\theta}s_t$, condition (26) is trivially satisfied.

Finally, let $\widetilde{\tau}_0^d$ satisfy

$$(1-\tau_0^d)(s_0 - w_0) = (1-\widetilde{\tau}_0^d)\frac{s_0}{\theta} \qquad (30)$$

Then, the intertemporal budget constraint is also satisfied. Now, note that for any sequence $\widetilde{w}_t$ that satisfies (26), the income tax rates and the dividend tax rate at the first period can be modified to obtain the same allocation. QED.

The proposition states that stabilization fiscal instruments are indeterminate in this sticky prices economy. Depending on how they are set, the resulting wages may be the ones of the flexible prices economy or not, without affecting the allocation. We have seen already that any allocation under flexible prices can be implemented in the sticky prices economy. The following corollary states the reverse.

Corollary 2 *If the allocation* $\{C_t^*, N_t^*\}_{t=0}^{\infty}$ *can be implemented as a competitive equilibrium with taxes in the SIA economy, it can also be implemented in the flexible prices economy.*

The proposition shows that any allocation that can be implemented in the sticky prices economy can be implemented using a policy that replicates the flexible prices wages. Trivially, that allocation together with that policy are a competitive equilibrium with taxes in the flexible prices economy. The proof of the proposition makes clear the equivalence between monetary policy inducing real wage departures from the flexible price wages - gaps - and other fiscal instruments - labor income taxes and profit taxes in that case. As this principle applies for any feasible allocation, and regardless of the welfare criteria, there are multiple (in fact a continuum) of optimal stabilization policies. In the next section, though, we show that this result crucially depends on the assumption that the restriction to set prices in advance applies to all the firms in the economy.

3 An economy with heterogenous price setting rules

In this section we modify the model and assume that a fraction α of the firms must set prices in advance while a fraction $1-\alpha$ can set state contingent prices. Given symmetry, all sticky firms will set the price P_t^S and all flexible firms set the common price P_t^F. Then, the price level is

$$P_t = \left[\alpha P_t^{S(1-\theta)} + (1-\alpha) P_t^{F(1-\theta)}\right]^{\frac{1}{1-\theta}}$$

Dividing by W_t

$$\frac{1}{w_t} = \left[\alpha\left(\frac{1}{w_t^S}\right)^{(1-\theta)} + (1-\alpha)\left(\frac{1}{w_t^F}\right)^{(1-\theta)}\right]^{\frac{1}{1-\theta}}$$

where $w_t^S = \frac{W_t^S}{P_t}$ and $w_t^F = \frac{W_t^F}{P_t}$. The pricing rule of the firms that set state contingent prices is

$$w_t^F = \frac{\theta - 1}{\theta} s_t$$

while

$$E_{t-1}\left[u_{Ct}\frac{w_t(1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)}\frac{1}{w_t^S}\left(\frac{\theta-1}{\theta} - \frac{w_t^S}{s_t}\right)y_t^S\right] = 0$$

describes the pricing rule of firms that set prices one period in advance. Note that the problem of the consumer is the same as before, so the intertemporal budget constraint is the same

$$E_0\sum_{t=0}^{\infty}\beta^t U_{Ct}\left(C_t - \frac{w_t(1-\tau_t^n)}{R_{t+1}(1+\tau_t^c)}N_t - \frac{D_t(1-\tau_t^d)}{W_t R_{t+1}}\frac{w_t}{(1+\tau_t^c)}\right) = 0$$

In this economy, total dividends are

$$D_t = \alpha y_t^S\left(P_t^S - \frac{W_t}{s_t}\right) + (1-\alpha)\, y_t^F\left(P_t^F - \frac{W_t}{s_t}\right)$$

Note that

$$\begin{aligned} & E_0\sum_{t=0}^{\infty}\beta^t U_{Ct}\frac{(1-\tau_t^d)}{(1+\tau_t^c)}\frac{D_t}{W_t}\frac{w_t}{R_{t+1}} \\ = \quad & E_0\sum_{t=0}^{\infty}\beta^t U_{Ct}\frac{(1-\tau_t^d)}{(1+\tau_t^c)}\frac{w_t}{R_{t+1}}\left(\alpha y_t^S\left(\frac{1}{w_t^S} - \frac{1}{s_t}\right) + (1-\alpha)\, y_t^F\frac{1}{\theta-1}\frac{1}{s_t}\right) \end{aligned}$$

The pricing rule of the sticky firms implies that

$$E_{t-1}\left[u_{Ct}\frac{w_t(1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)}\frac{1}{w_t^S}y_t^S\right] = E_{t-1}\left[\frac{\theta}{\theta-1}u_{Ct}\frac{w_t(1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)}\frac{1}{s_t}y_t^S\right]$$

Thus

$$E_0\left[u_{Ct}\frac{w_t(1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)}\frac{1}{w_t^S}y_t^S\right]=E_0\left[u_{Ct}\frac{w_t(1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)}\frac{\theta}{\theta-1}\frac{1}{s_t}y_t^S\right]$$

so, replacing above

$$\begin{aligned}&E_0\sum_{t=0}^{\infty}\beta^t U_{Ct}\frac{(1-\tau_t^d)}{(1+\tau_t^c)}\frac{w_t}{R_{t+1}}\frac{D_t}{W_t}\\ =\ &E_0\sum_{t=0}^{\infty}\beta^t U_{Ct}\frac{(1-\tau_t^d)}{(1+\tau_t^c)}\frac{w_t}{R_{t+1}}\frac{1}{s_t}\left(\frac{1}{\theta-1}\right)\left(\alpha y_t^S+(1-\alpha)\,y_t^F\right)\end{aligned}$$

Thus, the implementability conditions become

$$E_0\sum_{t=0}^{\infty}\beta^t U_{Ct}\left(C_t-\frac{w_t(1-\tau_t^n)}{R_{t+1}(1+\tau_t^c)}N_t-\frac{(1-\tau_t^d)}{(1+\tau_t^c)}\frac{w_t}{R_{t+1}}\frac{1}{s_t}\left(\frac{1}{\theta-1}\right)\left(\alpha y_t^S+(1-\alpha)\,y_t^F\right)\right)=0$$

$$\frac{u_{1-Nt}}{u_{Ct}}=w_t\frac{(1-\tau_t^n)}{R_{t+1}(1+\tau_t^c)} \tag{31}$$

$$c_t^S+g_t^S=y_t^s \tag{32}$$

$$c_t^F+g_t^F=y_t^F \tag{33}$$

$$\frac{c_t^S}{c_t^F}=\left(\frac{w_t^F}{w_t^S}\right)^{-\theta}$$

$$\frac{g_t^S}{g_t^F}=\left(\frac{w_t^F}{w_t^S}\right)^{-\theta}$$

$$C_t=\left[\alpha c_t^{S\frac{\theta-1}{\theta}}+(1-\alpha)\,c_t^{F\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},\theta>1. \tag{34}$$

$$G_t=\left[\alpha g_t^{S\frac{\theta-1}{\theta}}+(1-\alpha)\,g_t^{F\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},\theta>1. \tag{35}$$

$$\frac{1}{w_t}=\left[\alpha\left(\frac{1}{w_t^S}\right)^{(1-\theta)}+(1-\alpha)\left(\frac{1}{w_t^F}\right)^{(1-\theta)}\right]^{\frac{1}{1-\theta}}$$

$$\alpha y_t^S + (1-\alpha)\, y_t^F = s_t N_t \tag{36}$$

$$w_t^F = \frac{\theta - 1}{\theta} s_t$$

$$E_{t-1}\left[u_{Ct}\frac{w_t(1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)}\frac{1}{w_t^S}y_t^S\right] = E_{t-1}\left[\frac{\theta}{\theta-1}u_{Ct}\frac{w_t(1-\tau_t^d)}{R_{t+1}(1+\tau_t^c)}\frac{1}{s_t}y_t^S\right]$$

Lemma 3 *In a competitive equilibrium with taxes and* $\alpha \in (0,1)$ $w_t^S \neq w_t^F \Leftrightarrow C_t + G_t < s_t N_t$*, and* $w_t^S = w_t^F \Leftrightarrow C_t + G_t = s_t N_t$.

Corollary 4 *For any* $0 < \alpha < 1$*, the sets of allocations that can be implemented with* $w_t^S = w_t^F$ *and with* $w_t^S \neq w_t^F$ *are mutually exclusive.*

Proof:

$$c_t^S + g_t^S = y_t^s$$
$$c_t^F + g_t^F = y_t^F$$
$$\alpha y_t^s + (1-\alpha)\, y_t^F = s_t N_t$$

As $\frac{\theta-1}{\theta} < 1$,

$$\begin{aligned}
\left[\alpha\left(c_t^S\right)^{\frac{\theta-1}{\theta}} + (1-\alpha)\left(c_t^F\right)^{\frac{\theta-1}{\theta}}\right] &< \left[\alpha c_t^S + (1-\alpha)\, c_t^F\right]^{\frac{\theta-1}{\theta}} \text{ iff } w_t^S \neq w_t^F \\
\left[\alpha\left(c_t^S\right)^{\frac{\theta-1}{\theta}} + (1-\alpha)\left(c_t^F\right)^{\frac{\theta-1}{\theta}}\right] &= \left[\alpha c_t^S + (1-\alpha)\, c_t^F\right]^{\frac{\theta-1}{\theta}} \text{ iff } w_t^S = w_t^F
\end{aligned}$$

Similarly

$$\begin{aligned}
\left[\alpha\left(g_t^S\right)^{\frac{\theta-1}{\theta}} + (1-\alpha)\left(g_t^F\right)^{\frac{\theta-1}{\theta}}\right] &< \left[\alpha g_t^S + (1-\alpha)\, g_t^F\right]^{\frac{\theta-1}{\theta}} \text{ iff } w_t^S \neq w_t^F \\
\left[\alpha\left(g_t^S\right)^{\frac{\theta-1}{\theta}} + (1-\alpha)\left(g_t^F\right)^{\frac{\theta-1}{\theta}}\right] &= \left[\alpha g_t^S + (1-\alpha)\, g_t^F\right]^{\frac{\theta-1}{\theta}} \text{ iff } w_t^S = w_t^F
\end{aligned}$$

Then, using (34) and (35)

$$\begin{aligned}
C_t + G_t &< \left[\alpha c_t^S + (1-\alpha)\, c_t^F\right] + \left[\alpha g_t^S + (1-\alpha)\, g_t^F\right] \text{ iff } w_t^S \neq w_t^F \\
C_t + G_t &= \left[\alpha c_t^S + (1-\alpha)\, c_t^F\right] + \left[\alpha g_t^S + (1-\alpha)\, g_t^F\right] \text{ iff } w_t^S = w_t^F
\end{aligned}$$

or, using (32), (33) and (36)

$$\begin{aligned} C_t + G_t &< \alpha y_t^S + (1-\alpha)\, y_t^F = s_t N_t \text{ iff } w_t^S \neq w_t^F \\ C_t + G_t &= \alpha y_t^S + (1-\alpha)\, y_t^F = s_t N_t \text{ iff } w_t^S = w_t^F \end{aligned}$$

This lemma implies that the aggregates when $w_t^S \neq w_t^F$ do not belong to the production possibilities frontier, while aggregates when $w_t^S = w_t^F$ do[4].

Proposition 5 *If the social welfare function is increasing on C_t and decreasing on N_t, the optimal allocation will exhibit $w_t^S = w_t^F$.*

Proof. When $w_t^S = w_t^F$,the equilibrium conditions for the aggregates $\{C_t, N_t,\}_{t=0}^{\infty}$ are given by (31) , the intertemporal budget constraint that, using equation (36) becomes

$$E_0 \sum_{t=0}^{\infty} \beta^t U_{Ct} \left(C_t - \frac{w_t(1-\tau_t^n)}{R_{t+1}(1+\tau_t^c)} N_t - \frac{(1-\tau_t^d)}{(1+\tau_t^c)} \frac{w_t}{R_{t+1}} \left(\frac{1}{\theta - 1} \right) N_t \right) = 0$$

and the aggregate consistency condition

$$C_t + G_t = s_t N_t$$

When $w_t^S \neq w_t^F$, the equations are the same except for the aggregate consistency condition that becomes

$$C_t + G_t < s_t N_t$$

By setting $w_t^S = w_t^F$, the planner maximizes aggregate consumption given a value for aggregate labor. ■

Thus, if, for instance, we assume a benevolent government, then the optimal allocation will exhibit no distortion between the different varieties. This is an application of the well known result of Diamond and Mirlees, since the varieties are intermediate inputs that enter in a constant returns to scale fashion into the "production" of the final good.

Note also that once we focus on allocations that satisfy $w_t^S = w_t^F$, the equilibrium conditions become independent of α. The following corollaries follow.

[4] Note that by setting $w_t^F = w_t^S$, we obtain the same allocations as in the particular cases of $\alpha = 0$ (flexible prices) and $\alpha = 1$ (SIA prices) analyzed in the previous section.

Corollary 6 *The optimal allocation is independent of α.*

Corollary 7 *(Adao, Correia and Teles,(1999)) The optimal monetary policy does not depend on α. There is a unique optimal monetary policy that involves setting the money supply so that the price does not react to contemporaneous information and $w_t^S = w_t^F$*

4 Conclusions

The purpose of this paper is to discuss optimal fiscal and monetary policy in economies with different price setting behavior. The main contribution of our paper is to study the relationship between policies and allocations in the dynamic Ramsey tradition, so the optimal fiscal and monetary policies are analyzed in an integrated approach. However, we focus our analysis on feasible sets, so our results are independent of government objectives.

The economies that we compare share several features: monopolistic competition, no capital and money introduced via a cash-in-advance constraint. We then compare economies with differing degrees of price stickiness, measured as the share of firms that are restricted to set the price in advance.

The main finding of our paper is that the set of feasible allocations is invariant to the price setting behavior of firms. Thus, we conclude, sticky prices are redundant. In addition, we derive two simple rules for the conduct of optimal policy: fiscal policy should be set as if all prices were flexible, and monetary policy must replicate the flexible prices allocation, as if all prices were sticky.

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