

Investment Decisions in Retirement: The Role of Subjective Expectations

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Abstract

The rapid transition from defined benefit (DB) pension plans to defined contribution (DC) plans has a potential benefit of offering pension holders greater control over how their pension accumulations are invested. If pension holders are willing to take some risk, investments in the stock market could increase their economic preparation for retirement, and, indeed, economic theory as well as the typical advice of financial advisors calls for stock market investments. Yet, the rate of stock holding is much below what theory suggests it should be, undoing any benefit associated with the greater control coming from DC plans. The leading explanations for this under-investing include excessive risk aversion, costs of entry, and misperceptions about possible returns in the stock market. We show that excessive risk aversion is not able to account for the low fraction of stockholding. However, a model with heterogeneous subjective expectations about stock market returns is able to account for low stock market participation, and tracks the share of risky assets conditional on participation reasonably well. Based on the model with subjective expectations, we estimate a consumption loss of 7–9% compared to investment under rational expectations. The policy implication is that there is considerable scope for welfare improvement as a result of consumer education regarding stock market returns.

1 Introduction

The lifecycle model is the standard tool for prospectively evaluating possible Social Security policy changes. This model assumes that individuals are forward looking and seek to maximize lifetime utility. When appropriately calibrated to capture the economic incentives of Social Security programs, it gives realistic predictions of household savings and wealth accumulation. On the other hand, it tends to generate investment patterns that are grossly at odds with observed portfolio choices, both inside and outside defined contribution (DC) pension plans (e.g., Hung, Meijer, Mihaly, & Yoong, 2009). According to the model (and to financial experts), retirement-age individuals should hold a substantial fraction of their wealth in risky assets. Furthermore, they should gradually reduce the share of wealth held in stocks as they age and the risk of facing high medical expenses increases and the risk of losses in the stock market

cannot be as easily compensated by labor income (e.g., Cocco, Gomes, & Maenhout, 2005). The empirical evidence is very different. In the U.S. less than 30% of retirement-age individuals hold stocks (Lee, Kapteyn, Meijer, & Yang, 2010). Moreover, conditional on stock holding, the mean (median) share of risky assets is around 25% (18%) and there is no evidence of a gradual reduction in portfolio shares with age (Hurd, 2002; Korniotis & Kumar, 2011).

Different reasons have been put forward to explain such observed patterns. For example, Heaton and Lucas (2000), Hochguertel (2003), Rosen and Wu (2004), and Curcuru, Heaton, Lucas, and Moore (2010) mention excessive risk aversion and background risks (e.g., medical costs), Alan (2006) points at startup costs of starting stock market participation, Haliassos and Michaelides (2003) study borrowing constraints, and Hong, Kubik, and Stein (2004) and Guiso, Sapienza, and Zingales (2008) propose social interaction and trust as explanations. While each of these have their merits, they do not explain empirical patterns satisfactorily.

Recently, authors have focused on subjective equity return expectations to explain the observed low rates of stock holding (Hurd, Van Rooij, & Winter, 2011; Kézdi & Willis, 2008). Individual beliefs may diverge substantially from estimates of returns based on historical series. In particular, low average expectations, coupled with high uncertainty and large heterogeneity in beliefs may be the reason why so many individuals are reluctant to invest in stocks. Data eliciting subjective equity return expectations seem to support this hypothesis. Individuals on average have very pessimistic expectations about the stock market, but beliefs vary substantially across groups with different education, permanent income, cognitive functioning, numerical ability, and health status.

Assessing the importance of the various proposed reasons has important policy implications. If excessive risk aversion is the major cause, there is no obvious public policy lever that would increase participation. However, were misperceptions to be an important barrier to investing, a possible remedy would be education about the risks and rewards.

This paper quantifies the importance of knowledge about stock market rates of return in reducing stock market participation and how economic preparation for retirement would increase were individuals fully informed about the distribution of stock market returns. We estimate the costs of misperceptions in terms of foregone rates of return and in terms of loss of lifetime utility. These costs will give an indication of the benefit that public policy might achieve. In doing so, we study an important issue that has not been addressed in the literature, namely whether a standard lifecycle model of saving and portfolio choice incorporating subjective expectations can reproduce observed investment profiles. Clearly the model would predict that individuals holding negative equity return expectations (Kézdi & Willis, 2008) should exclusively allocate their wealth to risk-free assets. A more interesting question is which parameters of the equity return distribution would reconcile theoretical and observed portfolio patterns conditional on participation. We quantify what the mean and the variance of future returns are that would rationalize the observed low fractions of wealth that are invested in risky assets and how these compare with those inferred from survey questions (Dominitz & Manski, 2007; Hurd et al., 2011; Kézdi & Willis, 2008) and with estimates based on historical returns. We also compute to what extent they induce poor portfolio choices and what the consequences are for economic wellbeing.

We limit ourselves to individuals who are fully retired. The advantage of this is that retirees do not face the income risk (job loss, uncertainty about promotions, and generally future labor

income) that workers face (Guiso, Jappelli, & Terlizzese, 1996), which simplifies the model and the estimation considerably, and avoids incorrectly attributing patterns due to income risk to misperceptions about stock market risk.

The outline of the paper is as follows. In section 2, we describe the lifecycle model we use. The data, the estimation of auxiliary processes, and the sources of the baseline versions of the parameter estimates are described in section 3. The predictions of the resulting baseline model are evaluated in section 4. This section also explores to what extent different values of the parameters, such as risk aversion or expected stock market returns, are able to match observed patterns in the data. Section 5 discusses the survey questions that elicit subjective beliefs on stock market returns, and computes the resulting subjective distributions of stock market returns. Subsequently, section 6 computes the patterns of stock holding, wealth, and consumption that are generated by the lifecycle model with the subjective expectations governing investment decisions, and compares these with the predictions of the model using historical returns and the patterns observed in the data. Section 7 concludes.

2 Model

In this section we describe a stylized model of life-cycle saving and portfolio decisions for retirees. Agents in the model choose consumption and allocate their savings between risky and risk-free assets seeking to maximize expected utility over a finite time horizon. The stock of savings comprises private wealth and non-annuitized pension wealth held in Individual Retirement Accounts (IRAs).

Preferences Age is denoted by t . We consider individuals who are retired, which allows us to abstract from labor income risk. They enter the model at age $t = t_0 \in \{60, \dots, 64\}$ and live up to age T_M . Total lifetime T_M is stochastic, but in the model the largest allowed value of T_M is $T = 100$. Preferences are described by the time-separable iso-elastic utility function with coefficient of relative risk aversion γ and discount factor β . Thus, the individual's expected remaining lifetime utility at age t is

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} + E_t \left(\sum_{j=t+1}^{T_M} \beta^{j-t} \frac{C_j^{1-\gamma}}{1-\gamma} \right), \quad (1)$$

where C_t is consumption and E_t indicates that the expectation is taken at age t , conditional on all information available at that age. In addition to own consumption while alive, many lifecycle models employ a utility function with a utility term reflecting a bequest motive, which depends on the amount of wealth left when the individual dies. Lockwood (2012) finds that a bequest motive may explain why few individuals buy annuities. However, Hurd (1989) and De Nardi, French, and Jones (2010) find little evidence for a bequest motive and their results are hardly affected by including it. Therefore, we do not include it in our model.

Health status and survival Health status at age t is denoted by H_t . The individual can be either in good health, $H_t = \text{good}$, or in bad health, $H_t = \text{bad}$. Survival and health status evolve according to a first-order Markov process with transition probabilities that depend on age and previous year's health status. Formally,

$$s_t^h \equiv \Pr(\text{alive}_t \mid \text{alive}_{t-1}, H_{t-1} = h) \quad (2)$$

$$\phi_t^h \equiv \Pr(H_t = \text{good} \mid H_{t-1} = h, \text{alive}_t), \quad (3)$$

where $h \in \{\text{good}, \text{bad}\}$. If we define

$$S(j, g \mid t, h) \equiv \Pr(\text{alive}_j, H_j = g \mid \text{alive}_t, H_t = h),$$

then survival and health status probabilities are found by recursively applying $S(j, \text{good} \mid j-1, h) = s_j^h \phi_j^h$ and $S(j, \text{bad} \mid j-1, h) = s_j^h(1 - \phi_j^h)$. Expected remaining lifetime utility (1) can then be expressed in the form

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} + \sum_{j=t+1}^T \beta^{j-t} \sum_{g \in \{\text{good}, \text{bad}\}} S(j, g \mid t, H_t) E_t \left(\frac{C_j^{1-\gamma}}{1-\gamma} \mid \text{alive}_j, H_j = g \right). \quad (4)$$

Retirement income In retirement, individuals receive a constant income flow, consisting of Social Security benefits, defined-benefit (DB) pensions and annuities. Retirement income is denoted by

$$P_t = P, \quad \forall t. \quad (5)$$

Income is different for different individuals, but we suppress individual identifiers in the notation. In the current U.S. Social Security system, individuals can start receiving retirement benefits as early as age 62. However, financial incentives provided by private pension plans as well as health conditions may induce earlier withdrawals from the labor force. When taking our model to the data, we will only select fully retired individuals age 60 and older. This approach greatly simplifies the model's specification. It comes at the cost of ignoring that workers on the verge of retirement may switch from full-time employment to part-time bridge jobs while receiving pension benefits.

Medical expenses In every period, agents receive health shocks and incur out-of-pocket medical expenses. The logarithm of health costs is modeled as a function of the logarithm of retirement income, age, and health status,

$$\ln \text{HC}_t = \delta_0 + \delta_1 \ln P + f(t, H_t) + \eta_t, \quad \eta_t \sim \mathcal{N}(0, \sigma_\eta^2). \quad (6)$$

In order to save on one additional state variable and keep the dynamic programming problem tractable, the innovations η_t are assumed to be i.i.d., except that we allow the variance σ_η^2 to depend on health status. Hence, persistence in medical expenses is only attributable to persistence in health status and persistence in income. French and Jones (2004) show that the stochastic process for health care costs is well represented by the sum of a white noise term and a quite persistent AR(1) component. Our i.i.d. assumption may, therefore, imply an undervaluation of the duration of health cost shocks and, consequently, of the background risk associated with uncertain medical expenses.

Wealth and taxes For simplicity, we assume the existence of a single saving habitat, containing both conventional savings accounts and investments and Individual Retirement Accounts (IRAs). In the U.S. tax code, returns on IRA wealth accrue tax free, while withdrawals are taxed, whereas by combining private and IRA savings in a single stock of financial wealth, we will apply the same tax regime to both forms of savings. Our model assumes that returns are taxed at the source, but no taxes are paid when resources are used for consumption. As in the current U.S. tax code, taxes are paid on nominal rather than real returns. The marginal tax rate τ_r applies to nominal investment returns and the marginal tax rate τ_i applies to noninvestment (pension) income. Since we do not distinguish between capital gains and yield on risky assets, there is no differential taxation on dividend and capital gains.

Portfolio choice The investment set is constant and consists of two financial assets: a risk-free one that can be thought of as bonds, T-bills or cash, and a risky one, representing the stock market. In each period, individuals choose the share of wealth to hold in risky assets, denoted by α_t . There is no entry fee to participate in the stock market or cost associated with portfolio rebalancing.¹ Short sales are not allowed, and thus the following constraint is imposed:

$$\alpha_t \in [0, 1]. \quad (7)$$

Wealth dynamics We assume a constant inflation rate denoted by π . The risk-free asset yields a constant real return r and thus a real after-tax return

$$\tilde{r} = \frac{1 + [(1 + r)(1 + \pi) - 1](1 - \tau_r)}{1 + \pi} - 1. \quad (8)$$

The equity portfolio delivers a stochastic excess real return

$$r_t^e - r = \mu + \epsilon_t, \quad (9)$$

where ϵ_t is i.i.d. $\mathcal{N}(0, \sigma_\epsilon^2)$. Hence, the real after-tax return on the risky asset is

$$\tilde{r}_t^e = \frac{1 + [(1 + r_t^e)(1 + \pi) - 1](1 - \tau_r)}{1 + \pi} - 1. \quad (10)$$

Denoting wealth with W_t , its evolution over time is described by

$$W_{t+1} = [\alpha_t(1 + \tilde{r}_{t+1}^e) + (1 - \alpha_t)(1 + \tilde{r})] \{W_t + (1 - \tau_i)P - C_t - HC_t\}. \quad (11)$$

¹Market frictions in the form of fixed entry costs are often mentioned as plausible, although partial, explanations for the observed limited stock market participation. Vissing-Jørgensen (2002), Alan (2006), and Paiella (2007) all find evidence of nonnegligible entry costs. While introducing a fixed entry cost in a conventional investment account, Gomes, Michaelides, and Polkovnichenko (2009) consider stock market participation as costless in a retirement account. The idea is that informational and set-up costs associated with direct stockholding are bypassed in the latter. As the present model focuses on relatively older investors, we assume that agents have already sustained such costs earlier in their life.

A borrowing constraint is present so that:

$$W_t \geq 0 \quad \forall t. \quad (12)$$

Because of the stochastic health costs, this borrowing constraint cannot be strictly enforced unless we allow negative consumption or censor the distribution of health costs. In practice, individuals who face large health costs and have run down their assets may be covered by Medicaid or other government or charity programs. In our model, this is implemented in the form of a consumption floor of 1% of retirement income (i.e., $0.01P$).

Recursive form Define cash-on-hand as the sum of liquid assets and retirement income, net of out-of-pocket medical expenses:

$$X_t = W_t + (1 - \tau_i)P - \text{HC}_t$$

The life-cycle maximization problem under consideration involves two continuous state variables—cash-on-hand and retirement income—and two discrete state variables—age and health status. As shown in the appendix, it is possible to reduce the dimensionality of the problem by dividing all variables by the constant flow of retirement income (Carroll, 1992, 1997).

Let lowercase letters indicate the ratios of the original variables to the level of retirement income (e.g., $x_t = X_t/P$). The individual's problem can now be written in the recursive form of the *Bellman equation* (e.g., Christensen & Kiefer, 2009, p. 28; Adda & Cooper, 2003, p. 26; after Bellman, 1957)

$$v_t(x_t, H_t) = \max_{c_t, \alpha_t} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + \beta E_t [v_{t+1}(x_{t+1}, H_{t+1})] \right\}, \quad (13)$$

subject to

$$x_{t+1} = \left[\alpha_t(1 + \tilde{r}_{t+1}^e) + (1 - \alpha_t)(1 + \tilde{r}) \right] (x_t - c_t) + (1 - \tau_i) - \text{hc}_{t+1}. \quad (14)$$

Note that both x_{t+1} and H_{t+1} are stochastic functions of $\{x_t, H_t, c_t, \alpha_t\}$, and the survival probability is also part of the expectation.

Solution The model is solved by backward induction. The state space is given by $\{t, x_t, H_t\}$. We discretize the continuous state variable, x_t , by defining an equally spaced grid and we use two-dimensional cubic spline interpolation to construct the value function $v_t(x_t, H_t)$ at all points in the state space. Numerical integrations are performed by Gaussian quadrature. A solution consists of a set of policy functions for consumption and portfolio composition: $c_t^* = c(x_t, H_t)$ and $\alpha_t^* = \alpha(x_t, H_t)$. Optimal values for the choice variables are located using a combination of a root-finding algorithm and standard grid search. See the appendix for more technical details.

Optimal saving is increasing and convex in normalized cash-on-hand, which mirrors the consumption rules in Gourinchas and Parker (2002). The optimal portfolio share of the risky asset is decreasing in normalized cash-on-hand. This is because at lower levels of cash-on-hand, the risk-free asset position represented by the constant retirement income stream is relatively high and agents can tilt their portfolios more heavily towards risky investments. As cash-on-hand

increases, the relative importance of retirement income on total wealth decreases and so does the optimal share of risky asset.² It should be noted that the optimal share of risky assets is not defined for very low values of wealth at which saving is zero and no resources are available for investment.

At older ages, the optimal share of risky assets conditional on normalized cash-on-hand is lower. Since retirement benefits will be received over a shorter time horizon, the net present value of the risk-free income stream is lower and so is the willingness to bear financial risks in the form of risky investments. Also, at older ages, the risk of high out of pocket medical expenses is higher, which also leads to a less risky portfolio composition.

3 Data and estimation of auxiliary processes

3.1 Data

We use the Health and Retirement Study (HRS; Juster & Suzman, 1995). The HRS is a longitudinal biennial survey of individuals over the age of 50 in the U.S. and their households, which represents the primary source of information about the elderly and future elderly in the U.S. The data contain extensive information on household economic condition and demographics, individual employment history, retirement planning, pensions, and health status. We mainly use the RAND HRS (St.Clair et al., 2011), which is a postprocessed, more user-friendly version of the raw HRS data. We also add variables that are not included in the main RAND HRS file, such as subjective expectations about stock market returns (which are in the RAND-enhanced FAT files) and detailed pension data (from the RAND income and wealth imputations files). We use waves 5–10, which cover the period from 2000 to 2010.

The model as presented in section 2 is a model for the individual. In contrast with this, information on wealth and asset allocation is at the household level in the HRS. Thus, in our estimation and simulation exercises, we use the household as our unit of analysis. Wealth and income is reported by the financial respondent, who is the one deemed most knowledgeable about the household's finances. Accordingly, for some inherently individual-level variables (age, mortality), we use the variables for the financial respondent. We define an individual to be in bad health if he or she reports his or her health status as "fair" or "poor", and in good health otherwise. Because health costs are borne by the household, and economic decisions are made at the household level, the risk of health costs related to the spouse's health must be taken into account. Therefore, in our analyses, we use an indicator of bad health at the household level. The household is defined to be in bad health if either the respondent or the spouse (if any), or both are in bad health.

Out-of-pocket medical expenses comprise the costs incurred in a year for hospital, nursing home, doctor visits, dentist, outpatient surgery, average monthly prescription drug, home health care, and special facilities. Household out-of-pocket medical expenses are obtained summing the costs reported by the financial respondent and, if present, his or her spouse.

²As cash-on-hand increases, the optimal share of risky asset asymptotes to the value of $\mu/\gamma\sigma_e^2$. This is the constant, optimal share of risky assets generated by a portfolio model with CRRA preferences when markets are frictionless and complete and/or agents face no uncertainty about available future resources (Samuelson, 1969; Merton, 1969).

Household retirement income includes pension benefits, annuities, and Social Security retirement received by the financial respondent and his or her spouse in each wave. For each household, we first compute retirement income in each wave and then a constant flow of retirement income by averaging retirement income over all periods when it is observed.

For total wealth, we use the most comprehensive total wealth measure (HwATOTB) from the RAND HRS. This includes net housing wealth (primary and secondary residence, other real estate, minus balances of mortgages and other loans), vehicles, personal items of value, and net financial wealth (checking and savings accounts, certificates of deposit, bonds, stocks, businesses, IRAs, minus non-housing debt). We take stocks as the risky asset and everything else as the risk-free asset. We include both wealth in direct stock holding and stock holding in IRAs. We do not include balances in defined contribution (DC) pension plans in total wealth, nor the stock fraction of these balances in stock wealth, because these balances are not readily available. Gustman, Steinmeier, and Tabatabai (2010) construct measures of DC (and DB) pension wealth and the fraction of that invested in stocks. The latter is only available for 2006, in which among over 6400 fully retired financial respondents 60 and over, less than 1% had stocks in a DC account and therefore this omission does not influence our results.³

We compute household cash-on-hand by adding household retirement income to household wealth. We divide household cash-on-hand by household retirement income to obtain the measure of normalized cash-on-hand which we use to solve and simulate the model.

We restrict our analysis to financial respondents aged 60 and older who are fully retired. We exclude households whose reported retirement income in a given wave is less than 5,000 or more than 1,000,000 dollars per year and whose total wealth is less than 10,000 or more than 3,000,000 dollars.

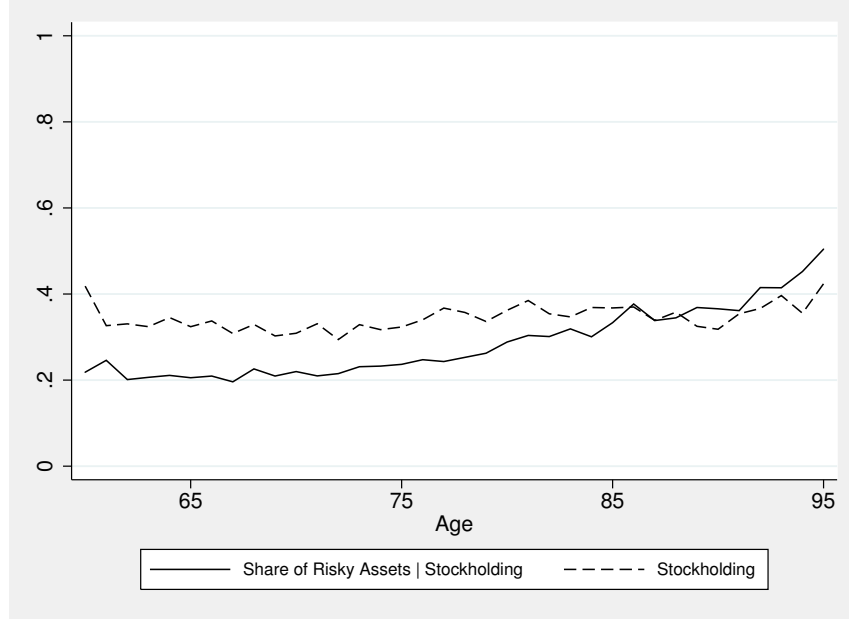
Figure 1 shows the fraction of individuals in our sample who hold stocks, and among those who hold stocks, what the average fraction of their wealth in stocks is. Stock holding is relatively constant at slightly below 40%, whereas the share of risky assets slopes upward a little with increasing age. In the following sections, we will present analogous graphs based on the model predictions and compare them to this graph.

3.2 Survival and health transition probabilities

The model has two interrelated transition processes: mortality and health transitions. We follow De Nardi et al. (2010) and others and assume that survival from age $t - 1$ to age t , and health status at age t conditional on survival, depend on health status at age $t - 1$. We cannot directly estimate the one-year survival and health transition probabilities (2)–(3) from the HRS, because the HRS interviews individuals every two years and thus we do not observe one-year transitions. This is only a minor nuisance, because probabilities of two-year transitions follow from the probabilities

³The reasons for this are arguably that in this age group, relatively few individuals have DC plans and that upon retirement, many of those who do have a DC plan roll over their balances into an IRA or take out the pension in the form of a lump sum.

Figure 1: Stock holding and share of risky asset conditional on stock holding among retired individuals age 60 and older in the HRS 2000–2010



of one-year transitions:

$$\begin{aligned}\Phi_t(\text{good} | h) &\equiv \Pr(\text{alive}_t, H_t = \text{good} | \text{alive}_{t-2}, H_{t-2} = h) \\ &= s_{t-1}^h [\phi_{t-1}^h s_t^{\text{good}} \phi_t^{\text{good}} + (1 - \phi_{t-1}^h) s_t^{\text{bad}} \phi_t^{\text{bad}}]\end{aligned}\quad (15a)$$

$$\begin{aligned}\Phi_t(\text{bad} | h) &\equiv \Pr(\text{alive}_t, H_t = \text{bad} | \text{alive}_{t-2}, H_{t-2} = h) \\ &= s_{t-1}^h [\phi_{t-1}^h s_t^{\text{good}} (1 - \phi_t^{\text{good}}) + (1 - \phi_{t-1}^h) s_t^{\text{bad}} (1 - \phi_t^{\text{bad}})]\end{aligned}\quad (15b)$$

$$\begin{aligned}\Phi_t(\text{dead} | h) &\equiv \Pr(\text{dead}_t | \text{alive}_{t-2}, H_{t-2} = h) \\ &= 1 - \Phi_t(\text{good} | h) - \Phi_t(\text{bad} | h).\end{aligned}\quad (15c)$$

The left-hand sides of these expressions are observable in the HRS and thus can be used to estimate the one-year transition probabilities. However, the number of deaths in the HRS is insufficient to reliably estimate conditional survival rates directly. Therefore, as in De Nardi et al. (2010), we combine information from the data with unconditional survival probabilities from a life table and Bayes' rule to estimate conditional survival probabilities.

Specifically, we start with the 2007 Actuarial Life Table from the Social Security Administration (<http://www.ssa.gov/oact/STATS/table4c6.html>). We weight the male and female columns by the fraction of males and females among 60-year olds in the HRS, and then add them to obtain a combined life table, from which we compute the unconditional one-year survival probabilities, $s_t = \Pr(\text{alive}_t | \text{alive}_{t-1})$. Conditional survival probabilities can now be expressed

using Bayes' rule as

$$\begin{aligned}
s_t^{\text{good}} &= \Pr(\text{alive}_t \mid H_{t-1} = \text{good}; \text{alive}_{t-1}) = \frac{\Pr(H_{t-1} = \text{good} \mid \text{alive}_t)}{\Pr(H_{t-1} = \text{good} \mid \text{alive}_{t-1})} \times \Pr(\text{alive}_t \mid \text{alive}_{t-1}) \\
&= \frac{\Pr(H_{t-1} = \text{good} \mid \text{alive}_t)}{\Pr(H_{t-1} = \text{good} \mid \text{alive}_{t-1})} \times s_t, \tag{16}
\end{aligned}$$

and analogously for survival conditional on bad health. Thus, we estimate the numerator and denominator probabilities in (16) and compute s_t^{good} and s_t^{bad} using (16) and the results from the lifetable. It follows that we estimate four probabilities: the health transition probabilities ϕ_t^{good} and ϕ_t^{bad} from (3) and the health status probabilities $\Pr(H_{t-1} = \text{good} \mid \text{alive}_t)$ and $\Pr(H_{t-1} = \text{good} \mid \text{alive}_{t-1})$ from (16).

To obtain smooth estimates, we specify each of these four probabilities as a logit model with a cubic polynomial in age as covariates. We insert the resulting expressions (3) and (16) into (15) and estimate all four submodels jointly by maximum likelihood in Stata. The estimation results are given in Table 1.

Figure 2 plots the resulting one-year conditional survival probabilities s_t^h and health transition probabilities ϕ_t^h . As expected, survival probabilities drop with age, especially after age 80 or so (note that these are one-year survival probabilities, not cumulative survival probabilities), and survival probabilities are lower for bad health than for good health. The probability of staying in good health decreases a little as well, and the probability of being in good health is much lower if the individual (or household, rather) was in bad health in the previous year. Remarkably, the probability of transitioning from bad health to good health increases a little at higher ages. We suspect that this is due to relatively healthier individuals (within the “bad health” category) living longer.

3.3 Out-of-Pocket Medical Expenses

As the structural model is solved in terms of normalized variables, we subtract the logarithm of retirement income from both sides of (6) and estimate a process for the logarithm of the ratio of household medical expenses to household retirement income: $\ln hc_t = \ln(\text{HC}_t/P)$. By assuming $\delta_1 = 1$, P drops out of the right-hand side of the model. We specify the function $f(\cdot)$ in (6) to be a third-order polynomial in age and an indicator for bad health status interacted with age. We also allow the variance of the unobserved component η_t to depend on health status in order to capture differential risks for households with different health.

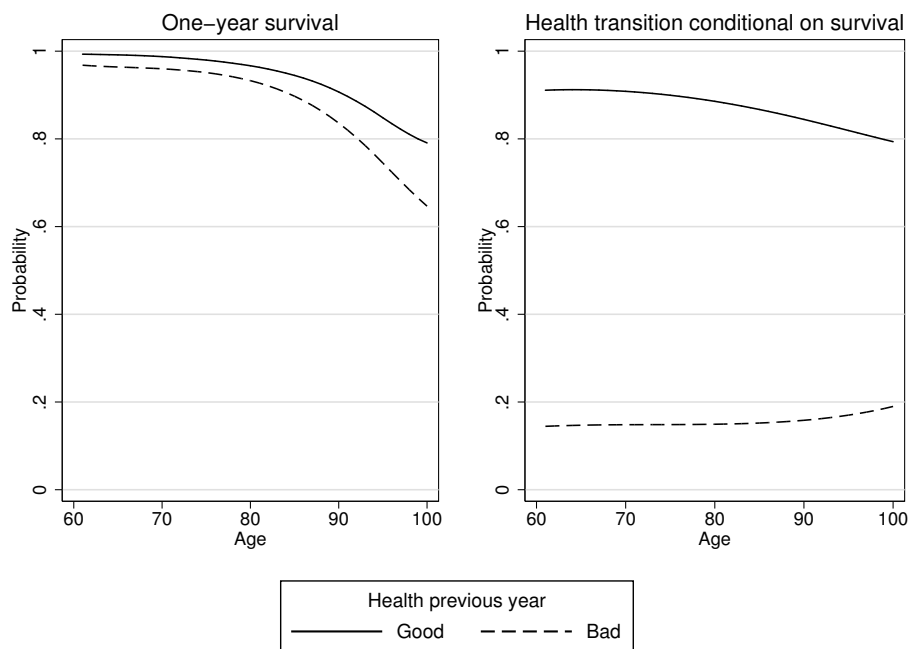
We include fixed effects in the model for the medical costs process to control for the potentially biasing effect of changes in sample composition with age due to differential mortality. Suppose, for example, that only wealthier households are observed at very old ages and that their ratio of medical expenses to retirement income is relatively low because of more generous pension allowances. Estimating the age profile of medical costs by pooled OLS would understate the extent to which the fraction of resources spent in health care increases with age. The fixed-effects estimator overcomes this problem. By using the time variation within each cross section, in fact, it allows us to correctly infer the age profile of household medical expenses. This also has the

Table 1: Parameter estimates for the transition models

Covariate	Coef.	s.e.
<i>Prob. good health(t)</i>		
$a = (\text{Age} - 80)/10$	-.7057**	(.3232)
a^2	.1641	(.3240)
a^3	-.0202	(.1871)
Constant	.2063	(.2897)
<i>Prob. good health(t) given survival until t + 1</i>		
a	-.6811**	(.3292)
a^2	.1851	(.3261)
a^3	-.0143	(.1890)
Constant	.2443	(.2923)
<i>Prob. good health(t + 1) given survival and good health(t)</i>		
a	-.3316***	(.0606)
a^2	-.0481	(.0358)
a^3	.0199	(.0268)
Constant	2.0140***	(.0369)
<i>Prob. good health(t + 1) given survival and bad health(t)</i>		
a	.0306	(.0802)
a^2	.0350	(.0386)
a^3	.0149	(.0345)
Constant	-1.7380***	(.0418)

** $p < .05$; *** $p < .01$.

Figure 2: Conditional one-year survival probabilities and health transitions



advantage of capturing cohort effects, which may be important for the variable of interest. If not appropriately accounted for, systematic differences in health care behavior among generations may lead to biased assessments of the age effect on household medical costs. The regressions also include year dummies, which pick up variations in medical prices that differ from inflation and variations in medical costs due to other unmodeled causes, such as the business cycle.

The estimates are presented in Table 2. Figure 3 presents the estimated age profiles for the ratio of medical expenses to retirement income by health status. This is the prediction from the model except for the fixed effects and the year dummies, and reflects the predicted median health costs in 2000 by age and health status. Median health costs amount to less than 20% of income among individuals in their early 60s. They reach 30% for individuals in their late 80s and rise sharply thereafter. The effect of health status on medical costs is modest until 85, but becomes substantial in the later part of the life span. These results are in line with what has been found in other studies. For example, De Nardi et al. (2010) estimate that single individuals at the median permanent income incur medical costs of roughly \$1,700 (in 2000 dollars) at age 70 and \$4,500 at age 90. Assuming that the typical household has two members and knowing that the median (constant) household retirement income is \$21,000, the implied ratios of medical expenses to permanent income are 16% at age 70 and 42% at age 90. The estimates in the last two rows of Table 2 indicate that households in bad health face a significantly higher risk of incurring high medical costs.

Table 2: Estimates of the medical expenses process

Covariate	Coef.	s.e.
Age	0.761***	(0.255)
Age ² /100	-0.973***	(0.332)
Age ³ /1000	0.044***	(0.014)
Bad Health × Age/100	0.374***	(0.029)
Year 2002	0.177***	(0.058)
Year 2004	0.211**	(0.102)
Year 2006	-0.017	(0.149)
Year 2008	-0.218	(0.197)
Year 2010	-0.168	(0.254)
Constant	-23.017***	(6.676)
Residual s.d. (σ_{η}^2):		
Good health	1.033***	(0.027) ^a
Bad health	1.383***	(0.031) ^a
Sample size	26,050	
F-statistic fixed effects	3.15***	

** $p < .05$; *** $p < .01$.

^a Standard errors computed by bootstrap using 1000 replications.

Figure 3: Medical expenses process by health status

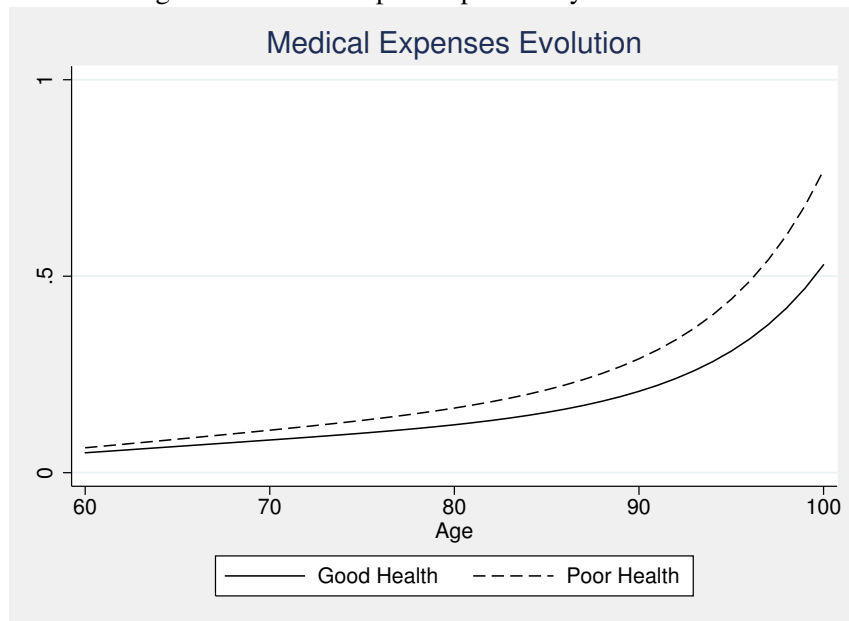


Table 3: Baseline model’s parameters

Initial age	60
Final age (T)	100
Discount factor (β)	0.96
Relative risk aversion (γ)	4
Real risk-free return (r)	1.6%
Mean real equity premium (μ)	5.0%
Standard deviation of real risky asset returns (σ_ϵ)	0.173
Inflation (π)	4.0%
Pension income tax rate (τ_i)	20%
Investment income tax rate (τ_r)	15%

3.4 Net asset returns and preferences

We assume that the tax rate on retirement income (τ_i) is 20%. The U.S. income tax code is progressive, and thus not every household faces the same average and marginal tax rates. Furthermore, tax rates change over time. Thus, our model (as most economic models) is a simplification. The 20% is a rate that is close to the median or mean federal income tax rate in our sample period (CBO, 2010; Piketty & Saez, 2007). In our model, we use a tax rate on nominal asset returns (τ_r) of 15%, which is the maximum rate on capital gains and dividends (Piketty & Saez, 2007).

We base our parameters for asset returns and inflation on the outcomes of the past 50 years (01/01/1961–12/31/2011). From the CPI (all urban consumers, series CUUR0000SA0; <http://www.bls.gov/cpi/>), we derive that the average inflation rate between January 1961 and December 2011 was about 4%, so in the model we take $\pi = 4\%$.

For the risk-free nominal asset return, we use the returns on 1-year Treasury Bills (series RIFSGFSY01_N.A; <http://www.federalreserve.gov/releases/H15/data.htm>), which had an average real return of $r = 1.6\%$.

For the risky asset returns, we use the total returns—including dividends—of the S&P500 (data available at <http://www.econ.yale.edu/~shiller/data.htm>), which had an average real return of 7.0% and, hence, a real risk premium of 5.39%. In our model, we use $\mu = 5\%$ as opposed to the historical 5.39% to take into account transaction costs such as mutual fund and brokerage fees (Gomes et al., 2009). The standard deviation of the risk premium is $\sigma_\epsilon = 0.173$, which comprises the observed historical variation in both the real risk-free and risky assets returns.

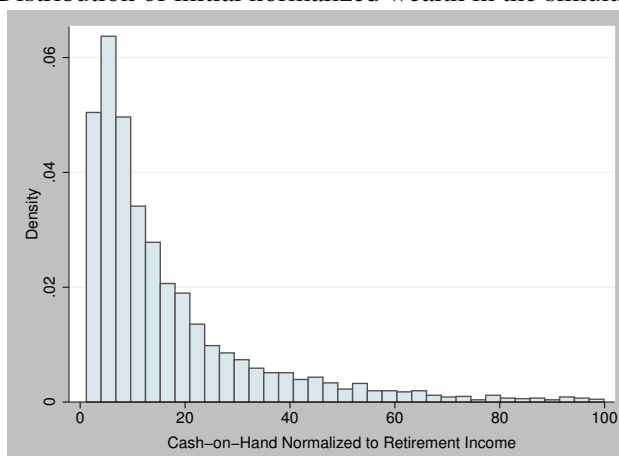
Preference parameters are taken from previous studies using life cycle models of saving and portfolio choice. In particular, the discount factor, β , is set to 0.96, as in Cocco et al. (2005), while the coefficient of relative risk aversion, γ , to 4, as in De Nardi et al. (2010).

Table 3 summarizes the benchmark values for the model’s parameters.

Table 4: Initial age and health distributions in the simulation sample

Age		Health	
Value	Fraction (%)	Value	Fraction (%)
60	9.95	Good	64.07
61	13.72	Bad	35.93
62	17.60		
63	27.03		
64	31.69		

Figure 4: Distribution of initial normalized wealth in the simulation sample



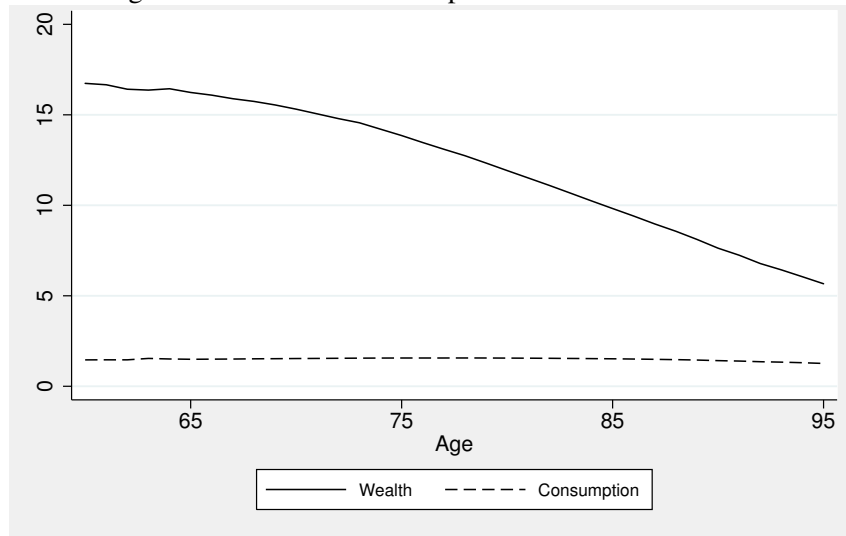
4 Model predictions with historical expectations

In this section we describe the wealth accumulation and investment patterns obtained when agents hold rational expectations and base their estimates of future market gains on historical returns. We thus solve and simulate the model under the assumptions that (1) individuals expect that returns in the future are characterized by the parameters in section 3.4 and (2) that their expectations are fulfilled, that is, future realizations will be in line with the historical trend.

We start with the sample described in section 3.1 and select the individuals between 60 and 64 years of age. This leaves us with a pool of 3,607 individuals, from which we sample with replacement 10,500 times to obtain our simulation sample. The age and health distributions in the simulation sample are reported in Table 4. The distribution of cash-on-hand normalized to retirement income is depicted in Figure 4.

We then simulate an outcome path for each individual in the sample, where the individual uses the optimal decision rules (described above) that solve the model. Below we report cross-sectional means of relevant variables from age 60 to age 95. Wealth and consumption are both normalized to retirement income. Investment behavior is illustrated by the fraction of individuals who hold

Figure 5: Wealth and consumption in the baseline model



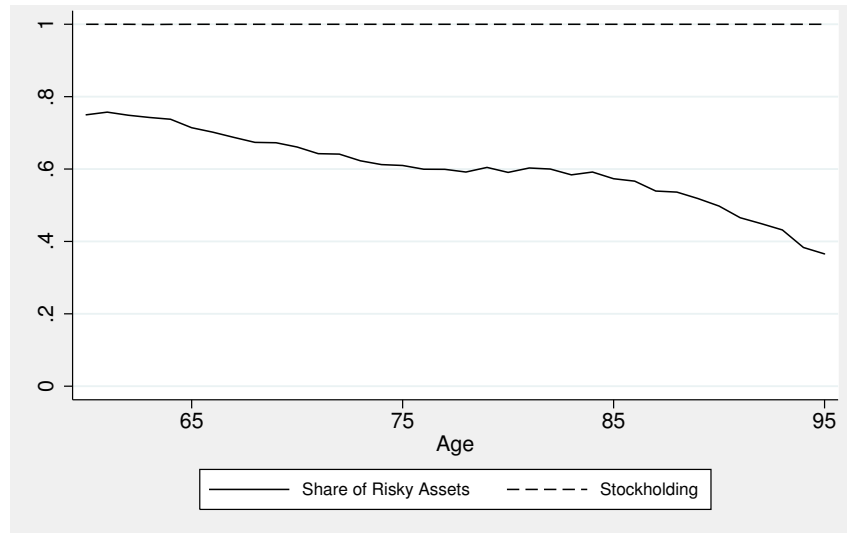
any wealth in stocks, and by the share of wealth held in the risky asset conditional on holding stocks. It should be noted that in our computations we exclude those individuals who do not have enough wealth to invest and for whom the share of wealth held in the risky asset is not defined. All individuals in our simulation sample start off with enough wealth to invest. For all these individuals, the portfolio problem is correctly defined until very old ages, when asset decumulation combined with high medical expenses force some to rely on the consumption floor and, consequently, to save and invest zero.

Figure 5 shows the evolution of (normalized) consumption and wealth over the life cycle. Consumption is perfectly smooth at about 1.5 times retirement income. The initial stock of household wealth in the simulation sample amounts to about 17 times retirement income. Total wealth declines rather modestly early into retirement and starts to be largely run down in the late 70s. This is consistent with the data, where the elderly are observed to reduce their asset holdings only slightly until very late in life. As in De Nardi et al. (2010), the increase risk of unexpected high out-of-pocket medical expenses induces older households to save more for precautionary reasons and significantly slows down asset decumulation.

Figure 6 shows the fraction of retirees holding stocks and the share of wealth held in risky assets conditional on holding stocks. With the historical equity premium and its volatility, the model predicts that everyone should invest in stocks, which is greatly at odds with the stock market participation rate of less than 40% that we observed in the HRS (Figure 1). This finding, however, is not surprising and confirms the well-documented “stock-market participation puzzle” (Haliassos & Bertaut, 1995): under the assumption of rational expectations based on historical returns, a standard life-cycle portfolio model cannot replicate the observed low rate of household stock market participation unless it features extremely high values of risk aversion or substantial market frictions in the form of large entry and transactions costs (Alan, 2006).

The model also predicts that individuals keep most of their wealth (75%) in risky assets at age

Figure 6: Stock holding and share of risky assets conditional on stock holding in the baseline model

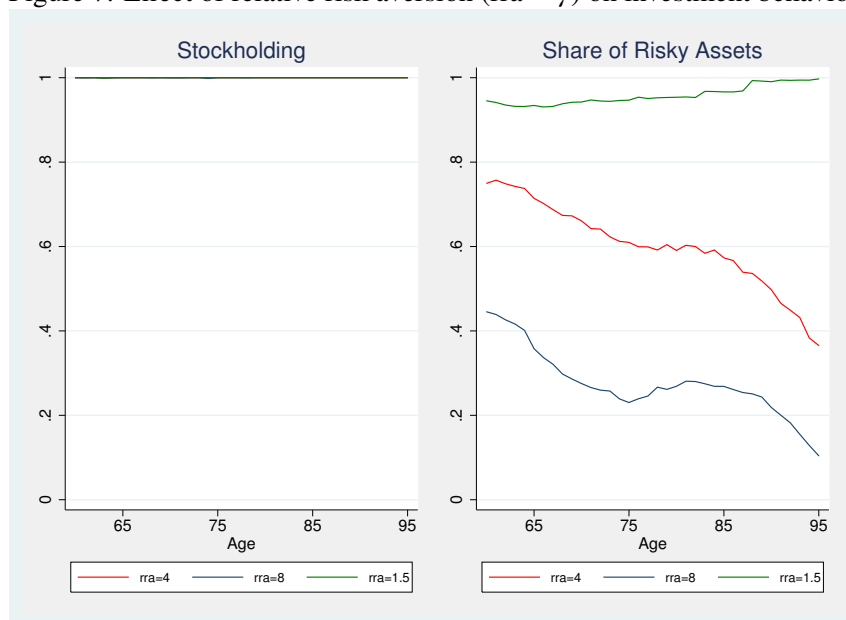


60 and gradually move towards more conservative portfolios as they age. This is in line with the typical financial planners' advice of shifting investments away from stocks in retirement (Cocco et al., 2005, e.g.). The reason for this is that younger individuals are better able to overcome negative portfolio returns using retirement income over a longer horizon. Because the optimal share of risky asset decreases in normalized wealth, asset decumulation towards the end of the life cycle could induce a higher share of the risky asset. This mechanism is, however, counteracted by the need for hedging against the risk of large medical expenses at old ages. The net effect is a sharp decline of the fraction of wealth held in risky assets from age 85 to 95. Again, the investment path predicted by the model is different from the empirical evidence in Figure 1. HRS respondents tend to keep around 25% of their wealth in risky assets throughout their retirement and increase this fraction to about 40% at the very end of the life cycle.⁴

The model approximates the portfolio profile observed in the HRS better with alternative parameter values. For instance, Figure 7 shows that if individuals are twice as risk averse as in the baseline model, the predicted share of risky assets is about 25% on average, which is much closer to the empirical pattern in the data. The need for precautionary savings increases with the degree of prudence Kimball (1990), which, with a CRRA utility function, is directly linked to the degree of risk aversion. For higher values of risk aversion, the preference for a buffer stock of wealth increases, inducing a slower asset run down at old ages and a lower level of consumption. Such a high value of risk aversion, however, appears to be in contrast with empirical estimates. Hurd (1989), and Attanasio and Weber (1993, 1995), among others, find evidence of relative risk aversion as low as 1.

⁴Several reasons may explain the observed increase in the share of risky assets at very old ages. For example, retirees may spend down their non-equity wealth before tapping their stock market investments. Also, mortality bias may play a role, as richer and more educated individuals are more likely to hold stocks and live longer.

Figure 7: Effect of relative risk aversion ($rra = \gamma$) on investment behavior



The empirical patterns can also be better approximated should the risk premium or its volatility differ from their historical values. Figures 8 and 9 show that if the expected and realized risk premium is half the historical one or if expected and realized return volatility is double the historical one, then the predicted share of risky assets is also about 25% on average. This exercise indicates the potential of using return expectations not based on historical returns to bring the model's predictions closer to observed investment patterns. We explore this possibility in the following sections.

However, as these figures show, even with different parameter values the model is not able to produce the low fraction of stock holding that we see in the data, but predicts that all individuals should participate in the stock market. In what follows, we will document that a substantial fraction of individuals with very pessimistic beliefs about the stock market, combined with fractions of individuals with moderate or optimistic beliefs, leads to predictions that resemble the data more closely on both the intensive and extensive margins.

5 Subjective beliefs about the distribution of stock market returns

Since 2002, the HRS has elicited respondents' beliefs about stock market returns. The basic question, asked in each wave, is the following:

By next year at this time, what is the percent chance that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more than they are today?

Figure 8: Effect of the risk premium ($rp = \mu$) on investment behavior

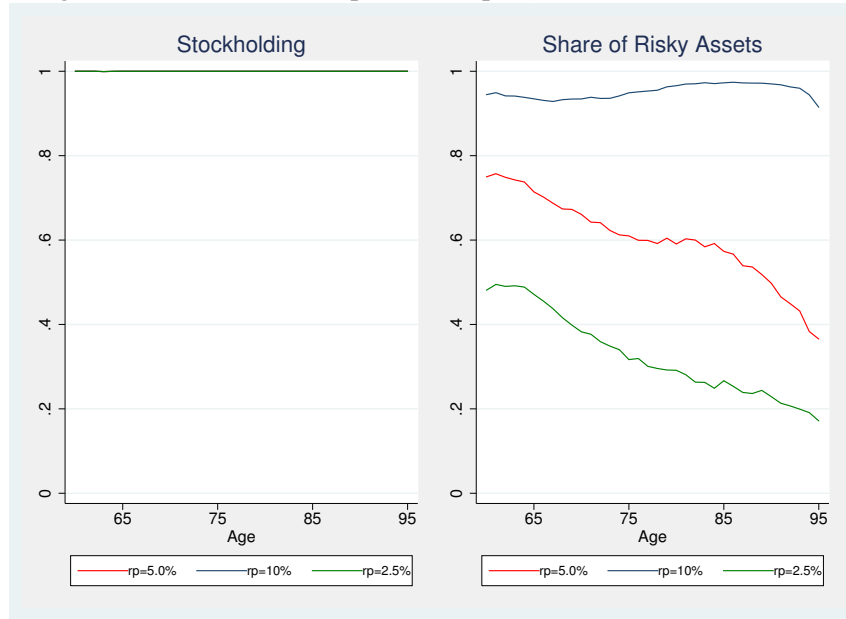
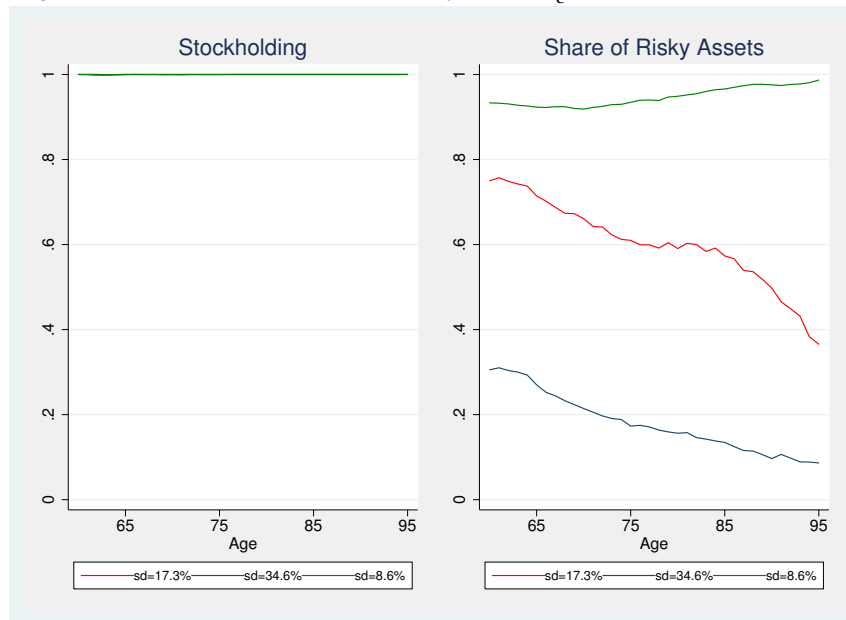


Figure 9: Effect of the return volatility ($sd = \sigma_\epsilon$) on investment behavior



We call this question P0. In the different waves, there are different additional questions. In 2004, only P0 was asked. In 2006 and later years, there was a follow-up question to understand whether a 50% answer expressed epistemic uncertainty, and in 2008, there were also follow-up questions after a 0% or 100% answer to P0 to elicit whether these were rounded (in which case they were probed again to give a continuous answer) or actual beliefs of certainty. We use all this information in cases where this leads to a more informative answer to P0. In 2002, there was one additional question, which we call P10:⁵

By next year at this time, what is the chance they will have grown by 10 percent or more?

In 2010, two similar questions were asked (P20 and P-20), in which the percentage was 20% or more gain or 20% or more loss. In 2008, respondents were randomized into one of 8 follow-up questions, with the same structure, with gains of 10, 20, 30, or 40% (P10–P40) and losses of 10, 20, 30, or 40% (P[-10]–P[-40]). So each respondent answered (at most) two questions, but overall there were nine questions. In 2006, there were additional questions about 10 years in the future and about the past 10 years. Because these ask about a different distribution, we do not use the latter.

Summarizing, for the distribution of the one-year ahead stock market return, respondents gave one probability in 2004 and 2006, up to two probabilities in 2002 and 2008, and up to three probabilities in 2010. Under suitable assumptions, we can use these self-reported probabilities to compute a individual-specific distributions of stock market returns. Throughout, we assume that these questions elicit beliefs about real before-tax returns.

Dominitz and Manski (2007) studied the 2004 data, which only had P0. They assumed that respondents' subjective distributions of stock market returns are normal distributions, and that the standard deviation of this distribution is the same as the standard deviation of historical nominal returns (for which they use 0.206). These assumptions can be used to compute the subjective mean. Let m_i be respondent i 's subjective mean, $s_i = 0.206$ be respondent i 's subjective standard deviation, and p_{0i} be the answer to P0 divided by 100. Then $m_i = -s_i \Phi^{-1}(1 - p_{0i})$, where $\Phi(\cdot)$ is the standard normal cumulative distribution function. This method can be used for the 2004 and 2006 data. If there is more information, this method can be generalized. In years where the respondent gave two answers, we can drop the assumption about s_i and solve the system of two equations in two unknowns. Let the two probabilities be denoted by p_{xi} and p_{yi} , corresponding to a stock market gain of at least x and y (e.g., $x = 0$ and $y = 0.10$ in 2002). In the case of losses, first transform the answer from $P[-x]$ to $100 - P[-x]$. Then

$$s_i = \frac{y - x}{\Phi^{-1}(1 - p_{yi}) - \Phi^{-1}(1 - p_{xi})} \quad (17a)$$

$$m_i = x - s_i \Phi^{-1}(1 - p_{xi}) = \frac{x \Phi^{-1}(1 - p_{yi}) - y \Phi^{-1}(1 - p_{xi})}{\Phi^{-1}(1 - p_{yi}) - \Phi^{-1}(1 - p_{xi})}. \quad (17b)$$

⁵The order of P0 and P10 was randomized, which requires slightly different wording for the two randomization groups.

With three answers, we have a system of three equations in two unknowns, and we have to compute some kind of compromise value. A simple method would be to compute the m_i and s_i resulting from applying (17) to the smallest two values of x (i.e., P0 and the loss question) and compute another m_i and s_i resulting from applying (17) to the largest two values of x (i.e., P0 and the positive gain question) and taking the average of the two.

Hurd and Rohwedder (2011) take a different approach. They observe that if the respondent answers k questions, we have the probabilities associated with $k + 1$ mutually exclusive intervals that span the whole real line. They proceed to assume that the subjective distribution *within* each interval coincides with the historical distribution within that interval. For example, suppose the respondent answers only P0. From this we can deduce $\Pr_i(n < 0)$ and $\Pr_i(n \geq 0)$, where n is the nominal return and the subscript i denotes respondent i 's subjective distribution. Then

$$\begin{aligned} E_i(n) &= \Pr_i(n < 0) E(n | n < 0) + \Pr_i(n \geq 0) E(n | n \geq 0) \\ E_i(n^2) &= \Pr_i(n < 0) E(n^2 | n < 0) + \Pr_i(n \geq 0) E(n^2 | n \geq 0). \end{aligned}$$

The conditional expectations on the right-hand side are taken from historical distributions, and the subjective mean and standard deviations are obtained as $m_i = E_i(n)$ and $s_i = [E_i(n^2) - m_i^2]^{1/2}$. This method immediately generalizes to any number of intervals.

We have experimented with both approaches and found that they give very similar results. Here, we only present results using the Hurd-Rohwedder approach.⁶

After computing subjective means and standard deviations of real returns, we transform them by correcting for the historical risk-free return so as to obtain the subjective real risk premium and subjective standard deviation of real excess returns. We then discretize their distributions by first taking terciles of the risk premium and collapsing the distributions to the interval midpoints. Within each of these three groups we then take terciles of the standard deviation and again collapse distributions to the interval midpoints. The advantage of having a discrete distribution with a small number of distinct values is that we do not have to add a state variable to the model, but can run the model for each group separately and aggregate the results afterwards, which is much less burdensome computationally. Because of ties, groups are not always exactly 1/9th of the sample, and we do not always have nine different groups. We have collapsed all groups with zero or negative subjective risk premia, because their optimal decisions are equal and consist of never investing in stocks.

The resulting distributions are presented in Table 5. A large fraction of respondents is very pessimistic about stock market returns and expects them to earn less than the risk-free rate. The

⁶We use basic probability rules to identify inconsistent answers. First, we drop self-reported probabilities that are outside the $[0, 1]$ interval. It should be noted that the procedure suggested by Dominitz and Manski (2007) implies that only values strictly greater than 0 and strictly less than 1 can be used. Given that a significant fraction of the respondents clusters at 0 or 1, we approximate a “zero probability” with 0.001 and a “one probability” with 0.999. This allows us to retain most of the selected individuals in the sample when estimating subjective distributions of returns. The Hurd-Rohwedder approach does not require this approximation as it can be implemented for probabilities taking value 0 or 1. Second, when two or more probabilities are elicited, we define as inconsistent all answers that do not conform to the probabilities of the single events and their union. If, for instance P0 and P10 are asked, we define as inconsistent all answers for which $P0 \leq P10$. In this case, we only use P0 and ignore P10. The inequality above is required to be strict under the Dominitz-Manski approach, but not under the Hurd-Rohwedder approach.

Table 5: Estimated distributions of subjective risk premium and volatility

2002			2004–2006			2008			2010		
RP	SD	Fraction	RP	SD	Fraction	RP	SD	Fraction	RP	SD	Fraction
≤ 0	15.1	54.0	≤ 0	15.8	38.4	≤ 0	17.7	45.5	≤ 0	17.2	57.1
.03	14.9	19.1	1.5	18.7	37.0	.24	15.6	21.2	1.7	14.6	4.3
							23.0	4.0		20.8	4.4
										29.1	4.3
3.5	9.7	2.5	4.2	17.6	4.1	4.4	14.9	3.4	6.7	14.9	3.4
	17.8	2.7		18.2	0.8		19.1	3.4		22.2	3.4
	19.3	2.3					24.3	3.4		28.1	3.3
8.1	11.8	3.5	7.4	16.4	7.0	8.9	11.2	4.2	10.9	12.4	3.3
	16.6	3.3		17.1	5.7		16.7	2.5		20.7	3.3
	18.3	3.2					22.4	3.3		27.6	3.3
15.1	8.0	3.6	12.1	9.6	3.7	17.6	10.2	3.4	19.3	6.5	4.1
	11.4	2.8		13.3	2.7		16.8	2.7		14.1	2.5
	14.9	3.0		15.2	0.6		24.2	3.0		22.9	3.3

RP = subjective risk premium (μ), SD = subjective volatility (σ_e), Fraction = fraction of the population

fraction varies between waves, though, and appears to be related to the business cycle, with higher fractions being pessimistic in recessions. The distributions are also skewed to the right, with noticeable fractions of respondents expecting risk premia of up to 20% (even more so in the nondiscretized data). Our findings—more pessimistic expectations about stock market gains than historical averages, high degree of heterogeneity in subjective expectations, and reported beliefs linked to the recent stock market performance—are in line with those documented by Hurd (2009).

6 Model predictions with subjective expectations

We now analyze the extent to which replacing rational expectations based on historical returns with subjective return expectations affects the model's predictions about household stock holding at both the intensive and extensive margins. We will also gauge the robustness of such predictions to different values of key preference parameters (e.g., degree of risk aversion) and quantify the cost of suboptimal portfolio choices induced by inaccurate beliefs about future market returns. In the simulations in this section, we use the subjective expectations for computing the decision rules of the individuals, but the historical returns for generating actual returns.

We present results using the combined 2004–2006 subjective expectations and results using the 2010 subjective expectations. These are the most optimistic and pessimistic ones in the sense of having the lowest and highest fractions of individuals who expect a zero or negative risk premium. We have also computed the results using the expectations from 2002 and 2008. These are similar

to the ones shown here and will not be discussed further.

Figure 10 shows predicted stock holding and share of risky assets conditional on stock holding using the more optimistic 2004–2006 expectations distribution. Stock holding is now substantially below 1, but still substantially higher than in the data (which are averaged over 2000–2010). The share of risky assets is now matched more closely, especially with the higher value of risk aversion (4). However, the model does not predict an increasing fraction of risky assets with increasing age. Figure 11 is the analogous graph using the 2010 data. Now stock holding is matched more closely with the data (though still slightly higher), and the share of risky assets also fits slightly better.

Figures 12 and 13 show the effects on consumption and wealth. With a relatively high relative risk aversion of 4, consumption is below optimal at almost all ages, and the overall loss in consumption is substantial. Wealth levels are also noticeably below optimal. With a lower risk aversion of 1.5, consumption is above optimal up to age 73 or 74 and below optimal thereafter. The total amount of consumption is not that different from optimal, but welfare loss is suffered because of the lack of consumption smoothing. Wealth is below optimal by a large fraction.

Figures 12 and 13 show the effects on consumption and wealth. With a coefficient of relative risk aversion of 4, wealth levels are significantly below the level implied by investment choices under rational expectations. Overall, the retirement wealth loss associated with inaccurate return expectations is 21%, when subjective expectations are estimated using data from 2004–2006, and 28%, when subjective expectations are estimated using data from 2010. In both cases, consumption under subjective expectations is 9% below the level predicted by the model with rational expectations. With a degree of risk aversion of 1.5, individuals are substantially less prudent and tend to run down their assets relatively quickly. In this scenario, inaccurate beliefs about future risk premia induce a wealth loss of around 20% in both Figure 12 and 13. Consumption tracks wealth quite closely and decreases steadily over time. It is on average 7% below the level predicted by the model with rational expectations based on historical returns. In general, the consumption loss associated with inaccurate return expectations is higher, the higher the degree of risk aversion. This is because higher risk aversion implies more prudent behaviors and a higher preference for accumulating a buffer stock to hedge against background risks. As a consequence, any missed investment opportunity due to overpessimistic expectations and unrealized investment returns due to overoptimistic beliefs translate into a higher welfare cost due to the need of reducing consumption and maintaining a sufficiently high level of wealth.

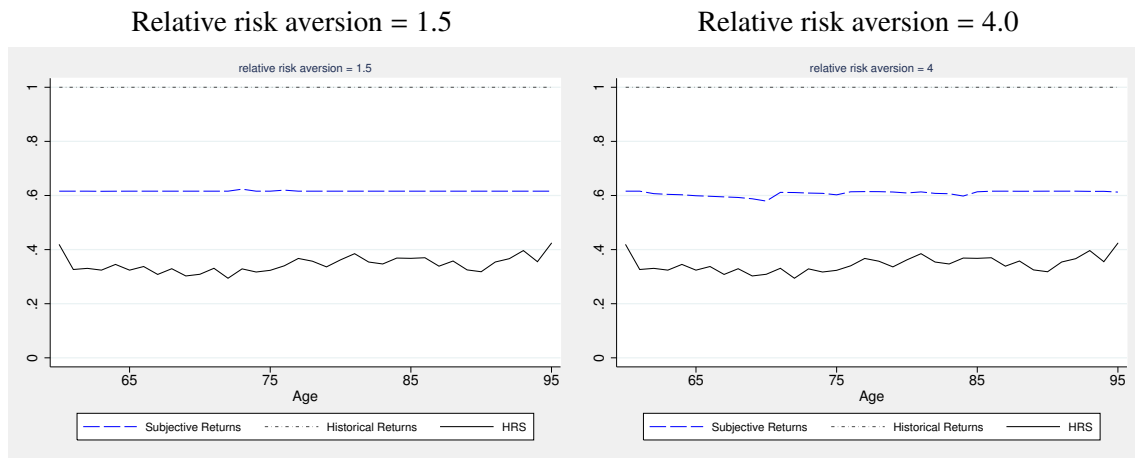
7 Discussion

Standard life cycle models of economic behavior predict that essentially everybody should invest in risky assets to some extent, which is greatly at odds with the less than 40% of households that are observed to own stocks in the data. Moreover, conditional on stock holding, standard models with realistic values of the parameters predict much higher fractions of wealth invested in risky assets than we observe in the data. Several explanations have been put forward in the literature, but these are not able to explain observed patterns.

In this paper, we explore to what extent subjective expectations about stock market returns

Figure 10: Fraction of stockholding and share of risky assets under subjective and historical returns (subjective expectations from 2004–2006)

Stockholding



Share of risky assets, conditional on stockholding

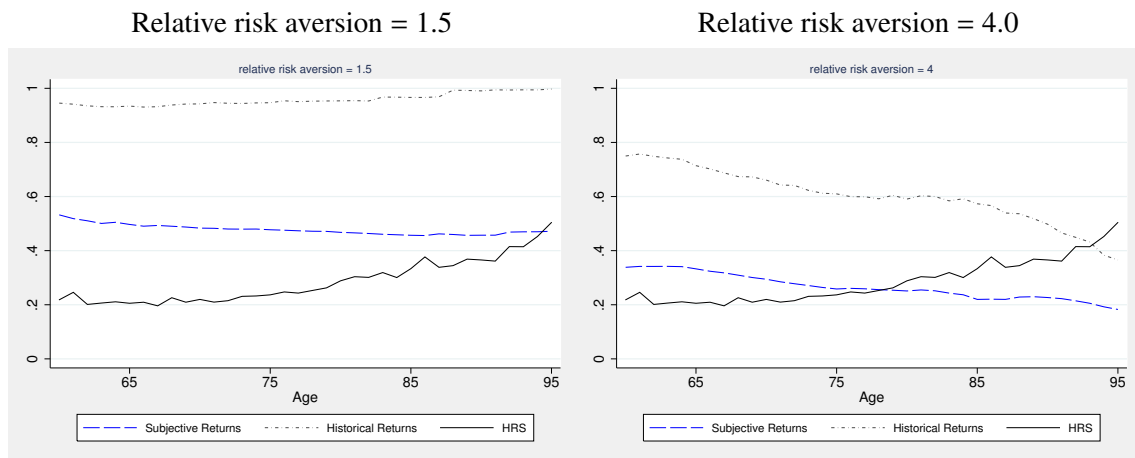
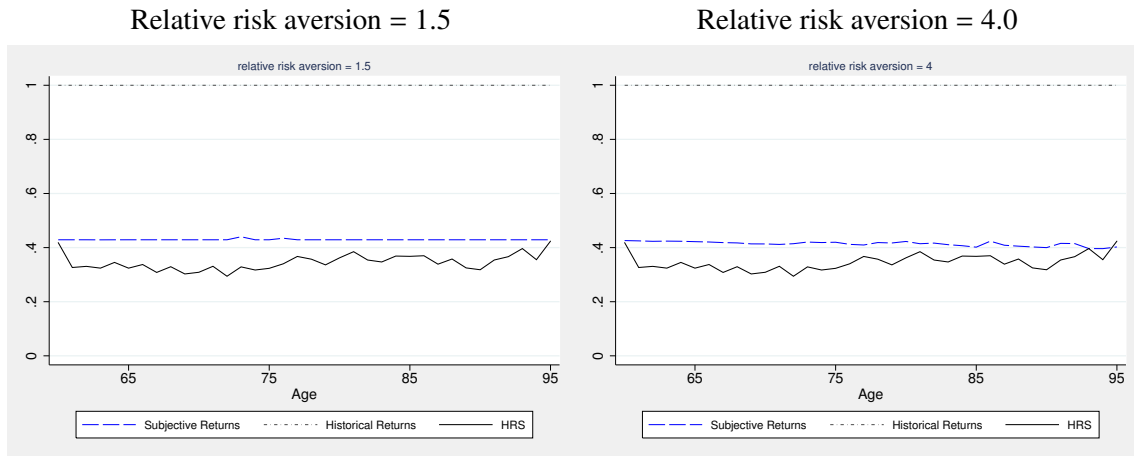


Figure 11: Fraction of stockholding and share of risky assets under subjective and historical returns (subjective expectations from 2010)

Stockholding



Share of risky assets, conditional on stockholding

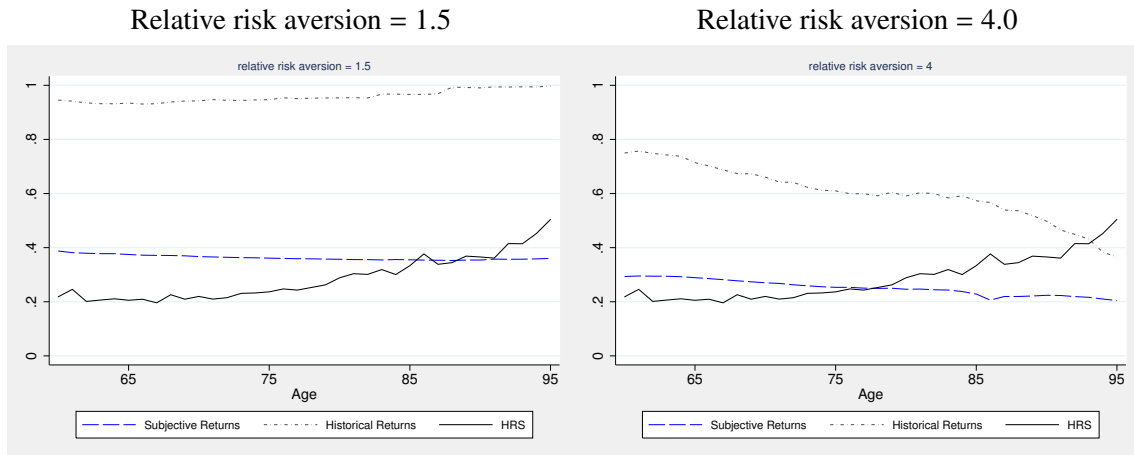
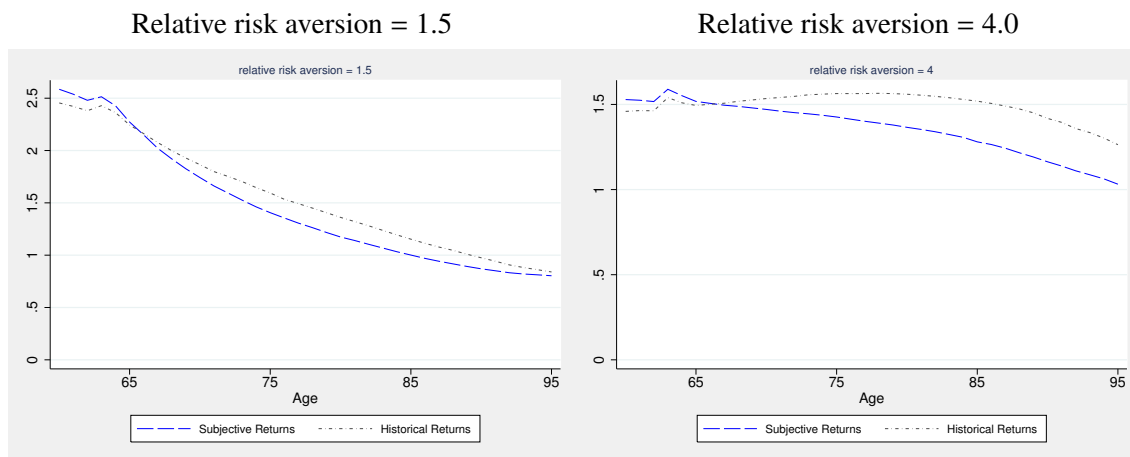


Figure 12: Consumption and wealth profiles under subjective and historical returns (subjective expectations from 2004–2006)

Consumption



Wealth

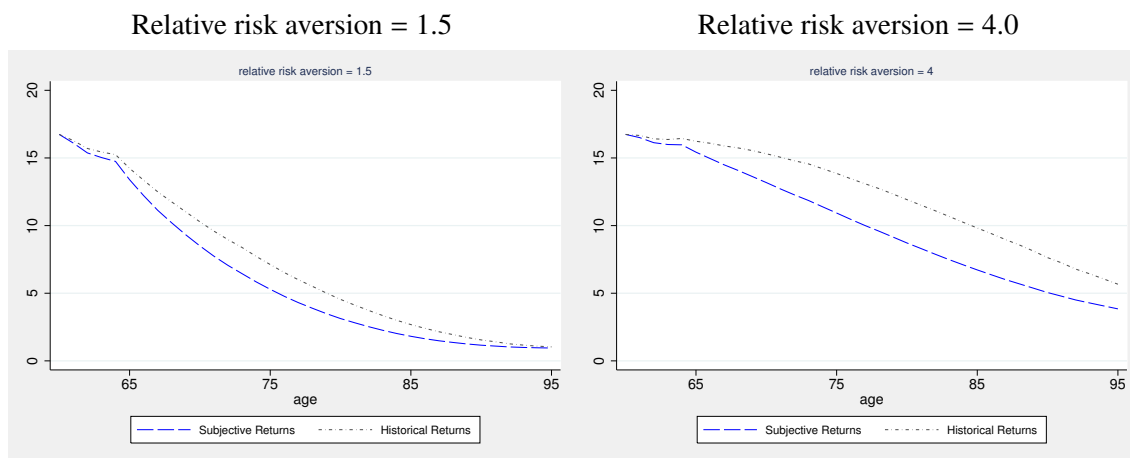
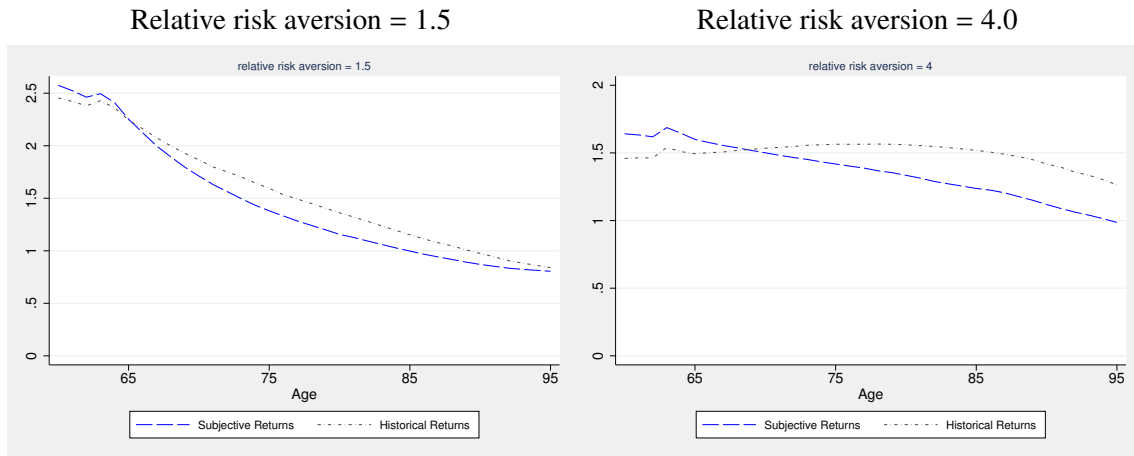
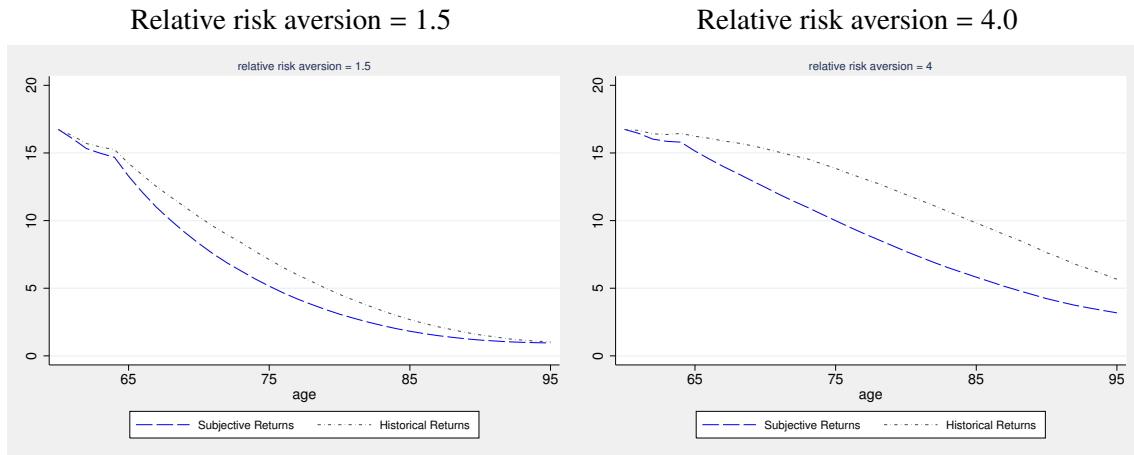


Figure 13: Consumption and wealth profiles under subjective and historical returns (subjective expectations from 2010)

Consumption



Wealth



that differ from historical returns can explain empirical patterns. We embedded subjective expectations in an otherwise relatively standard economic life cycle model. Using data on subjective expectations from the Health and Retirement Study, we show that a large fraction of the population holds beliefs that are so pessimistic that they should never invest in stocks. Of the remaining individuals, a nonnegligible fraction is very optimistic and they should invest almost all of their wealth in risky assets. A “moderate” middle group rounds out the distribution. This groups should invest moderate amounts of wealth in risky assets. The model with these heterogeneous subjective expectations is remarkably well able to match the empirical patterns, although there are still a few unresolved issues. The main empirical issue is that the fraction of wealth invested in stocks slightly increases with age in the data, whereas the model predict a gradual decrease. Most likely this is a selection effect due to wealth, stock holding, and mortality being correlated.

Our model simulations also show that individuals pay for their suboptimal behavior through a large welfare loss. We estimate that their consumption is on average 7–9% lower than when using correct expectations. The loss is larger if individuals are more risk averse.

These results have profound policy implications. Misperceptions about stock market returns can be attacked by various forms of education, information, and exemplary behavior. Although misperceptions are likely to be persistent and such a program will not reach everyone, there is a large scope for such programs. Not only government and non-profit organizations may be interested in this, but also commercial enterprises, in particular financial institutions.

An important aspect that is still missing from the model is the evolution of subjective expectations. The current model assumes that different individuals may have different expectatiobns, but they keep the same expectations throughout their lifetimes. In practice, optimistic individuals who consume much and expect the asset returns to support their life style may notice a systematic rundown of their assets caused by lower returns than expected. They may respond to this by updating their expectations and behaving more cautiously in later life. Moreover, we have seen that in the data, subjective expectations vary between waves and may be related to the business cycle. This suggests a role for the business cycle in changing expectations as well.

Our model contains risks of medical expenses as estimated from the HRS data. If individuals hold beliefs about their likelihood of being in good health or about the risks of large medical expenses conditional on health status that are different from historical distributions, these may also impact individuals’ behavior. We have performed limited experiments with subjective distributions of health transitions, but these did not have much effect on the results. However, it is worthwhile to study this more systematically and focus on expectations about out-of-pocket medical expenses directly.

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Appendix

A Normalization

Consider the model set-up of section 2. Define $c_t = C_t/P$. Then expected remaining lifetime utility (1) is

$$U_t = P^{1-\gamma} \left[\frac{c_t^{1-\gamma}}{1-\gamma} + E_t \left(\sum_{j=t+1}^{T_M} \beta^{j-t} \frac{c_j^{1-\gamma}}{1-\gamma} \right) \right],$$

and, using the definition of cash on hand X_t in the text, and $x_t = X_t/P$ and $hc_t = HC_t/P$, the wealth transition (11) can be rewritten in the form (14). Clearly, P does not play any role in the utility function, and it is eliminated from the wealth transition equation. By assuming that $\delta_1 = 1$ in the health cost model (6), this model can also be expressed in terms of the normalized health cost hc_t , with P not playing an independent role. Survival and health transition equations and other restrictions also do not depend on P . Thus, exploiting the scale-independence of the original maximization problem, we can rewrite all variables as ratios to the constant flow of retirement income so to reduce the dimensionality of the problem from 3 state variables to 2. This renormalization, introduced by Carroll (1992, 1997), makes the numerical problem more tractable as it significantly decreases the computational burden. Carroll (1992, 1997) shows that this normalization can even be done if income is variable and uncertain, at least within the income transition model he considers, but our model is much simpler because we only consider retirees and assume they have a constant stream of income.

B Numerical solution

The model is solved via backward induction. Let $u_t = U_t/P^{1-\gamma}$ be the normalized expected remaining lifetime utility and define the *value function*

$$v_t(x_t, H_t) \equiv \max u_t,$$

where the maximization is done over all decision variables. In the final period $T = 100$, expected remaining lifetime utility is $u_T = c_T^{1-\gamma}/(1-\gamma)$. This is maximized when c_t is maximized. With the restriction $W_{T+1} \geq 0$ and income, taxes, and health costs also being zero, it follows from (14) that utility is maximized by $c_T = x_T$ and the value function is

$$v_T(x_T, H_T) = \frac{x_T^{1-\gamma}}{1-\gamma}.$$

Now consider the period $T - 1$ problem. Normalized cash on hand x_{T-1} and health status H_{T-1} are given and the agent maximizes

$$u_{T-1} = \frac{c_{T-1}^{1-\gamma}}{1-\gamma} + \beta \sum_{g \in \{\text{good}, \text{bad}\}} S(T, g | T-1, H_{T-1}) E_{T-1} \left(\frac{c_T^{1-\gamma}}{1-\gamma} \middle| \text{alive}_T, H_T = g \right).$$

Whatever the choice at age $T - 1$ and realizations of the health transition and health cost processes, if the individual is still alive at age T , he or she will choose $c_T = x_T$ and remaining lifetime utility will be $v_T(x_T, H_T)$. Thus, the problem at age $T - 1$ becomes the problem of maximizing

$$u_{T-1} = \frac{c_{T-1}^{1-\gamma}}{1-\gamma} + \beta E_{T-1} [v_T(x_T, H_T)]$$

over the decision variables c_{T-1} and α_{T-1} . Note that the expectation now includes the survival probability. The maximized value of u_{T-1} is $v_{T-1}(x_{T-1}, H_{T-1})$. Walking backwards, the argument can now be repeated for $T - 2, \dots, t_0$, and the problem at time t can be rephrased recursively as

$$v_t(x_t, H_t) = \max_{c_t, \alpha_t} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + \beta E_t [v_{t+1}(x_{t+1}, H_{t+1})] \right\}.$$

The value function at age $t + 1$ can be used to compute the optimal decision rules in period t . This procedure is iterated backwards to the initial period $t = 60$.

In order to implement this solution algorithm, we discretize the continuous state variable, normalized cash-on-hand, by defining an equally spaced grid, $\{x_k\}_{k=1}^K$. The upper bound is chosen to be nonbinding in all periods, specifically $x_K = 99$. The lower bound is set to 0.01. This can be interpreted as a minimum level of consumption guaranteed by government transfers in all periods. By imposing $x_t \geq 0.01$ for all t , the model solution implicitly accounts for transfers from welfare programs that equal to 1% of retirement income plus medical expenses minus all available resources, if that amount is positive, and zero otherwise.

The optimization procedure combines a root-finding algorithm (Brent's method; Brent, 1973, chapter 4) and standard grid search. For given values of x_t and H_t , the value function is concave with respect to c_t . Thus, optimal normalized consumption can be computed using Brent's method, significantly improving on computational efficiency. It is not known whether the value function is concave in portfolio share α_t . Hence, to avoid the potential danger of selecting local optima, we optimize over the space of the remaining decision variable using a standard grid search. For this purpose, we discretize the share of risky assets, α , to take 11 equally spaced values in the interval $[0, 1]$.

Interpolation plays a crucial role in the solution of the dynamic programming problem at hand. We use two-dimensional cubic splines interpolation to evaluate the value function between points on the cash-on-hand grid. Cubic splines have the advantage of being twice continuously differentiable with a nonzero third derivative. These properties preserve the *prudence* of the utility function, which plays an important role in precautionary savings and the effect of background risks on individual decisions (Eeckhoudt & Kimball, 1992; Kimball, 1990). The strictly positive

lower bound on the grid for cash-on-hand implies that the value function is also bounded from below. This makes the spline interpolation work very well as long as the discretization of the state space is sufficiently fine. Numerical integrations are performed by Gaussian quadrature.

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