Tracking the New Economy:
Using Growth Theory to Detect Changes in Trend Productivity†

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October 29, 2003

Abstract

The acceleration of productivity since 1995 has prompted a debate over whether the economy's underlying growth rate will remain high. In this paper, we draw on growth theory to identify variables other than productivity—namely consumption and labor compensation—to help estimate trend productivity growth. We treat that trend as a common factor with two "regimes" high-growth and low-growth. Our analysis picks up striking evidence of a switch in the mid-1990s to a higher long-term growth regime, as well as a switch in the early 1970s in the other direction. In addition, we find that productivity data alone provide insufficient evidence of regime changes; corroborating evidence from other data is crucial in identifying changes in trend growth. We also argue that our methodology would be effective in detecting changes in trend in real time: In the case of the 1990s, the methodology would have detected the regime switch within one quarter of its actual occurrence according to subsequent data.

Keywords: Productivity Growth, Regime-Switching, Neoclassical Growth Model, Factor Model

JEL Codes: O4, O51, C32

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1. Introduction

Discerning the underlying trend in productivity growth has long been a goal of both policymakers and economists. At least since Solow's (1956) pioneering work on long-term growth, economists have understood that sustained productivity growth is the only source of long-term growth in living standards. It is also important for short-term policy analysis, as any assessment of "output gaps" or growth "speed limits" ultimately derives from some understanding of the trend. It is widely believed, for example, that the difficulty of detecting a change in trend growth contributed significantly to the economic instability of the 1970's, as policymakers were unaware of the slowdown in productivity growth for many years. Only much later were they able to date the slowdown at approximately 1973.[1] This resulted in overestimating potential GDP (at least so the conventional wisdom goes) and setting interest rates too low, and double-digit inflation followed not long after.

On a quarterly basis, however, measured productivity growth is extremely volatile. Over the postwar period the average quarterly growth rate of nonfarm productivity has been 2.2 percent (annualized), but with a standard deviation of 3.9 percent. Moreover, the volatility is not confined to high frequency fluctuations. Productivity growth is also cyclical, typically declining at the onset of a recession and rising during a recovery. Thus it is often only years after the fact that any change in its long-term trend will be apparent.

In recent years, attention has turned once again to productivity because of speculation that its trend growth rate may be picking up again. The growth rate of nonfarm output per hour increased by approximately 1 percent beginning in 1996 relative to the period 1991-1995, and by about 1.3 percent relative to 1973-1995. The acceleration of productivity puts its growth rate during this 5-year period close to where it was during the most recent period of strong growth, from roughly 1948 to 1973. This has provoked a debate over whether we can expect an extended period of more rapid productivity growth. Robert Gordon (2000), for example, attributes about half of the acceleration to a "cyclical" effect. Others (e.g. Stiroh, 2002) find evidence that productivity growth has spilled over into other sectors through capital deepening.

Much of the difficulty in evaluating the arguments in this debate relates to the issue of separating permanent and transitory movements in the data, particularly toward the end of a

[1] See, for example, Sims (2001), who writes that during the 1970's, "unemployment rose and inflation rose because of real disturbances that lowered growth. . . . Since such 'stagflation' had not occurred before on such a scale, they faced a difficult inference problem, which it took them some years to unravel."

sample, or in real time before subsequent data shed light on any given episode. In this paper we attack this problem by drawing on neoclassical growth theory to help identify variables other than productivity itself—namely consumption and labor compensation—that should help to estimate the trend in long-term growth. We treat that trend as a stochastic process whose mean growth rate has two "regimes," high and low, with some probability of switching between the two at any point in time. We model the business cycle as a second process common to all of the variables in the analysis, also with two regimes of its own, based on the so-called "plucking" model of Friedman (1969,1993).

There are several advantages to this approach. First, we show that aggregate productivity data alone do not provide as clear or as timely a signal of changes in trend growth as does the joint signal from the series we examine. Second, we do not have to choose break dates *a priori*, as we let the data speak for themselves. Third, the model not only provides information about when regime switches occurred, it also provides estimates of how long the regimes are likely to last. This last property contrasts with even the most sophisticated structural break tests, such as those described by Bai et al. (1998) and Hansen (2001).

Also worth emphasizing is that the use of theory enables us to restrict our analysis to a low dimensional system of variables and to impose parameter restrictions in the estimation procedure. Thus our approach contrasts with atheoretical applications of factor models that involve a large number of variables or that do not place theory-based restrictions on estimated coefficients.[2] Here there are both advantages and disadvantages: Our model may not provide as tight a fit to the data as would a more eclectic approach, but it is likely to be more robust to structural changes in the economy.

Our analysis picks up striking evidence of a switch in the mid-1990's to a higher long-term growth regime, some 25 years after a switch from higher to lower growth in the early 1970's. While these findings themselves may not be surprising, our results point to further conclusions as well. First, one could not decisively conclude that there was a return to a higher growth regime on the basis of productivity data alone, or even with the addition of a second variable to control for the business cycle. Only the corroborating evidence from other cointegrated series can swing the balance strongly in favor of a regime switch. Second, our approach appears effective in detecting changes in trend in real time: In the case of the 1990s, the

[2] See for example, Stock and Watson (1989, 2002), Kim and Piger (2002).

methodology would have detected the regime switch within one quarter of its actual occurrence according to subsequent data.

The remainder of the paper proceeds as follows. Section 2 motivates the theoretical restrictions based on a variant of the neoclassical growth model. Section 3 describes the statistical model and the data. Section 4 presents the parameter estimates and the estimated common factors, and compares the findings with various alternative specifications. Section 5 concludes.

2. Implications of the Neoclassical Growth Model

2.1. Background

Over forty years ago, Nicholas Kaldor (1961) established a set of stylized facts about economic growth that have guided empirical researchers ever since. His facts are: (1) labor and capital's income shares are relatively constant; (2) growth rates and real interest rates are relatively constant; (3) the ratio of capital to labor grows over time, and at roughly the same rate as output per hour, so that the capital-output ratio is roughly constant. To these facts, more recent research has added another: (4) measures of work effort show no clear tendency to grow or shrink over time on a per capita basis. The important implication of this additional fact is that wealth and substitution effects roughly offset each other. This means, for example, that a permanent change in the level of labor productivity has no permanent impact on employment.

Of course, closer inspection suggests that none of the above "stylized facts" is literally true. Indeed the premise of much work on U.S. productivity is that productivity growth was systematically higher from 1948-1973 than it was over the subsequent 20-plus years. We will also see that work effort per capita has been anything but stationary since World War II, and that there have been large shifts in capital-output ratios. But Kaldor's facts still provide a starting point for modeling economic growth, particularly since there may be reasonable explanations for departures from those facts that do not require discarding the framework that they inspired. We begin in this section with a neoclassical growth model consistent with the Kaldor facts, but then relax all but the first fact. We then examine the implications of the generalized model for empirical efforts to assess growth trends.

2.2. A Growth Model with Nonstationary Labor Supply

In our analysis we allow for exogenous changes in preferences between consumption and leisure to account for long-term movements in work effort (as measured by hours) that show up in the data. Specifically, let C denote aggregate consumption, Y aggregate output, N population (measured in person-hours and growing at rate n), K capital, X effective labor per unit of labor input, and L aggregate labor input (in hours). We also assume that there is a production function

$$Y_t = K_{t-1}^{\alpha}\left(L_t A_t\right)^{1-\alpha} \tag{1}$$

where A represents permanent technological progress and has a unit root. Preferences are defined in terms of a present discounted value of single-period utility

$$U(C_t / N_t, \ell_t) = \Lambda_t \ln\left(C_t / N_t\right) + v\left(1-\ell_t\right), \tag{2}$$

where $\ell \equiv L/N$ represents the proportion of available hours devoted to work. The marginal rate of substitution between consumption and leisure is $\Lambda^{-1}(C/N)v'(1-\ell)$, where v is a concave differentiable function, v' is strictly decreasing, and Λ is a taste parameter that can shift over time. Note that while Λ is modeled as a preference shock, it could reflect taxation or other labor market distortions (see Mulligan, 2002), as well as demographic shifts.

We will also allow Λ to be non-stationary. For the sake of exposition we will specify it as a unit root process with zero drift, though it could also be a deterministic function of time, or a combination of the two. This is in recognition of the fact that there is significant low-frequency variation of work effort in postwar U.S. data, as seen in the behavior of per capita hours in the non-farm sector since 1947 (Figure 1).[3] Apart from the large middle frequency fluctuations associated with business cycles, there are clear secular changes. There was a decline of roughly 15 percent between the end of World War II and the early 1960s, followed by an increase of about 20 percent from the mid-1960s to the present. Studies that have assumed that aggregate per capita output, along with consumption and investment, have the same permanent component as (i.e. are cointegrated with) labor productivity implicitly assume that hours per capita is a stationary time series.[4] We argue below that the stochastic trend in per capita output is better described as two separate trend components, one demographic (i..e. labor supply), the other technological (i.e. labor demand), and that by doing so we are able to identify regime shifts in the latter that otherwise would be obscured by movements in the former.

[3] Per capita variables are obtained by dividing by the total resident population, averaging the monthly data to obtain a quarterly series, and extrapolating to extend the series beyond 2001. Note that the share of non-farm to total employment has varied only slightly over the sample period and is not responsible for the low frequency movements visible in Figure 1.

We assume that the economy evolves as if a planner solves the following problem:

$$\max E_t\left\{\sum_{t=0}^{\infty}\beta^t U(C_t/N_t, L_t/N_t)\right\}, \tag{3}$$

subject to

$$C_t + P_t I_t \le K_{t-1}^{\alpha}\left(L_t A_t\right)^{1-\alpha} \tag{4}$$

$$K_t \le I_t + \left(1-\delta\right)K_{t-1}, \tag{5}$$

where I_t is investment (in efficiency units), P_t the price of investment goods in terms of consumption, and β is a discount factor. We assume that P_t, which is inversely related to the efficiency level of new investment, varies exogenously, with an average growth rate of $-\nu$. A_t, which measures the level of disembodied technical progress, is also exogenous, with an average growth rate of *g*.

Let $Z_t = L_t A_t P_t^{-\alpha/(1-\alpha)}$. Also, let *W* denote labor compensation, and note that $W_t = \left(1-\alpha\right)K_{t-1}^{\alpha}\left(L_t X_t\right)^{1-\alpha}$. It is straightforward to show that the variables $c_t \equiv C_t / Z_t$, $y_t \equiv Y_t / Z_t$, $w_t \equiv W_t / Z_t$, $P_t k_{t-1} \equiv P_t K_{t-1} / Z_t$, and $h_t \equiv h\left(\ell_t, \Lambda_t\right) = \Lambda_t^{-1} v'\left(1-\ell_t\right)\ell_t$ are stationary along a balanced growth path. Thus the economy will grow on average at the rate $g + \nu\alpha/\left(1-\alpha\right)$. This rate reflects the two components of technical progress, disembodied (*g*) and embodied (ν). Note, however, that aggregate or per capita quantities such as *Y* or *Y*/*N* will have a common stochastic trend made up of two components, the trend in *L* (arising from the nonstationarity of the labor supply trend Λ) and the trend in technology, which we will denote by $X_t = A_t P_t^{-\alpha/(1-\alpha)}$. Aggregate variables normalized by hours, on the other hand, will have a common trend that is stripped of the labor supply component and is driven only by disturbances to the composite technology trend *X*.

We also see that taking embodied progress into account illustrates that capital *K* and investment *I* do not in general share a common trend with output and consumption, as has been assumed in a number of related studies.[5] If we could observe the relative price of capital goods *P*, or, equivalently, the rate of embodied progress, then *PK* would be cointegrated with *Y*, *C*, and *W*. While annual estimates of capital equipment prices (often assumed to be where progress is

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[4] For example, Bai, Lumsdaine, and Stock (1998).
[5] See, for example, King, et al. (1991), Kim and Piger (2002).

embodied) are available (see Cummins and Violante, 2002), there remain a number of other issues with capital that make incorporating it into this cointegration framework problematic.[6]

The result that *Y*/*L*, *W*/*L*, and *C*/*L* have a common permanent component is robust to other generalizations of the model, provided they are consistent with the same balanced growth path, i.e. so long as they result in only transitory deviations from the steady state. For example, the variables may be measured with error, or may have transitory dynamics that reflect imperfect information, adjustment costs, or other rigidities. So long as such deviations (which we will allow for in the estimation) are transitory, the four ratios should be cointegrated.

We should note that our focus on labor productivity rather than on total factor productivity (TFP) is intentional. Anything that *permanently* raises output per hour will enter our estimated "technology" component, whether it be capital deepening, growth in human capital, or TFP. Of course growth theory suggests that capital deepening is unlikely to be an *independent* contributor to sustained growth. Rather, it is a symptom of underlying technological progress and/or growth in human capital. Thus, for example, the capital deepening of the late 1990's, much of which can be attributed to computer and related high-tech investment, ultimately reflects TFP in the sectors that produce that equipment.[7]

One final issue: Can per capita hours (ℓ) really be non-stationary? Certainly a bounded variable cannot follow a simple linear process with a unit root. In our model, however, it is log(Λ), not ℓ, that follows such a process. Consequently, only a transformation of ℓ does so. For example, if $v(\ell) = \log(\ell)$, then $\log(\ell/(1-\ell))$ is cointegrated with $\log\Lambda$ and ranges over the entire real line, and a permanent shift in $\log\Lambda$ changes $\log\ell$ by approximately $d\left(\log\Lambda\right)\left(1-\ell\right)^2/\ell$. In any case, the long-run implications of non-stationary ℓ are not the issue. Rather, if ℓ exhibits non-stationary behavior in the sample we have, i.e. post-war U.S. data, then treating it as stationary may give misleading results, as we shall see.

2.3. Summary

The upshot of this foray into a neoclassical stochastic growth model is that under plausible assumptions, labor compensation per hour, consumption divided by hours, and output per hour should have a common permanent component. It is natural to think of this component

[6] For example, depreciation rates have not remained constant over time as the composition of the capital stock has evolved.

[7] See, for example, Gordon (2000), Stiroh (2002).

as "technology," i.e. that its driving force is technological progress. Hours of work per capita may have its own permanent component that is unrelated to technology, but instead is driven by policy and demographics. Normalizing *Y*, *W*, and *C* by hours rather than by population should, however, have the effect of neutralizing the impact of permanent shifts in *L*/*N*.

In effect, the theory tells us that as far as low frequency behavior is concerned, we can divide output per capita *Y*/*N* into *Y*/*L* (labor demand) and *L*/*N* (labor supply). We refer to *Y*/*L* as labor demand because with Cobb-Douglas technology it is proportional to the marginal product of labor (MPL). Standard assumptions about preferences and technology imply that long-run labor demand is horizontal, while long-run labor supply is vertical. Thus a permanent shift in, say, labor supply (as represented by *L*/*N*) does not lead in the long run to any change in the other quantity (MPL). Similarly, a permanent increase in *Y*/*L* (i.e. in the MPL), should have no permanent impact on *L*/*N*.

3. A Common Factor Model

3.1. The Regime-Switching Dynamic Factor Model

Our estimation strategy draws upon the regime-switching dynamic factor model recently proposed by Kim and Murray (2002) and Kim and Piger (2002), among others. The essence of this approach is to examine a number of related economic time series and to use their comovements to identify two shared factors: a common permanent component and a common transitory component. In addition, we follow these authors in allowing for regime changes in both components. The regime-switching aspect of the model has several attractive features. First, in the permanent component it allows us to account for sustained changes in trend growth without making the growth process itself nonstationary. Second, in the transitory component it allows for asymmetries in business cycles that others using this methodology have found significant.[8] These regime changes in the transitory component capture the idea proposed in Friedman's (1964, 1993) "plucking model" model that economic fluctuations are largely permanent during expansions and transitory during recessions. Third, Perron (1989) has argued that regime-changes can cause testing procedures to indicate non-stationarity, i.e. failure to allow for them could lead one erroneously to infer the presence of a unit root. Finally, the regime-switching specification is a straightforward way of estimating both the timing and expected duration of periodic changes in the processes generating the two components.

[8] See, for example, Kim and Piger (2002), Kim, Piger, and Startz (2002), and Beaudry and Koop (1993).

If the three "per hour" variables *Y*/*L*, *W*/*L*, and *C*/*L* have some independent sources of measurement error, there may be some gain to using all three to extract the best estimate of the common permanent component.[9] Consequently we adopt a multivariate, common factor approach in estimating the single permanent component of the three variables. We also use that approach to estimate a common transitory component, although without the benefit of any theoretical restrictions.

Following Kim and Murray (2002), we can describe the regime-switching dynamic factor model as follows.[10] Suppose we consider a number of time series indexed by *i*. Let Q_{it} denote logarithm of the *i*th individual time series. It is assumed that the movements in each series are governed by the following process:

$$Q_{it} = \gamma_i X_t + \lambda_i x_t + z_{it}\,, \tag{6}$$

where X_t denotes a permanent component that is common to all series, x_t denotes a common transitory component, and z_{it} is an idiosyncratic error term. The parameter γ_i (the permanent "factor loading") indicates the extent to which the series moves with the common permanent component. Similarly, the parameter λ_i indicates the extent to which the series is affected by the transitory component.

The common permanent component is assumed to be difference stationary, but subject to the type of regime-switching proposed by Hamilton (1989) in which there are periodic shifts in its growth rate:

$$\Delta X_t = \mu(S_{1t}) + \phi_1 \Delta X_{t-1} + \ldots + \phi_p \Delta X_{t-p} + \upsilon_{t,} \upsilon_t \sim iidN(0,1) \tag{7}$$

$$\mu(S_{1t}) = \begin{cases} \mu_0 & if \quad S_{1t} = 0 \\ \mu_1 & if \quad S_{1t} = 1 \end{cases}, \tag{8}$$

$$\Pr[S_{1t} = 0 \mid S_{1,t-1} = 0] = q_{1,} \qquad \Pr[S_{1t} = 1 \mid S_{1,t-1} = 1] = p_1 \tag{9}$$

where S_{1t} is an index of the regime for the common permanent component. The transition probabilities p_1 and q_1 indicate the likelihood of remaining in the same regime. Under these

[9] This is not to say that the information is completely independent, and indeed the source of errors in one series may be present in the other series as well. For example, an inaccurate price deflator could result in common mismeasurement across multiple series. Nonetheless, the theory suggests that considering these series together may provide better information about underlying trends than consideration of any of them in isolation.

[10] Additional details are in the Appendix.

assumptions, the common permanent component X_t grows at the rate $\mu_0/(1-\phi_1-...-\phi_p)$ when S_{1t}= 0, and at the rate $\mu_1/(1-\phi_1-...-\phi_p)$ when S_{1t}=1.

The common transitory component x_t is stationary in levels, but also subject to regime-switching:

$$x_t = \tau(S_{2t}) + \phi_1^* x_{t-1} + \phi_2^* x_{t-2} + ... + \phi_p^* x_{t-p} + \varepsilon_t, \varepsilon_t \sim iidN(0,1) \tag{10}$$

$$\tau(S_{2t}) = \begin{cases} 0 & if \quad S_{2t} = 0 \\ \tau & if \quad S_{2t} = 1 \end{cases}, \tag{11}$$

$$\Pr[S_{2t} = 0 \mid S_{2,t-1} = 0] = q_2 \qquad \Pr[S_{2t} = 1 \mid S_{2,t-1} = 1] = p_2 \tag{12}$$

where S_{2t} is an index of the regime for the common transitory component, with transition probabilities p_2 and q_2. The parameter τ represents the size of the "pluck," with $\tau < 0$ implying that the common transitory component is plucked down during a recession.

The permanent and transitory regimes are assumed to be independent of each other. While the two regimes are not directly observable, it is nevertheless possible to estimate the parameters of the model and to extract estimates of the common components.[11] An important byproduct from the estimation procedure is that we can draw inferences about the likelihood that each common component is in a specific regime at a particular date. The restriction of unit variance for the error terms of the two processes is an identifying restriction, since X and x are of indeterminate scale.

Finally, the idiosyncratic components are assumed to have the following structure:

$$z_{it} = \psi_{i1} z_{i,t-1} + \psi_{i2} z_{i,t-2} + ... + \psi_{ip} z_{i,t-p} + \eta_{it,} \ \eta_{it} \sim iidN(0,\sigma_i^2), \ i = 1,...,4 \tag{13}$$

where all innovations in the model are assumed to be mutually and serially uncorrelated at all leads and lags. We assume that z_{it} is stationary, i.e. the roots of $1-\Psi_i(L)$ all lie inside the unit circle, so these are transitory shocks to the levels of the variables.

To relate all of this back to the growth model from Section 2 of the paper, the permanent component X corresponds to the stochastic trend term from the growth model, which we saw is common to *Y*/*L*, *C*/*L*, and *W*/*L*. The theory implies, therefore, that the factor loadings on the permanent component should satisfy $\gamma_1 = \gamma_2 = \gamma_3$. The transitory component *x* reflects the direct impact of transitory disturbances as well as transition dynamics from all shocks, but only to the extent they are linearly related across all four series. The idiosyncratic component includes what is left of the transitory movements after the common component is subtracted. It will include measurement error and model noise. For example, a literal reading of the model is that $W/L = (1-\alpha)Y/L$, so the two series should be perfectly correlated. But *W* and *Y* are measured (to some extent) independently, and with error, and moreover the assumption of a Cobb-Douglas production function with constant parameters is undoubtedly not literally accurate. These factors will also contribute to idiosyncratic variation.

3.2. Data

Our data consist of quarterly observations of non-farm sector output, labor productivity, real compensation per hour (nominal compensation relative to the nonfarm output deflator), and hours of work. The non-farm sector was chosen because of the availability of consistent data for all of these series. We also use aggregate data on real consumption expenditures. While a series that converted expenditures on durables to service flows would be preferable, at the time of these computations such a series was not available for the whole sample period, and where it was it exhibited very similar behavior to total consumer expenditures. Another issue with using this consumption series is that it is for the entire U.S. economy, whereas the other series represent the nonfarm sector only. This will only create a problem if there are significantly different trends, but the results did not appear to be sensitive to these choices. Unless stated otherwise, all variables are in logarithms, multiplied by 100, with first differences interpreted as quarterly growth rates in percent. For calculating per capita quantities we used the resident population (interpolated from annual to quarterly).

As we have seen, the growth model from Section 2 implies that *C*/*L*, *Y*/*L*, *K*/*L*, and *W*/*L* have a common permanent component that corresponds to *X*, the technology factor, and are unaffected by any stochastic trend in labor supply. There is no analogous set of variables that can be used to help estimate a stochastic trend in labor supply—if *L*/*N* is nonstationary, theory

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[11]Kim and Murray (2002) discuss how the regime-switching dynamic factor model can be cast in a state-space representation and estimated using the Kalman filter. More details of how we applied their methods to the present study are in Kahn and Rich (2003).

provides no other variables with which it should be cointegrated.[12] Thus the common factor model will not be of direct use for estimating it. But since higher frequency movements in hours of work would presumably be useful for capturing the transitory component, we simply detrend the hours series using the Hodrick-Prescott (1980) filter and include it in our system with a zero loading on the permanent component.[13] Thus our benchmark specification has $Q_1 = Y/L$, $Q_2 = W/L$, $Q_3 = C/L$, and $Q_4 = \hat{L}$, where the "^" indicates the the H-P filtered series.

That *Y/L*, *W/L*, and *C/L* are cointegrated is illustrated by Figure 2, which plots *Y/C*, *Y/W* and *Y/K*. *Y/C* and *Y/W* appear stationary, which supports the notion that *Y*, *C*, and *W* have a single common permanent component, from which it would follow that *Y/L*, *C/L*, and *W/L* do as well (to the extent they have any permanent component at all). As mentioned earlier, it is equally clear that *Y/K* is not stationary, which is consistent with the presence of a stochastic trend in *P*.

To examine the cointegration properties of *Y/L*, *W/L*, and *C/L* more rigorously we conducted multivariate unit root tests based on the procedure developed by Johansen (1991, 1995). Table 1 describes the results, based on quarterly data over the sample period 1947:Q1 to 2002:Q2, including a constant in the cointegrating relationship. The combination of trace and maximal eigenvalue tests suggests that there are two cointegrating equations, implying a single common trend, as the theory suggests. Letting $Q = (Q_1\, Q_2\, Q_3)$, and letting $\beta^i = (\beta_1^i\, \beta_2^i\, \beta_3^i)$, i = 1,2 denote the cointegrating vectors, we estimated the two cointegrating relationships $Q'\beta^1$ and $Q'\beta^2$ and found:

$$\begin{aligned} \beta^1 &= (1.0 \quad 0.0 \quad -1.027)' \\ \beta^2 &= (0.0 \quad 1.0 \quad -0.986)', \end{aligned} \qquad (14)$$

where we normalized the equations with respect to β_1^1 and β_2^2. Because the estimates of β_3 differ from -1, we also conducted tests of the null hypothesis that the two cointegrating equations are $(1.0 \quad 0.0 \quad -1.0)$ and $(0.0 \quad 1.0 \quad -1.0)$. The calculated value of the χ^2 statistic with two degrees of freedom is 8.92, with an associated p-value of 0.0115. Thus although the results confirm the theory qualitatively, the quantitative implications that $\beta_3 = -1$ in both vectors

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[12] There could be another demographic variable besides population that would be cointegrated with *L*, such as population over 16, or even a policy variable such as an average marginal tax rate. We have not pursued this yet.
[13] Using the first difference of hours (in logs) rather than the H-P filtered series yielded very similar results.

is rejected. This does not necessarily mean, however, that we will reject $\gamma_1 = \gamma_2 = \gamma_3$ in (6), which is the real implication of the theory, and one that we will test when we estimate the model.

Initial estimates of more general specifications suggested that the common permanent component should include one lagged value of ΔX_t, the common transitory component should include two lagged values of x_t, and that the idiosyncratic component should include one lagged values of z_{it} for each series. We restricted the estimated factor loadings on the permanent component for productivity, real compensation per hour, and consumption per hour to be equal (i.e. we set $\gamma_1 = \gamma_2 = \gamma_3$), and set the value of the permanent factor loading for detrended hours, γ_4, equal to zero.

We also considered two alternative specifications. In one, we eliminated the regime shift in the transitory component, thereby imposing symmetry on business cycles. We will refer to this as the "no pluck" specification. In the other, we used per capita rather than per hour variables, i.e. we set $Q_1 = Y/N$, $Q_2 = W/N$, $Q_3 = C/N$. We will refer to this as the "per capita" model, as opposed to the benchmark or "per hour" specification. As discussed in more detail below, if L/N were stationary, the per capita and per hour specifications would yield similar estimates of the permanent component.

One final modeling issue relates to the synchronization of the data. The three trending variables were selected primarily on the basis of their having a common permanent component. They are not, however, necessarily "coincident indicators" with respect to the transitory component. In theory it would be possible to allow for a more general lead/lag structure in our system, but this would greatly increase the number of parameters to estimate. As an alternative, we first examined the cross-correlations of the four series. We found that the first two variables (productivity and labor compensation per hour) both tended to lead the other two series by about three quarters. To capture this asynchronization in the estimation we lagged Q_{1t} and Q_{2t} by three quarters in the system described above.[14]

4. Results

4.1. Parameter Estimates

[14] A minor drawback to this procedure is that the last three observations of variables 1 and 2 are not used in estimating the parameters of the model. We do incorporate them in the period-by-period assessments of the state variables, as described below.

The first column of Table 2 provides the parameter estimates for our benchmark model with the four variables as described above.[15] The data cover 1947:Q1-2002:Q2, though because the first two variables are lagged three quarters, their growth rates cover the period 1947:Q2-2001:Q3, while the growth rates of consumption and detrended hours variables run from 1948:Q1-2002:Q2 .

The model yields precise estimates of most of the parameters of interest: The factor loadings on both the permanent and transitory components, the transition probabilities, and the shift parameters associated with the regimes (μ_0, μ_1, τ) all enter significantly. The difference between the low- and high-growth regimes works out to be $\gamma(\mu_0 - \mu_1)/(1-\phi) = 0.365$. This corresponds to approximately 1.46 percent on an annualized basis, very close to the difference between the 1948-73 and 1973-96 growth rates of productivity.

The transition probabilities for the permanent regimes imply an expected duration of $1/(1-p_1) = 25$ years for the high-growth regime, and 17.9 years for the low-growth. They also imply that the unconditional probability of being in the high growth regime is $(1-q_1)/(2-p_1-q_1) = 0.593$, suggesting that the economy was in the high growth regime on the order of 60 percent of the time between 1947 and the present. The AR coefficient on the permanent component is estimated to be –0.440, suggesting that growth innovations in one quarter tend to get partially offset in the following one.

The transitory process is estimated to be a "hump-shaped" autoregressive process typical of the business cycle (see, e.g., Blanchard, 1981), but with a statistically significant negative pluck, i.e. a relatively short-lived reduction in the level of the transitory component presumably associated with recessions. The magnitude of this downward shift is related to $\lambda_i\tau$. Productivity, for example, would decline by $\lambda_1\tau = 0.597$, which corresponds to about 2.4 percent annualized. The expected duration of the pluck regime is given by $1/(1-q_2)$, or 2.34 quarters. The transition probabilities imply that the pluck regime occurs less than four percent of the time. Since about one-sixth of the quarters since 1947 have been in recessions, and the average recession has lasted roughly 3.5 quarters, these results suggest that not all recessions are associated with transitory regime shifts.

The four λ_i coefficients, which represent the factor loadings on the transitory process, are all estimated very precisely, and have the expected signs. Note that λ_3, the loading factor for

[15] Estimates of the idionsyncratic variances are omitted from Table 2 for the sake of brevity.

consumption/hours, is negative, reflecting the fact that hours are more cyclical than consumption. The positive estimates for the factor loadings on real hourly compensation and productivity indicate that those two variables are positively related to the transitory component, albeit leading by three quarters relative to the other two variables (since they enter the system lagged by three quarters).

Finally, we test the restriction $\gamma_1 = \gamma_2 = \gamma_3$ by estimating the unrestricted model and conducting a likelihood ratio test. The test statistic has a value of 0.525, and is asymptotically distributed as χ^2 with 2 degrees of freedom. The critical value for rejecting the hypothesis at a 10 percent significance level is 4.61. This suggests that the basic theoretical implications with respect to the common permanent component are confirmed by the data, notwithstanding the estimates of the cointegrating vectors described in Section 3.2.

4.2. Growth Regime Assessments

Before further describing the permanent and transitory components of productivity growth, however, it is instructive to examine the inferred probability of being in the high- or low-growth state over time. There are two ways to examine this. The first is in "real time": At each point in time, using only data through that point in time, what probability would one have assigned to being in the high-growth state? The second way is retrospectively: Given what we now know has happened through 2002:Q2, what can we say looking back over time about the likelihood of being in the high growth state? (Obviously the two assessments coincide at the end, i.e. as of 2002:Q2.)[16]

Even within the "real time" approach, two practical issues arise. First, there is the matter of data revisions. The historical series we have today have been revised numerous times over the years, so the data truncated at some date does not correspond to what anyone would actually have known. We ignore this problem in the present study, but plan to address it in subsequent work. Second, it is not practical to have rolling estimates of parameters, especially if regime shifts occur infrequently. Over a shorter sample few if any regime switches will be observed, making it impossible to estimate transition probabilities. Other parameters will also be difficult

[16] Hamilton (1994) refers to these two approaches as using a "zero-lag" versus "full-sample smoother."

to estimate precisely as well. Consequently in looking at real time assessments of the state vector we rely on parameter estimates from the full sample.[17]

There is also a technical issue related to the timing of the data series. Since ΔQ_1 and ΔQ_2 are lagged by three periods, if we were simply to assess the state variable given data through a given observation number, we would be ignoring the most recent three data points for those two series. For example, a standard application of the Kalman filter at date *t* would yield an assessment of the state given data through period *t*,

$$\hat{\xi}_{t|t} = E(\xi_t \mid Q_{1t}, Q_{2t}, Q_{3t}, Q_{4t}; Q_{1t-1}, Q_{2t-1}, Q_{3t-1}, Q_{4t-1}; \ldots) . \tag{15}$$

In terms of the actual underlying data in our study, however, this would be expressed as

$$E(\xi_t \mid \tilde{Q}_{1t-3}, \tilde{Q}_{2t-3}, \tilde{Q}_{3t}, \tilde{Q}_{4t}; \tilde{Q}_{1t-4}, \tilde{Q}_{2t-4}, \tilde{Q}_{3t-1}, \tilde{Q}_{4t-1}; \ldots) , \tag{16}$$

where the "~" denotes the variable indexed by its true time period. Fortunately it is relatively straightforward to "partially update" the state and regime assessments. This involves conditioning on three subsequent observations of Q_1 and Q_2 at each point in time to get

$$\begin{aligned} \hat{\xi}_{t|t+3'} &= E(\xi_t \mid Q_{1t+3}, Q_{2t+3}, Q_{3t}, Q_{4t}; Q_{1t+2}, Q_{2t+2}, Q_{3t-1}, Q_{4t-1.}; \ldots) \\ &= E(\xi_t \mid \tilde{Q}_{1t}, \tilde{Q}_{2t}, \tilde{Q}_{3t}, \tilde{Q}_{4t}; \tilde{Q}_{1t-1}, \tilde{Q}_{2t-1}, \tilde{Q}_{3t-1}, \tilde{Q}_{4t-1}; \ldots) \end{aligned} . \tag{17}$$

Here we use t+3′ to denote Q_1 and Q_2 observed through t+3, and Q_3 and Q_4 observed through time *t*. This simply undoes the staggering so that the information set is appropriately aligned. (Additional details on the partial updating procedure are provided in the Appendix.)

These two regime assessments are plotted in Figure 3. The vertical axis is the probability of being in the high-growth regime. The retrospective assessment presents a very clear picture: The economy was in a high-growth state until the early 1970's, followed by a roughly 20-year low-growth regime, followed by a switch back to high-growth in the second half of the 1990's. Perhaps the only surprise here is how unambiguous the assessment as of 2002:Q2 was. The probability that the economy was in the high-growth regime surpassed 0.95 in 1998:Q4, and has not subsequently fallen below 0.9. Given the 3-quarter lead for productivity, this dates the

[17] We experimented with rolling parameter estimates going back five years and found that the results were not significantly different.

acceleration of productivity growth at 1998:Q1. If we use a more lenient 0.5 threshold, we get the high-growth regime probability first exceeding this level in 1998:Q2, we would date the acceleration of productivity at 1997:Q3.

As the previous discussion makes clear, however, the retrospective estimates do not mean that as of 1998:Q2 we actually could have made such an optimistic assessment. With data only through 1998:Q2 the high-growth probability assessment would have been 0.45 rather than 0.65. But in fact the "real time" assessment lags the retrospective one by only about one quarter: by 1998:Q3 it is 0.63. Thus hindsight provides us in this instance with only a small advantage in dating the shift. On the other hand, Figure 3 makes clear that in earlier episodes there were a number of "false positives," post-1973 episodes when it appeared that productivity growth might shift into high gear but the higher growth was not sustained. Notwithstanding our attempt to control for the business cycle, these tended to occur during recoveries, when productivity growth did in fact increase, and when it was too soon to tell whether the higher growth rate would be sustained.[18]

Nonetheless, even in real time it would have been clear from these techniques with data through 1999:Q1 (when the probability assessment first topped 0.9) that there had been a change in regime back to stronger productivity growth beginning in 1998:Q2. While certainly the idea of a "new economy" with strong productivity growth had gained many adherents well before 1999, there were also plenty of nay-sayers, and few of the optimists would have ventured to base their views on objective statistical analysis.[19]

It is worth noting that the partial updating methodology described earlier can make a substantial difference in assessing current conditions. For example, without it (that is, ignoring the last three observations of output and labor compensation per hour), the estimated probability that the economy is in the high-growth regime falls from 0.997 in 2001:Q4 to 0.9057 in 2002:Q2. Adding back the information contained in those observations results in a 2002:Q2 estimate of 0.9740.

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[18] Of course a truly real-time analysis would probably not have produced such strong false positives. The parameters from the full sample reflect the reemergence of high growth in the last five years of the sample. This would have been viewed as much less likely from the perspective of 1986 or 1992.

[19] Indeed, optimistic views go back as early as 1997 (see, for example, *The New York Times*, August 2, 1997, "Measuring Productivity in the 90's: Optimists vs. Skeptics," by Louis Uchitelle). The optimism appears, however, to have been based on something other than the productivity data themselves, about which there was much skepticism (see Corrado and Slifman, 1999). Of course, pessimists have also based their views on skepticism about the data, e.g. Roach (1998).

We can also examine the assessment of the transitory regimes. The probability of being in the "plucked down" state is plotted in Figure 4. Here the probability assessments are a little more ambiguous. While the more prominent spikes all coincide with NBER-defined recessions, in only two cases does the probability of a negative pluck exceed 0.5. Moreover, several recessions (most notably the 1990-91 recession) are missed entirely. It is perhaps instructive that the 1990-91 recession does not register in this picture. The idea of a pluck is a sharp downturn followed by an equally sharp recovery sufficient to get the economy back to trend. The 1990-91 recession was characterized by a relatively mild downturn followed by an unusually slow and gradual recovery.

4.3. Do Business Cycles Have Permanent Effects?

If we examine the common permanent component, in comparison to labor productivity, for example (Figure 5), we see it clearly indicates changes in trend in the early 1970's and mid-1990's, with little apparent cyclical residue. This contrasts with many recent estimates of the permanent component of output. For example, Kim and Murray's (2002) estimated permanent aggregate component shows substantial downward movement during recessions. Kim and Piger (2002) equate the permanent component of output to consumption of nondurables and services, which is much more cyclical and volatile than our permanent component. Kim, Piger, and Startz (2002) estimate a common permanent component for output and consumption, with regime-switching, and argue that the permanent component contributes meaningfully to business cycle fluctuations.

Other work, notably Beaudry and Koop (1993), has argued that if asymmetries in business cycles are properly taken into account, the permanent impact of business cycles (and recessions in particular) is greatly diminished. Our Figure 4, however, suggests that asymmetries may not be very important, at least in data that are detrended by hours of work. To examine this further, we estimated our model without the transitory regime-switching component, and obtained results that are virtually indistinguishable from our first set of estimates, particularly with regard to the estimated permanent component. The parameter estimates from this specification are given in second column of Table 2.

If, on the other hand, we estimate the model with output, compensation, and consumption on a per capita (instead of per hour) basis, we get very different results, as indicated in the third column of Table 2. Under this specification, the growth regime shift disappears (note that the estimated values of μ_0 and μ_1 are identical, so the transition probabilities are indeterminate). To understand where this result is coming from, note that the dependent variables in the per capita specification differ by the logarithm of hours per capita. We can see how the models are related if we begin with the equation for output per hour from the per hour specification, and then add the equation for detrended hours:

$$Y_t - L_t = \gamma X_t + \lambda_1 x_t + z_{1t} \qquad (18)$$

$$\hat{L}_t = \lambda_4 x_t + z_{4t} \qquad (19)$$

$$Y_t - \left(L_t - \hat{L}_t\right) = \gamma X_t + \left(\lambda_1 + \lambda_4\right) x_t + z_{1t} + z_{4t} \qquad (20)$$

The resulting dependent variable is output relative to the trend in hours. That trend in turn can be divided into population growth and the low-frequency movement in Λ (i.e. the trend in *L*/*N*) depicted earlier in Figure 1, which we will denote by $\overline{\Lambda}_t$. Consequently output per capita should be

$$Y_t - N_t = \overline{\Lambda}_t + \gamma X_t + \left(\lambda_1 + \lambda_4\right) x_t + z_{1t} + z_{4t} \qquad (21)$$

where $\overline{\Lambda}_t$ is just $L_t - \hat{L}_t - N_t$. And just as $W_t - L_t$ and $C_t - L_t$ have the same form as (18), $W_t - N_t$ and $C_t - N_t$ have the same form as (21).

Thus the permanent component in the per capita specification is $\overline{\Lambda}_t + \gamma X_t$ rather than just γX_t, i.e. it is the sum of the labor supply and technology trends. While there are other differences between the two specifications, a plausible explanation for the absence of a detectable regime shift in the permanent component of the per capita specification is simply that movements in $\overline{\Lambda}_t$ obscure the regime shifts in X_t. That in itself is not a problem if the estimated permanent component is similar to the one obtained from combining the separate labor supply trend $\overline{\Lambda}_t$ (i.e. the HP filter of hours of work less population depicted in Figure 1) and permanent component γX_t from the per hour specification. This does not turn out to be the case, however, as Figure 6 illustrates. The estimated trends in per capita output from the two specifications clearly differ qualitatively, with the trend from the per capita specification showing much greater cyclical movement.

Our findings also relate to Perron's (1989) argument that failures to reject unit roots can be a consequence of series with occasional structural breaks. The idea is that if your estimation procedure misses a structural break in a linear trend, for example, then deviations from a linear trend may be so persistent as to suggest unit root behavior. In the present example, the per capita specification fails to find a regime switch in its permanent component. Our conjecture is that the resulting linear trend gives rise to greater persistence, and results in some transitory movements being labeled as permanent by the Kalman filter. In support of this it is worth noting that the estimate of the AR(1) parameter ϕ_1 goes from –0.440 from the per hour specification to 0.577 with the per capita model. The persistence of the idiosyncratic components is higher as well.

These findings also suggest an explanation for Hamilton's (1989) finding that growth regime shifts characterize business cycles rather than longer-term trends. Our results suggest that the treatment of hours of work is crucial. When we filter the data so to remove the influence of any stochastic trend in hours, growth regime shifts are no longer cyclical phenomena, but instead reflect changes in long-term trends. This finding is strengthened by looking at common trends in a multivariate analysis.

Of course none of this proves that the per hour specification results in "better" estimates of the permanent component. Certainly the per capita model explains more of the variance of per capita output in terms of the two common factors, as the transitory components are similar across the specifications. We would argue that this is because it overfits the data, i.e. it attributes too much of the variation to the permanent component. Of course the ultimate test would be out-of-sample performance, which is beyond the scope of this paper. But the growth model implies that low frequency variation in *L/N* on the one hand, and in *Y/L*, *W/L*, and *C/L* on the other, are driven by completely different fundamentals. We would argue that using the theory to estimate the two permanent components separately leads to better results than estimating a hodgepodge of two completely different concepts.

We should also note the reasons for the asymmetric treatment in our approach of the technology trend, which we estimate using a Kalman filter, versus the labor supply component, for which we simply fit an H-P trend to hours of work and adjust for population. As previously noted, there is no obvious analog to the cointegrating relationships for the technology trend that would provide additional information about the labor supply trend. That is, there are no other quantities that the neoclassical growth model suggests should share a common trend with *L*/*N*. Hence there is no particular advantage to estimating the labor supply trend as part of the system.[20]

Thus using the per hour specification we are able to characterize aggregate output as made up of three distinct and more or less independent components: a transitory and essentially symmetric "business cycle" component, a permanent "technology trend" component and a low frequency "labor supply" component. The business cycle component exhibits the standard hump-shaped behavior emphasized by Blanchard (1982). The technology component is close to piecewise linear, with two breaks, one in around 1973, and the other around 1996. The labor supply component is J-shaped, declining in the post-war era until the early 1960s and then rising from the mid-1960s till the present. This itself actually reflects the sum of three underlying trends: a decline in male labor force participation, an increase in female participation, and a decline in average weekly hours of employed workers. Naturally the same argument we have made in favor of examining the technology and labor supply trends separately applies to the labor supply trend itself, namely that a more reliable assessment of that trend (particularly for extrapolation) might emerge from examining its components separately.

4.4. The Relative Importance of Additional Variables

Finally, we have also stressed the importance of information gleaned from several series in identifying the common permanent component. It is reasonable to ask how important this consideration actually is. To answer this question, we estimated the same econometric model, but with only two series, nonfarm output per hour and detrended hours (variables 1 and 4 from the previous analysis). Thus we are looking to estimate trend growth with productivity data alone, using the detrended hours series to control for the business cycle. The result of this exercise is the final set of estimates in Table 2. Note first that the estimates of the transition probabilities are very similar to the earlier estimates, suggesting that the fundamental properties of the regime-switching dimension of the model are similar.

[20] There are undoubtedly deeper explanations for the trend depicted in Figure 1, related to separate treatment of men's and women's labor supply, the return to schooling, the structure of retirement benefits, and so on. These are obviously beyond the scope of this paper.

We also estimated two additional specifications, adding back alternately either C/L or W/L to the 2-equation system. The results are illustrated in Figure 7, in which all four specifications' retrospective regime assessments are plotted against each other. There appears to be a big impact from adding either variable to the system, with the labor compensation variable clearly outperforming consumption/hours in terms of approaching the 4-equation results. The change in going from three to four equations is relatively modest, especially if the fourth variable is consumption/hours. Note, however, that in the more recent data it is the consumption/hours variable that is giving the stronger indication that the economy is in the high-growth regime, and comes closest to matching the assessment based on all four series.

The value of the multivariate approach is easiest to see in relation to a univariate analysis, which would suggest that productivity growth is close to i.i.d., especially in data since the early 1980s. (Earlier data suggests some negative autocorrelation.) By itself this would imply that productivity innovations have largely permanent effects. The multivariate results are saying, in contrast, that productivity movements associated with business cycles are largely transitory, insofar as the common trend with consumption and labor compensation appears to exhibit little cyclical variation.[21]

On the other hand, real time probability assessments for the 2-equation model exhibit fewer and smaller false positives—episodes where the probability that a regime shift has occurred jumps up, only to quickly reverse itself as more data come in. Perhaps the four-variable system is "overfitting" the data, i.e. it does better than the two-variable system in avoiding "type 2" errors (the error of failing to detect a regime change), but it makes more "type 1" errors (detecting a regime change that in fact never took place).

To address this question we use the 4-equation and 2-equation models to forecast productivity growth at one- and four-quarter horizons. The idea here is that if the four-equation system were overfitting the data, the root mean square error of its forecasts would be larger than that of the two-variable system. We constructed forecasts beginning in 1972:Q1 using the formulas outlined in Hamilton (1994). Consequently, the sample for the one-quarter forecast horizon covers the period 1971:Q2 - 2002:Q2, while the sample for the four-quarter forecast horizon covers the period 1972:Q1-2002:Q2. Table 3 provides the results of this exercise. For both forecast horizons, the root mean square error of the two-variable system is slightly larger,

[21] Of course, to assess the contribution of technology shocks to business cycle fluctuations really requires a structural model such as an identified structural VAR. We are pursuing this in another paper, Kahn and Rich (2003).

suggesting that the four-variable system is doing a better job, notwithstanding the fact that its primary intention is not short-term forecasting.

Table 3 also provides the model's forecasts of productivity growth for 2002:Q3-2003:Q2. The four-variable system predicts substantially stronger productivity growth for all four quarters, primarily because of its assessment of a relatively high probability of currently being in a high-growth regime. Even four quarters out, the model forecasts growth substantially above the sample mean of 2.16 percent, notwithstanding its built-in (albeit slow) mean reversion. In fact, just the possibility of a switch back to a lower-growth regime builds in approximately a 0.08 percent per quarter decline in growth. Thus conditional on no regime change, the forecasted growth rates would be about 4.1, 3.2, 3.2, and 3.1 percent for the four quarters. The long-run growth rate conditional on remaining in the high-growth regime is roughly 2.9 percent.

Of course productivity growth is a very noisy series, so a forecast for any given quarter cannot be made with much confidence. But the difference between the two sets of forecasts largely reflects different assessments of current underlying trends, and provides some idea of the potential usefulness of the approach adopted in this paper. In fact, as the table indicates, over the subsequent four quarters our model's relatively optimistic out-of-sample forecast of 3.2 percent productivity growth turned out to be unduly pessimistic, although much closer to the mark than the forecast based on the 2-equation model.

5. Conclusions

The view that higher productivity growth is likely to be sustained has only really gained something approaching a consensus with the recent recession. Prior to 2001, one could more easily argue that the increased growth rates experienced since 1995 were merely cyclical or otherwise ephemeral. That lack of agreement not only reflects the difficulty of separating a time series into its trend and cycle, but also the sensitivity of the results to various assumptions used in the decomposition. In the case of productivity, the problematic nature of the decomposition is only likely to be exacerbated by the inherent volatility of the series. Policymakers faced the same difficulty (albeit in the opposite direction) in the mid-1970's when the dramatic slowing of productivity growth coincided with a severe recession.

We explore the issue of the long-term trend in productivity by adopting a modeling strategy that integrates both theoretical considerations and recently developed statistical methods. We undertake a multivariate analysis in which we exploit information from additional variables that growth theory implies should be helpful in the identification of the trend and cycle in productivity. Specifically, we extend the data set to include consumption and labor compensation as well as detrended hours of employment.

For our empirical framework we adopt the regime-switching dynamic factor model recently proposed by Kim and Murray (2002). This approach has a number of attractive features. First, it allows for the estimation of a common permanent component and a common transitory component, consistent with our interest in the trend and cycle in the productivity data. Second, the model allows for rich dynamics and can account for periodic changes in the underlying processes generating the common components. This latter consideration is not only important for providing a better characterization of the data, but is central to any discussion about a possible shift in the secular growth rate of productivity. Last, the nature and timing of the regime changes is determined as an outcome of the estimation procedure rather than imposed *a priori*. In fact, one could view the implied regime changes and their reasonableness as an additional metric by which to judge the adequacy of our approach.

We find strong support in the data for the notion that the economy (and productivity growth in particular) switched from a relatively low-growth to a high-growth regime in the mid-1990's. The annualized difference between the mean growth rates in the two regimes is estimated to be approximately 1.5 percent. We also show that these techniques could have provided conclusive signals of the regime shift by early 1999. Finally, from a methodological standpoint we argue that the incorporation of additional information from other time series is crucial to the strength of our conclusions.

We also find that taking account of low-frequency movements in labor supply is crucial for detecting the regime shift in the permanent "technology" component, as an alternative specification based on the assumption that hours of work per capita is stationary fails to find any regime shift in the permanent component. Thus in the end we characterize aggregate output as made up of three distinct and more or less independent components: a transitory and essentially symmetric "business cycle" component, a permanent "technology trend" component and a low frequency "labor supply" component. The business cycle component exhibits the standard hump-shaped behavior emphasized by Blanchard (1982). The technology component is close to piecewise linear, with two breaks, one in around 1973, and the other around 1996.

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Appendix

A1. State Space Model

We employ the following state-space representation for our model:

Measurement Equation: $\Delta Q_t = H'\xi_t$, $\Delta Q_t \equiv (\Delta Q_{1t},...,\Delta Q_{4t})'$

Transition Equation: $\xi_t = \alpha(S_t) + F\xi_{t-1} + V_t$,

with $E(V_t V_t') = \Sigma$,

and where (after we restrict $\gamma_1 = \gamma_2 = \gamma_3 \equiv \gamma$, and set $\gamma_4 = 0$)

$$H' = \begin{bmatrix} \gamma & \lambda_1 & -\lambda_1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma & \lambda_2 & -\lambda_2 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ \gamma & \lambda_3 & -\lambda_3 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & \lambda_4 & -\lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\xi_t = \begin{bmatrix} \Delta X_t \\ x_t \\ x_{t-1} \\ z_{1t} \\ z_{1t-1} \\ z_{2t} \\ z_{2t-1} \\ z_{3t} \\ z_{3t-1} \\ z_{4t} \\ z_{4t-1} \end{bmatrix}, \qquad \alpha(S_t) \equiv \alpha(S_{1t}, S_{2t}) = \begin{bmatrix} \mu_0(1-S_{1t}) + \mu_1(S_{1t}) \\ \tau S_{2t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad V_t = \begin{bmatrix} \upsilon_t \\ \varepsilon_t \\ 0 \\ \eta_{1t} \\ 0 \\ \eta_{2t} \\ 0 \\ \eta_{3t} \\ 0 \\ \eta_{4t} \\ 0 \end{bmatrix}$$

$$F=\begin{bmatrix}\phi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \phi_1^* & \phi_2^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \psi_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \psi_{21} & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{31} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{411} & \psi_{412}\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\end{bmatrix},\quad \Sigma=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \sigma_1^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \sigma_2^2 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_3^2 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_4^2 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}.$$

We next provide a brief overview of a filter developed by Kim (1994) that can be used for approximate maximum likelihood estimation of the state-space model with Markov switching. We focus our attention on the issue of drawing inferences about the unobserved regimes. For further details, interested readers are referred to Kim and Murray (2002).

To facilitate the discussion, we will represent the two unobserved Markov-switching variables S_{1t} and S_{2t} by a single Markov-switching variable defined such that:

$$\begin{aligned}S_t &= 1 \text{ if } S_{1t}=0 \text{ and } S_{2t}=0\\ S_t &= 2 \text{ if } S_{1t}=0 \text{ and } S_{2t}=1\\ S_t &= 3 \text{ if } S_{1t}=1 \text{ and } S_{2t}=0\\ S_t &= 4 \text{ if } S_{1t}=1 \text{ and } S_{2t}=1\end{aligned}$$

with

$$\Pr[S_t=j\,|\,S_{t-1}=i]=p_{ij}$$

and

$$\sum_{j=1}^{4} p_{ij}=1$$

Conditional on $S_t=j$ and $S_{t-1}=i$, the Kalman filter equations are given by:

$$\begin{aligned}\xi_{t|t-1}^{(i,j)} &= \alpha(S_j)+F\xi_{t-1|t-1}^{i}+V_t\\ P_{t|t-1}^{(i,j)} &= FP_{t-1|t-1}^{i}F'+\Sigma\\ \eta_{t|t-1}^{(i,j)} &= \Delta Q_t-H'\xi_{t|t-1}^{(i,j)}\\ f_{t|t-1}^{(i,j)} &= HP_{t|t-1}^{(i,j)}H'\\ \xi_{t|t}^{(i,j)} &= \xi_{t|t-1}^{(i,j)}+P_{t|t-1}^{(i,j)}H'[f_{t|t-1}^{(i,j)}]^{-1}\eta_{t|t-1}^{(i,j)}\\ P_{t|t}^{(i,j)} &= (I-P_{t|t-1}^{(i,j)}H'[f_{t|t-1}^{(i,j)}]^{-1})HP_{t|t-1}^{(i,j)}\end{aligned}$$

where $\xi_{t|t}^{(i,j)}$ and $\xi_{t|t-1}^{((i,j)}$ are, respectively, an inference on ξ_t based on information through time period t (Ω_t) and t-1 (Ω_{t-1}), given $S_t = j$ and $S_{t-1} = i$; $P_{t|t}^{(i,j)}$ and $P_{t|t-1}^{(i,j)}$ are, respectively, the mean squared error matrix of $\xi_{t|t}^{(i,j)}$ and $\xi_{t|t-1}^{(i,j)}$, given $S_t = j$ and $S_{t-1} = i$; $\eta_{t|t-1}^{(i,j)}$ is the conditional forecast error of ΔQ_t based on information through time period t-1, given $S_t = j$ and $S_{t-1} = i$; and $f_{t|t-1}$ is the conditional variance of the forecast error $\eta_{t|t-1}^{(i,j)}$.

To keep the Kalman filter from becoming computationally infeasible, the following approximations are introduced to collapse the posteriors terms $\xi_{t|t}^{(i,j)}$ and $P_{t|t}^{(i,j)}$ into the posterior terms $\xi_{t|t}^{j}$ and $P_{t|t}^{j}$:

$$\xi_{t|t}^{j} = \frac{\sum_{i=1}^{4} \Pr[S_{t-1} = i, S_t = j \mid \Omega_t]\xi_{t|t}^{(i,j)}}{\Pr[S_t = j \mid \Omega_t]}$$

and

$$P_{t|t}^{j} = \frac{\sum \Pr[S_{t-1} = i, S_t = j \mid \Omega_t]\{P_{t|t}^{(i,j)} + (\xi_{t|t}^{j} - \xi_{t|t}^{(i,j)})(\xi_{t|t}^{j} - \xi_{t|t}^{j})'\}}{\Pr[S_t = j \mid \Omega_t]}$$

The approximations result from the fact that $\xi_{t|t}^{(i,j)}$ does not calculate $E[\xi_t \mid S_{t-1} = i, S_t = j, \Omega_t]$ and $P_{t|t}^{(i,j)}$ does not calculate $E[(\xi_t - \xi_{t|t}^{(i,j)})(\xi_t - \xi_{t|t}^{(i,j)})' \mid S_{t-1} = i, S_t = j, \Omega_t]$ exactly. This is because ξ_t conditional on Ω_{t-1}, $S_t = j$, and $S_{t-1} = i$ is a mixture of normals for t > 2.

To obtain the probability terms necessary to construct the approximations, the following three-step procedure is employed.

<u>Step 1:</u>

At the beginning of the t^th iteration, given $\Pr[S_{t-1} = i \mid \Omega_{t-1}]$, we can calculate:

$$\Pr[S_t = j, S_{t-1} = i \mid \Omega_{t-1}] = \Pr[S_t = j \mid S_{t-1} = i] \times \Pr[S_{t-1} = i \mid \Omega_{t-1}]$$

where $\Pr[S_t = j \mid S_{t-1} = i]$ is a transition probability.

<u>Step 2:</u>

We can then consider the joint density of $\Delta Q_t, S_t$ and S_{t-1}:

$$f(\Delta Q_t, S_t = j, S_{t-1} = i \mid \Omega_{t-1}) = f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1}) \times \Pr[S_t = j, S_{t-1} = i \mid \Omega_{t-1}]$$

and then obtain the marginal density of ΔQ_t as:

$$f(\Delta Q_t \mid \Omega_{t-1}) = \sum_{i=1}^{4} \sum_{i=1}^{4} f(\Delta Q_t, S_t = j, S_{t-1} = i \mid \Omega_{t-1})$$

$$= \sum_{i=1}^{4} \sum_{i=1}^{4} f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1}) \times \Pr[S_t = j, S_{t-1} = i \mid \Omega_{t-1}]$$

where the conditional density $f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1})$ is obtained using the prediction error decomposition:

$$f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1}) = (2\pi)^{\frac{T}{2}} \mid f_{t|t-1}^{(i,j)} \mid^{-\frac{1}{2}} \exp\{-\tfrac{1}{2}\eta_{t|t-1}^{(i,j)}{}' f_{t|t-1}^{(i,j)-1}\eta_{t|t-1}^{(i,j)}\}$$

A byproduct of this step is that we can obtain the log likelihood function:

$$\ln L = \sum_{t=1}^{T} \ln(f(\Delta Q_t \mid \Omega_{t-1}))$$

which can be maximized with respect to the parameters of the model.

Step 3:

We can then update the probability terms after observing ΔQ_t and the end of period t:

$$\Pr[S_t = j, S_{t-1} = i \mid \Omega_t] = \Pr[S_t = j, S_{t-1} = i \mid \Delta Q_t, \Omega_{t-1}]$$

$$= \frac{f(S_t = j, S_{t-1} = i, \Delta Q_t \mid \Omega_{t-1})}{f(\Delta Q_t \mid \Omega_{t-1})}$$

$$= \frac{f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1}) \times \Pr[S_t = j, S_{t-1} = i \mid \Omega_{t-1}]}{f(\Delta Q_t \mid \Omega_{t-1})}$$

with

$$\Pr[S_t = j \mid \Omega_t] = \sum_{i=1}^{4} \Pr[S_t = j, S_{t-1} = i \mid \Omega_t]$$

The last term provides the "real-time" inference about the unobserved regimes conditional on only contemporaneously available information.

We can also derive smoothed values of ξ_t and S_t using all available information through period T. That is, we can construct $\xi_{t|T}$ as well as $\Pr[S_t = j \mid \Omega_T]$which represent the "retrospective" assessments of the state vector and unobserved regimes. Because the inferences about the unobserved regimes do not depend on the state vector, we can first calculate smoothed probabilities. The smoothed probabilities can then be used to generate the smoothed estimates of the state vector.

The smoothing algorithm for the probabilities will involve the application of approximations similar to those introduced in the basic filtering. The procedure can be understood by considering the following derivation of the joint probability that $S_{t+1} = k$ and

$S_t = j$ conditional on full information:

$$\Pr[S_{t+1}=k, S_t=j \mid \Omega_T] = \Pr[S_{t+1}=k \mid \Omega_T] \times \Pr[S_t=j \mid S_{t+1}=k, \Omega_T]$$
$$\approx \Pr[S_{t+1}=k \mid \Omega_T] \times \Pr[S_t=j \mid S_{t+1}=k, \Omega_t]$$
$$= \frac{\Pr[S_{t+1}=k \mid \Omega_T] \times \Pr[S_{t+1}=k, S_t=j \mid \Omega_t]}{\Pr[S_{t+1}=k \mid \Omega_t]}$$
$$= \frac{\Pr[S_{t+1}=k \mid \Omega_T] \times \Pr[S_t=j \mid \Omega_t] \times \Pr[S_{t+1}=k \mid S_t=j]}{\Pr[S_{t+1}=k \mid \Omega_t]}$$

and

$$\Pr[S_t=j \mid \Omega_T] = \sum_{i=1}^{4} \Pr[S_{t+1}=k, S_t=j \mid \Omega_T]$$

The actual construction of the smoothed probabilities requires running through the basic filter and then storing the sequences $P_{t|t-1}^{(i,j)}, P_{t|t}^{j}, \Pr[S_t=j \mid \Omega_{t-1}]$ and $\Pr[S_t=j \mid \Omega_t]$. For $t = T - 1$, $T - 2, \dots, 1$, the above formulas define a backwards recursion that can be used to derive the full-sample smoothed probabilities. It should be noted that the starting value for the smoothing algorithm is $\Pr[S_t=j \mid \Omega_T]$, which is given by the final iteration of the basic filter.

A2. Partial Updating

Suppose that additional observations become available, but only for some subset of the four data series represented by Q. Specifically, suppose that for the subset Q^1, data are available for periods 1 through T+3, whereas for Q^2, observations are only available through T. Let $T+1'$ denote the augmented information available through $T+1$, i.e. including Q_{T+1}^1 but not Q_{T+1}^2.

Through T the standard Kalman updating algorithm applies (ignoring the regime-related term $\alpha(S_t)$ for brevity's sake), i.e.

$$\hat{\xi}_{t+1|t} = F\hat{\xi}_{t|t-1} + FP_{t|t-1}H(H'P_{t|t-1}H)^{-1}(\Delta Q_t - H'\hat{\xi}_{t|t-1})$$
$$P_{t+1|t} = F[P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H)^{-1}H'P_{t|t-1}]F' + \Sigma$$

for $t \le T$.

To update further, we simply recognize that

$$\Delta Q_t^i = H^{i}{}'\xi_t, \qquad i = 1, 2$$

where H^i is the appropriate submatrix of H. We then iterate beginning at T+1 according to

$$\hat{\xi}_{T+1|T+1'} = \hat{\xi}_{T|T-1} + P_{T|T-1}H^1(H^1{'}P_{T+1|T}H^1)^{-1}(\Delta Q^1_{T+1} - H^1{'}\hat{\xi}_{T+1|T})$$
$$P_{T+1|T+1'} = P_{T+1|T} - P_{T+1|T}H^1(H^1{'}P_{T+1|T}H^1)^{-1}H^1{'}P_{T+1|T}$$

and then

$$\hat{\xi}_{T+2|T+1'} = F\hat{\xi}_{T+1|T+1'}$$
$$P_{T+2|T+1'} = FP_{T+1|T+1'}F' + \Sigma \, .$$

The iteration can then proceed forward if subsequent observations on Q^1 become available.

In our case we essentially have three additional observations on two of the variables, since they appear lagged by three quarters. To take them into account at any point in time t, we can compute $\hat{\xi}_{t|t+3'}$, that is, the assessment of the state vector given the three additional observations of the two series that would otherwise be ignored because they are lagged by three quarters. To obtain $\hat{\xi}_{t|t+3'}$, we iterate forward to get $\hat{\xi}_{t+3|t+3'}$ and $P_{t+3|t+3'}$, and then iteratively "smooth" backwards using, e.g.

$$\hat{\xi}_{t+2|t+3'} = \hat{\xi}_{t+2|t+2'} + P_{t+2|t+2'}H^1(H^1{'}P_{t+3|t+2'}H^1)^{-1}(\hat{\xi}_{t+3|t+3'} - \hat{\xi}_{t+3|t+2'}) \, .$$

The result is an improved estimate of the state vector, one that incorporates the most recent values of all of the variables.

Table 1: Unrestricted Cointegration Rank Test

Hypothesized Number of Cointegrating Equations	Eigenvalue	Trace Statistic†	5 Percent Critical value	1 Percent Critical value
None**	$\hat{\lambda}_1 = 0.172154$	70.85945	29.68	35.65
At most 1**	$\hat{\lambda}_2 = 0.109881$	29.10624	15.41	20.04
At most 2	$\hat{\lambda}_3 = 0.015186$	3.381938	3.76	6.65

Hypothesized Number of Cointegrating Equations	Eigenvalue	Maximal Eigenvalue Statistic††	5 Percent Critical value	1 Percent Critical value
None**	$\hat{\lambda}_1 = 0.172154$	41.75320	20.97	25.52
At most 1**	$\hat{\lambda}_2 = 0.109881$	25.72431	14.07	18.63
At most 2	$\hat{\lambda}_3 = 0.015186$	3.381938	3.76	6.65

† $\lambda_{trace} = -T\sum \ln(1-\hat{\lambda}_i)$

†† $\lambda_{\max} = -T\sum \ln(1-\hat{\lambda}_{r+1})$

Table 2: Estimation of Model

Coefficient	4 equation system	No pluck system	Per capita system	2 equation system
p_1	0.990 (0.011)	0.991 (0.010)	***	0.994 (0.007)
q_1	0.986 (0.013)	0.985 (0.013)	***	0.994 (0.008)
p_2	0.985 (0.012)	—	0.976 (0.017)	0.953 (0.026)
q_2	0.573 (0.227)	—	0.579 (0.236)	0.621 (0.130)
ϕ	-0.440 (0.132)	-0.450 (0.151)	0.577 (0.097)	-0.746 (0.133)
ϕ_1^*	1.416 (0.089)	1.488 (0.067)	1.400 (0.077)	1.253 (0.097)
ϕ_2^*	-0.552 (0.080)	-0.605 (0.066)	-0.538 (0.068)	-0.443 (0.085)
ψ_{11}	0.879 (0.042)	0.871 (0.046)	0.922 (0.040)	-0.905 (0.045)
ψ_{21}	0.876 (0.055)	0.891 (0.055)	0.897 (0.043)	—
ψ_{31}	-0.562 (0.087)	-0.565 (0.109)	-0.242 (0.144)	—
ψ_{41}	1.454 (0.055)	1.461 (0.055)	1.705 (0.174)	1.585 (0.140)
ψ_{42}	-0.528 (0.040)	-0.533 (0.040)	-0.726 (0.148)	-0.628 (0.111)
γ	0.262 (0.042)	0.260 (0.044)	0.416 (0.045)	0.314 (0.122)
λ_1	0.207 (0.046)	0.235 (0.051)	0.662 (0.058)	0.128 (0.041)
λ_2	0.121 (0.030)	0.133 (0.034)	0.240 (0.037)	—
λ_3	-0.483 (0.042)	-0.547 (0.043)	0.596 (0.045)	—
λ_4	0.443 (0.044)	0.497 (0.047)	0.770 (0.046)	0.429 (0.120)
μ_0	0.906 (0.218)	0.889 (0.226)	0.0002 (0.359)	0.851 (0.417)
μ_1	-1.099 (0.254)	-1.117 (0.270)	0.0002 (0.911)	-0.873 (0.441)
τ	-2.883 (0.749)	—	-2.539 (0.578)	-3.507 (1.099)

Note: The estimation also produces estimates of the variances of the idiosyncratic errors, not reported here.
*** indicates parameters that could not be estimated because they are not identified.

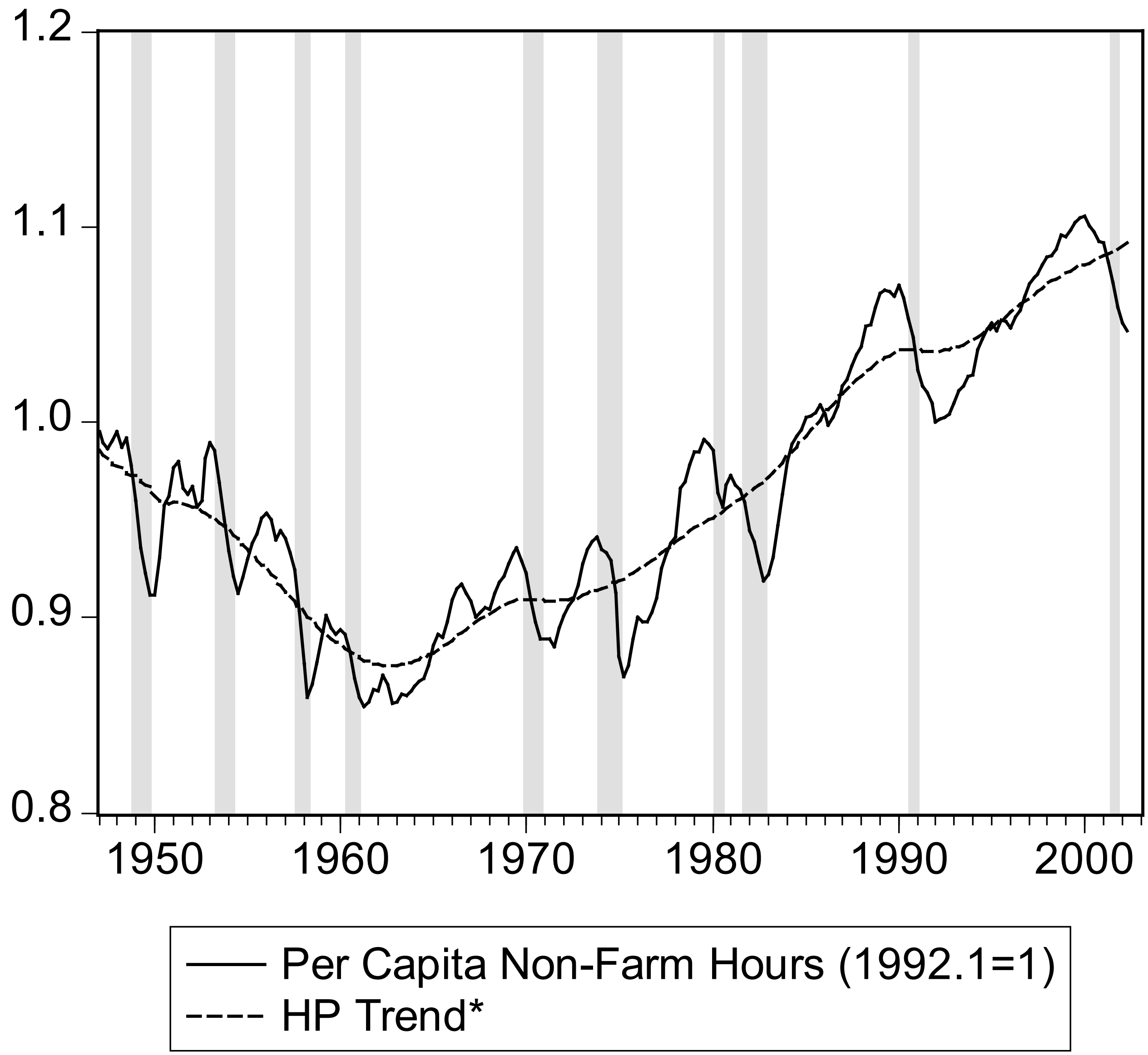
Table 3: Forecast Performance

	2-variable system	4-variable system
	Root Mean Square Errors	
1-quarter horizon	3.024	2.966
1-year horizon	3.000	2.992

	Forecast (data through 2002:Q2)		Actual
2002:Q3	3.14%	4.03%	5.78%
2002:Q4	2.28	3.04	1.59
2003:Q1	2.58	2.99	2.22
2003:Q2	2.16	2.75	6.58
4-quarter average	2.54	3.20	4.04

Note: All figures are at annualized percentage rates. The average annualized growth rate of productivity (log differences) over the full sample is 2.16 percent, with a standard deviation of 3.61.

Figure 1: Hours of Work Per Capita in the Postwar U.S.



*obtained from log(hours) using a parameter of 9600, then subtracting log(population)

Figure 2: Cointegration Properties of Y, W, C, and K

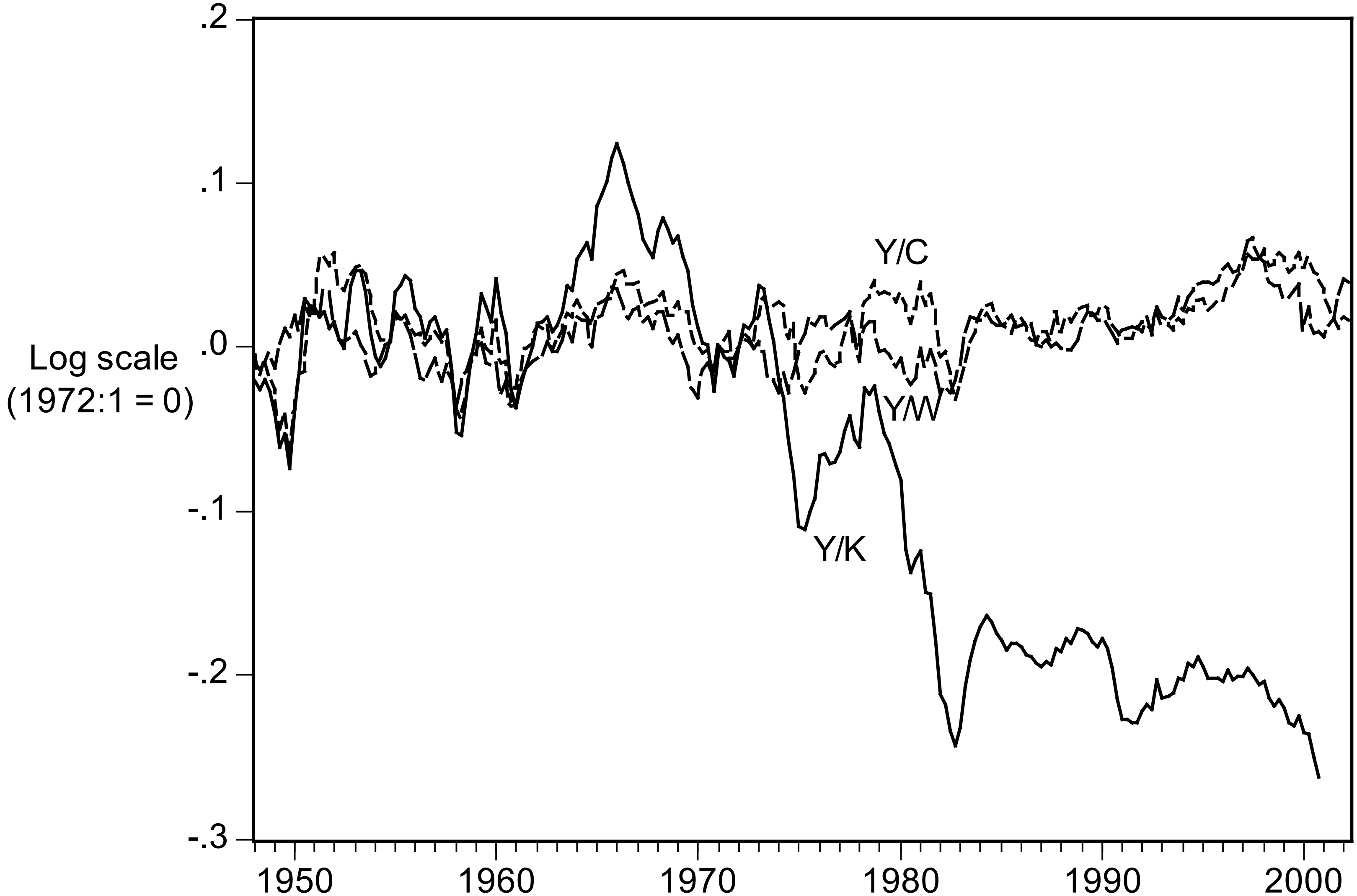


Figure 3: Regime Assesments

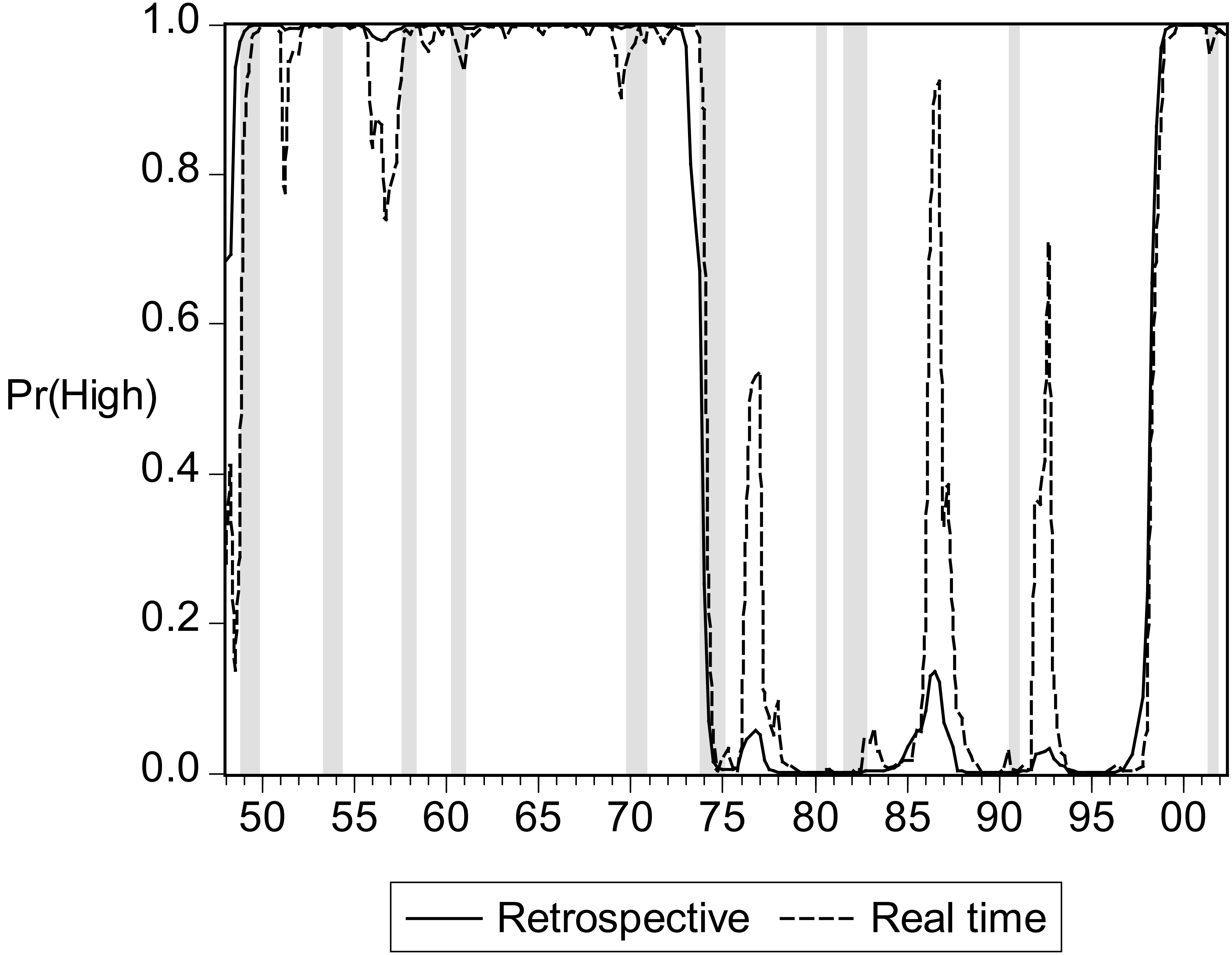


Figure 4: Full-Sample Assessments of Transitory Regimes

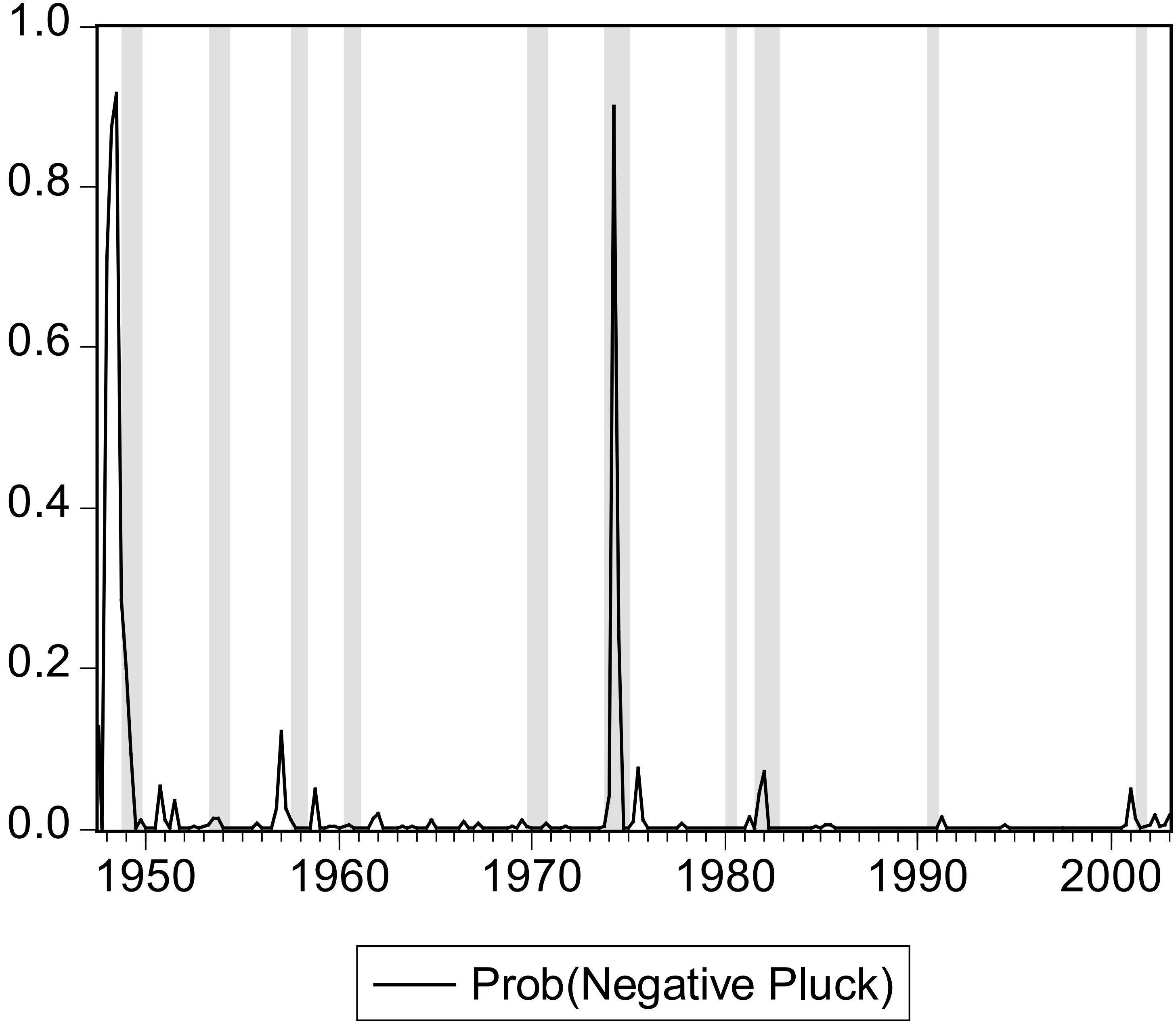


Figure 5: Trend Productivity

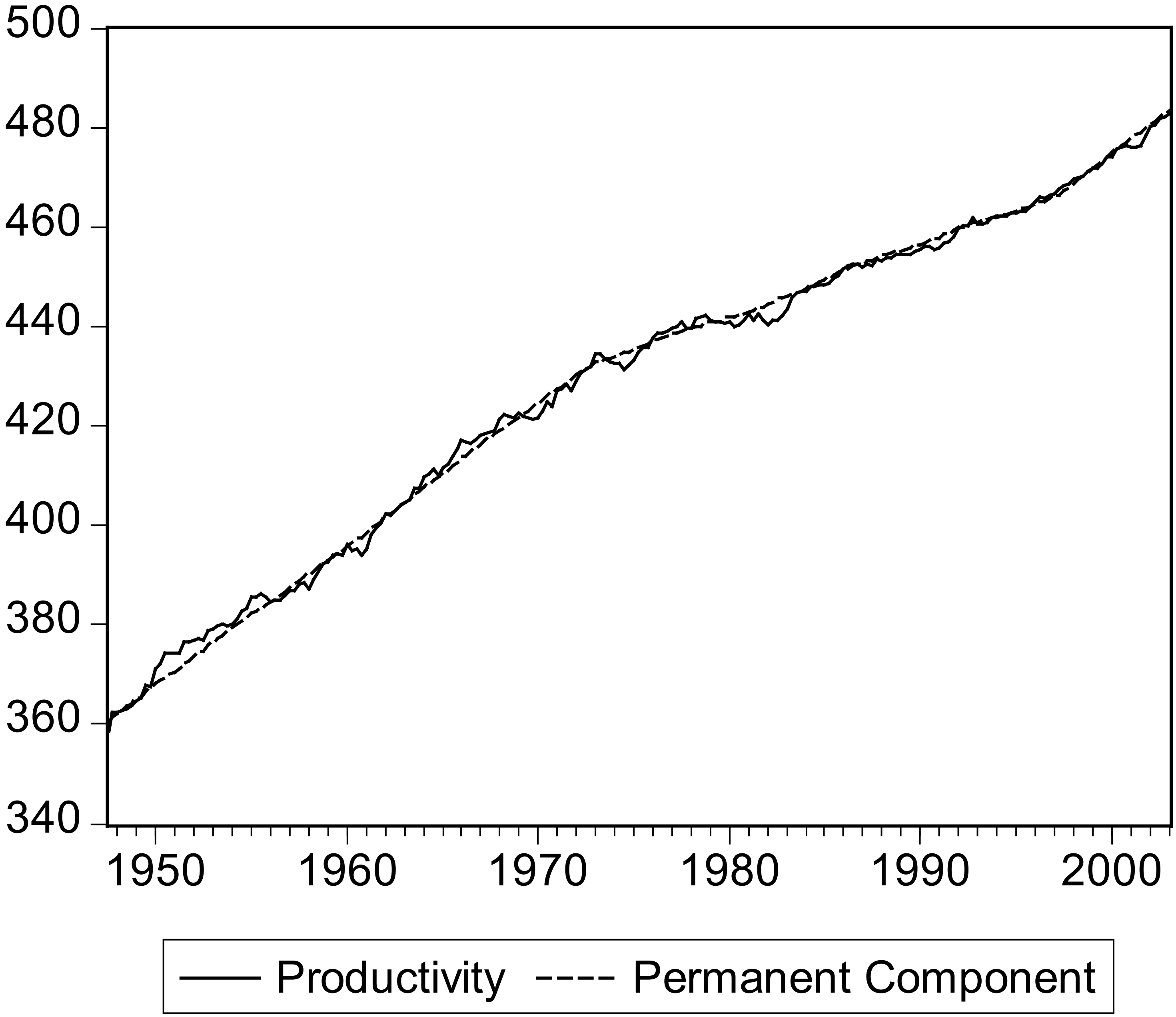
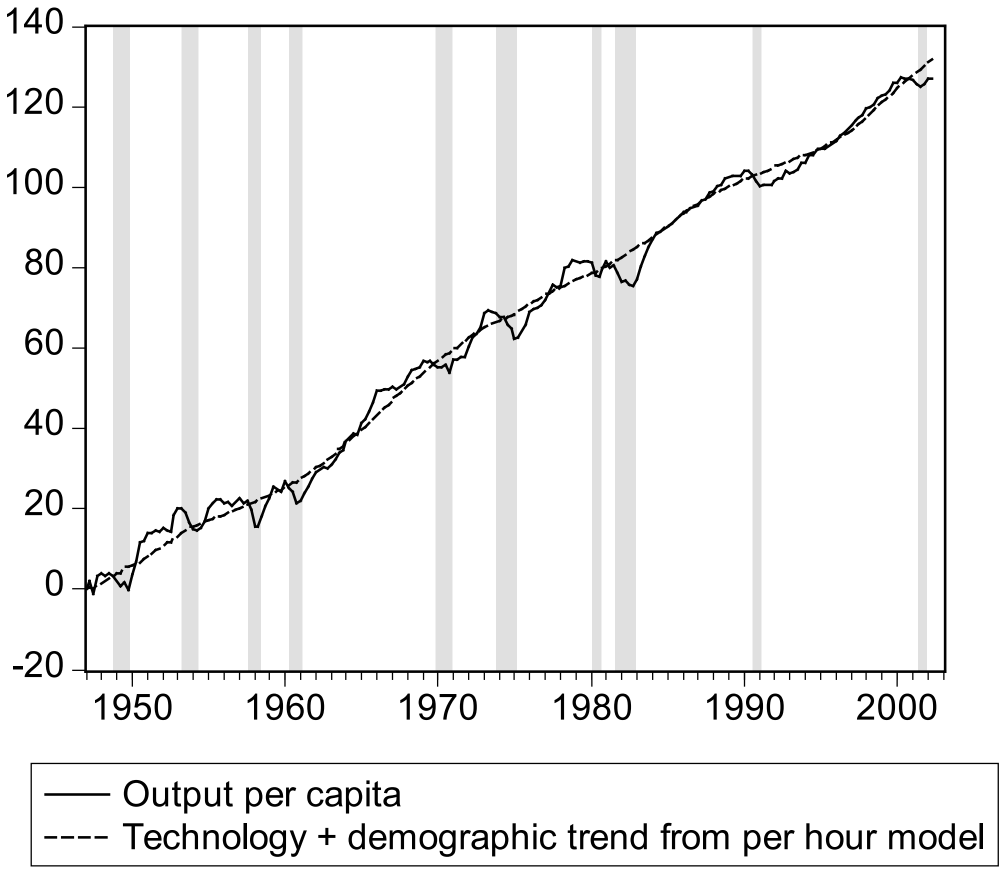


Figure 6: Comparing the Estimated Trends in Nonfarm Output per Capita



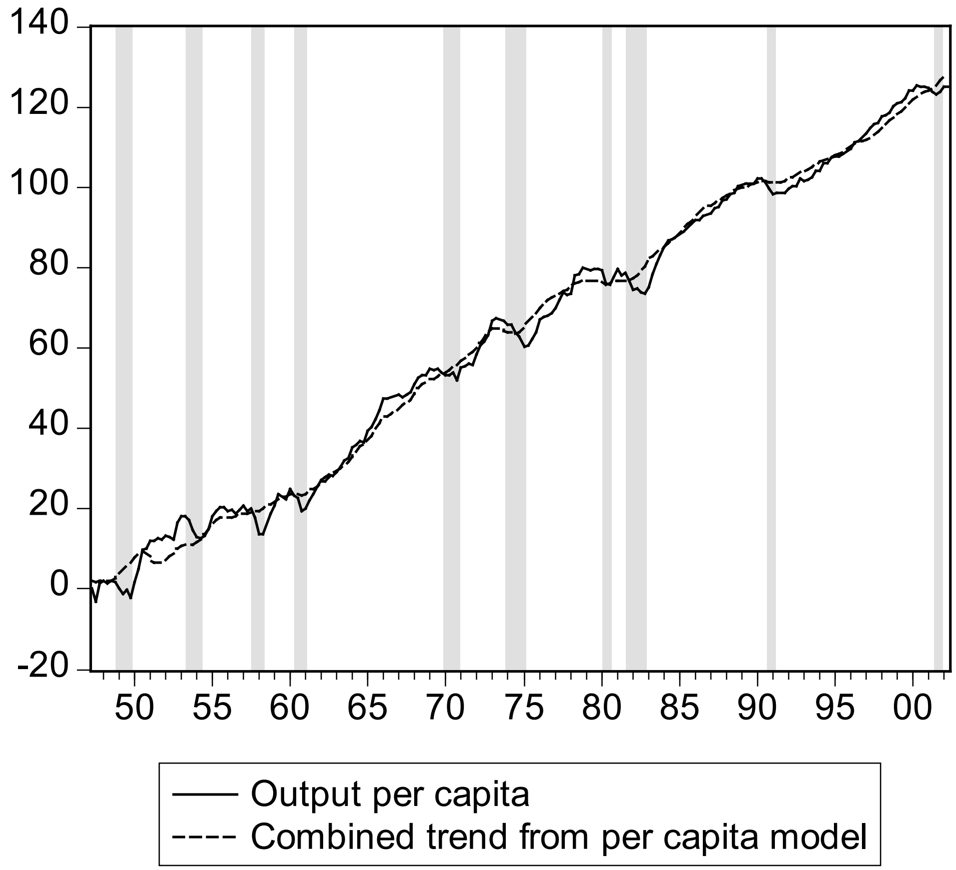


Figure 7: Relative Importance of Variables
for Regime Assessments