Financial Innovation, Macroeconomic Stability and Systemic Crises*

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Abstract

We present a general equilibrium model of intermediation designed to capture some of the key features of the modern financial system. The model incorporates financial constraints and state-contingent contracts, and contains a clearly defined pecuniary externality associated with asset fire sales during periods of stress. If a sufficiently severe shock occurs during a credit expansion, this externality is capable of generating a systemic financial crisis that may be self-fulfilling. Our model suggests that financial innovation and greater macroeconomic stability may have made financial crises in developed countries less likely than in the past, but potentially more severe.

Keywords: Systemic Financial Crises; Financial Innovation; Macroeconomic Stability; Modern Financial Systems; Fire Sales.
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"When [financial] innovation ... takes place in a period of generally favorable economic and financial conditions, we are necessarily left with more uncertainty about how exposures will evolve and markets will function in less favorable circumstances. The past several years of exceptionally rapid growth in credit derivatives and the larger role played by non-bank financial institutions, including hedge funds, has occurred in a context of ... relatively strong and significantly more stable economic growth, less concern about the level and volatility in future inflation, and low expected volatility in many asset prices. Even if a substantial part of these changes prove durable, we know less about how these markets will function in conditions of stress..." (Geithner, 2006)

1 Introduction

Systemic financial crises often occur when investment booms and rapid credit expansions collapse because the expectations of high future returns that drove them are not fulfilled (Borio and Lowe, 2002; Eichengreen and Mitchener, 2003). But while investment booms and busts have been an important part of recent financial crises in emerging market economies, their impact on financial stability in the advanced economies has been less marked. Greater macroeconomic stability and the growing sophistication of financial intermediation appear to have reduced the incidence of crisis. Increasingly, however, policymakers have become concerned that while these factors may have helped to reduce the likelihood of systemic crises, their impact, should one occur, could be on a significantly larger scale than hitherto (see, for example, Rajan, 2005 and Tucker, 2005).

It is difficult to make judgments on such issues without formally modelling the underlying externalities associated with systemic financial crises. One strand of the literature (e.g. Aghion, Banerjee and Piketty, 1999; Aghion, Bacchetta and Banerjee, 2001) draws on Kiyotaki and Moore (1997) to highlight credit frictions arising from enforcement problems.[1] These papers illustrate how endogenous balance sheet constraints, and financial development more generally, contribute to financial instability. But since these papers do not permit state-contingent financial contracts, the extent to which the underlying externality drives their results is unclear. By contrast, in existing models with state-contingent contracts (e.g. Kehoe and Levine, 1993; Krishnamurthy, 2003; Lorenzoni, 2005; Gai, Kondor and Vause, 2005), agents do not liquidate and crises never occur. Moreover, these papers do not consider the effects of financial innovation or changes in macroeconomic volatility.

This paper seeks to bridge this gap. We develop a general equilibrium model of inter-

[1] An alternative strand of the literature highlights coordination problems amongst financial market participants as the key externality driving financial crises. See, for example, Diamond and Dybvig (1983), Obstfeld (1996), and Morris and Shin (1998).

mediation with financial constraints and state-contingent contracts. Systemic financial crises are generated through a clearly defined pecuniary externality associated with asset 'fire sales' during periods of stress. Moreover, the potential for instability is present *ex ante* and does not rely on sunspots or other undefined factors external to the model.

In our setup, consumers channel funds through collateral-constrained financial intermediaries to firms operating in more-productive sectors of the economy. Firms manage investment projects but intermediaries retain financial control over them. Even though financial contracts can be made contingent on the aggregate state, enforcement problems mean that insurance opportunities for intermediaries are limited. As a result, adverse aggregate shocks to the productive sectors of the economy may force intermediaries to sell capital to less-productive sectors to remain solvent. In the spirit of Shleifer and Vishny (1992), this distress selling is associated with reduced asset prices. In turn, this creates a feedback to net worth which affects the balance sheets of all intermediaries, potentially leading to further asset sales. Since intermediaries do not internalise the effect on asset prices of their own sales, the competitive equilibrium is constrained inefficient. In extreme cases, it is this externality which can result in a systemic financial crisis that may be self-fulfilling.

Our model has some similarities to Holmstrom and Tirole (1998) and builds on Lorenzoni's (2005) analysis of lending under endogenous financial constraints and asset prices. It differs in two key respects. First, we show how multiple equilibria and systemic crises can arise in such a model. Second, we capture some of the key features of intermediation in the modern financial system: though our model also applies to traditional banks, it is especially relevant to the activities of hedge funds, private equity firms, and other non-bank financial institutions. These developments allow us to model the effects of financial innovation and greater macroeconomic stability on the likelihood and potential scale of systemic crises.

The analysis points to a range of possible outcomes. Since expected future returns in productive sectors are high, initial investment is always strong and associated with a large credit expansion. Provided that there is *no adverse shock*, investment and credit growth remain robust, and there are no asset sales. For *mild negative shocks*, firms and intermediaries liquidate some of their assets. However, since intermediaries remain solvent and firms continue to operate in productive sectors, this outcome can be viewed as a 'recession' rather than a systemic crisis.

For *more severe shocks*, multiple equilibria can arise, with (*ex ante*) beliefs determining the actual equilibrium which results. Multiplicity can occur in bad states because the supply of capital by intermediaries during fire sales is downward sloping in price, since the lower the price, the more capital they will have to sell to remain solvent. If agents have 'optimistic' beliefs about how the economy will evolve under stress, there will only be a partial liquidation of assets, as in the 'recession' case. But if beliefs are 'pessimistic', a systemic financial crisis occurs. Moreover, for *extremely severe shocks*,

a crisis is inevitable, regardless of beliefs. Under this scenario, asset prices are driven down to such an extent that all intermediaries and firms are forced to liquidate all of their assets – a full-blown financial crisis occurs, intermediaries shut down, and the closure of firms means that there are no investment opportunities in the more-productive sectors of the economy.

The financial system has been changing rapidly in recent years: resale markets for capital have deepened; and sophisticated financial products and contracts, such as credit derivatives and asset-backed securities, have mushroomed (White, 2004; Plantin, Sapra and Shin, 2005; Allen and Carletti, 2006; Allen and Gale, 2006). Our model suggests that these developments may have made economies less vulnerable to crises as they widen access to liquidity and allow assets to be traded more easily during periods of stress. But by relaxing financial constraints facing borrowers, they imply that, should a crisis occur, its impact could be more severe than previously.[2]

We demonstrate how these effects may be reinforced by greater macroeconomic stability.[3] Our model predicts that mean preserving reductions in volatility make crises less likely since severe shocks occur less frequently. However, greater stability also makes 'recession' states less likely. As a result, consumers are more willing to lend, allowing intermediaries to increase their borrowing and initial investment. But if a crisis does then ensue, losses will be greater. Overall, our findings thus make clear how financial innovation and increased macroeconomic stability may serve to reduce the likelihood of crises in developed countries, but increase their potential impact.

The paper is structured as follows. Section 2 presents the basic structure of the model, while section 3 solves for equilibrium and discusses how multiplicity and systemic financial crises arise. Section 4 considers the effects of financial innovation and changes in macroeconomic volatility on the likelihood and potential scale of financial crises. A final section concludes.

2 The Model

The economy evolves over three periods ($t = 0, 1, 2$) and has two goods, a consumption good and a capital good. Consumption goods can always be transformed one for one into capital goods, but not vice versa. Since there is a large supply of the consumption good in every period (see section 2.1), the price of the capital good in terms of the consumption good (the asset price), q, is one if capital goods are not being sold. But

[2] Allen and Carletti (2006) also assess the systemic effects of financial innovation. But they have a specific focus on credit risk transfer between banks and insurance companies, and on how its effects differ according to the type of liquidity risk that banks face. In particular, their model highlights how, in some circumstances, credit risk transfer can create the potential for contagion from the insurance sector to the banking sector, and thus be detrimental. By contrast, we consider the more general consequences of financial innovation through its broader impact on financial constraints and the depth of resale markets.

[3] A range of empirical studies (e.g. Benati, 2004; Stock and Watson, 2005) find that output and inflation volatility have fallen in many developed countries in recent years.

because of the irreversibility of investment, q may be less than one in the event of asset sales – this is one of the key drivers of our results.

2.1 Financial Intermediaries and Other Agents

The economy is composed of consumers, financial intermediaries, and firms, with large numbers of each type of agent. All agents are risk-neutral and identical within their grouping, and there is no discounting.

Consumers aim to maximise total consumption, $c_0 + c_1 + c_2$, where c_t is consumption in period t. They each receive a large endowment, e, of the consumption good in every period. Since they are only able to produce using a relatively unproductive technology operating in the traditional sector of the economy, they channel funds through intermediaries to firms operating in the more-productive sector of the economy.[4]

Intermediaries in the model are best viewed as operating in the modern financial system: they could be interpreted as traditional banks, but our model is also designed to apply to the activities of hedge funds, private equity firms, and other non-bank financial institutions. They borrow from consumers and invest in firms in order to maximise total profits, $\pi_0 + \pi_1 + \pi_2$, where profits and consumption goods are assumed to be interchangeable. However, their wealth is relatively limited: although they receive an endowment, n_0, of the consumption good in period 0 (this may be thought of as their initial net worth), this is assumed to be very small relative to e. We also assume that intermediaries are unable to trade each other's equity due to limited commitment, though relaxing this assumption does not affect our qualitative results.

Firms have no special role in our setup. They are agents with no net worth who manage investment projects in exchange for a negligible payment – this could be viewed as following from perfect competition amongst firms. Since this implies that intermediaries effectively have complete control over investment projects, we abstract from the behaviour of firms in all of what follows, and simply view intermediaries as having direct access to the productive technology.

The assumption that intermediaries have financial control over firms may appear somewhat extreme. But it embeds some of the recent developments in financial markets in a simple way. In particular, as Plantin, Sapra and Shin (2005) stress, the greater use of sophisticated financial products such as credit derivatives, and the deepening of resale markets for capital have made it easier for intermediaries to trade their assets (i.e.

[4] Although intermediaries clearly have an important role in practice, there is nothing in the structure of our model which precludes consumers from investing directly in firms. We could formally motivate the existence of intermediaries by, for example, introducing asymmetric information or, more specifically, following Diamond and Dybvig (1983) or Holmstrom and Tirole (1998). But this would significantly complicate the analysis without changing our main results. Therefore, for simplicity and transparency, we simply assume that consumers can only invest in the more-productive sector through intermediaries. Indeed, the involvement of intermediaries in investment projects in the more-productive sector could be interpreted as partially driving the higher returns in that sector relative to the traditional sector.

their loans / investments in firms). This especially applies to non-traditional financial intermediaries.

2.2 Production Opportunities

Figure 1 depicts the timing of events. Intermediaries can invest in the productive sector in periods 0 and 1. Since there is no depreciation, an investment of i_0 in period 0 delivers i_0 units of capital in period 1. We also suppose it delivers xi_0 units of the consumption good (profit) in period 1, where x is a common aggregate shock with distribution function $H(x)$. The realisation of x is revealed to all agents in period 1, depends on the aggregate state, s, and can be contracted upon. Intuitively, the shock represents the per unit surplus (positive x) or shortfall (negative x) in period 1 revenue relative to (future) operating expenses.[5] Let $E\{x\} = \mu > 0$, so that early investment in period 0 is expected to be profitable. If x turns out to be negative, the intermediary has two options: it can either incur the cost xi_0 (possibly by selling a portion of its capital to consumers) and continue with the investment project; or it can go into liquidation, abandoning the project and selling all of its capital to consumers.[6] In the latter case, it receives zero profit in period 2 but does does not need to pay xi_0. In what follows, we associate total liquidation by the representative intermediary as reflecting a systemic financial crisis.[7]

In period 1, intermediaries can either sell k^S units of capital to consumers or make an additional investment, $i_1 \geq 0$. Therefore, they enter period 2 owning a total capital stock of:

$$k_s = i_0 - k_s^S + i_{1s} \tag{1}$$

Invested in the productive sector, this capital yields Ak_s units of the consumption good in period 2, where A is a constant greater than one.

If consumers acquire capital from intermediaries in period 1, they can also use it to produce consumption goods in period 2, but they only have access to a less-productive technology operating in the perfectly competitive traditional sector of the economy. In particular, the production function in the traditional sector, $F(k^T)$, displays decreasing returns to scale, with $F'(k^T) > 0$ and $F''(k^T) < 0$. For simplicity, $F'(0) = 1$, implying that there is no production in the traditional sector unless $q < 1$ (i.e. unless intermediaries sell capital in period 1). To aid intuition, we assume the specific form:

$$F(k^T) = k^T\left(1 - \alpha k^T\right) \tag{2}$$

[5] Alternatively, a positive x could be viewed as an early return on investment and a negative x as a restructuring cost or an additional capital cost which must be paid to continue with the project.

[6] Since intermediaries are homogeneous and unable to trade each other's equity, there is no scope for them to sell capital to each other following a negative aggregate shock.

[7] As financial contracts are fully state-contingent in this model (see section 2.3), they will be specified so that repayments from intermediaries to consumers are zero in states in which intermediaries are solvent but in severe distress. Since this implies that intermediaries never default on their contractual liabilities to consumers, it makes sense to associate systemic financial crises with total liquidation.

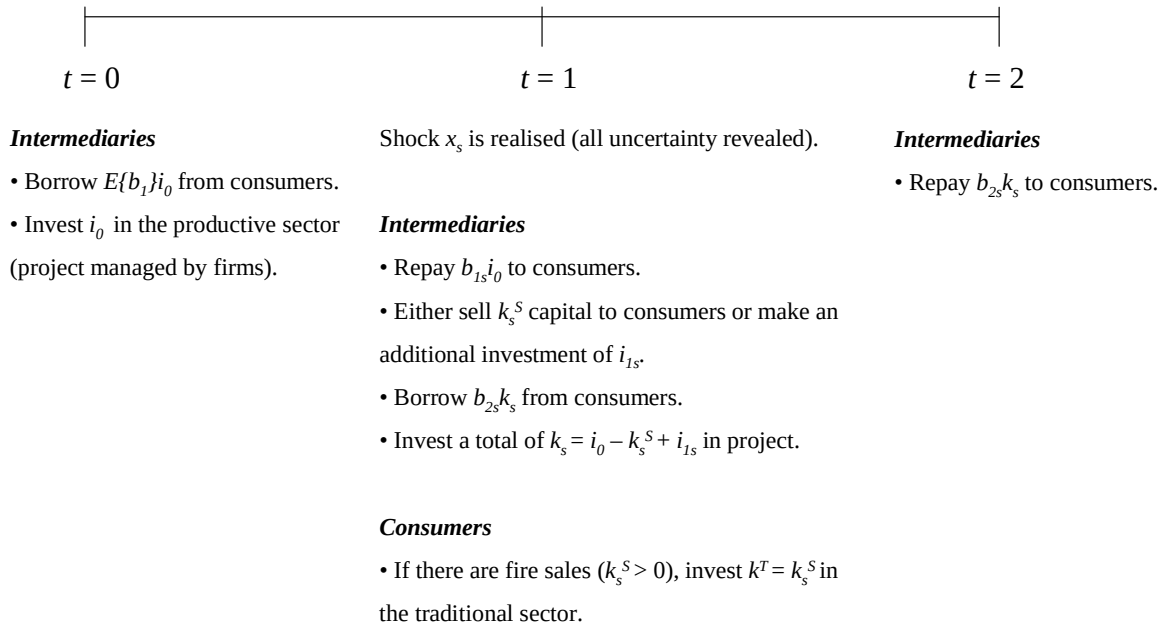


Figure 1: Timeline of Events

where $2\alpha k^T < 1$.

The diminishing returns embedded in the production function are designed to capture the link, highlighted by Shleifer and Vishny (1992), between distress selling of capital and reduced asset prices. As they argue, many physical assets (e.g. oil tankers, aircraft, copper mines, laboratory equipment etc.) are not easily redeployable, and the portfolios of intermediaries, many of which contain exotic tailor-made assets, are similar in this regard. Therefore, if an aggregate shock hits an entire sector, participants in that sector wishing to sell assets may be forced to do so at a substantial discount to industry outsiders.

The parameter α reflects the productivity of second-hand capital. Although this partly depends on the underlying productivity of capital in alternative sectors, it also captures the effectiveness with which capital is channelled into its most effective use when it is sold. As such, it is likely to be decreasing in financial market depth (note that $\alpha = 0$ corresponds to constant returns to scale in the traditional sector). Since increased market participation, greater global mobility of capital, and the development of sophisticated financial products may all serve to deepen resale markets, α is likely to have fallen in recent years.

2.3 Financial Contracts and Constraints

Intermediaries partially finance investment projects by borrowing. At date 0, they offer a state-contingent financial contract to consumers. As shown in the timeline, this specifies repayments in state s of $b_{1s}i_0$ in period 1 and $b_{2s}k_s$ in period 2, and borrowing

of $E\{b_1\} i_0$ in period 0 and $b_{2s}k_s$ in period 1 and state s, where b is the repayment / borrowing ratio. Since period 1 repayments to consumers on period 0 lending are state-contingent, this has some features of an equity contract. In particular, the contract is capable of providing intermediaries with some insurance against aggregate shocks.

Although this contract is fully contingent on the aggregate state, it is subject to limited commitment and potential default. This friction is fundamental to the model: without it, the competitive equilibrium would be efficient and systemic financial crises would never occur. Its significance lies in the borrowing constraints which it imposes on financial contracts:

$$(b_{1s}i_0 - b_{2s}k_s) + b_{2s}k_s \geq 0 \qquad \forall s \tag{3}$$

$$b_{2s}k_s \geq 0 \qquad \forall s \tag{4}$$

$$b_{1s}i_0 \leq \theta q_{1s}i_0 \qquad \forall s \tag{5}$$

$$b_{2s}k_s \leq \theta q_{2s}k_s \qquad \forall s \tag{6}$$

where q_{ts} is the asset price in period t and state s, and $\theta \leq 1$ is the fraction of the asset value that can be used as collateral.

The first two constraints, (3) and (4), reflect limited commitment on the consumer side. In particular, they imply that net future repayments to consumers must be non-negative. In other words, regardless of the state, consumers cannot commit to make net positive transfers to intermediaries at future dates. Constraint (3) relates to net future repayments as viewed in period 0 (for which additional intermediary borrowing in period 1 must be taken into account); constraint (4) relates to future repayments as viewed in period 1. These constraints follow from assuming that the future income of consumers cannot be seized – consumers can always default on their financial obligations.[8]

The final two constraints, (5) and (6), specify that intermediaries can only borrow up to a fraction, θ, of the value of their assets in each period, where we define θ to be the maximum loan-to-value ratio. Jermann and Quadrini (2006, Appendix B) present a simple model which motivates constraints such as these. In particular, they link an equivalent parameter to θ to the value of capital recovered upon default relative to its original value when held by the borrower, and to the relative bargaining power of borrowers and lenders. Importantly, if the recovery rate is less than one, the maximum loan-to-value ratio will also be less than one. As argued by Gai, Kondor and Vause (2005), recovery rates below one may reflect transaction costs built into the specifics of collateral arrangements, such as dispute resolution procedures. Alternatively, there may be human capital loss associated with default.

[8]Collectively, it would be in the interests of consumers to commit to make net positive transfers to intermediaries in certain states at future dates. But such a commitment is not incentive compatible since consumers each have an individual incentive to renege *ex post*. Limited commitment on the consumer side can thus also be viewed as stemming from the lack of a suitable commitment device amongst consumers.

We regard the maximum loan-to-value ratio as being linked to the level of financial market development. It seems likely that financial innovation may have increased θ in recent years. Deeper resale markets may have reduced the human capital loss associated with default, and could have enabled sellers of assets seized upon default to pass on a larger proportion of the resale transaction costs to buyers than previously.[9] More generally, the greater use of credit derivative and syndicated loan markets may have increased recovery rates for lenders. Alternatively, as highlighted by Jermann and Quadrini (2006), the development of more sophisticated asset-backed securities may have made it easier for borrowers to pledge their assets as collateral to lenders. All of these factors may have made investors willing to accept higher loan-to-value ratios, thus raising θ.

It is clear that some of these factors relate to the depth of secondary markets. As such, increases in θ may be closely tied to reductions in α. This concurs with broader theoretical arguments linking the debt capacity of investors to the liquidity and depth of the secondary markets for assets used as collateral for that debt. For example, Williamson (1988) and Shleifer and Vishny (1992) discuss how the redeployability of assets is a key factor in determining their liquidation value and that this, in turn, affects investors' debt capacity. More recently, Brunnermeier and Pedersen (2006) have studied the relationship between the leverage capacity of traders and financial market liquidity, demonstrating that they are likely to be positively correlated and, importantly, that causality can run both ways.

3 Equilibrium

We now solve for equilibrium, focusing primarily on the competitive outcome. Since consumers expect investment in the productive sector of the economy to be profitable, and since they have very large endowments relative to financial intermediaries, they always meet the borrowing demands of intermediaries provided that constraints (3)-(6) are satisfied. Meanwhile, as noted above, firms simply manage investment projects for a negligible wage. Therefore, we can solve for the competitive equilibrium by considering the optimisation problem of the representative intermediary.

3.1 The Representative Intermediary's Optimisation Problem

The representative intermediary's optimisation problem is given by:

$$\max_{\pi_0,\{\pi_{1s}\},i_0,\{k_s\},\{b_{1s}\},\{b_{2s}\}} E_0\left\{\pi_0+\pi_1+\pi_2\right\}$$

[9] The latter point could potentially be modelled formally in a Nash bargaining framework – for a related model in this spirit, see Duffie, Garleanu and Pedersen (2005).

subject to:

$$\pi_0 + q_0 i_0 = n_0 + E\{b_1\} i_0 \tag{7}$$

$$\pi_{1s} + q_{1s} k_s = q_{1s} i_0 + x_s i_0 - b_{1s} i_0 + b_{2s} k_s \qquad \forall s\text{: no liquidation} \tag{8}$$

$$\pi_{1s} = q_{1s} i_0 - b_{1s} i_0 \qquad \forall s\text{: liquidation in period 1} \tag{8L}$$

$$\pi_{2s} = A k_s - b_{2s} k_s \qquad \forall s\text{: no liquidation} \tag{9}$$

$$\pi_{2s} = 0 \qquad \forall s\text{: liquidation in period 1} \tag{9L}$$

$$0 \leq b_{1s} \leq \theta q_{1s} \qquad \forall s \tag{10}$$

$$0 \leq b_{2s} \leq \theta q_{2s} \qquad \forall s \tag{11}$$

Equation (7) represents the intermediary's period 0 budget constraint: investment costs and any profits taken by the intermediary in period 0 must be financed by its endowment (initial net worth) and borrowing from consumers.[10] In period 1, provided that the investment project is continued (i.e. provided that the intermediary does not go into liquidation), the intermediary's budget constraint is given by (8): financing is provided by start of period assets at their market value ($q_{1s} i_0$) and net period 1 borrowing ($b_{2s} k_s - b_{1s} i_0$), adjusted for the revenue surplus or shortfall, $x_s i_0$. Period 2 profits in this case are then given by (9). By contrast, if the intermediary goes into liquidation in period 1, it sells all of its capital at the market price, yielding $q_{1s} i_0$ in revenue. Therefore, its period 1 profits are given by (8L), while period 2 profits are zero (equation (9L)). Finally note that (10) and (11) simply represent combined and simplified versions of the borrowing constraints, (3)-(6).

This optimisation problem can immediately be simplified. Since expected returns on investment are always high, it is clear that the intermediary will never take any profits until period 2 unless it goes into liquidation. Therefore $\pi_0 = 0$ in (7) and $\pi_{1s} = 0$ for all s in (8). Moreover, given that it is certain, the high return between periods 1 and 2 also implies that intermediaries wish to borrow as much as possible in period 1. So (11) binds at its upper bound and $b_{2s} = \theta q_{2s}$. Finally, since the asset price only differs from one if capital goods are being sold, and since the structure of the model implies that this can only ever occur in period 1, $q_0 = 1$ and $q_{2s} = 1$ for all s. Therefore, we can rewrite the intermediary's optimisation problem as:

$$\max_{i_0, \{k_s\}, \{b_{1s}\}} E_0\{\pi_1 + \pi_2\}$$

[10] Both this and the other budget constraints must bind by local non-satiation.

subject to:

$$i_0 = n_0 + E\{b_1\} i_0 \tag{12}$$

$$q_{1s} k_s = q_{1s} i_0 + x_s i_0 - b_{1s} i_0 + \theta k_s \quad \forall s\text{: no liquidation} \tag{13}$$

$$\pi_{1s} = q_{1s} i_0 - b_{1s} i_0 \quad \forall s\text{: liquidation in period 1} \tag{8L}$$

$$\pi_{2s} = A k_s - \theta k_s \quad \forall s\text{: no liquidation} \tag{14}$$

$$\pi_{2s} = 0 \quad \forall s\text{: liquidation in period 1} \tag{9L}$$

$$0 \leq b_{1s} \leq \theta q_{1s} \quad \forall s \tag{10}$$

3.2 Multiple Equilibria and Systemic Crises: Intuition

Before solving the intermediary's optimisation problem, we graphically illustrate how multiple equilibria and systemic financial crises arise in the model. Faced with a negative realisation of x, intermediaries may be forced to sell a portion of their capital to the traditional sector in period 1 to remain solvent. In these fire sale states, $i_{1s} = 0$ and, using (1), $k_s = i_0 - k_s^S = i_0 - k_s^T$, where $k_s^S = k_s^T \leq i_0$. Provided that intermediaries remain solvent, we can substitute this expression into (13) and rearrange to obtain the inverse supply function for capital in the traditional sector:

$$q_{1s} = \frac{(b_{1s} - x_s - \theta) i_0}{k_s^T} + \theta \tag{15}$$

From (15), it is clear that the supply function is downward sloping and convex. The intuition for this is that when the asset price falls, intermediaries are forced to sell more capital to the traditional sector to remain solvent; the more the asset price falls, the more capital needs to be sold to raise a given amount of liquidity. Equation (15) holds for all $k_s^T < i_0$. But if intermediaries sell all of their capital and go into liquidation, the supply of capital to the traditional sector is simply given by:

$$\left(k_s^T\right)^L = i_0 \tag{16}$$

Meanwhile, since the traditional sector is perfectly competitive, the inverse demand function for capital sold by intermediaries follows directly from (2):

$$q = F'(k^T) = 1 - 2\alpha k^T \tag{17}$$

This function is downward sloping and linear due to linearly decreasing returns to scale in the traditional sector. Combining (15), (16) and (17) yields the equilibrium asset price(s) in fire sale states.

The supply and demand functions are sketched in $\left(q, k^T\right)$ space in Figure 2. As can be seen, there is the potential for multiple equilibria in fire sale states. In particular, if

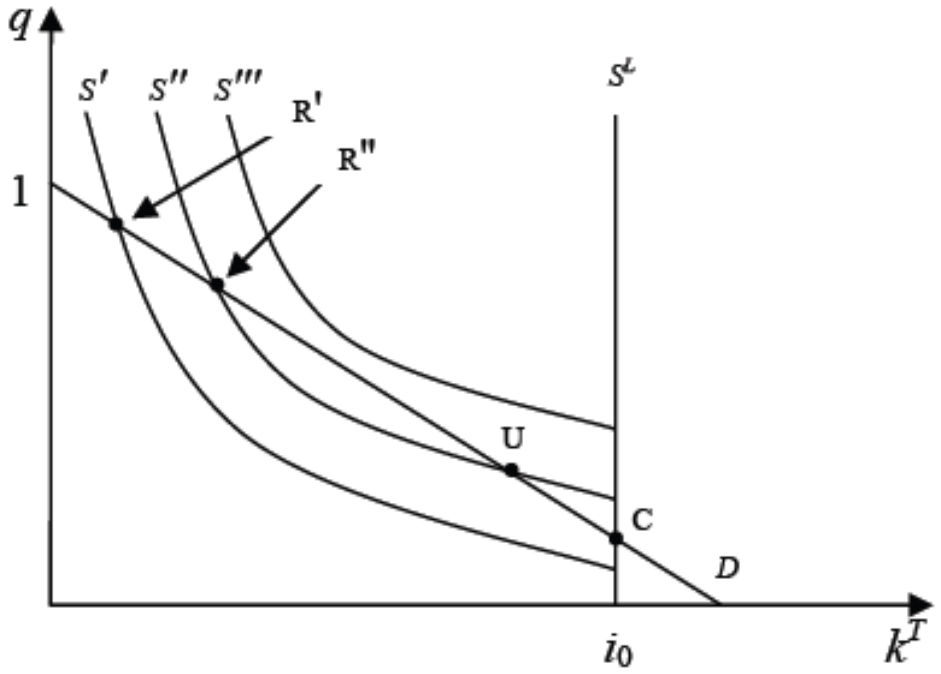


Figure 2: Demand and Supply for Capital in the Traditional Sector

the (non-liquidation) supply schedule is given by S'', there are three equilibria: R$''$, U and C. From (15), $S(0) > 1$ for all supply schedules. Therefore, U is unstable but the other two equilibria are stable. Point C corresponds to a crisis: intermediaries go into liquidation, firms shut down, and all capital is sold to the traditional sector, causing the asset price to fall substantially. By contrast, at R$''$, fire sales are limited and the asset price only falls slightly – we view this as a 'recession' equilibrium since intermediaries remain solvent and firms continue to operate in the productive sector.

The actual outcome between R$''$ and C is determined solely by beliefs: if intermediaries believe *ex ante* (before the realisation of the shock) that there will be a systemic crisis in states for which there are multiple equilibria, a crisis will indeed ensue in those states; if they believe *ex ante* that there will only be a 'recession' in those states, then that will be the outcome. Moreover, their *ex ante* investment and borrowing decisions depend on their beliefs. Therefore, multiple equilibria arise *ex ante*: after beliefs have been specified (at the start of period 0), investment and borrowing decisions will be made contingent on those beliefs and the period 1 equilibrium will be fully determinate, even in states for which there could have been another equilibrium.

However, multiple equilibria and systemic crises are not always possible in fire sale states. Specifically, if the supply schedule is given by S', R$'$ is the unique equilibrium and there can never be a systemic crisis, regardless of beliefs. From (15), it is intuitively clear that this is more likely to be the case when the negative x shock is relatively mild. By contrast, if the shock is extremely severe, a crisis could be inevitable – supply schedule S''' depicts this possibility.

3.3 The Competitive Equilibrium

We now proceed to solve the model for both 'optimistic' and 'pessimistic' beliefs. Suppose that all agents form a common exogenous belief at the start of period 0 about what equilibrium will arise when multiple equilibria are possible in period 1: if beliefs are 'optimistic', agents assume that there will not be a crisis unless it is inevitable (i.e. unless the supply schedule resembles S'''); if beliefs are 'pessimistic', agents assume that if there is a possibility of a crisis, it will indeed happen. Then, as shown in Appendix A, the competitive equilibrium is characterised by the following repayment ratios associated with each possible state, x_s, where the precise thresholds ($\widehat{x}$, $\widehat{x} - \theta\widehat{q}$ and x^C) depend on beliefs and the distribution of shocks:

$$\text{if } \widehat{x} < x_s, \text{ then } b_{1s} = \theta q_{1s} \tag{18}$$

$$\text{if } \widehat{x} - \theta\widehat{q} < x_s < \widehat{x}, \text{ then } b_{1s} = \theta\widehat{q} - (\widehat{x} - x_s) \tag{19}$$

$$\text{if } x^C < x_s < \widehat{x} - \theta\widehat{q}, \text{ then } b_{1s} = 0 \tag{20}$$

$$\text{if } x_s < x^C, \text{ then } b_{1s} = \theta q^C = \max\left\{\theta\left(1 - 2\alpha i_0\right), 0\right\} \tag{21}$$

Expressions (18)-(20) correspond to similar expressions in Lorenzoni (2005), though the actual thresholds differ. However, (21) is unique to our model and reflects the possibility of systemic financial crises in our setup.

Apart from noting that $\widehat{x} \leq 0$ (since intermediaries will never choose to borrow less than the maximum against states where the realised x is positive), relatively little can be said about the precise location of the thresholds without specifying how the shock is distributed. Section 4 determines these thresholds, initial investment, and the state-contingent asset price for a specific distribution.

3.4 Discussion of the Competitive Equilibrium

Since expected future returns are positive, the competitive equilibrium always exhibits a high level of credit-financed investment in period 0. As summarised in Table 1, subsequent outcomes depend on the realisation of x. In 'good' states, x is positive, investment and credit growth remain strong in period 1, and the economy benefits from high returns in period 2. Of more interest for our analysis are the 'recession' and 'crisis' states in which x is negative. To further clarify what happens in these cases, we sketch the period 1 repayment ratio, b_1, and asset price, q_1, against x in Figures 3 and 4 respectively. For illustrative purposes, we present the cases of 'optimistic' and 'pessimistic' beliefs on the same diagram, adding an additional threshold, x^M, to reflect the range of x for which multiple equilibria are possible.[11] However, it is important to bear in mind that

[11] As for the other thresholds, the location of x^M cannot be computed without specifying the distribution of the shock. However, Figure 2 and the associated discussion clearly illustrate how multiple equilibria are only possible over a certain range of x.

State	Realisation of x_s	Description of Outcome
'Good'	$x_s > 0$	Intermediaries do not sell any capital. There is no production in the traditional sector.
'Recession'	x^C or $x^M \leq x_s \leq 0$	Intermediaries sell a portion of their capital but remain solvent (i.e. there are only limited fire sales). Firms continue to operate in the productive sector, but with a lower capital stock than in 'good' states. There is some production in the traditional sector.
'Crisis'	$x_s < x^C$ or x^M	Intermediaries sell all of their capital and go into liquidation. Firms operating in the productive sector shut down. Production only takes place in the traditional sector.

Table 1: Summary of Outcomes

the thresholds themselves are endogenous to beliefs.

To explain the repayment ratio function in Figure 3, consider what happens when there is a negative x shock (for positive x, $q_1 = 1$, implying that $b_1 = \theta$). As noted above, if the intermediary goes into liquidation as a result of the shock (i.e., if $x_s < x^C$ or x^M, depending on beliefs), it does does not need to pay the cost xi_0. In this case, it sells all of its capital at the prevailing market value and repays this 'scrap value' to consumers. Although it may seem unusual that repayments are positive in 'crisis' states (and potentially higher than in 'recession' states), this is entirely optimal. Intuitively, intermediaries have no need for liquidity in 'crisis' states because they shut down and do not pay the cost xi_0. By increasing repayments to consumers in these states, they are able to increase their period 0 borrowing. Since period 0 investment is expected to be profitable, it is, therefore, optimal for intermediaries to promise to repay the entire 'scrap value' of the project to consumers in 'crisis' states.

If, however, the intermediary wants to avoid liquidation following a negative shock, it must find a way of financing the cost xi_0. Given that it always chooses to borrow the maximum amount it can between periods 1 and 2, the cost can be financed either by reducing repayments to consumers in adverse states or by selling a portion of its capital.

The first option reduces expected repayments to consumers (i.e. $E\{b_1\}$), lowering the amount that the intermediary can borrow in period 0 (see equation (12)) and therefore reducing returns in 'good' states. The expected cost associated with doing this is constant. By contrast, the cost of the second option increases as the asset price falls. So, for mild negative shocks in region F of Figure 3, it is better to sell capital because the asset price remains relatively high. The borrowing / repayment ratio in these states remains at its maximum, but this maximum falls slowly as the asset price falls (see equations (5) and (18)).

However, when shocks are more severe and fall in region G, the costs of selling capital are so high that it becomes better to reduce repayments to consumers than to sell

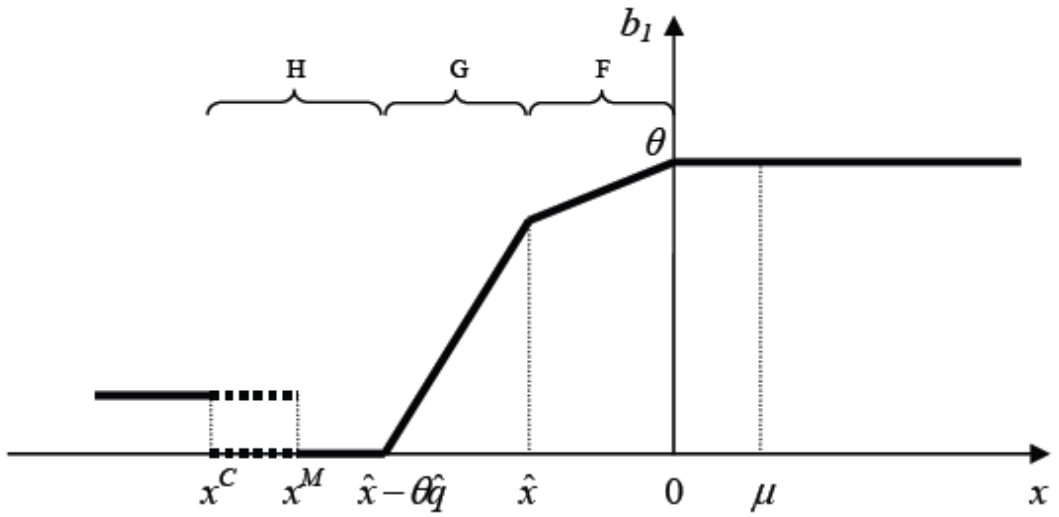


Figure 3: The Repayment Ratio as a Function of the Shock

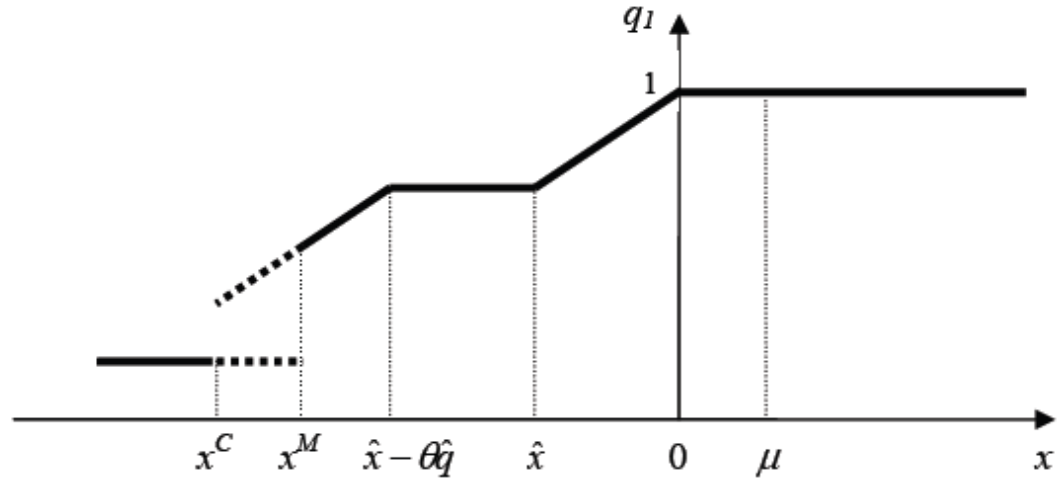


Figure 4: The Asset Price as a Function of the Shock

further capital – this is reflected in (19). Eventually, however, the scope for reducing repayments is fully exhausted and the only way to finance the cost is to sell further capital even though the asset price is relatively low (region H). It is at this point that the $b_{1s} > 0$ constraint bites: intermediaries would ideally like to receive payments from consumers in these extremely bad states but are prevented from doing so by limited commitment on the consumer side.[12]

Since the asset price, q_1, only changes when the amount of capital being sold changes, the intuition behind Figure 4 follows immediately. For positive x, no capital is ever sold, so the asset price remains at one. However, for negative (but non-crisis) values of x, the asset price falls over those ranges for which intermediaries finance xi_0 by selling addi-

[12]Since early investment is expected to be profitable, intermediaries have no incentive to set aside liquid resources in period 0 to self-insure against extremely bad states in period 1. But even if some self-insurance were optimal, asset sales would still be forced for sufficiently severe shocks.

tional capital (i.e. for $\widehat{x} < x_s < 0$ and $x_s < \widehat{x} - \theta\widehat{q}$). Meanwhile, in crises, intermediaries sell all of their capital and the asset price is determined by substituting (16) into (17), which gives $q^C = \theta\left(1 - 2\alpha i_0\right)$. If this expression is negative, returns to capital in the traditional sector fall to zero before all the available capital is being used. In this case, the leftover capital has no productive use in the economy, and $q^C = 0$.

3.5 The Constrained Efficient Equilibrium, Efficiency, and the Source of the Externality

We can show that the competitive equilibrium is constrained inefficient by solving the problem faced by a social planner who maximises the same objective function as intermediaries and is subject to the same constraints, but who does not take prices as given. Under certain mild conditions (see Appendix B), the social planner can obtain a welfare improving allocation by reducing intermediaries' borrowing and investment. More specifically, the social planner implements a reduction in borrowing against certain states that has no direct effect on intermediaries' welfare. But it has a potentially important indirect effect: by reducing investment, the amount of capital that has to be sold in fire sale states is reduced, and this both reduces the negative effects of asset price falls, and lowers the likelihood and severity of crises.

The competitive equilibrium thus exhibits over-borrowing and over-investment relative to the constrained efficient equilibrium. In particular, if we view the situation with no frictions (i.e. without borrowing constraints (3)-(6)) as corresponding to the first-best outcome and the constrained efficient equilibrium as the second-best, then the competitive allocation is fourth-best. This is because policy intervention could feasibly achieve a third-best outcome even if the second-best allocation cannot be attained.

As noted earlier, the limited commitment and potential default to which financial contracts are subject is the key friction in this model. It is straightforward to show that the critical constraint is (3): if this were relaxed, the competitive equilibrium would be efficient and there would never be systemic crises because intermediaries would be able to obtain additional payments from consumers in times of severe stress (i.e. when $x_s < \widehat{x} - \theta\widehat{q}$) rather than being forced to sell capital. However, when coupled with decreasing returns to capital in the traditional sector, the presence of this constraint introduces an asset fire sale externality: intermediaries do not internalise the negative effects on asset prices that their own fire sales have. By tightening their budget constraints further, these asset price falls force other intermediaries to sell more capital than they would otherwise have to. In extreme cases, this externality is the source of systemic crises.

4 Comparative Statics

We now analyse the effects of financial innovation and changes in macroeconomic volatility on the likelihood and potential scale of systemic crises. This necessitates an assumption about beliefs so that the cut-off value of x below which crises occur is determinate. Accordingly, we suppose that agents have 'optimistic' beliefs, so that crises only occur when they are inevitable.[13]

The shock x is assumed to be normally distributed with mean μ and variance σ^2, where $\mu > 0$. Since analytical solutions for thresholds are unavailable, we present the results of numerical simulations. Robustness checks were performed by varying the parameters over a range of values. Unless stated otherwise, the comparative static results are not sensitive to the particular parameter values used.[14]

We measure the likelihood of a crisis by $H(x^C) = \Pr\left[x < x^C\right]$ and its scale (impact) in terms of the asset price, q^C, which prevails in it. Lower values of q^C correspond to more serious crises. To motivate q^C as a measure of the impact of crises, recall that in period 0, consumption goods are turned into capital goods one for one. If some capital goods end up being used in the less-productive sector to produce consumption goods (as happens in a crisis), fewer consumption goods can be produced than were used to buy those capital goods initially. Since a lower q corresponds to reduced returns on the marginal unit of capital in the traditional sector and hence less production of the consumption good from the marginal capital good, the loss associated with a crisis increases as q^C falls. Moreover, lower values of q^C correspond to greater asset price volatility in the economy, further suggesting that it may be an appropriate measure of the scale of systemic instability.

4.1 Changes in Macroeconomic Volatility

We interpret a change in macroeconomic volatility as affecting σ. Since x is linked to revenue shortfalls and surpluses, it is reasonable to assume that a reduction in output and inflation volatility (as is likely to be associated with a general reduction in macroeconomic volatility) corresponds to a fall in the standard deviation of x.

Intuitively, a reduction in σ will lower the probability of crises since extreme states become less likely. This is borne out in Figure 5(a). However, provided that the mean, μ, is sufficiently above zero and the variance is not too large, a lower standard deviation also makes states 'recession' states less likely to occur. As a result, expected repayments to consumers, $E\{b_1\}$, are higher, meaning that intermediaries can borrow more in period 0. Therefore, initial investment, i_0, is higher. But this means that if a crisis then does

[13] All of our qualitative results continue to hold if agents have 'pessimistic' beliefs.

[14] In our baseline analysis, we assume the following parameter values: $A = 1.5$; $n_0 = 1$; $\mu = 0.5$; $\sigma = 0.5$; $\theta = 0.75$; $\alpha = 0.05$. We then consider the effects of varying σ, θ and α. The relevant *Matlab* code is available on request from the authors.

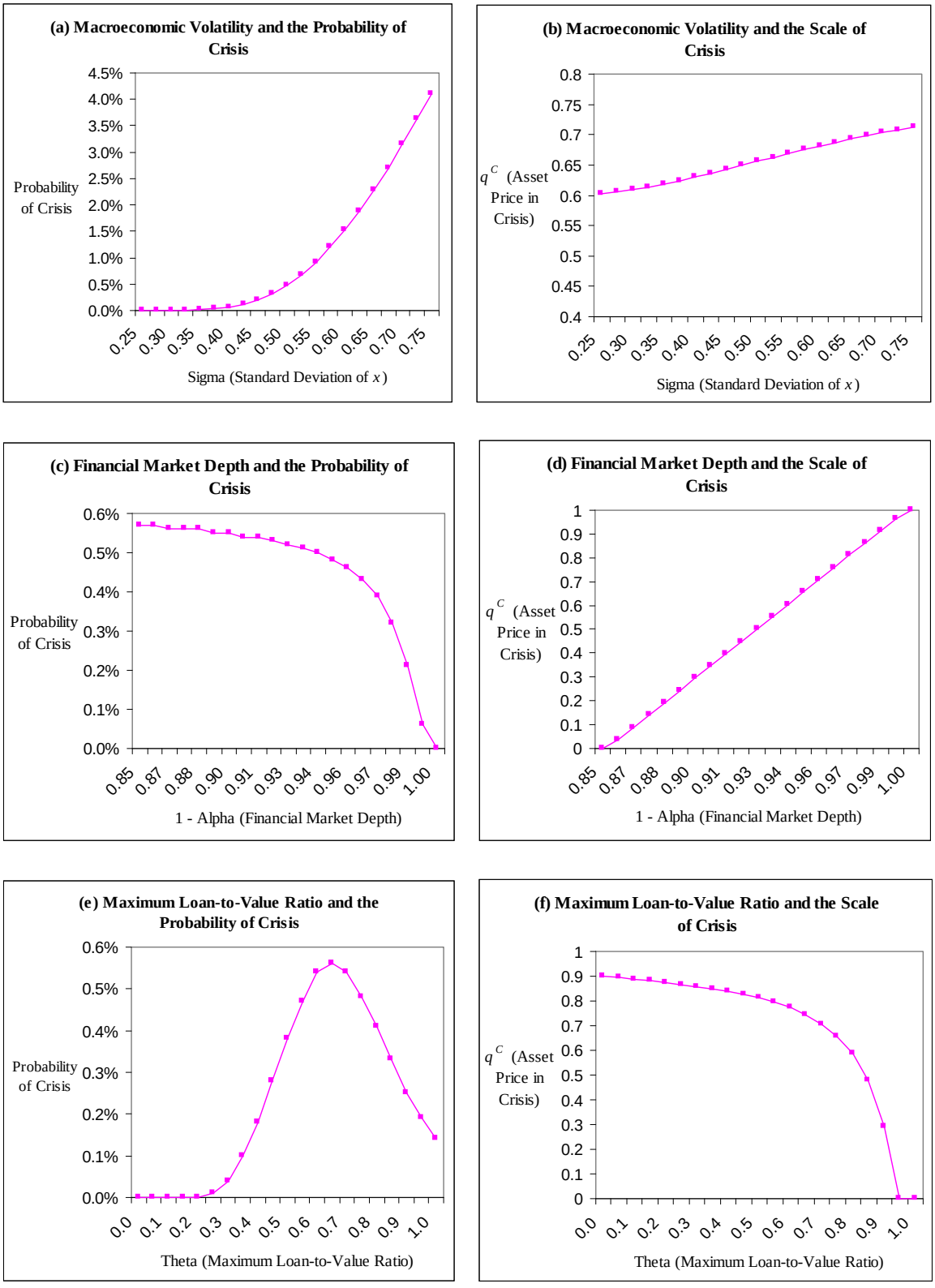


Figure 5: Comparative Static Results

arise, more capital will be sold to the traditional sector, the asset price will be driven down further, and the crisis will have a greater impact. This is shown in Figure 5(b) and can also be seen by considering a rightward shift of S^L in Figure 2.[15]

4.2 The Impact of Financial Innovation

We have already argued that financial innovation and recent developments in financial markets can be interpreted as driving higher maximum loan-to-value ratios (higher values of θ) and greater financial market depth (lower values of α). Assuming that the initial value of θ is not particularly low, Figure 6(a) illustrates how these changes have made crises less likely (darker areas in the chart correspond to a higher crisis frequency). But from Figure 6(b), it is apparent that the severity of crises may have increased (darker areas correspond to a more severe crisis).

To understand the intuition behind these results, we isolate the individual effects of changes in α and θ. Figures 5(c) and 5(d) suggest that a reduction in α reduces both the likelihood and scale of crises. This is intuitive. If the secondary market for capital is deeper, shocks can be better absorbed and, in the context of Figure 2, the demand curve in the traditional sector is flatter. As a result, crises are both less likely and less severe.[16]

By contrast, Figures 5(e) and 5(f) suggest that an increase in θ increases the severity of crises and has an ambiguous effect on their probability. Intuitively, a rise in θ enables intermediaries to borrow more. Therefore, i_0 is higher, and crises will be more severe if they occur. Greater borrowing in period 0 clearly serves to increase the probability of crises as well (consider what would happen with $\theta = 0$ and no borrowing in period 0). However, a rise in θ also means that intermediaries have greater access to liquidity in period 1: specifically, they have more scope to reduce period 1 repayments to consumers. This effect means that they are less likely to go into liquidation, making crises less likely.

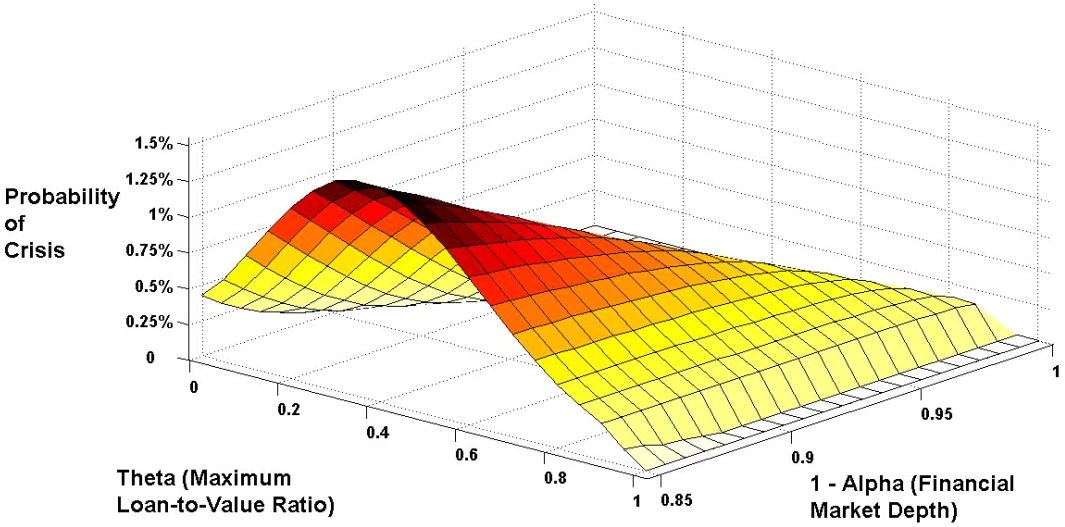
Figure 5(e) shows that crises are most frequent for intermediate values of θ, suggesting that middle-income emerging market economies may be most vulnerable to systemic instability.[17] By contrast, countries with extremely well-developed or very underdeveloped financial sectors, with high / low maximum loan-to-value ratios, are probably less

[15] If μ is very close to zero and/or σ is very large, it is possible for a reduction in σ to make 'recession' states more likely. This can potentially lead to a reduction in $E\{b_1\}$ and hence i_0, thus reducing the impact of crises upon occurrence. Since the numerical results suggest that this only happens for fairly extreme combinations of the mean and variance, we view the case discussed in the main text as being more likely. However, this feature does have the interesting implication that crises could be most severe in fairly stable and extremely volatile economies.

[16] This analysis assumes that secondary markets continue to function with the onset of a crisis. However, α itself could be endogenous and change during periods of stress. So reductions in α in benign times may have little effect on the severity of crises.

[17] Aghion, Bacchetta and Banerjee (2004) present a similar result but their approach is quite different, focussing on the effects of fluctuating real exchange rates and international capital flows in a small open economy model.

(a) Maximum Loan-to-Value Ratio, Financial Market Depth and the Probability of Crisis



(b) Maximum Loan-to-Value Ratio, Financial Market Depth and the Scale of Crisis

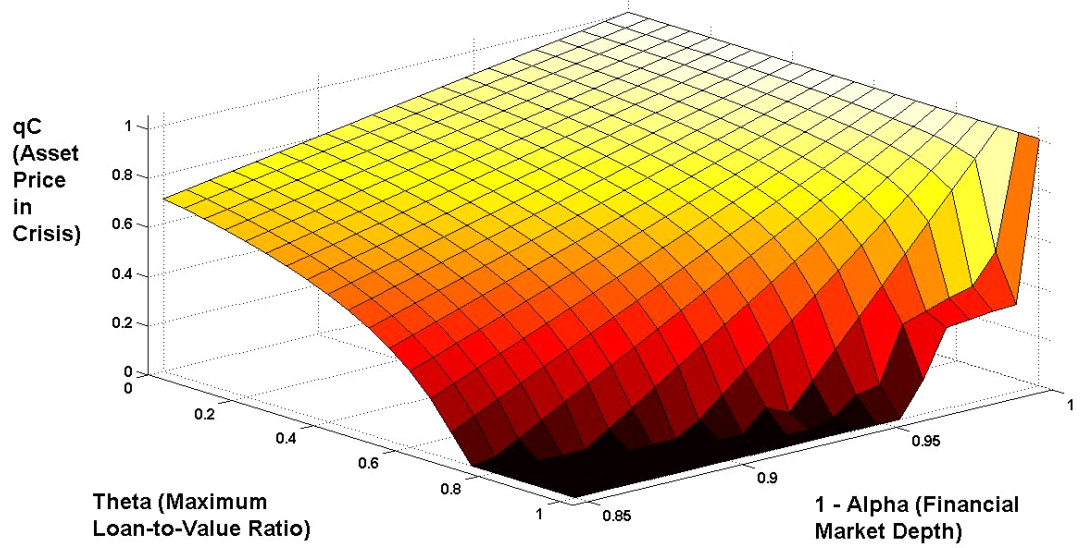


Figure 6: Financial Innovation and the Probability and Scale of Crises: 3D Charts

vulnerable to crises.

5 Conclusion

This paper analysed a theoretical general equilibrium model of intermediation with financial constraints and state-contingent contracts containing a clearly defined pecuniary externality associated with asset fire sales during periods of stress. After showing that this externality was capable of generating multiple equilibria and systemic financial crises, we considered the effects of changes in macroeconomic volatility and developments in financial markets on the likelihood and severity of crises. Together, our results suggest how greater macroeconomic stability and financial innovation may have reduced the probability of systemic financial crises in developed countries in recent years. But these developments could have a dark side: should a crisis occur, its impact could be greater than was previously the case.

The paper sheds interesting light on cross-country variation in the likelihood and scale of financial crises. Macroeconomic volatility is generally higher in developing countries than in advanced economies but maximum loan-to-value ratios are invariably lower. Given this, our results predict that crises in emerging market economies should be more frequent but less severe than in developed countries. The first of these assertions is clearly borne out by the data (Caprio and Klingebiel, 1996, Table 1; Demirguc-Kunt and Detragiache, 2005, Table 2). Although the second is more difficult to judge given the rarity of financial crises in developed countries in recent years, the length and depth of the Japanese financial crisis of the 1990s suggests that such intuition is plausible. Moreover, in terms of output losses, Hoggarth, Reis and Saporta (2002) find that crises in developed countries do indeed tend to be more costly than those in emerging market economies.

Appendix A: The Competitive Equilibrium

In this appendix, we solve the model for the competitive equilibrium when all agents have 'optimistic' beliefs about what equilibrium will arise in states in which multiple equilibria are possible. Specifically, they believe that crises only happen when they are inevitable and never occur when there are multiple equilibria. If agents have 'pessimistic' beliefs, the derivation proceeds along very similar lines.

Conditional on beliefs, the equilibrium is unique, and can be fully characterised by the three cut-off values for the aggregate shock x shown in expressions (18)-(21). These cut-offs determine four intervals in the distribution of x (i.e. in the distribution of possible states). In each of these intervals, intermediaries' incentives to protect their net worth, and hence their decisions about optimal repayments, will be different. We show

how the equilibrium can be fully characterised by these three cut-off points and how, conditional on beliefs, it is unique.

Define the subset $\mathbf{C}$ as the (endogenous) set of states where there is a crisis. Then the return, z_s, that intermediaries obtain in period 2 in state s by investing one unit of their net worth in state s in period 1 is given by:

$$z_s = \begin{cases} \frac{A-\theta}{q_{1s}-\theta} & \forall s \notin \mathbf{C} \\ 1 & \forall s \in \mathbf{C} \end{cases} \tag{22}$$

To derive this expression, note that in non-crisis states in period 1, a given amount of net worth, n_1, can be leveraged to obtain a total investment by intermediaries of $q_{1s}k_s = n_1 + \theta k_s$. In other words, each unit of net worth is leveraged by a factor of $1/(q_{1s} - \theta)$. Since the return per unit of capital after payment of liabilities is $A - \theta$ (recall that $b_{2s} = \theta$), return per unit of net worth in non-crisis states is therefore $(A - \theta) / (q_{1s} - \theta)$. By contrast, in crisis states, intermediaries do not invest, so the marginal return to net worth is one.

Meanwhile, the return, z_0, that intermediaries obtain in period 2 by investing one unit of their net worth in period 0 is given by:

$$z_0 = E_{s\notin\mathbf{C}}\left\{z\frac{x+q-b_1}{1-E\{b_1\}}\right\}\Pr[s\notin\mathbf{C}] + E_{s\in\mathbf{C}}\left\{\frac{q-b_1}{1-E\{b_1\}}\right\}\Pr[s\in\mathbf{C}] \tag{23}$$

This is the expected value of the product of period 1 and period 2 returns. The period 1 return may be explained along similar lines to the period 2 return. The factor by which intermediaries leverage one unit of period 0 net worth to purchase capital is $1 - E\{b_1\}$. In non-crisis states, the return per unit of capital is $x_s + q_{1s} - b_{1s}$. However, since intermediaries that liquidate do not pay the cost $x_s i_0$, the return per unit of capital in crisis states is $q_{1s} - b_{1s}$.

States can be divided into four sets: $S_1 = \{s : 1 < z_s < z_0\}$; $S_2 = \{s : z_s = z_0\}$; $S_3 = \{s : z_s > z_0\}$; and $\mathbf{C} = \{s : z_s = 1 < z_0\}$.We want to show that these sets cover the whole distribution of x, with S_1 covering states from $+\infty$ to $\widehat{x}(< 0)$, S_2 from $\widehat{x}$ to $\widehat{x} - \theta\widehat{q}$, S_3 from $\widehat{x} - \theta\widehat{q}$ to x^C, and $\mathbf{C}$ from x^C to $-\infty$.

Consider a state s that belongs to S_1. We want to show that if $x_{s'} > x_s$, then $s' \in S_1$. In state $s \in S_1$, borrowing will be at its maximum possible level in period 0 ($b_{1s} = \theta q_{1s}$) because $z_0 > z_s$, and the price of capital will satisfy $q_{1s} = F'[\max\{k_s^T, 0\}]$. If $x_{s'} > x_s > 0$, then there are no fire sales and $q_{1s} = q_{1s'} = 1$, and $z_s = z_{s'}$. If $0 > x_{s'} > x_s$, then $k_{s'}^T < k_s^T$, $q_{1s} < q_{1s'}$ and $z_{s'} < z_s$. In both cases, $z_{s'} < z_0$ and hence s' belongs to S_1.

The threshold for x that separates S_1 and S_2 is $\widehat{x}$. It is the value for which, in equilibrium, $z_0 = z_s$ and there is maximum borrowing ($q_{1s} = \widehat{q}$ is the equilibrium price in that state). For all states in $S_2 = \{s : z_s = z_0\}$, q_{1s} has to be constant, and given that i_0 is constant in all states in S_2, the amount borrowed in each state is pinned down and

given by $b_{1s} = \theta\widehat{q} - (\widehat{x} - x_s)$. The second cut-off, $\widehat{x} - \theta\widehat{q}$, is the value of x for which $b_{1s} = 0$ and $z_s = z_0$. As x decreases beyond $\widehat{x} - \theta\widehat{q}$, the repayment / borrowing ratio cannot be reduced any further. Therefore, more capital is sold in the secondary market, implying that $q_{1s} < \widehat{q}$ and hence $z_s > z_0$. Following the same logic as when we show that all values above $\widehat{x}$ belong to S_1, it is straightforward to show that all values below $\widehat{x} - \theta\widehat{q}$ but above the crisis threshold, x^C, belong to S_3. (It is important to note at this point that we are assuming that whenever it is possible to have multiple equilibria, 'optimistic' self-fulfilling beliefs imply that the 'recession' equilibrium arises rather than the 'crisis' equilibrium. We do not specify the precise set of multiple equilibria states, as this set is itself endogenous and a function of beliefs.)

To complete the characterisation, we need to show that there is a threshold, x^C, below which crises are unavoidable, and find conditions under which this threshold is lower than $\widehat{x} - \theta\widehat{q}$. The solution for x^C is obtained by solving the system of two equations that results from equating the demand and supply curves and their slopes. It is given by:

$$x^C = -\left[\frac{(1-\theta)^2}{8\alpha i_0} + \theta\right] \tag{24}$$

An exact analytical condition for x^C to be lower than $\widehat{x} - \theta\widehat{q}$ requires an assumption about the distribution of x. In our numerical exercises we check that this condition is satisfied, finding that it is for most parameter values.

Appendix B: The Social Planner's Solution

The social planner's optimisation problem is given by:

$$\max_{i_0,\{k_s\},\{b_{1s}\}} E_0\left\{\pi_1 + \pi_2\right\} = \\ E_{s\notin\mathbf{C}}\left\{\frac{A-\theta}{q-\theta}(x+q-b_1)i_0\right\}\Pr\left[s \notin \mathbf{C}\right] + E_{s\in\mathbf{C}}\left\{(q-b_1)i_0\right\}\Pr\left[s \in \mathbf{C}\right] \tag{25}$$

subject to:

$$i_0 = n_0 + E\left\{b_1\right\} i_0 - \tau \tag{26}$$

$$k_s^T q_{1s} = -(x_s - b_{1s})i_0 - (i_0 - k_s^T)\theta \qquad \forall s\text{: no liquidation } (s \notin \mathbf{C}) \tag{27}$$

$$0 \leq b_{1s} \leq \theta q_{1s} \qquad \forall s \tag{10}$$

and:

$$E\left\{3e + \tau + F(k^T) - qk^T - c\right\} \geq U^{CE} \tag{28}$$

where $\mathbf{C}$ is the set of crisis states, U^{CE} is the utility of consumers under the competitive equilibrium, τ is a transfer from intermediaries to consumers, $F(k^T) - qk^T$ represents

profits to consumers from production in the traditional sector, $c = -\lambda x$ is the cost of a financial crisis to consumers, and $0 < \lambda < 1$.

Condition (28) requires that consumers are at least as well off in the constrained efficient equilibrium as in the competitive equilibrium. To satisfy this condition, the social planner implements any necessary transfer, τ, from intermediaries to consumers in period 0. The key difference between the social planner and representative intermediary problems is that the social planner does not take the asset price, q_{1s}, as given.

Since $q_{1s} = F'(k_s^T)$ and since $k^T = i_0$ in crisis states, the social planner's problem can be rewritten as:

$$\max_{i_0,\{k_s\},\{b_{1s}\}} E_0\left\{\pi_1 + \pi_2\right\} =$$
$$E_{s\notin\mathbf{C}}\left\{\frac{A-\theta}{F'(k^T)-\theta}(x + F'(k^T) - b_1)i_0\right\}\Pr\left[s \notin \mathbf{C}\right] + E_{s\in\mathbf{C}}\left\{(F'(i_0) - b_1)i_0\right\}\Pr\left[s \in \mathbf{C}\right]$$

subject to:

$$i_0 = n_0 + E\left\{b_1\right\} i_0 - \tau \tag{26}$$
$$k_s^T F'(k_s^T) = -(x_s - b_{1s})i_0 - (i_0 - k_s^T)\theta \qquad \forall s\text{: no liquidation } (s \notin \mathbf{C}) \tag{29}$$
$$0 \leq b_{1s} \leq \theta F'(k_s^T) \qquad \forall s \tag{30}$$

and:

$$E\left\{3e + \tau + F(k^T) - F'(k^T)k^T - c\right\} \geq U^{CE} \tag{31}$$

To show that the competitive allocation is not constrained efficient, it is sufficient to show that the social planner can increase welfare by decreasing borrowing and investment in period 0. Such a change has several effects:

1. It reduces welfare by lowering the level of *ex ante* investment, i_0.
2. It increases welfare by reducing liabilities, b_{1s}, in certain states.
3. It reduces the amount of capital that has to be sold in fire sale states, increasing the asset price in those states.
4. It reduces the likelihood of a crisis.

We wish to determine when the net effect on welfare is positive. The positive contributions to welfare arise directly from the lower level of asset sales in fire sale states, and indirectly from a decrease in the likelihood of a crisis. We derive a condition under which the direct mechanism alone gives a positive net effect. Considering the indirect effect would strengthen our results but the analysis depends on the specific distributional assumptions taken and there is generally no closed-form solution.

Starting from the competitive allocation, suppose the social planner reduces *ex ante* investment by Δi_0 and reduces borrowing by the same amount against states in which $z_0 = z_s$ (z_0 and z_s are *ex ante* and *ex post* returns, as defined in Appendix A). First note that reducing borrowing against these states has no negative welfare effect on intermediaries since they are indifferent between investing *ex post* in them and *ex ante* in general. Therefore, to determine whether the reduction in i_0 is welfare-improving, we simply need to consider whether the welfare cost to consumers can be fully compensated for by any gain to intermediaries.

Differentiating the market clearing condition for used capital (which is obtained by equating supply, (15), and demand (17)), we can see that the reduction in i_0 decreases the amount of capital sold in 'recession' states by:

$$\frac{dk_s^T}{di_0} = \frac{x_s + \theta - b_{1s}}{[F'(k^T) - \theta] + F''(k^T)k^T} \tag{32}$$

The profit consumers obtain from operating their technology is $F(k^T) - F'(k^T)k^T$. Therefore, in 'recession' states, the reduction in i_0 has a direct welfare cost to consumers of:

$$\rho_s = \frac{d[F(k^T) - F'(k^T)k^T]}{dk_s^T}\frac{dk_s^T}{di_0} = -\frac{x_s + \theta - b_{1s}}{[F'(k^T) - \theta] + F''(k^T)k^T}F''(k^T)k^T \tag{33}$$

Intuitively, ρ_s represents the amount of goods transferred in 'recession' states from consumers to intermediaries as a result of the social planner's implementation of an equilibrium with lower borrowing than the competitive equilibrium. Intermediaries have to transfer at least this amount to consumers (in period 0, when they have resources to do so) to compensate them for this loss. What needs to be shown is that the net effect of this transfer is positive for intermediaries.

This will be the case if:

$$E\{\rho\} z_0 < E\{\rho z\} \tag{34}$$

The left hand side of (34) is the cost of the transfer to intermediaries and the right hand side is the benefit. In period 0, intermediaries transfer $E\{\rho\}$ goods to consumers, which they could have invested at a return z_0. On the other hand, intermediaries now have extra resources of ρ_s in each 'recession' state in period 1. Since returns on additional capital in period 1 are z_s, the expected benefit from these extra resources is $E\{\rho z\}$ .Without specifying the distribution of x and the parameter values, we cannot be specific about when this inequality is satisfied. However, provided that the distribution of x has sufficient variance, so that states in which $z_s > z_0$ are not very isolated events, it is generally satisfied (note that the positive correlation between ρ and z helps it to be satisfied). If this is the case, welfare is unambiguously higher under the social planner's allocation than under the competitive equilibrium.

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