Is the Potential for International Diversification Disappearing?*

Peter Christoffersen   Vihang Errunza   Kris Jacobs   Hugues Langlois

University of Toronto   McGill University   University of Houston   McGill University

November 18, 2011

Abstract

Quantifying the evolution of security co-movements is critical for asset pricing and portfolio allocation, hence we investigate patterns and trends in correlations and tail dependence for developed markets (DMs) and emerging markets (EMs). We use the standard DCC and DECO correlation models, and we also develop a nonstationary DECO model as well as a novel dynamic skewed t-copula to allow for dynamic and asymmetric tail dependence. We show that it is possible to characterize co-movements for many countries simultaneously. We find that correlations have significantly trended upward for both DMs and EMs, but correlations between EMs are much lower than between DMs. Tail dependence has also increased but its level is still very low for EMs as compared to DMs. Thus, while the correlation patterns suggest that the diversification potential of DMs has reduced drastically over time, our findings suggest that EMs offer significant diversification benefits, especially during large market moves.

JEL Classification: G12

Keywords: Asset allocation, dynamic correlation, dynamic copula, asymmetric dependence.

1   Introduction

Understanding and quantifying the evolution of security co-movements is critical for asset pricing and portfolio allocation. The traditional case for international diversification benefits has relied largely on the existence of low cross-country correlations. Initially, the literature studied developed markets, but over the last two decades much of the focus has shifted to the diversification benefits offered by emerging markets.[1] Two critical questions, with important implications for asset allocation and international diversification, are of special interest for academics and practitioners alike.

First, how have cross-country correlations changed through time? It is far from straightforward to address this ostensibly simple question without making additional assumptions. Computing rolling correlations is subject to well-known drawbacks. Multivariate GARCH models, as for example in Longin and Solnik (1995), seem to provide a solution, but the implementation of these models using large numbers of countries is subject to well known dimensionality problems, as discussed by Solnik and Roulet (2000). As a result, most of the available evidence on the time-variation in cross-country correlations is based on factor models.[2] In a recent paper, Bekaert, Hodrick, and Zhang (2009) convincingly argue that the evidence from this literature is mixed at best and state that (see p. 2591): "It is fair to say that there is no definitive evidence that cross-country correlations are significantly and permanently higher now than they were, say, 10 years ago." Bekaert, Hodrick, and Zhang (2009) proceed to investigate international stock return co-movements for 23 DMs during 1980-2005, and find an upward trend in return correlations only among the subsample of European stock markets, but not for North American and East Asian markets.

The second question is whether correlation is a satisfactory measure of dependence in international markets, or if we need to consider different measures, notably those that focus on the dependence between tail events? This question is related to the analysis of correlation asymmetries, and changes in correlation as a function of business cycle conditions or stock market performance. Following the seminal paper by Longin and Solnik (2001) and the corroborating evidence of Ang

---

[1] For early studies documenting the benefits of international diversification, see Solnik (1974) for developed markets and Errunza (1977) for emerging markets. For more recent evidence, see for example Erb, Harvey and Viskanta (1994), DeSantis and Gerard (1997), Errunza, Hogan and Hung (1999), and Bekaert and Harvey (2000).

[2] King, Sentana, and Wadhwani (1994) do not find evidence of increasing cross-country correlations for 16 developed markets during the period 1970-1988, except around the market crash of 1987. Carrieri, Errunza, and Hogan (2007) do not find a common pattern in the correlation trend for eight emerging markets (EMs) during 1977-2000. Eiling and Gerard (2007) find an upward time trend in co-movements between 24 developed markets but not between 26 emerging markets over the period 1973-2005. Goetzmann, Li, and Rouwenhorst (2005) document substantial changes in the correlation structure of world equity markets over the past 150 years. Baele and Inghelbrecht (2009) report increasing correlations over the period 1973-2007 for their sample of 21 DMs. See also Karolyi and Stulz (1996), Forbes and Rigobon (2002), Brooks and Del Negro (2003), Lewis (2006), and Rangel (2011).

and Bekaert (2002) and Ang and Chen (2002), the hypothesis that cross-market correlations rise in periods of high volatility has been supplanted by the notion that correlations increase in down markets, but not in up markets.[3] Longin and Solnik (2001) use extreme value theory in bivariate monthly models for the U.S. with either the U.K., France, Germany, or Japan during 1959-1996. Ang and Bekaert (2002) develop a regime switching dynamic asset allocation model, and estimate it for the U.S., U.K., and German system over the period 1970-1997. Both papers estimate return extremes at predetermined threshold values, i.e. they define the tail observations ex ante, and then compute unconditional correlations for the tail for a small sample of developed markets.[4]

This paper substantially contributes to our understanding of both these important questions. Regarding the patterns and trends in correlations over time, we argue that recent advances make it feasible to overcome dimensionality and optimization problems in international finance applications. We characterize time-varying correlations using weekly returns during the 1973-2009 period for a large number of countries (either thirteen or seventeen EMs, sixteen DMs, as well as combinations of the EM and DM samples), without relying on a factor model. We implement models that overcome the dimensionality problems, and that are easy to estimate. To do so, we rely on the variance targeting idea in Engle and Mezrich (1996) and the numerically efficient composite likelihood procedure proposed by Engle, Shephard and Sheppard (2008). The composite likelihood estimation procedure is essential for estimating dynamic correlation models on large sets of weekly international equity data such as ours. We use the flexible dynamic conditional correlation (DCC) model of Engle (2002) and Tse and Tsui (2002), as well as the dynamic equicorrelation (DECO) model of Engle and Kelly (2009) that can be estimated on large sets of assets using conventional maximum likelihood estimation. We thus demonstrate that it is possible to estimate correlation patterns in international markets using large numbers of countries and extensive time series, without relying on a factor model that may bias inference. Our implementation is relatively straightforward and computationally fast, which allows us to report results using several estimation approaches, while assessing the robustness of our findings.

Regarding the second question, the DECO and DCC correlation models with normal innovations do not generate the levels of tail dependence required by the data, nor do they generate asymmetries in correlations. Hence, we introduce copula approaches to capture nonlinear dependence across markets. We fit the tails of the marginal distributions using the Generalized Pareto distribution (GP), and the joint distribution is modeled using time-varying copulas. We develop a novel skewed

---

[3]On tail dependence, see also Poon, Rockinger, and Tawn (2004). On the related topic of contagion, see for example Forbes and Rigobon (2002), Bekaert, Harvey, and Ng (2005), and Bae, Karolyi, and Stulz (2003).

[4]A related literature explores the relationship between industrial structure and the dynamics of equity market returns and cross-country correlations. See for instance Roll (1992), Heston and Rouwenhorst (1994), Griffin and Karolyi (1998), Dumas, Harvey and Ruiz (2003), and Carrieri, Errunza and Sarkissian (2007).

dynamic t copula which allows for asymmetric and dynamic tail dependence in large portfolios.

Our results based on DCC and DECO models are extremely robust and suggest that correlations have been significantly trending upward for both DMs and EMs. However, the correlation between DMs has been higher than the correlation between EMs at all times in our sample. For developed markets, the average correlation with other developed markets is higher than the average correlation with emerging markets. For emerging markets, the correlation with developed markets is generally somewhat higher than the correlation with the other emerging markets, but the differences are small. When dividing our sample into four regions: EU and developed non-EU, Latin America, and Emerging Eurasia, we find that the correlation between all four regions have gone up, and so has the average correlation within each region. While the range of correlations for DMs has narrowed around the increasing trend in correlation levels, this is not the case for EMs. Emerging markets thus still offer substantial correlation-based diversification benefits to investors.

Our robust finding of an upward trend in correlations is all the more remarkable because the parametric models we use enforce mean-reversion in volatilities and correlation, and we estimate the models using long samples of weekly returns. The data clearly pull the models away from the average correlation in the samples we investigate. In order to explicitly address the issue of nonstationarity in correlations, we develop a new two-component correlation model which includes a nonstationary long-run correlation component. We refer to this model as Spline DECO. Its estimates confirm the upward trends in correlation across DMs and EMs.

We find overwhelming evidence that the assumption of multivariate normality is inappropriate. Results from the dynamic t copula indicate substantial tail dependence. Moreover, tail dependence as measured by the skewed t copula is asymmetric and increasing through time for both EMs and DMs. We demonstrate that the skewed t copula can capture the empirical asymmetries in threshold correlations. However, the most striking finding is that the level of the tail dependence is still very low at the end of the sample period for EMs as compared to DMs. Our findings on tail dependence thus suggest that EMs have offered diversification benefits during large market moves. The underlying intuition for this finding is that while financial crises in EMs are frequent, many of them are country-specific. Thus, although the benefits of international diversification might have lessened both for DMs and EMs, a strong case can still be made for EMs, and the diversification benefits from adding emerging markets to a portfolio appear to be significant.

We contribute to the literature in several ways. At the methodological level, we demonstrate that it is possible to model correlation dynamics and tail dependence in international equity markets using large samples, without relying on factor models. We build a new correlation model with a nonstationary low-frequency component, as well as a new fully-specified dynamic model that can capture nonlinear and asymmetric dependence in a large number of equity markets.

4

From an empirical perspective, we document several important stylized facts. First, we demonstrate that measures of international dependence have increased significantly over the course of our sample. This is of course a purely descriptive statement, and does not imply that correlations will remain high. Second, we document the inadequacy of the multivariate normality assumption for modeling international equity returns, and we provide a genuinely multivariate characterization of asymmetries in international equity markets. We also document asymmetric threshold correlation patterns for EMs, and find that they differ from those for DMs. Longin and Solnik (2001) document asymmetric threshold correlation patterns for the United States vis-a-vis other DMs, but to the best of our knowledge the literature does not contain evidence on EMs. We demonstrate that our multivariate asymmetric model can capture the threshold correlation patterns observed in DMs and EMs. Third, we extend existing results on dependence to a more recent period characterized by significant liberalizations for the EM sample, as well as substantial market turmoil during 2007-2009, which helps identify tail dependence. These results also allow us to elaborate on existing findings and further investigate if correlations for EMs are impacted by measures of market openness. Fourth, we use our estimates to compute a measure of conditional diversification benefits, and we find that diversification benefits decreased over our sample period. Fifth, we investigate the relationships between correlations and volatilities. Our model does not assume a factor structure but we do find a significant positive association between correlations and volatilities.

The paper proceeds as follows. Section 2 provides a brief outline of DCC and DECO correlation models, with special emphasis on the estimation of large systems. Section 3 presents the data, as well as empirical results on time variation in linear correlations. Sections 4 and 5 build and estimate a new set of copula models with dynamic tail dependence, asymmetry and dynamic copula correlations. Section 6 investigates the linear correlations further, computes threshold correlations and develops the new two-component correlation model that includes a nonstationary long-run component. Section 7 concludes.

2   Dynamic Linear Dependence Models for Many Equity Markets

This section outlines the various models we use to capture dynamic dependence across equity markets. We describe how the dynamic conditional correlation model of Engle (2002) and Tse and Tsui (2002) can be implemented simultaneously on many assets.

2.1 The Dynamic Conditional Correlation Approach

In the existing literature, the scalar BEKK model has been the standard econometric approach for capturing dynamic dependence.[5] Implementations of multivariate GARCH models have traditionally used a limited number of countries because of dimensionality problems.[6] Further, the defining characteristic of the scalar BEKK model is that the parameters are identical across all conditional variance and covariance dynamics. This common persistence across all variances and covariances is clearly restrictive. Cappiello, Engle and Sheppard (2006) have found that the persistence in correlation differs from that in variance when looking at international stock and bond markets.[7]

Equally important is the restriction that the functional form of the variance dynamic is required to be identical to the form of the covariance dynamic. This rules out for example asset-specific leverage effects in volatility, which has been found to be an important stylized fact in equity index returns (see for example Black, 1976, and Engle and Ng, 1993). The leverage effect is an asymmetric volatility response that captures the fact that a large negative shock to an equity market increases the equity market volatility by much more than a positive shock of the same magnitude.

Hence, we implement the flexible dynamic conditional correlation (DCC) model of Engle (2002) and Tse and Tsui (2002).[8] Allowing for a leverage effect in conditional variance, we assume that the return on asset i at time t follows an Engle-Ng (1993) dynamic

$$R_{i,t} = \mu_{i,t} + \varepsilon_{i,t} = \mu_{i,t} + \sigma_{i,t} z_{i,t} \tag{2.1}$$

$$\sigma_{i,t}^2 = \omega_i + \alpha_i \left(\varepsilon_{i,t-1} - \theta_i \sigma_{i,t-1} \right)^2 + \beta_i \sigma_{i,t-1}^2. \tag{2.2}$$

Because the covariance is just the product of correlations and standard deviations, we can write

$$\Sigma_t = D_t \Gamma_t D_t \tag{2.3}$$

where D_t has the standard deviations $\sigma_{i,t}$ on the diagonal and zeros elsewhere, and where Γ_t has ones on the diagonal and conditional correlations off the diagonal.

We implement the modified DCC model discussed in Aielli (2009), in which the correlation

---

[5] The BEKK model is most often used to estimate factor models with a GARCH structure. See for instance DeSantis and Gerard (1997, 1998), and Carrieri, Errunza, and Hogan (2007) for examples. See Ramchand and Susmel (1998), Baele (2005), and Baele and Inghlebrecht (2009) for more general multivariate GARCH models with regime switching.

[6] See for instance Solnik and Roulet (2000), Longin and Solnik (1995) and Karolyi (1995) for early examples of bivariate models.

[7] See Kroner and Ng (1998) and Solnik and Roulet (2000) for a more elaborate discussion of the restrictions imposed in the first generation of multivariate GARCH models.

[8] Our main finding of an upward trend in correlation in our samples is confirmed when using the BEKK approach. Results for the BEKK model are available upon request.

dynamics are driven by the cross-products of the return shocks

$$\tilde{\Gamma}_t = \Omega_\Gamma + \beta_\Gamma \tilde{\Gamma}_{t-1} + \alpha_\Gamma \tilde{z}_{t-1} \tilde{z}_{t-1}^\top \qquad (2.4)$$

where $\tilde{z}_{i,t} = z_{i,t}\sqrt{\tilde{\Gamma}_{ii,t}}$. These cross-products are used to define the conditional correlations via the normalization

$$\Gamma_{ij,t}^{DCC} = \tilde{\Gamma}_{ij,t}/\sqrt{\tilde{\Gamma}_{ii,t}\tilde{\Gamma}_{jj,t}}. \qquad (2.5)$$

This normalization ensures that all correlations remain in the -1 to 1 interval.

If N denotes the number of equity markets under study then the DCC model has $N(N-1)/2+2$ parameters to be estimated. Below we will study up to 17 emerging markets and 16 mature markets, thus $N = 33$ and so the DCC model will have 530 parameters. It is well recognized in the literature that it is impossible to estimate these parameters reliably due to the need to use numerical optimization techniques, see for instance Solnik and Roulet (2000) for a discussion. In order to operationalize estimation, we follow DeSantis and Gerard (1997) who rely on the targeting idea in Engle and Mezrich (1996).

Taking expectations on both sides of (2.4) and solving for the unconditional correlation matrix $\tilde{\Gamma}$ of the vector $\tilde{z}_t$, yields

$$\tilde{\Gamma} = \Omega_\Gamma/\left(1 - \alpha - \beta\right). \qquad (2.6)$$

Note that this relationship enables us to rewrite the DCC model in a more intuitive form

$$\tilde{\Gamma}_t = \left(1 - \alpha_\Gamma - \beta_\Gamma\right)\tilde{\Gamma} + \beta_\Gamma \tilde{\Gamma}_{t-1} + \alpha_\Gamma \tilde{z}_{t-1}\tilde{z}_{t-1}^\top \qquad (2.7)$$

which shows that the conditional correlation in DCC is a weighted average of the long-run correlation, yesterday's conditional correlation, and yesterday's innovation cross-product.

Now, if we use the sample correlation matrix, $\hat{\Gamma} = \frac{1}{T}\sum_{t=1}^{T} \tilde{z}_t\tilde{z}_t^\top$ as an estimate of the unconditional correlation matrix, $\tilde{\Gamma}$, then the numerical optimizer only has to search in two dimensions, namely over α_Γ and β_Γ, rather than in the original 530 dimensions. Note that this implementation also ensures that the estimated DCC model yields a positive semi-definite correlation matrix, because $\tilde{z}_t\tilde{z}_t^\top$ and thus $\hat{\Gamma}$ is positive semi-definite by construction. Appendix A contains more details on the implementation of correlation targeting in the DCC model.

The standard DCC model is symmetric in the sense that a negative pair of asset return shocks impact correlation in the same way as do a positive pair of return shocks of the same magnitude. One may reasonably wonder if such symmetry is empirically valid. We therefore consider the asymmetric

DCC model in Cappiello, Engle and Sheppard (2006) in which

$$\tilde{\Gamma}_t = (1 - \alpha_\Gamma - \beta_\Gamma)\tilde{\Gamma} + \beta_\Gamma \tilde{\Gamma}_{t-1} + \alpha_\Gamma \tilde{z}_{t-1}\tilde{z}_{t-1}^\top + \delta_\Gamma \left(\tilde{\xi}_{t-1}\tilde{\xi}_{t-1}^\top - E\left[\tilde{\xi}_{t-1}\tilde{\xi}_{t-1}^\top\right] \right)$$

where $\tilde{\xi}_{t-1} = \tilde{z}_{t-1}I(\tilde{z}_{t-1} < 0)$. In our application the empirical support for the correlation asymmetry parameter, δ_Γ, turned out to be weak and so we only report results for the symmetric DCC model below.

Even when using correlation targeting, estimation is cumbersome in large-dimensional problems due to the need to invert the N by N correlation matrix, Γ_t, on every day in the sample for every likelihood evaluation. The likelihood in turn must be evaluated many times in the numerical optimization routine. More importantly, Engle, Shephard, and Sheppard (2008) find that in large-scale estimation problems, the parameters α and β which drive the correlation dynamics are estimated with bias when using conventional estimation techniques. They propose an ingenious solution based on the composite likelihood defined as

$$CL(\alpha, \beta) = \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j>i} \ln f(\alpha_\Gamma, \beta_\Gamma; z_{it}, z_{jt}) \tag{2.8}$$

where $f(\alpha_\Gamma, \beta_\Gamma; z_{it}, z_{jt})$ denotes the bivariate normal distribution of asset pair i and j, and where correlation targeting is imposed.

The composite log-likelihood is thus based on summing the log-likelihoods of pairs of assets. Each pair yields a valid (but inefficient) likelihood for α and β, but summing over all pairs produces an estimator which is relatively efficient, numerically fast, and free of bias even in large-scale problems. We use the composite log-likelihood in all our estimations below. We have found it to be very reliable and robust, effectively turning a numerically impossible task into a manageable one. The composite likelihood procedure allows us to estimate dynamic correlations in larger systems of international equity data using longer time series of returns than previously done in the literature. This is important because long time series on large sets of countries are needed for the identification of variance and covariance dynamics.

2.2 The Dynamic EquiCorrelation Approach

The dynamic equicorrelation (DECO) model in Engle and Kelly (2009) can be viewed as a special case of the DCC model in which the correlations are equal across all pairs of countries but where this common so-called equicorrelation is changing over time. The resulting dynamic correlation can be thought of as an average dynamic correlation between the countries included in the analysis.

8

Following Engle and Kelly (2009), we parameterize the dynamic equicorrelation matrix as

$$\Gamma_t^{DECO} = (1 - \rho_t)I_N + \rho_t J_{N \times N}$$

where ρ_t is a scalar, I_N denotes the n-dimensional identity matrix and $J_{N \times N}$ is an $N \times N$ matrix of ones.

The scalar dynamic equicorrelation, ρ_t, is obtained by taking the cross-sectional average each period of the DCC conditional correlation matrix in (2.5)

$$\rho_t = \frac{1}{N(N-1)} \left(J_{1 \times N} \Gamma_t^{DCC} J_{N \times 1} - N \right). \tag{2.9}$$

Note that subtracting N eliminates the trivial term arising from the ones on the diagonal of Γ_t^{DCC}.

The determinant of the DECO correlation matrix is simply

$$\left| \Gamma_t^{DECO} \right| = (1 - \rho_t)^{N-1} \left(1 + (N-1)\rho_t \right)$$

and from this we can derive the inverse correlation matrix as

$$\left(\Gamma_t^{DECO} \right)^{-1} = \frac{1}{(1 - \rho_t)} \left[I_N - \frac{\rho_t}{1 + (N-1)\rho_t} J_{N \times N} \right].$$

The simple structure of the inverse correlation matrix ensures that the model can be estimated on large sets of assets using conventional maximum likelihood estimation. The dynamic correlation parameters α_Γ and β_Γ embedded in ρ_t will not be estimated with bias even when N is large.

2.3 Measuring Conditional Diversification Benefits

If correlations are changing over time, then the benefits of portfolio diversification will be changing as well. We therefore need to develop a dynamic measure of diversification benefits.[9] First, let us define portfolio volatility $\sigma_{PF,t}$ generically as

$$\sigma_{PF,t} \equiv \sqrt{w_t^\top \Sigma_t w_t} = \sqrt{w_t^\top D_t \Gamma_t D_t w_t}$$

where w_t is the vector of portfolio weights at time t and D_t is the diagonal matrix of volatilities as in (2.3).

Consider then the extreme case of a portfolio without any diversification benefits, that is, the

---

[9]Our dynamic measure is related to the static measure in Choueifaty and Coignard (2008).

correlation matrix Γ_t is a matrix of ones. The portfolio volatility at time t can be expressed in this case as

$$\bar{\sigma}_{PF,t} = \sqrt{w_t^\top D_t J_{N\times N} D_t w_t} = w_t^\top \sigma_t$$

where σ_t denotes the vector of individual asset volatilities at time t.

The opposite extreme would correspond to each pair of assets having a correlation of -1 in which case it is possible to find a long-only portfolio such that the portfolio volatility $\sigma_{PF,t}$ is zero.

Using these upper and lower bounds on portfolio volatility, we define the conditional diversification benefit as

$$CDB_t = \frac{\bar{\sigma}_{PF,t} - \sigma_{PF,t}}{\bar{\sigma}_{PF,t}} = 1 - \frac{\sqrt{w_t^\top \Sigma_t w_t}}{w_t^\top \sigma_t}. \tag{2.10}$$

This measure describes the level of diversification benefits in a concise manner. It is increasing as the correlations decrease, and it is normalized to lie between zero and one: The portfolio volatility in the numerator has a lower bound of zero and the denominator is always positive in a long-only portfolio.

When computing CDB_t one must first decide on the portfolio weights, w. One approach is to construct the minimum variance portfolio each week and compute the CDB_t value corresponding to this portfolio. Alternatively, we could choose the weights that maximize CDB_t.[10] We follow the second approach. We further impose that the weights sum to one and we rule out short-selling.

In order to assess how much of the conditional diversification benefit stems from active asset allocation, we also construct a CDB_t^{EW} measure for an equal-weighted portfolio. In this case

$$CDB_t^{EW} = 1 - \frac{\sqrt{w_t^\top \Sigma_t w_t}}{w_t^\top \sigma_t} = 1 - \frac{\sqrt{J_{1\times N}\Sigma_t J_{N\times 1}}}{J_{1\times N}\sigma_t}. \tag{2.11}$$

By definition CDB_t^{EW} will be less than or equal to the optimal CDB_t at any point in time. The difference between the CDB_t and CDB_t^{EW} measures will tell us about the extent to which changing volatilities and correlations can potentially be exploited via dynamic asset allocation and about the optimality (or lack thereof) of an equal-weighted portfolio over time.[11]

3   Empirical Correlation Analysis

This section contains our empirical findings on correlation patterns. We first describe the different data sets that we use and briefly discuss the univariate results. We then analyze the time-variation

---

[10]The two approaches will coincide only when the volatilities are identical across assets.

[11]DeMiguel, Garlappi and Uppal (2009) and Tu and Zhou (2011) analyze the relative performance of equal-weighted versus optimally-weighted portfolios in an unconditional setting.

in linear correlations. Subsequently we measure the dispersion in correlations across pairs of assets at each point in time and check if this dispersion has changed over time.

3.1   Data and Univariate Models

We employ the following three data sets:

First, from DataStream we collect weekly closing U.S. dollar returns for the following 16 developed markets: Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Ireland, Italy, Japan, Netherlands, Singapore, Switzerland, U.K., and U.S. This data set contains 1,901 weekly observations from January 12, 1973 through June 12, 2009.

Second, from Standard and Poor's we collect the IFCG weekly closing U.S. dollar returns for the following 13 emerging markets: Argentina, Brazil, Chile, Colombia, India, Jordan, Korea, Malaysia, Mexico, Philippines, Taiwan, Thailand, and Turkey. This data set contains 1,021 weekly observations from January 6, 1989 through July 25, 2008.

Third, from Standard and Poor's we collect the weekly closing investable IFCI U.S. dollar returns for the following 17 emerging markets: Argentina, Brazil, Chile, China, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Peru, Philippines, Poland, South Africa, Taiwan, Thailand, and Turkey. This data set contains 728 weekly returns from July 7, 1995 through June 12, 2009.

We use two emerging markets data sets because they have their distinct advantages. The IFCG data set spans a longer time period, and represents a broad measure of emerging market returns, but is not available after July 25, 2008. The IFCI data set tracks returns on a portfolio of emerging market securities that are legally and practically available to foreign investors. The index construction takes into account portfolio flow restrictions, liquidity, size and float. It continues to be updated but the sample period is shorter, which is a disadvantage in model estimation and of course in assessing long-term trends in correlation.

Table 1 contains descriptive statistics on the 1989-2008 data set. While the cross-country variations are large, Table 1 shows that the average annualized return in the developed markets was 12.06%, versus 17.68% in the emerging markets. This emerging market premium is reflective of an annual standard deviation of 33.63% versus only 18.41% in developed markets. Kurtosis is on average higher in emerging markets, indicating more tail risk. But skewness is slightly positive in emerging markets and slightly negative in mature markets, suggesting that emerging markets are not more risky from this perspective. The first-order autocorrelations are small for most countries. The Ljung-Box (LB) test that the first 20 weekly autocorrelations are zero is not rejected in most developed markets but it is rejected in most emerging markets. We will use an autoregressive model of order two, AR(2), for each market to pick up this return dependence. The Ljung-Box test that

the first 20 autocorrelations in absolute returns are zero is strongly rejected for all 29 markets. In the DECO and DCC models, we will employ a GARCH(1,1) model for each market to pick up this second-moment dependence. We use the NGARCH model of Engle and Ng (1993) found in equation (2.2) to account for asymmetries.

Table 2 reports the results from the estimation of the AR(2)-NGARCH(1,1) models on each market for the 1989-2008 data set. The results are fairly standard. The volatility updating parameter, α, is around 0.1, and the autoregressive variance parameter, β, is around 0.8. The parameter θ governs the volatility asymmetry and is also known as the leverage effect. It is commonly found to be large and positive in developed markets and we find that here as well. Austria is the only outlier in this regard. Interestingly, the average leverage effect is much closer to zero in the emerging markets. The slightly negative average is driven largely by the unusual estimate of -3.38 for Jordan. The model-implied variance persistence is high for all countries, as is commonly found in the literature.

The Ljung-Box (LB) test on the model residuals show that the AR(2) models are able to pick up the weak evidence of return predictability found in Table 1. Impressively, the GARCH models are also able to pick up the strong persistence in absolute returns found in Table 1. Note also that the GARCH model picks up much of the excess kurtosis found in Table 1. The remaining nonnormality will be addressed using copula modeling below.

We conclude from Tables 1 and 2 that the AR(2)-NGARCH(1,1) models are successful in delivering the white-noise residuals that are required to obtain unbiased estimates of the dynamic correlations. We will therefore use the AR(2)-NGARCH(1,1) model in the DECO and DCC applications.

3.2   Correlation Patterns Over Time

Table 3 reports the parameter estimates and log likelihood values for the DECO and DCC correlation models. We report results for the three data sets introduced above. For each set of countries we estimate two versions of each model: one version allowing for correlation dynamics and another where the correlation dynamics are shut down, and thus $\alpha_\Gamma = \beta_\Gamma = 0$. A conventional likelihood ratio test would suggest that the restricted model is rejected for all sets of countries, but unfortunately the standard chi-squared asymptotics are not available for composite likelihoods.

The correlation persistence $(\alpha_\Gamma + \beta_\Gamma)$ is close to one in all models, implying very slow mean-reversion in correlations. In the DECO model, persistence is estimated to be essentially one, reflecting the upward trend in correlations which we now discuss.

We present time series of dynamic equicorrelations (DECOs) for several samples. The left panels

in Figure 1 present results for twenty-nine developed and emerging markets for the sample period January 20, 1989 to July 25, 2008. As explained in Section 3.1, sixteen of these markets are developed and thirteen are emerging markets. We also present DECOs for each group of countries separately. We refer to this sample as the 1989-2008 sample.

The right panels in Figure 1 present results for thirty-three developed and emerging markets for the sample period July 21, 1995 to June 12, 2009. This sample contains the same sixteen developed markets, and seventeen emerging markets. There is considerable overlap between this sample of emerging markets and the one used in the left panels of Figure 1. Section 3.1 discusses the differences. We refer to this sample as the 1995-2009 sample.

The top left-hand panel in Figure 2 contains the time series of DECOs for the group of sixteen developed markets between January 26, 1973 and June 12, 2009. We refer to this sample as the 1973-2009 sample. Figure 2 also shows results for the 1989-2008 and the 1995-2009 data for comparison.

These figures contain some of the main messages of our paper. The DECOs in Figures 1 and 2, which can usefully be thought of as the average of the pairwise correlations between all pairs of countries in the sample, fluctuate considerably from year to year, but have been on an upward trend since the early 1970s. Figure 2 shows that for the sixteen developed markets, the DECO increased from approximately 0.3 in the mid-1970s to between 0.7 and 0.8 in 2009. Figure 1 indicates that over the 1989-2009 period, the DECO correlations between emerging markets are lower than those between developed markets, but that they have also been trending upward, from approximately 0.1-0.2 in the early nineties to over 0.5 in 2009.

Because the DECO model assumes correlation is time-varying with a model-implied long-run mean, one may wonder whether the choice of sample period strongly affects inference on correlation estimates at a particular point in time. Figure 2 addresses this issue by reporting DECO estimates for the sixteen developed markets for three different sample periods. Whereas there are some differences, the correlation estimate at a particular point in time is remarkably robust to the sample period used, and the conclusion that correlations have been trending upward clearly does not depend on the sample period used. Comparing the left and right panels of Figure 1, it can be seen that a similar conclusion obtains for the emerging markets, even though this comparison is more tenuous, as the sample composition and the return data used for the emerging markets are somewhat different across panels.

3.3   Cross-Sectional Differences in Dependence

The DECO correlations give us a good idea of the evolution of correlation over time in a given sample of markets. They can usefully be thought of as an average of all possible permutations of pairwise correlations in the sample. The next question is how much cross-sectional heterogeneity there is in the correlations. The DCC framework discussed in Section 2.1 is designed to address this question. It yields a time-varying correlation series for each possible permutation of markets in the sample.

Reporting on all these time-varying pairwise correlation paths is not feasible, and we have to aggregate the correlation information in some way. Figures 2-5 provide an overview of the results. The right-side panels in Figure 2 provide the average across all markets of the DCC paths, and compare them with the DECO paths. The top-right panel provides the average DCC for the sixteen developed markets from 1973 through 2009. The middle-right panel provides the average DCC for the same sixteen markets for the 1989-2008 sample period, and the bottom-right panel for the 1995-2009 period. The left-side panels provide the DECO correlations. Figure 2 demonstrates that the DECO can indeed be thought of as an average of the DCCs. Moreover, Figure 2 demonstrates that the average DCC correlation at each point in time is robust to the sample period used in estimation, as is the case for the DECO.[12]

Figure 3 uses the 1989-2008 sample to report, for each of the twenty-nine countries in the sample, the average of its DCC correlations with all other countries using light grey lines. Figure 3.A contains the 16 developed markets and Figure 3.B contains the 13 emerging markets. While these paths are averages, they give a good indication of the differences between individual countries, and they are also informative of the differences between developed and emerging markets. In order to further study these differences, each figure also gives the average of the market's DCC correlations with all (other) developed markets using black lines and all (other) emerging markets using dark grey lines. Figure 3.A and 3.B yields some very interesting conclusions. First, the DCC correlation paths display an upward trend for all 29 countries, except Jordan. Second, for developed markets the average correlation with other developed markets is higher than the average correlation with emerging markets at virtually each point in time for virtually all markets. Third, for emerging markets the correlation with developed markets is generally higher than the correlation with other emerging markets. However, the difference between the two correlation paths is much smaller than in the case of developed markets. In several cases the average correlation paths are very similar.

---

[12]In Figure 3, and throughout the paper, we report equal-weighted averages of the pairwise correlations from the DCC models. Value-weighted correlations (not reported here) also display an increasing pattern during the last 10-15 years. Note that in the benchmark DECO model all pairwise correlations are identical and so the weighting is irrelevant.

Note that in Figure 3.A the trend patterns for European countries are also not very different from those for other DMs. Notice that, even if their level is still somewhat lower, the correlations for Japan and the US have increased just as for the European countries during the last decade. Inspection of the pairwise DCC paths, which are not reported because of space constraints, reveals that the trend patterns are remarkably consistent for almost all pairs of countries, and there is no meaningful difference between European countries and other DMs.

Figure 3 reports the average correlation between the DCC of each market and that of other markets. It could be argued that instead the correlation between each market and the average return of the other markets ought to be considered. We have computed these correlations as well. While the correlation with the average return is nearly always higher than the average correlation from Figure 3, the conclusion that the correlations are trending upwards is not affected. In order to save space we do not show the plots of the correlation with average returns on other markets.

We can use the correlation paths from the DCC model to assess regional patterns in correlation dynamics. Figure 4 does exactly this. We divide the 16 DMs into two regions (EU and non-EU) and we divide the 13 EMs into another two EM regions: Latin American and Emerging Eurasia.[13] We report in Figure 4 the average correlation within and across the four regions, based on the DCC model's country-specific correlation paths. Strikingly, Figure 4 shows that the increasing correlation patterns are evident within each of the four regions and also across all the six possible pairs of regions. The highest levels of correlation are found in the upper-left panel which shows the intra-EU correlations. The lowest level of correlations are found in the bottom-right panel which shows the intra Emerging Eurasia correlations. Emerging Eurasia in the right-most column generally has the lowest interregional correlations.

Figures 3 and 4 do not tell the entire story, because we have to resort to reporting correlation averages due to space constraints. Figure 5 provides additional perspective by providing correlation dispersions for the developed markets, emerging markets, and all markets respectively. In particular, at each point in time, the shaded areas in Figure 5 shows the range between the 10th and 90th percentile based on all pairwise correlations between groups of countries. The top panel considers the sixteen developed countries. The middle panel in Figure 5 reports the same statistics for the emerging markets for the 1989-2008 sample and the bottom panel shows all 29 markets together. While the increasing level of correlations is evident, the range of correlations seems to have narrowed for developed markets, widened a bit for emerging markets, and the range width seems to have stayed roughly constant for all markets taken together. The wide range of correlations found within

_____

[13]The European Union (EU) includes Austria, Belgium, Denmark, France, Germany, Ireland, Italy, Netherlands, and the UK. Developed Non-EU includes Australia, Canada, Hong Kong, Japan, Singapore, Switzerland, and the US. Latin America includes Argentina, Brazil, Chile, Colombia, and Mexico. Emerging Eurasia includes India, Jordan, Korea, Malaysia, Philippines, Taiwan, Thailand, and Turkey.

emerging markets again suggests that the potential for diversification benefits are greater here.

Figure 6 plots the conditional diversification benefit measures developed in equations (2.10) and (2.11) for developed, emerging, and all markets using the dynamic correlations from the DCC model. The CDB-optimal portfolio is depicted with a black line in Figure 6 and it shows a clearly decreasing trend in diversification benefits in DMs (top panel): Correlations have been rising rapidly and the benefits of diversification have been decreasing during the last ten years. Figure 6 shows that it is not possible to avoid the declining benefits from international diversification via active asset allocation. Diversification benefits have also somewhat decreased in emerging markets (middle panel) but the level of benefit is still much higher than in developed markets. When combining the developed and emerging markets (bottom panel), the diversification benefits are declining as well but the level is again much higher than when considering developed markets alone. Emerging markets thus still offer substantial correlation-based diversification benefits to investors.

The grey lines in Figure 6 show the benefits from diversification in an equal-weighted portfolio. In the case of DMs in the top panel it is striking how close the equal-weighted portfolio is to the CDB-optimal portfolio in terms of diversification benefits. In the case of EMs in the middle panel the differences between the two lines are a bit larger and in the bottom panel of Figure 6 the differences are the largest. This shows that when EMs are included in a DM portfolio, not only are the benefits of diversification much larger, the scope for active asset allocation is much greater as well.

4   Dynamic Nonlinear Dependence

We have relied on the multivariate normal distribution to implement the dynamic correlation models. The multivariate normal distribution is the standard choice in the literature because it is convenient, and because quasi maximum likelihood results ensure that the dynamic correlation parameters will be estimated consistently even when the normal distribution assumption is incorrect, as long as the dynamic models are correctly specified.

While the multivariate distribution is a convenient statistical choice, the economic motivation for using it is more dubious. It is well-known (see for example Longin and Solnik, 2001, and Ang and Bekaert, 2002) that international equity returns display threshold correlations not captured by the normal distribution: Large down moves in international equity markets are highly correlated, which is of course crucial for assessing the benefits of diversification. The dynamic correlation models considered above can generate more realistic threshold correlations, but likely not to the degree required by the data. Moreover, they are symmetric by design, and cannot accommodate Longin and Solnik's (2001) finding that returns are more correlated in down markets. In this section, we

therefore go beyond the dynamic multivariate normal distributions implied by the DCC and DECO models discussed above and introduce dynamic copula models which have the potential to generate empirically relevant levels of threshold correlations as well as asymmetric threshold correlations. We will continue to allow for the asymmetry arising from the leverage effect in variance as well as for an asymmetric marginal distribution in each country.

Copulas constitute an extremely convenient tool for building a multivariate distribution for a set of assets from any choice of marginal distributions for each individual asset.[14] From Patton (2006), who relies on Sklar (1959), we can decompose the conditional multivariate density function into a conditional copula density function and the product of the conditional marginal distributions

$$f_t\left(z_t\right) = c_t\left(F_{1,t}\left(z_{1,t}\right), F_{2,t}\left(z_{2,t}\right), ..., F_{N,t}\left(z_{N,t}\right)\right) \prod_{i=1}^{N} f_{i,t}\left(z_{i,t}\right).$$

From this the multivariate log-likelihood function can be constructed as

$$L = \sum_{t=1}^{T} \sum_{i=1}^{N} \log\left(f_{i,t}\left(z_{i,t}\right)\right) + \sum_{t=1}^{T} \log\left(c_t\left(F_{1,t}\left(z_{1,t}\right), F_{2,t}\left(z_{2,t}\right), ..., F_{N,t}\left(z_{N,t}\right)\right)\right)$$

The upshot of this decomposition is that we can make assumptions about the marginal densities that are independent of the assumptions made about the copula function. Below we will assume that the marginal densities differ across assets but are constant over time, $f_{i,t}\left(z_{i,t}\right) = f_i\left(z_{i,t}\right)$ and so of course $F_{i,t}\left(z_{i,t}\right) = F_i\left(z_{i,t}\right)$, and we will allow for the copula function to potentially be dynamic. We will also again rely on the composite likelihood approach when estimating the models.

It is of course crucial to first specify appropriate and potentially non-normal marginal distributions in order to ensure that the copula-based multivariate distribution will be well specified. This is the topic to which we now turn.

4.1   Building the Marginal Distributions

In order to allow for flexible marginal distributions (see Ghysels, Plazzi and Valkanov, 2011) we use a kernel approach to nonparametrically estimate the empirical cumulative distribution function (EDF) of each standardized return time series, $z_{i,t}$. Recall from (2.1) that

$$z_{i,t} = \frac{R_{i,t} - \mu_{i,t}}{\sigma_{i,t}}$$

where $\mu_{i,t}$ is obtained from an AR model.

---

[14]McNeil, Frey and Embrechts (2005) provide an authoritative review of the use of copulas in risk management.

Nonparametric kernel EDF estimates are well suited for the interior of the distribution where most of the data is found, but tend to perform poorly when applied to the tails of the distribution. Fortunately, a key result in extreme value theory shows that the Generalized Pareto distribution (GP) fits the tails of a wide variety of distributions. Thus we fit the tails of the marginal distributions using the GP.

The marginal densities are constructed by combining the kernel EDF for the central 80% of the distribution mass with the GP distribution for the two tails. We write the cumulative density function as

$$\eta_i = F_i(z_i) \tag{4.1}$$

We refer to McNeil (1999) and McNeil and Frey (2000) for more details on our approach.

4.2 Modeling Multivariate Nonnormality

The most widely applied copula function is built from the multivariate normal distribution and referred to as the Gaussian copula. Though convenient to use, it is not flexible enough to capture the tail dependence in asset returns. We therefore investigate the t copula which is constructed from the multivariate standardized student's t distribution. The t copula cumulative density function is defined as

$$C(\eta_1, \eta_2, ..., \eta_N; \Psi, \nu) = t_{\Psi,\nu}(t_\nu^{-1}(\eta_1), t_\nu^{-1}(\eta_2), ..., t_\nu^{-1}(\eta_N)) \tag{4.2}$$

where $t_{\Psi,\nu}(\cdot)$ is the multivariate standardized student's t density with correlation matrix Ψ and ν degrees of freedom. $t_\nu^{-1}(\eta_i)$ is the inverse cumulative density function of the univariate Student's t distribution, and the marginal probabilities $\eta_i = F_i(z_i)$ are from (4.1). More details on the t- copula are provided in Appendix B.

Note that the matrix Ψ captures the correlation of the fractiles $z_i^* \equiv t_\nu^{-1}(\eta_i)$ and not of the return shock z_i. We refer to Ψ as the copula correlation matrix in order to distinguish it from the conventional matrix of linear correlations studied above. Notice also that

$$z_i^* \equiv t_\nu^{-1}(\eta_i) = t_\nu^{-1}(F_i(z_i))$$

so that if the marginal distributions F_i are close to the t_v distribution, then z_i^* will be close to z_i and the copula correlations will be close to the conventional linear correlations.

18

4.3 Allowing for Dynamic Copula Correlations

We now combine copula functions with the dynamic correlation models considered above. We again rely on the parsimonious DCC and DECO approaches. Using the fractiles $z_i^* \equiv t_\nu^{-1}(\eta_i)$ instead of the return shock z_t in the DCC model yields dynamics for the conditional copula correlations, as follows

$$\tilde{\Psi}_t = \Omega_\Psi + \beta_\Psi \tilde{\Psi}_{t-1} + \alpha_\Psi \tilde{z}_{t-1}^* \tilde{z}_{t-1}^{*\top} \tag{4.3}$$

where $\tilde{z}_{i,t}^* = z_{i,t}^* \sqrt{\frac{v-2}{2} \tilde{\Psi}_{ii,t}}$ using the Aielli (2009) modification. These cross-products are used to define the conditional copula correlations via the normalization

$$\Psi_{ij,t}^{DCC} = \tilde{\Psi}_{ij,t} / \sqrt{\tilde{\Psi}_{ii,t} \tilde{\Psi}_{jj,t}}. \tag{4.4}$$

In the empirics below we will refer to the model combining the copula density in (4.2) and the copula correlation dynamics in (4.3) as the t DCC copula model. We also estimate the t DECO copula in which the dynamic copula correlations are identical across all pairs of assets. The parameters in these dynamic t copula models are easily estimated using the composite likelihood approach discussed above.

4.4 Allowing for Multivariate Asymmetries

The presence of asymmetry in the threshold correlations of international equity returns has long been established, see for example Longin and Solnik (2001) and Ang and Bekaert (2002). Unfortunately, the standard t copula model considered so far implies symmetric threshold correlations. To address this problem, we consider the skewed t distribution discussed in Demarta and McNeil (2005), which we use to develop an asymmetric t copula. In parallel with the symmetric t copula we can write the skewed t copula cumulative density function

$$C(\eta_1, \eta_2, ..., \eta_N; \Psi, \lambda, v) = t_{\Psi,\lambda,\nu}(t_{\lambda,\nu}^{-1}(\eta_1), t_{\lambda,\nu}^{-1}(\eta_2), ..., t_{\lambda,\nu}^{-1}(\eta_N))$$

where λ is an asymmetry parameter, $t_{\Psi,\lambda,\nu}(\cdot)$ is the multivariate asymmetric standardized student's t density with correlation matrix Ψ, and $t_{\lambda,\nu}^{-1}(\eta_i)$ is the inverse cumulative density function of the univariate asymmetric Student's t distribution. The univariate probabilities $\eta_i = F_i(z_i)$ are from (4.1) as before. The skewed t copula is built from the asymmetric multivariate t distribution and the symmetric t copula is nested when λ tends to zero. Appendix C provides the details needed to implement the skewed t copula. Note that the semiparametric approach to the marginal distributions captures any univariate skewness present in the equity returns. The λ parameter

captures multivariate asymmetry.

For the sake of parsimony in our high-dimensional applications, we report on a version of the skewed t copula where the asymmetry parameter λ is a scalar. It is straightforward to develop a more general version of the skewed t copula allowing for an N-dimensional vector of asymmetry parameters. But such a model is not easily estimated on a large number of countries.

Asymmetry in the bivariate distribution of asset returns has generally been modeled using copulas from the Archimedean family which include the Clayton, the Gumbel, and the Joe-Clayton specifications.[15] These models are rarely used in high-dimensional applications. The skewed t copula is parsimonious, tractable in high dimension, and flexible, allowing us to model non-linear and asymmetric dependence with the degree of freedom parameter, v, and the asymmetry parameter, λ, while retaining a dynamic conditional copula correlation matrix, Ψ. Figure A.1 plots probability contours for the bivariate case for two parameterizations of the skewed t copula, as well as the special cases of the t copula and the normal copula. The probability levels for each contour are kept the same for all four figures. The ability of the skewed t copula to generate substantial asymmetries with realistic parameter values is evident.

4.5 Allowing for Dynamic Degrees of Freedom

So far we have assumed that the degree of freedom parameter, v is constant over time. Allowing for dynamics in v and thus in the degree of nonnormality can be done in several ways. Inspired by Engle and Rangel (2008, 2010), we assume that the degree of freedom evolves as a quadratic trend

$$\nu_t = c^\nu \exp\left(w_0^\nu t + w_1^\nu (t - t_0)^2\right),$$

where we impose a lower bound on the dynamic so that the degree of freedom ν_t is above the number required for finite second moments, which is two in the symmetric case and four in the asymmetric case.[16]

5 Empirical Nonlinear Dependence Analysis

The empirical results in Section 3 demonstrate that it is feasible to characterize dynamic correlations between a large number of markets. While these results are of great interest, it is worthwhile keeping in mind that correlation is an inadequate dependence measure for analyzing financial markets,

---

[15]See for example Patton (2004, 2006).

[16]Engle and Rangel (2008, 2010) model multiple quadratic splines, thus allowing for structural breaks in the quadratic part of the trend. Our results are qualitatively similar when allowing for multiple splines functions.

because it relies on normality, and the deviations of normality for (international) stock returns are well documented. The methods developed in Section 4 show that it is feasible to analyze dependence more generally in international stock returns using a fully-specified conditional distribution model for a large number of markets.

When characterizing multivariate dependence using the DCC and DECO models, the normality assumption enters in two critical ways: First, the marginal distribution of returns for each country is assumed to be normal; Second, the joint distribution is also assumed to be normal. The t copula introduced in Section 4.2 and the skewed t copula introduced in Section 4.4 allow us to address the appropriateness of these assumptions.[17]

5.1   Model Estimates

Table 4 reports the parameter estimates and composite likelihood values of the different t copula models we consider. The top row shows the DCC copulas, the second row the DECO copulas, and the third row the DECO copulas with dynamic degree of freedom. The left column shows the symmetric t copulas and the right column shows the skewed t copulas. Note that the copula correlation persistence is–as was the case in Table 3–very close to one in all models.

Comparing the symmetric to the asymmetric version of the t copula, we observe that the introduction of the asymmetry parameter does not seem to impact the correlation parameters much, nor the estimated degrees of freedom. This suggests that the asymmetry parameter captures a different dimension of dependence.

5.2   Tail Dependence

The various t copula models developed above generalize the normal copula by allowing for non-zero dependence in the tails. One way to measure the lower tail dependence is via the probability limit

$$\tau_{i,j,t}^{L} = \lim_{\zeta \to 0} \Pr[\eta_{i,t} \leq \zeta | \eta_{j,t} \leq \zeta] = \lim_{\zeta \to 0} \frac{C_t(\zeta, \zeta)}{\zeta} \tag{5.1}$$

where ζ is the tail probability. The upper tail dependence at time t can similarly be defined by

$$\tau_{i,j,t}^{U} = \lim_{\zeta \to 1} \Pr[\eta_{i,t} \geq \zeta | \eta_{j,t} \geq \zeta] = \lim_{\zeta \to 1} \frac{1 - 2\zeta + C_t(\zeta, \zeta)}{1 - \zeta} \tag{5.2}$$

---

[17]We also estimated a Gaussian copula with dynamic correlation. We omit these results to save space, but they are available on request. Comparison with the DCC results and the t copula results indicates that marginal distributions are fairly close to normal, but normality is a poor assumption for the joint distribution.

The normal copula has the empirically questionable property that this tail dependence is zero, whereas it is positive in the various t copula models we develop.[18]

In the conventional symmetric t copula the lower and upper tail dependences are identical, that is $\tau_{i,j,t}^L = \tau_{i,j,t}^U$. Based on the work by Longin and Solnik (2001) and Ang and Bekaert (2002), we suspect that this symmetry is not valid in international equity index returns and we therefore investigate the upper and lower tail dependence separately using the skewed t copula model developed above.

Figure 7 plots the dynamic measure of tail-dependence in equations (5.1) and (5.2) for the skewed t copula for the DCC (left panels) and the DECO (right panels) models. We report the average of the bivariate tail dependence across all pairs of countries.[19] In each graph, the dark line depicts the evolution of the upper tail dependence, while the gray line is for the lower tail dependence.[20] The tail dependence measure depends on the degree of freedom, v, the copula correlation, $\Psi_{i,j}$, and the asymmetry parameter, λ. Figure 7 shows quite dramatic differences across markets. The tail dependence in developed markets has risen markedly during the last twenty years. Remarkably, while the emerging market tail dependence measures in the middle panel of Figure 7 have also increased, they remain very low compared to developed markets. When considering all markets in the bottom panel of Figure 7, we find that while the tail dependence is rising, it is still much lower than for the developed markets alone. From this perspective, the diversification benefits from adding emerging markets to a portfolio appear to be large compared to those offered by developed markets alone, even if these benefits have become smaller over time. In all cases, lower tail dependence is higher than upper tail dependence suggesting an important asymmetry in the multivariate distribution of international equity returns.

5.3   Threshold Correlations

Figure 7 suggests that the skewed t DCC copula model may be able to capture Longin and Solnik's (2001) finding that downside threshold correlations are much larger than their upside counterparts. Figure 8 explores this in more detail. We follow the empirical setup in Ang and Bekaert (2002), and compare the pattern in empirical threshold correlations with threshold correlations from data generated using the estimated model parameters. Specifically, for each pair of countries we compute

---

[18]See Patton (2006) for an application of the extreme dependence measure to exchange rates.

[19]The tail dependence concept introduced above is inherently bivariate and not easily generalized to the high-dimensional case. In higher dimensions, tail dependence is defined as the probability limit of all variables being below a threshold conditional on a subset of them being below the same threshold. However, in a portfolio context, it is not obvious how that conditioning subset should be defined. In order to convey the empirical evolution of tail dependence for many countries, we report the average of the bivariate tail dependence across all pairs of countries.

[20]To the best of our knowledge a closed form solution is not available for the tail dependence measure in the skewed t copula. We therefore approximate by simulation using $\zeta = 0.001$.

threshold correlations from the return shocks as follows

$$
\widetilde{\rho}_\xi(z_{i,t}, z_{j,t}) = \left\{ \begin{array}{ll} Corr\left(z_{i,t}, z_{j,t} | z_{i,t} \le \xi, z_{j,t} \le \xi\right) & \text{if } \xi \le 0, \\ Corr\left(z_{i,t}, z_{j,t} | z_{i,t} > \xi, z_{j,t} > \xi\right) & \text{if } \xi > 0. \end{array} \right.
$$

where $z_{i,t}$ is the shock for country i corresponding to the return standardized by an AR-NGARCH model as before. The correlations are computed for a grid of thresholds ξ (denoted in standard deviations) so long as at least 20 observations are available. We plot in Figure 8 the pairwise threshold correlations averaged across countries.

When considering monthly returns in the US versus other DMs Longin and Solnik (2001) found that the downside threshold correlations were much larger than their upside counterparts. The solid black line in the top panel of Figure 8 confirms and extends the findings of Longin and Solnik (2001): When computing the average of all possible pairwise threshold correlations for weekly returns in sixteen DMs we find that the downside threshold correlations indeed are much larger.

The bottom panel of Figure 8 shows the average threshold correlations for EMs. In this case we have a shorter sample period available and fewer countries and so we estimate the threshold correlations less precisely. When comparing the top and bottom panels of Figure 8 we see that the downside threshold correlations are higher for developed markets than for emerging markets which confirms our earlier findings.

As is well-known, asymmetric threshold correlations cannot be captured using a multivariate normal distribution: threshold correlations in the normal distribution are symmetric, and also decrease rather quickly in the tails. The dashed lines in Figure 8 show this. Figure 8 also indicates that the symmetric t DCC copula is successful in producing higher threshold correlations, but by design these correlations are of course also symmetric. The skewed t DCC copula clearly achieves its aim: it produces an asymmetric pattern in threshold correlations, with substantially higher downside threshold correlations. Indeed, the skewed t DCC copula fits the EM threshold correlations remarkably well. While the model is also very successful in capturing the level of the downside correlations for DMs, it generates upside correlations that are too high in this case. We hasten to add that the DCC copula models have been estimated to maximize the composite likelihood and not to minimize the distance between model based threshold correlations and their empirical counterpart. It is therefore not surprising that the models are not capturing the threshold correlation patterns perfectly.

When considering the results in Figure 8, it may seem surprising that we did not find evidence for the asymmetric DCC model mentioned in Section 2.1. Note, however, that the threshold correlations in Figure 8 capture asymmetric nonlinear dependence whereas the asymmetric DCC model captures the response of linear dependence to joint positive versus joint negative shocks. When we simulate

23

the asymmetric DCC model, we find that it produces a threshold correlation pattern which is very similar to that in the multivariate Gaussian distribution shown in Figure 8. We conclude that the asymmetric DCC model mentioned in Section 2.1 cannot capture the asymmetric threshold patterns found in the data, and that our asymmetric copula approach is preferable from this perspective.

6   Extensions and Robustness

We now further explore some of the implications of our empirical estimates.

First, most correlation estimates in the existing literature are obtained using factor models, and therefore a positive association between volatilities and correlations is built into the models. We do not use a factor model, and therefore our estimates are useful to further investigate the relationship between volatilities and correlations.

Second, our key finding above is that the benefits from diversification across developed markets have largely disappeared, but that the benefits from diversifying across emerging markets are still intact. To anticipate how these diversification benefits might change in the future, it is of interest to investigate the effects of the easing of cross-border capital flow controls and increasing levels of market integration on correlations.[21]

Third, we employ mean-reverting models in the analysis above. These models should bias the results against finding long-term trends in correlation. Nevertheless the sample-paths we extract from the models display increasing long-term trends in correlation. It is natural to wonder if these patterns are confirmed in a model that explicitly allows for non-stationarity in the correlation dynamics. We develop such a model below and find that it confirms the increase in correlations over the sample. Fourth, we investigate if a fully model-free (but ad-hoc) approach confirms the long-term upward trend in the correlation paths. It does.

6.1   Correlation and Volatility

Factor models typically imply a positive relationship between correlation and volatility. Because we obtain our results without the aid of a factor model, it is worth investigating if this positive relationship is confirmed by our results.

We consider three sets of regressions for EM countries and three for DM countries. For EM countries, we consider the average correlation with other EMs, the average correlation with DMs, and the average correlation with all other markets as regressands, and we do the same for DMs. For each of these six cases, we run two regressions. Consider for instance the case of EM countries, and

---

[21]See Bekaert et al (2011), and Carrieri et al (2010) for the evolution of market integration for EMs.

consider the average correlation with other EMs. We then estimate panel regressions of the form

$$\ln(\rho_{i,t}^{EM,EM}) \;=\; b_{i,0} + b_1 \tau + b_2 \ln\left(\sigma_{EM,t}\right) + \epsilon_{i,t}, \qquad i = 1, ..., N_{EM} \qquad (6.1)$$

$$\ln(\rho_{i,t}^{EM,EM}) \;=\; b_{i,0} + b_1 \tau + b_2 \ln\left(\sigma_{i,t}\right) + \epsilon_{i,t}, \qquad i = 1, ..., N_{EM} \qquad (6.2)$$

where ln( ) is the natural logarithm and $\rho_{i,t}^{EM,EM}$ is the average correlation between EM country i and all other EM countries in month t. On the right-hand-side $b_{i,0}$ is a country specific fixed effect, τ is a time trend, $\sigma_{EM,t}$ denotes the average volatility across EMs in month t, and $\sigma_{i,t}$ is EM country i's own volatility. We use weekly data, and use the longest time period available in each case. Following Petersen (2009), we compute White standard errors adjusted for within cluster (country) correlation. The other regressions for EMs and regressions for DM countries are similar.

Table 5 shows that the correlation time-trend is significantly positive at the 1% level for all specifications, confirming the visual impression of upward trending correlations in Figures 1-4. The average volatilities are always significantly positive, which is often found in the risk management literature: correlations tend to rise when volatility rises, which clearly lowers the benefits of diversification. The countries' own volatility is also always estimated with a positive coefficient, but the estimates are not always statistically significant.

In summary, we find that the EM correlations have clearly trended upwards in this period. We find robust evidence of a positive relationship between volatility and correlation. Because volatilities and correlations are highly autocorrelated, we also estimated these regressions in differences, and obtained similar results.

6.2   Correlation and Market Development

We now investigate the relationship between financial development and correlation in emerging markets.

In order to measure financial development, we first rely on Bekaert (1995) and Edison and Warnock (2003), who use a direct measure of *de jure* market openness. Their measure is defined as the ratio of the market capitalizations of the investable and global indexes from S&P/IFC, and we denote it by "MCR" below. The IFC Global (IFCG) index is designed to represent the market portfolio for each country, whereas the IFC Investable (IFCI) index is designed to represent a portfolio of domestic equities that are available to foreign investors. When the MCR measure is one, the market capitalization of the investable index is equal to that of the market-wide index indicating that all of that countries' stocks are available to foreign investors. We also consider a measure of emerging market integration based on Errunza and Losq (1985). This measure was most

recently used in Carrieri, Chaieb, and Errunza (2010) and we refer to it as "EMI" below. The correlation between the MCR and EMI measures is -0.10 on average, and therefore they clearly measure different aspects of emerging market development.

Due to MCR and EMI data availability, our sample is restricted to the period August 1995 to December 2006 for the 17 emerging markets in the IFCI index. As MCR and EMI are available on a monthly basis, we average the weekly GARCH volatilities and the DCC correlations each month. Table 6 reports regression results. As in Table 5, we use as regressands three sets of correlations for each EM country: The average correlation with other EMs, the average correlation with DMs, and the average correlation with all other markets. Using the case of correlations with other emerging markets as an example, we estimate the following panel regressions, using the same notation and panel setup as in Section 6.1

$$
\begin{align}
\rho_{i,t}^{EM,EM} &= b_{i,0} + b_1\tau + b_2 MCR_{i,t} + \epsilon_{i,t}, \qquad i = 1,...,N_{EM} \tag{6.3} \\
\rho_{i,t}^{EM,EM} &= b_{i,0} + b_1\tau + b_2 EMI_{i,t} + \epsilon_{i,t}, \qquad i = 1,...,N_{EM} \tag{6.4} \\
\rho_{i,t}^{EM,EM} &= b_{i,0} + b_1\tau + b_2 MCR_{i,t} + b_3 EMI_{i,t} + b_4 \log\left(\sigma_{EM,t}\right) + \epsilon_{i,t}, \qquad i = 1,...,N_{EM} \tag{6.5} \\
\rho_{i,t}^{EM,EM} &= b_{i,0} + b_1\tau + b_2 MCR_{i,t} + b_3 EMI_{i,t} + b_4 \log\left(\sigma_{i,t}\right) + \epsilon_{i,t}, \qquad i = 1,...,N_{EM} \tag{6.6}
\end{align}
$$

Overall, Table 6 indicates that the impact of the market cap ratio MCR is positive and significant at the 5% level, whereas the impact of the market integration indicator EMI is positive but not significantly estimated. This insignificance is not surprising as the theory underlying the EMI measure does not predict a relationship between correlation and market integration. The results also confirm the positive relationship between volatilities and correlations, but just as in Table 5, it is only when using cross-sectionally averaged volatilities as regressors that we obtain statistically significant results.

6.3 Nonstationary Correlation Dynamics

The correlation regressions in Tables 5 and 6 show a very clear pattern: The simple linear time-trend in correlation is positive and strongly significant in all cases. This finding suggests that the mean-reverting DCC and DECO models considered so far may be inadequate at fully describing the evolution of international equity index correlations over time. The mean-reverting models will try to pull the correlation path back down towards the unconditional mean even if the observed returns keep pushing the correlation paths higher. Even if the correlations were not trending up one could reasonably argue that a constant long-run correlation is unrealistic for the relatively long time-series that we are analyzing here.

In this section we therefore propose a new way to model a slowly varying long-run component in correlation. Engle and Rangel (2008) model low-frequency dynamics in volatility using an extended GARCH model that features a dynamic long-run component given by a quadratic exponential spline. Engle and Rangel (2010) develop a Factor Spline GARCH for covariance by using a quadratic spline for the market stationary variance and each asset's idiosyncratic long run risk. We try to avoid imposing a factor structure and instead use the Spline GARCH idea in a DECO correlation framework. We assume that the long-run component of correlation evolves as a quadratic trend

$$\rho_t^{LR} = \Lambda\left(c + w_0 t + \sum_{i=1}^{k} w_i \max(t - t_{i-1}, 0)^2\right) \tag{6.7}$$

where the double logistic function $\Lambda(x) = \frac{1-e^{-x}}{1+e^{-x}}$ is used to restrict ρ_t^{LR} to be between -1 and 1. The number of quadratic splines is denoted by k and the spline connection dates, t_i, are assumed to be equally spaced in time.

Our DECO correlation dynamics are built from the DCC model as before, but the constant matrix of unconditional correlations, $\tilde{\Gamma}$, is replaced by

$$\tilde{\Gamma}_t^{LR} = (1 - \rho_t^{LR})I_N + \rho_t^{LR} J_{N \times N}$$

so that we now have the dynamic

$$\tilde{\Gamma}_t = (1 - \alpha_\Gamma - \beta_\Gamma)\tilde{\Gamma}_t^{LR} + \beta_\Gamma \tilde{\Gamma}_{t-1} + \alpha_\Gamma \tilde{z}_{t-1}\tilde{z}_{t-1}^\top. \tag{6.8}$$

We also need the normalization in (2.5) and the DECO restriction in (2.9).

Following Engle and Rangel (2008), we use the BIC model selection criteria to chose the optimal number of splines, k. The BIC typically indicated a low number and so for transparency we simply set $k = 1$ everywhere, implying a quadratic trend with no breaks. The estimation results are reported in Table 7. Comparing log likelihoods with the left side of Panel B in Table 3, we see that the improvements in likelihood compared with the standard DECO model are quite modest. The simple DECO model with very high persistence seems to be able to adequately capture the correlation pattern over time. Comparing the Spline DECO likelihoods with the special case of no stochastics ($\alpha_\Gamma = \beta_\Gamma = 0$) shows that the spline function alone is capable of capturing the correlation dynamics quite well.

The right panels of Figure 9 shows the evolution of total correlation as well as the dynamic long-run correlation in the new Spline DECO model. For comparison, the basic DECO correlations from Figure 1 are repeated in the left panels of Figure 9. The dramatic upward trend in correlation

is clear in both models. It is quite striking that the flexible exponential-quadratic Spline DECO model we develop implies an almost linearly rising trend in correlation through the recent decade.

6.4 Model-Free Correlations

The Spline DECO model developed above is of course just one approach to capturing potential non-stationarity in the correlation dynamic. However, any parametric approach requires modeling decisions that may effect results. We therefore end our analysis with a completely model-free (but not assumption-free) alternative to correlation estimation.

Figure 10 plots the average (across all pairs of countries) model-free rolling correlations using a relatively short 6-month estimation window (denoted by grey lines) and using a relatively long 2-year estimation window (denoted by black lines). Both estimates use weekly returns to compute the rolling correlations.

Figure 10 shows that it is not the DECO model structure nor the Spline DECO model structure that are driving the upward-sloping trend result. The model-free estimates of dynamic correlation in Figure 10 show the same upward trend in correlation evident in Figures 1 and 9. The disadvantage of the model-free rolling estimates of dynamic correlation is that they depend greatly on the width of the data window chosen: A long window will result in stable but potentially biased estimates of the true dynamic correlation, whereas a very short window will result in very noisy estimates. The dynamic models we apply have the important advantage of letting the data choose–via maximum likelihood estimation–the optimal weights on past data points.

7   Summary and Conclusion

We characterize time-varying correlations using long samples of weekly returns for a large number of countries. We implement models that overcome econometric complications arising from the dimensionality problem, and that are easier to estimate, using variance targeting and the composite likelihood procedure.

Results based on the DCC and the DECO as well as on our new Spline DECO model are extremely robust and suggest that correlations have been significantly trending upward for both DMs and EMs. Correlations between DMs have exceeded correlations between EMs throughout the 1989-2009 period. Moreover, for developed markets, the average correlation with other developed markets is higher than the average correlation with emerging markets. For emerging markets, the correlation with developed markets is generally somewhat higher than the correlation with the other emerging markets, but the differences are small. While the range of correlation for DMs

has narrowed around the increasing trend in correlation levels, this is not the case for EMs, where the range instead appears to have widened. Overall the correlation analysis suggests that the diversification benefits have largely disappeared for DMs but that EMs still offer some correlation-based diversification benefits.

We develop a novel skewed dynamic t copula which allows for asymmetric and dynamic tail dependence in large portfolios. Results from the dynamic t copula indicate substantial and asymmetric tail dependence with lower tail dependence being larger than upper tail dependence. Moreover, tail dependence as measured by the t copula is large and increasing through time for DMs, but remains low for EMs. Our findings on tail dependence indicate significant diversification opportunities in EMs, due to the fact that while equity market crises in EMs are frequent, many of them are country-specific. From this perspective, our results suggest that although diversification benefits have lessened in the case of DMs, the case for EMs remains strong.

These results have very important implications for portfolio management, and it may prove interesting to explore them in future work. It may also prove useful to investigate the robustness of our findings to allowing for multiple regimes, or to the inclusion of multiple stochastic components, as for example in the model of Colacito, Engle, and Ghysels (2009). Our new Spline DECO model represents an initial step in this direction.

References

[1] Aielli, G.P., 2009, Dynamic Conditional Correlations: On Properties and Estimation, Working paper: Department of Statistics, University of Florence.

[2] Ang, A., and G. Bekaert, 2002, International Asset Allocation with Regime Shifts, Review of Financial Studies 15, 1137-1187.

[3] Ang, A. and J. Chen, 2002, Asymmetric Correlations of Equity Portfolios, Journal of Financial Economics 63, 443-494.

[4] Bae, K.-H., Karolyi, G.A., and R. Stulz, 2003, A New Approach to Measuring Financial Contagion, Review of Financial Studies 16, 717-763.

[5] Baele, L., 2005, Volatility Spillover Effects in European Equity Markets, Journal of Financial and Quantitative Analysis 40, 373–402.

[6] Baele, L., and K. Inghelbrecht, 2009, Time-Varying Integration and International Diversification Strategies, Journal of Empirical Finance 16, 368-387.

[7] Bekaert, G., 1995, Market integration and investment barriers in emerging equity markets, The World Bank Economic Review 1, 75-107.

[8] Bekaert, G., and C. Harvey, 2000, Foreign Speculators and Emerging Equity Markets, Journal of Finance 55, 565-613.

[9] Bekaert, G., Harvey, C., Lundblad, C. and S. Siegel, 2011, What Segments Markets? Working Paper, Columbia University.

[10] Bekaert, G., Harvey, C., and A. Ng, 2005, Market Integration and Contagion, Journal of Business 78, 39-69.

[11] Bekaert, G., Hodrick, R., and X. Zhang, 2009, International Stock Return Comovements, Journal of Finance 64, 2591-2626.

[12] Black, F., 1976, Studies of Stock Price Volatility Changes, In: Proceedings of the 1976 Meetings of the Business and Economic Statistics Section, American Statistical Association, 177-181.

[13] Brooks, R., and M. Del Negro, 2003, The Rise in Comovement Across National Stock Markets: Market Integration or IT Bubble? Journal of Empirical Finance 11, 659-680.

[14] Cappiello, L., Engle, R., and K. Sheppard, 2006, Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns, Journal of Financial Econometrics 4, 537-572.

[15] Carrieri, F., Chaieb, I., and V. Errunza, 2010, Do Implicit Barriers Matter for Globalization?, Working Paper, McGill University.

[16] Carrieri, F., Errunza, V., and K. Hogan, 2007, Characterizing World Market Integration Through Time, Journal of Financial and Quantitative Analysis 42, 915-940.

[17] Carrieri F., Errunza, V., and S. Sarkissian, 2007, Industrial Structure, Market Integration, and International Portfolio Diversification, Working Paper, McGill University.

[18] Choueifaty, Y., and Y. Coignard, 2008, Toward Maximum Diversification, The Journal of Portfolio Management, 35, 1, 40-51.

[19] Colacito, R., Engle, R., and E. Ghysels, 2009, A Component Model for Dynamic Correlations, Working Paper, New York University.

[20] Demarta, S., and A.J. McNeil, 2004, The t Copula and Related Copulas, International Statistical Review 73, 1, 111-129.

[21] DeMiguel, V., L. Garlappi and R. Uppal, 2009, Optimal versus Naive Diversification: How Inefficient Is the 1/N Portfolio Strategy?, Review of Financial Studies 22, 1915–1953.

[22] DeSantis, G., and B. Gerard, 1997, International Asset Pricing and Portfolio Diversification with Time-Varying Risk, Journal of Finance 52, 1881-1912.

[23] De Santis, G., and B. Gerard, 1998, How Big is the Premium for Currency Risk, Journal of Financial Economics 49, 375–412.

[24] Dumas, B., Harvey, C., and P. Ruiz, 2003, Are Correlations of Stock Returns Justified by Subsequent Changes in National Outputs? Journal of International Money and Finance 22, 777-811.

[25] Edison, H.J. and F.E. Warnock, 2003. A simple measure of the intensity of capital controls, Journal of Empirical Finance, 10, 81-103.

[26] Eiling, E., and B. Gerard, 2007, Dispersion, Equity Return Correlations and Market Integration, Working Paper, University of Toronto.

[27] Engle, R., 2002, Dynamic Conditional Correlation: A Simple Class of Multivariate GARCH Models, Journal of Business and Economic Statistics 20, 339-350.

[28] Engle, R., and B. Kelly, 2009, Dynamic Equicorrelation, Working Paper, New York University.

[29] Engle R., and J. Mezrich, 1996, GARCH for Groups, Risk 9, 36–40.

[30] Engle, R., and V. Ng, 1993, Measuring and Testing the Impact of News on Volatility, Journal of Finance 48, 1749-1778.

[31] Engle, R., and J. Rangel, 2008, The Spline GARCH Model for Low Frequency Volatility and its Global Macroeconomic Causes, Review of Financial Studies 21, 1187-1222.

[32] Engle, R., and J. Rangel, 2010, The Factor-Spline-GARCH Model for High and Low Frequency Correlations, Working Paper, New York University.

[33] Engle, R., Shephard, N., and K. Sheppard, 2008, Fitting Vast Dimensional Time-Varying Covariance Models, Working Paper, New York University.

[34] Erb, C., Harvey, C., and T. Viskanta, 1994, Forecasting International Equity Correlations, Financial Analysts Journal 4, 32-45.

[35] Errunza, V., 1977, Gains from Portfolio Diversification into Less Developed Countries' Securities, Journal of International Business Studies 55, 83-99.

[36] Errunza, V., and E. Losq, 1985, International Asset Pricing Under Mild Segmentation: Theory and Test, Journal of Finance, 40, 105-124.

[37] Errunza, V., Hogan, K., and M-W. Hung, 1999, Can the Gains from International Diversification be Achieved without Trading Abroad? Journal of Finance 54, 2075-2107.

[38] Forbes, K., and R. Rigobon, 2002, No Contagion, Only Interdependence: Measuring Stock Market Co-Movements, Journal of Finance 57, 2223-2261.

[39] Ghysels, E., A. Plazzi, and R. Valkanov, 2011, Conditional Skewness of Stock Market Returns in Developed and Emerging Markets and its Economic Fundamentals, Working Paper, University of North Carolina.

[40] Goetzmann, W., Li, L., and K. G. Rouwenhorst, 2005, Long-Term Global Market Correlations, Journal of Business 78, 1-38.

[41] Griffin, J., and A. G. Karolyi, 1998, Another Look at the Role of the Industrial Structure of Markets for International Diversification Strategies, Journal of Financial Economics 50, 351-373.

[42] Heston, S., and G. K. Rouwenhorst, 1994, Does Industrial Structure Explain the Benefits of International Diversification? Journal of Financial Economics 36, 3-27.

[43] Karolyi, G.A., 1995, A Multivariate GARCH Model of International Transmissions of Stock Returns and Volatility: The Case of the United States and Canada, Journal of Business and Economic Statistics 13, 11-25.

[44] Karolyi, G.A., and R. Stulz, 1996, Why do Markets Move Together? An Investigation of U.S.-Japan Stock Return Comovements, Journal of Finance 51, 951-986.

[45] King, M., Sentana, E., and S. Wadhwani, 1994, Volatility and Links Between National Stock Markets, Econometrica 62, 901-933.

[46] Kroner, K., and V. Ng, 1998, Modeling Asymmetric Comovement of Asset Returns, Review of Financial Studies 11, 817-844.

[47] Lewis, K.K., 2006, Is the International Diversification Potential Diminishing? Foreign Equity Inside and Outside the US, Working Paper, the Wharton School at the University of Pennsylvania.

[48] Longin, F., and B. Solnik, 1995, Is the Correlation in International Equity Returns Constant: 1960-1990?, Journal of International Money and Finance 14, 3-26.

[49] Longin, F., and B. Solnik, 2001, Extreme Correlation of International Equity Markets, Journal of Finance 56, 649-676.

[50] McNeil, A., 1999, Extreme Value Theory for Risk Managers, Internal Modeling CAD II, Risk Books, 93-113.

[51] McNeil, A., and R. Frey, 2000, Estimation of Tail-Related Risk Measures for Heteroscedastic Financial Time Series: an Extreme Value Approach, Journal of Empirical Finance 7, 271-300.

[52] McNeil, A., Frey, R., and P. Embrechts, 2005, Quantitative Risk Management, Princeton: Princeton University Press.

[53] Patton, A., 2004, On the out-of-sample importance of skewness and asymmetric dependence for asset allocation, Journal of Financial Econometrics, 2, 130-168.

[54] Patton, A., 2006, Modelling Asymmetric Exchange Rate Dependence, International Economic Review 47, 527-556.

[55] Petersen, M.A., 2009, Estimating Standard Errors in Finance Panel Data Sets: Comparing Approaches, Review of Financial Studies, 22, 435-480.

[56] Poon, S.-H., Rockinger, M., and J. Tawn, 2004, Extreme Value Dependence in Financial Markets: Diagnostics, Models, and Financial Implications, Review of Financial Studies 17, 581-610.

[57] Ramchand, L., and R. Susmel, 1998, Volatility and Cross Correlation Across Major Stock Markets, Journal of Empirical Finance 5, 397-416.

[58] Rangel, J., 2011, FX Comovements: Disentangling the Role of Market Factors, Carry-trades and Idiosyncratic Components, Working Paper, Banco de Mexico.

[59] Roll, R., 1992, Industrial Structure and the Comparative Behavior of International Stock Market Indices, Journal of Finance 47, 3-42.

[60] Sklar, A., 1959, Fonctions de répartition à n dimensions et leurs marges, Publications de l'Institut de Statistique de L'Université de Paris 8, 229-231.

[61] Solnik, B., 1974, The International Pricing of Risk: An Empirical Investigation of the World Capital Market Structure, Journal of Finance 29, 365-378.

[62] Solnik, B., and J. Roulet, 2000, Dispersion as Cross-Sectional Correlation, Financial Analysts Journal 56, 54-61.

[63] Tse, Y., and A. Tsui, 2002, A Multivariate GARCH Model with Time-Varying Correlations, Journal of Business and Economic Statistics 20, 351-362.

[64] Tu, J. and G. Zhou, 2011, Markowitz meets Talmud: A Combination of Sophisticated and Naive Diversification Strategies, Journal of Financial Economics, 99, 204-215.

Appendix

Appendix A. Correlation Targeting in DCC

Correlation targeting in the DCC model allows us to significantly reduce the number of parameters estimated via numerical optimization of the likelihood function. We need to estimate α_Γ, β_Γ, and $\tilde{\Gamma}$ in the DCC recursion

$$\tilde{\Gamma}_t = (1 - \alpha_\Gamma - \beta_\Gamma) \tilde{\Gamma} + \beta_\Gamma \tilde{\Gamma}_{t-1} + \alpha_\Gamma \tilde{z}_{t-1} \tilde{z}_{t-1}^\top. \tag{7.1}$$

But if we can target the long run correlation $\tilde{\Gamma}$ to its sample analogue $\hat{\Gamma} = \frac{1}{T} \sum_{t=1}^{T} \tilde{z}_t \tilde{z}_t^\top$ then we need only to estimate the scalars α_Γ and β_Γ in the numerical MLE procedure.

Recall that $\tilde{z}_{i,t} = z_{i,t} \sqrt{\tilde{\Gamma}_{ii,t}}$. A circularity problem is apparent because we need $\tilde{\Gamma}_{ii,t}$ to estimate $\tilde{\Gamma}$, which in turn is required to compute the time series of $\tilde{\Gamma}_{ii,t}$. Note however that $\tilde{\Gamma}$ is a correlation matrix, so that $\tilde{\Gamma}_{ii} = 1$, for all i, and note also that only the diagonal elements of $\tilde{\Gamma}_t$ are needed to compute $\tilde{z}_{i,t}$. Aielli (2009) therefore proposes to first compute equation (7.1) for the diagonal elements only, that is

$$\tilde{\Gamma}_{ii,t} = (1 - \alpha_\Gamma - \beta_\Gamma) + \beta_\Gamma \tilde{\Gamma}_{ii,t-1} + \alpha_\Gamma \tilde{z}_{i,t-1}^2$$

for all i and t. Having computed the $\tilde{\Gamma}_{ii,t}$, the sample correlation matrix of the $\tilde{z}_{i,t}$ can be obtained which in turn yields $\hat{\Gamma} = \frac{1}{T} \sum_{t=1}^{T} \tilde{z}_t \tilde{z}_t^\top$, and the recursion in (7.1) can now be run replacing $\tilde{\Gamma}$ by $\hat{\Gamma}$.

Appendix B. The t Copula

The conventional symmetric N-dimensional t distribution has the stochastic representation

$$X = \sqrt{W} Z \tag{7.2}$$

where W is an inverse gamma variable $W \sim IG\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$, Z is a normal variable $Z \sim N\left(0_N, \Psi\right)$, and where Z and W are independent.

The probability density function of the t copula defined from the t distribution is given by

$$c(u; v, \Psi) = \frac{\Gamma\left(\frac{\nu+N}{2}\right)}{|\Psi|^{\frac{1}{2}} \Gamma\left(\frac{\nu}{2}\right)} \left(\frac{\Gamma\left(\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu+1}{2}\right)}\right)^N \frac{\left(1 + \frac{1}{v} z^{*\top} \Psi^{-1} z^*\right)^{-\frac{\nu+N}{2}}}{\prod_{j=1}^{N}\left(1 + \frac{z_j^{*2}}{\nu}\right)^{-\frac{\nu+1}{2}}}$$

where $z^* = t_v^{-1}(u)$ and $t_v(u)$ is the univariate Student's t density function given by

$$t_v(u) = \int_{-\infty}^{u} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\,\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}} dx.$$

Appendix C. The Skewed t Copula

The skewed t distribution discussed in Demarta and McNeil (2005) has the more general stochastic representation

$$X = \sqrt{W} Z + \lambda W \tag{7.3}$$

where λ is the asymmetry parameter, W is again an inverse gamma variable $W \sim IG\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$, Z is a normal variable $Z \sim N\left(0_N, \Psi\right)$, and Z and W are again independent. The skewed t distribution generalizes the t distribution by adding a second term related to the same inverse gamma random variable which is scaled by an asymmetry parameter λ. Note that the conventional symmetric t distribution is nested when λ tends to zero.

The probability density function of the skewed t copula defined from the asymmetric t distribution is given by

$$
\begin{aligned}
c(u; \lambda, v, \Psi) &= \frac{2^{\frac{(\nu-2)(N-1)}{2}} K_{\frac{\nu+N}{2}}\left(\sqrt{\left(\nu + z^{*\top}\Psi^{-1}z^*\right)\lambda^2\Psi^{-1}}\right) e^{z^{*\top}\Psi^{-1}\lambda}}{\Gamma\left(\frac{\nu}{2}\right)^{1-N} |\Psi|^{\frac{1}{2}} \left(\sqrt{\left(\nu + z^{*\top}\Psi^{-1}z^*\right)\lambda^2\Psi^{-1}}\right)^{-\frac{\nu+N}{2}} \left(1 + \frac{1}{v}z^{*\top}\Psi^{-1}z^*\right)^{\frac{\nu+N}{2}}} \\
&\times \prod_{j=1}^{N} \frac{\left(\sqrt{\left(\nu + (z_j^*)^2\right)\lambda^2}\right)^{-\frac{\nu+1}{2}} \left(1 + \frac{1}{v}\left(z_j^*\right)^2\right)^{\frac{\nu+1}{2}}}{K_{\frac{\nu+1}{2}}\left(\sqrt{\left(\nu + (z_j^*)^2\right)\lambda^2}\right) e^{z_j^*\lambda}}
\end{aligned}
\tag{7.4}
$$

where $K(\cdot)$ is the modified Bessel function of third kind, and where the fractiles $z^* = t_{\lambda,v}^{-1}(u)$ are defined from the asymmetric univariate student t density defined by

$$t_{\lambda,v}(u) = \int_{-\infty}^{u} \frac{2^{1-\frac{\nu+N}{2}} K_{\frac{\nu+1}{2}}\left(\sqrt{\left(\nu + x^2\right)\lambda^2}\right) e^{x\lambda}}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi\nu}\left(\sqrt{\left(\nu + x^2\right)\lambda^2}\right)^{-\frac{\nu+1}{2}} \left(1 + \frac{x^2}{\nu}\right)^{\frac{\nu+1}{2}}} dx. \tag{7.5}$$

The asymmetric Student t quantile function, $t_{\lambda,v}^{-1}(u)$, is not known in closed form but can be well approximated by simulating 100,000 replications of equation 7.3. Note that we constrain the copula to have the same asymmetry parameter, λ, across all assets.

36

The moments of the W variable are given by $m_i = \nu^i / \left(\prod_{j=1}^{i}(\nu - 2j) \right)$, and from the the normal mixture structure of the distribution, we can derive the expected value

$$E\left[X\right] = E\left(E\left[X|W\right]\right) = E(W)\lambda = \frac{\nu}{\nu - 2}\lambda$$

and the variance-covariance matrix

$$
\begin{aligned}
Cov\left(X\right) &= E\left(Var(X|W)\right) + Var\left(E\left[X|W\right]\right) \\
&= \frac{\nu}{\nu - 2}\Psi + \frac{2\nu^2\lambda^2}{(\nu - 2)^2(\nu - 4)}.
\end{aligned}
\tag{7.6}
$$

Notice that the covariances are finite if $\nu > 4$. These moments provide the required link between the multivariate asymmetric t distribution and the copula correlation matrix Ψ.

Figure 1: Dynamic (DECO) Correlations for Developed, Emerging, and All Markets.
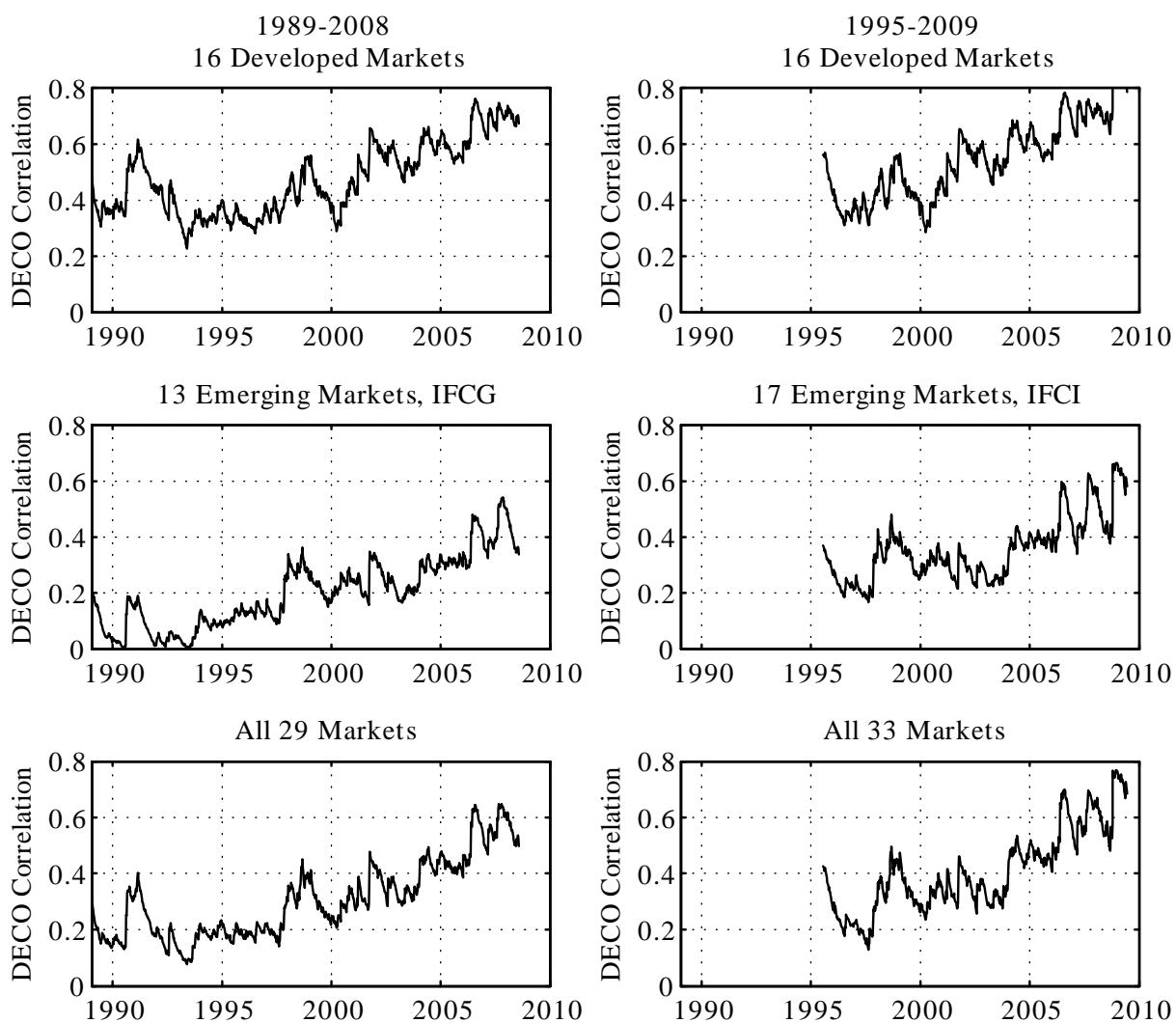
Notes to Figure: We report dynamic equicorrelations (DECOs) for two sample periods. The left-side panels report on the period January 20, 1989 to July 25, 2008. The right-side panels report on the period July 21, 1995 to June 12, 2009. The top panels report on developed markets, the middle panels report on emerging markets, and the bottom panels report on samples consisting of developed and emerging markets.

Figure 2: Comparing DECO and DCC Correlations. Developed Markets. Various Sample Periods.
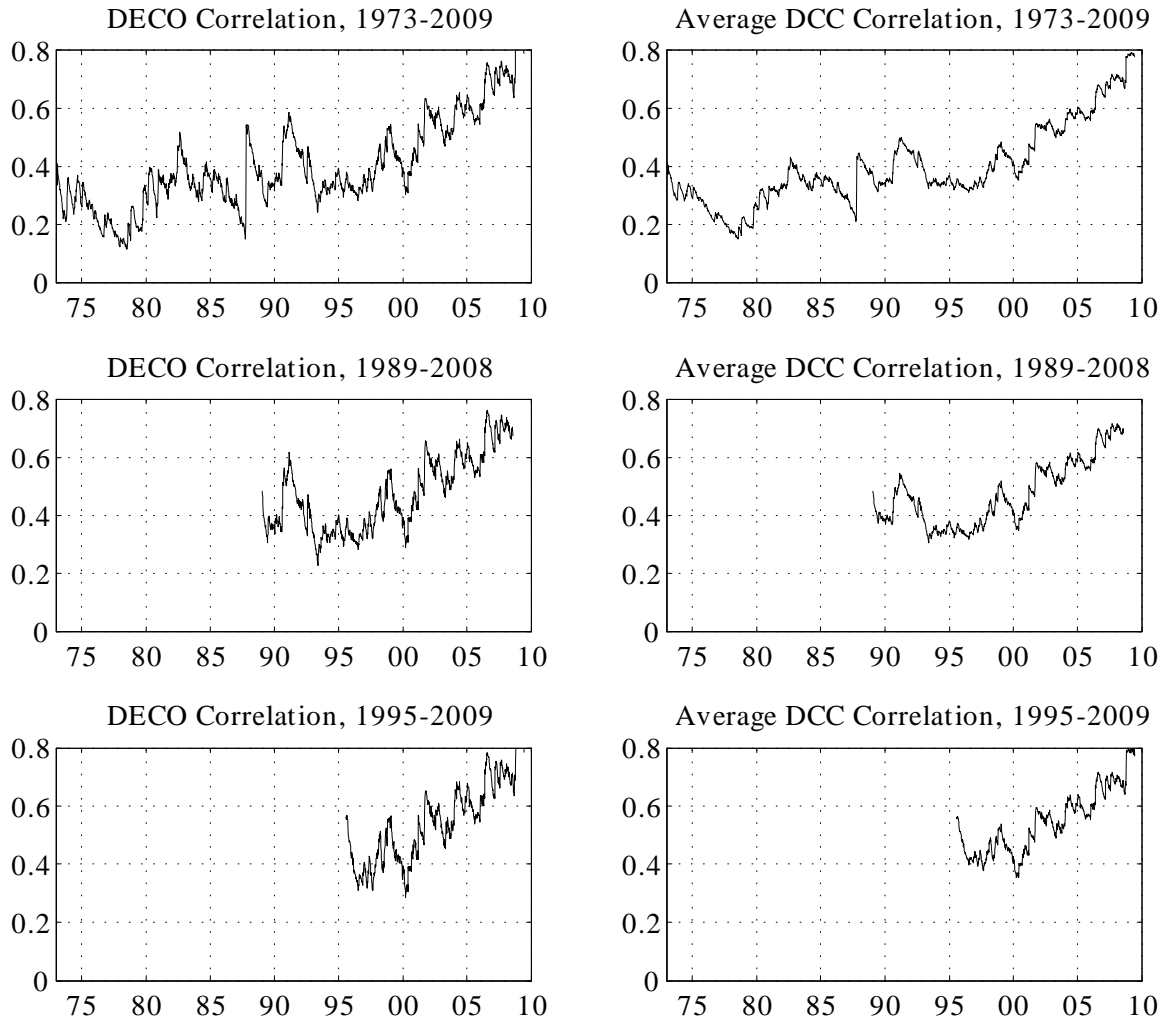
Notes to Figure: We report dynamic equicorrelations (DECOs) and dynamic conditional correlations (DCCs) for sixteen developed markets for three sample periods. The top panels report on the period January 26, 1973 to June 12, 2009. The middle panels report on the period January 20, 1989 to July 25, 2008. The bottom panels report on the period July 21, 1995 to June 12, 2009.

Figure 3.A: Correlations for Each Developed Market.
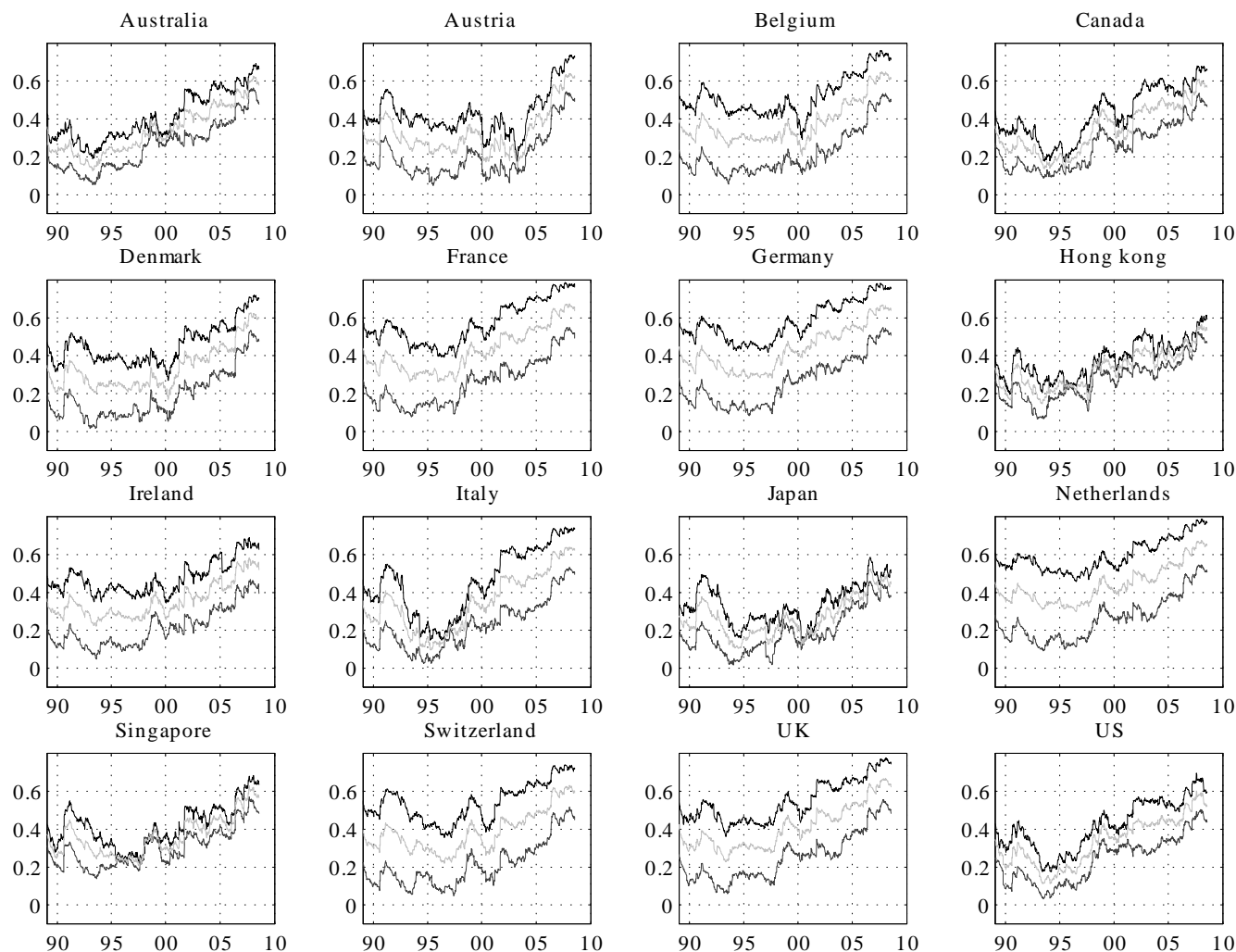
Notes to Figure: We report dynamic conditional correlations for sixteen developed markets for the period January 20, 1989 to July 21, 2008. For each country, at each point in time we report three averages of conditional correlations with other countries: the average of correlations with the fifteen other developed markets (black line), with the thirteen emerging markets (dark grey line), and with the fifteen developed and thirteen emerging markets (light grey line).

Figure 3.B: Correlations for each Emerging Market.
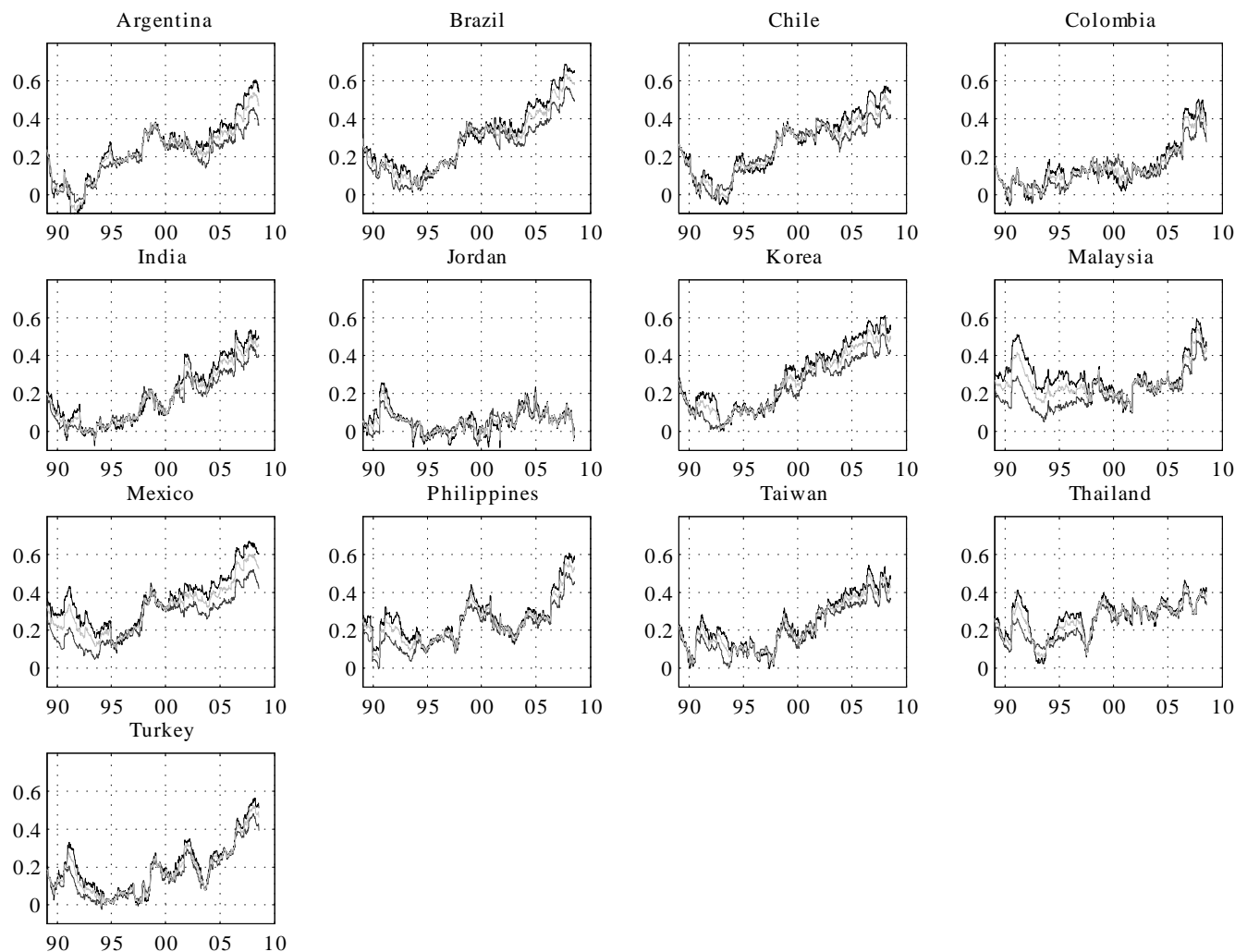
Notes to Figure: We report dynamic conditional correlations for thirteen emerging markets for the period January 20, 1989 to July 25, 2008. For each country, at each point in time we report three averages of conditional correlations with other countries: the average of correlations with sixteen developed markets (black line), with the twelve other emerging markets (dark grey line), and with the sixteen developed and twelve emerging markets (light grey line).

Figure 4: Regional Correlation Patterns.
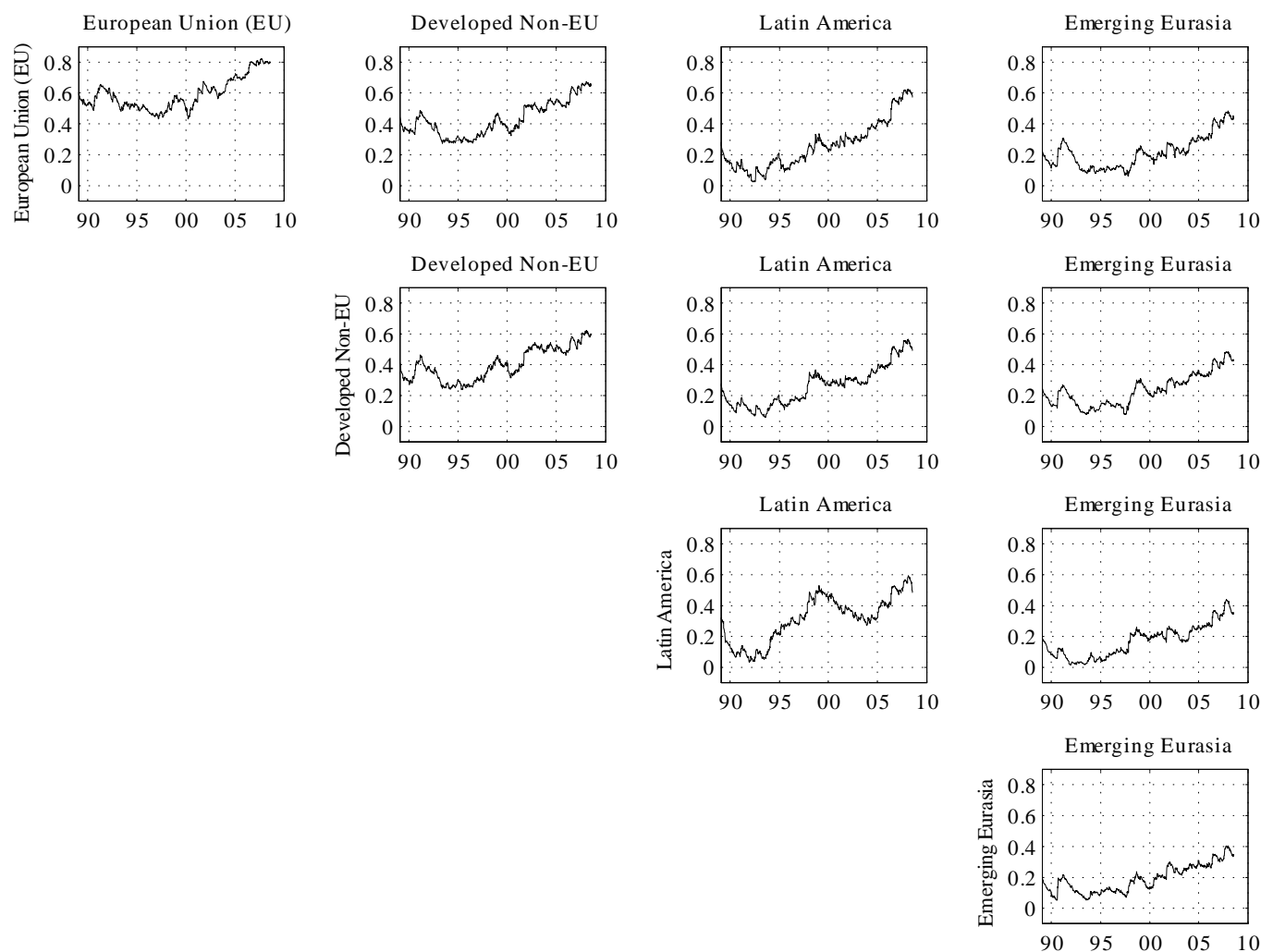
Notes to Figure: We use the DCC model to plot the average correlation within and across four regions. The European Union (EU) includes Austria, Belgium, Denmark, France, Germany, Ireland, Italy, Netherlands, and the UK. Developed Non-EU includes Australia, Canada, Hong Kong, Japan, Singapore, Switzerland, and the US. Latin America includes Argentina, Brazil, Chile, Colombia, and Mexico. Emerging Eurasia includes India, Jordan, Korea, Malaysia, Philippines, Taiwan, Thailand, and Turkey.

Figure 5: Correlation Range (90th and 10th Percentile). Developed, Emerging and All Markets.
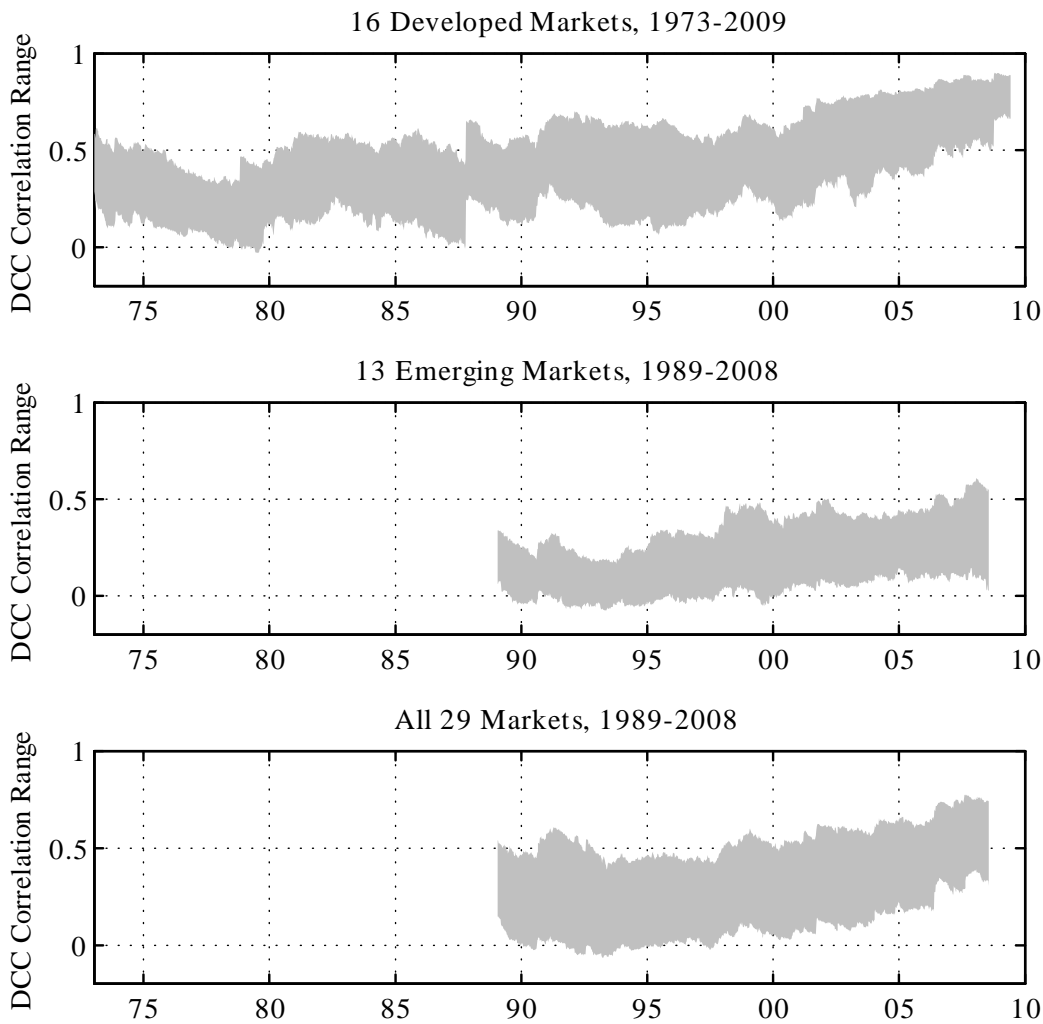
Notes to Figure: The shaded areas show the correlation range between the 90th and 10th percentiles for DCCs. The top panels report on sixteen developed markets for the period January 26, 1973 to June 12, 2009. The middle panels report on thirteen emerging markets for the period January 20, 1989 to July 25, 2008. The bottom panels report on sixteen developed and thirteen emerging markets for the period January 20, 1989 to July 25, 2008.

Figure 6: Conditional Diversification Benefits (CDB) using the DCC Model.
Developed, Emerging and All Markets.
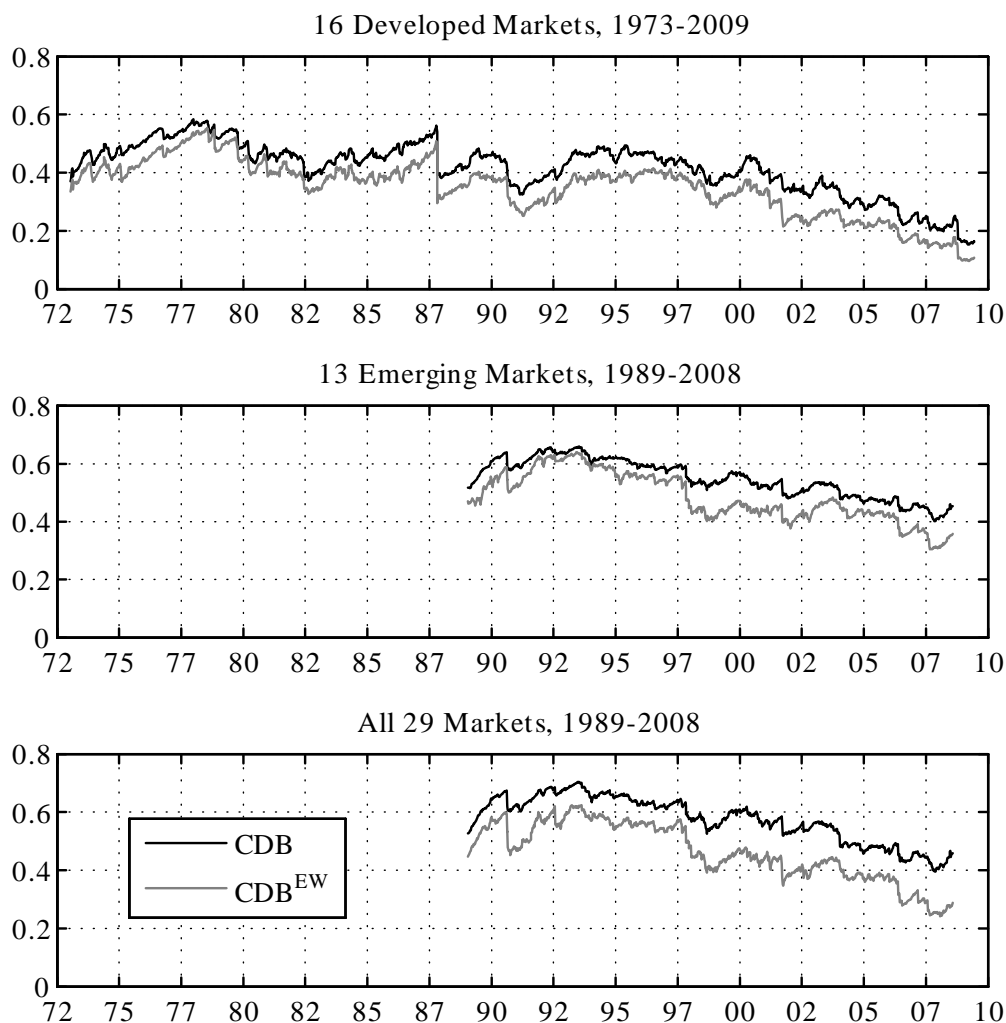
Notes to Figure: Each week and for each set of countries, we use the dynamic conditional correlation (DCC) model to compute the conditional diversification benefit (CDB) as defined in (2.10). The dark line denotes the CDB computed using optimal portfolio weights and the gray line represents the CDB^{EW} measure for an equal-weighted portfolio.

Figure 7: Dynamic Average Bivariate Tail Dependence in Skewed t Copula. Constant Degree of Freedom.
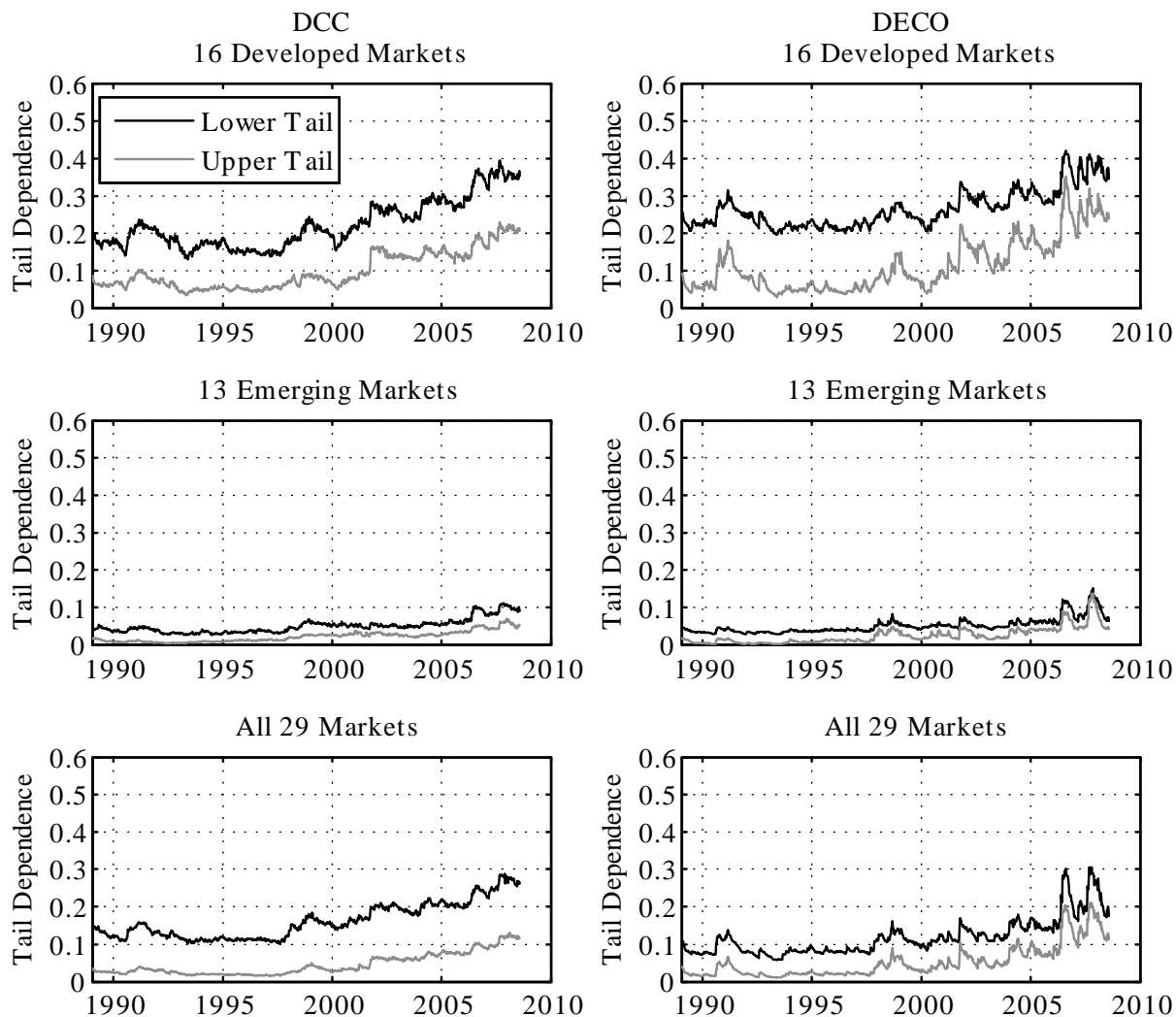
Notes to Figure: We report estimated average bivariate tail dependence for the DCC (left panels) and DECO (right panels) constrained skewed t copula with constant degree of freedom. The black line is the left tail dependence. The gray line is the tail dependence for the right tail. The top panels report on sixteen developed markets, the middle panels report on thirteen emerging markets, and the bottom panels report on sixteen developed and thirteen emerging markets for the period January 20, 1989 to July 25, 2008.

Figure 8: Threshold Correlations.

16 Developed Markets
1973-2009

13 Emerging Markets
1989-2008
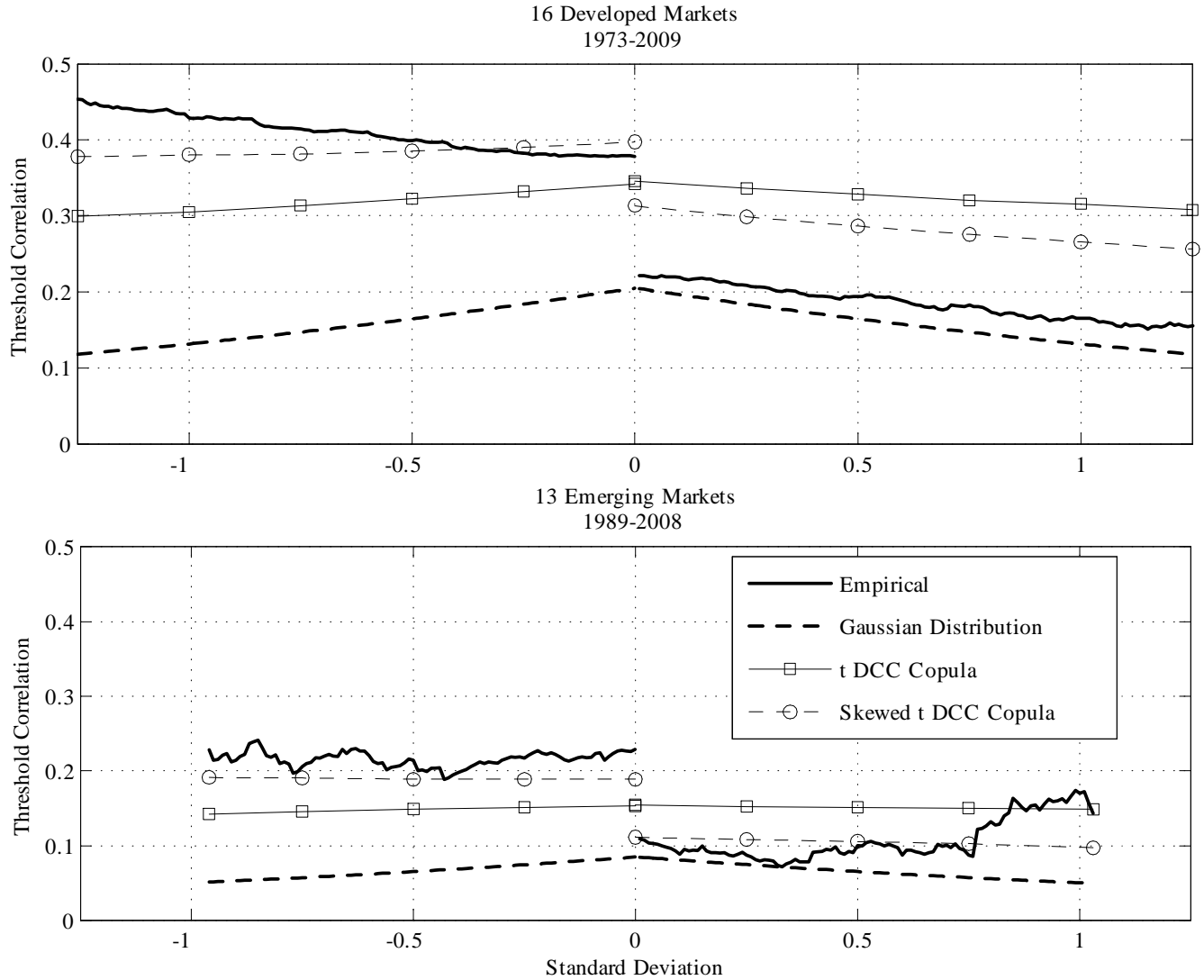
Notes to Figure: The top panel reports the average pairwise threshold correlations for sixteen developed markets for the period January 26, 1973 to June 12, 2009. The bottom panel reports the average pairwise threshold correlations for thirteen emerging markets for the period January 20, 1989 to July 25, 2008. We compute threshold correlations using the empirical return shocks as well as threshold correlations based on generated data from three models: the multivariate Gaussian distribution, the t DCC copula, and the skewed t DCC copula.

Figure 9: DECO and Spline DECO Correlations for Developed, Emerging, and All Markets. 1989-2008.
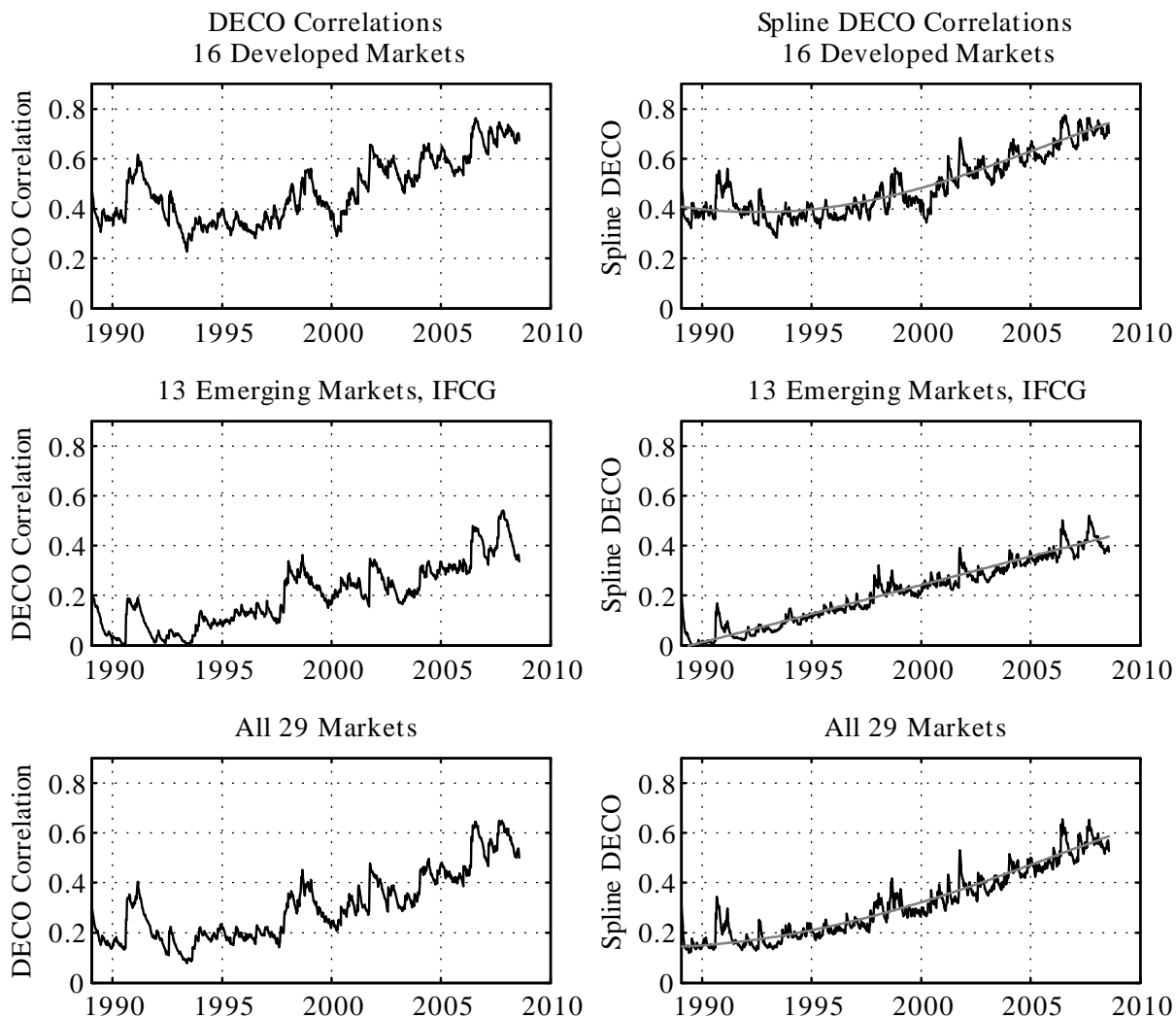
Notes to Figure: We report the DECO and spline DECO correlations for the period January 20, 1989 to July 25, 2008. The left panels correspond to the DECO model with a fixed long run average, and the right panels are equicorrelations from the Spline DECO. The top panel reports on developed markets, the second panel reports on emerging markets, the third on all markets. In the right panels, the black line shows the total correlation while the gray line shows the long-run correlation component.

Figure 10: Rolling Correlations for Developed, Emerging, and All Markets. Two Estimates.
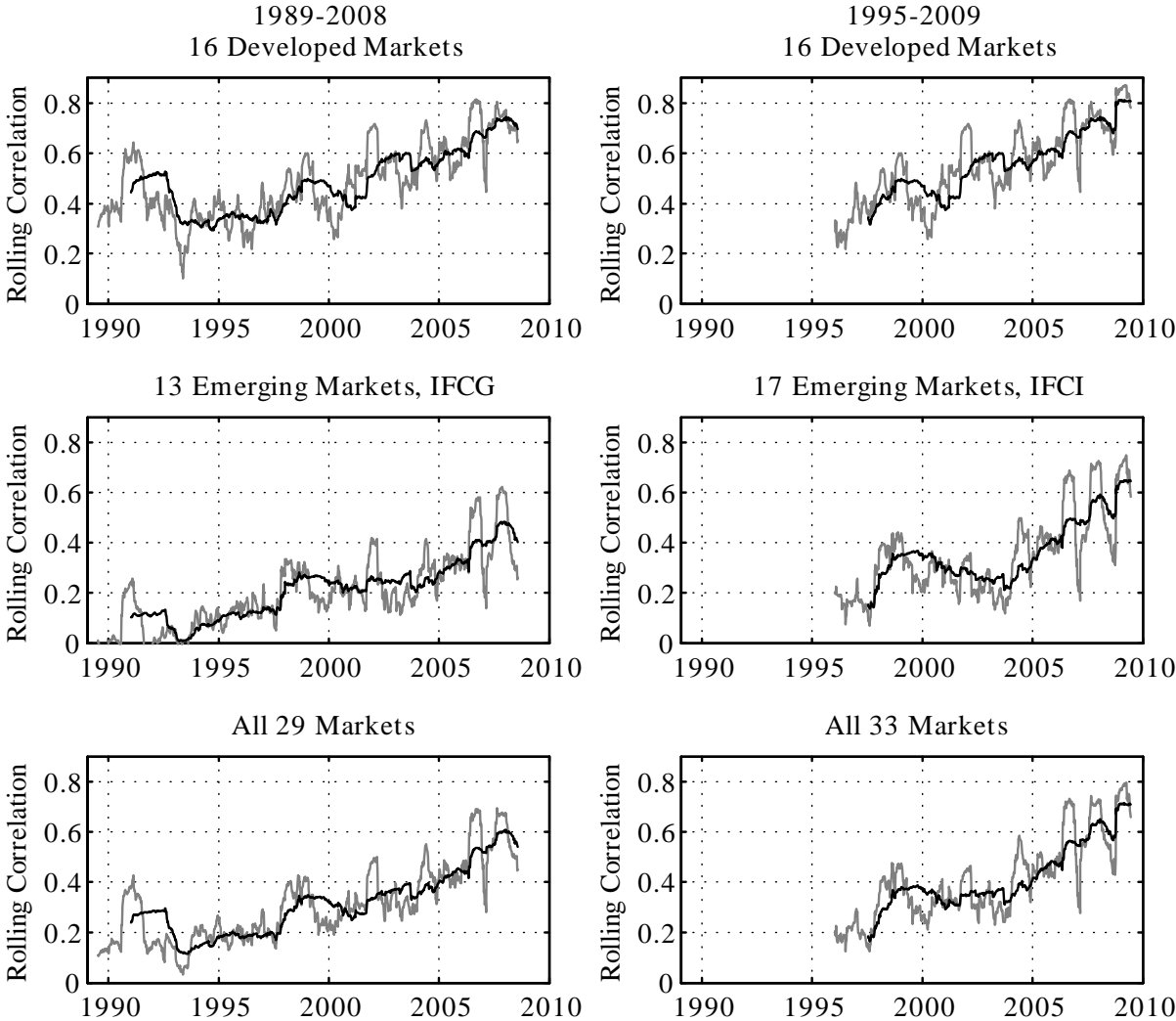
Notes to Figure: We report rolling correlations for two sample periods. The left-side panels report on the period January 20, 1989 to July 25, 2008. The right-side panels report on the period July 21, 1995 to June 12, 2009. The top panels report on developed markets, the middle panels report on emerging markets, and the bottom panels report on samples consisting of developed and emerging markets. We use 6-month (grey lines) and 2-year (black lines) windows to estimate rolling correlations for each pair of markets which are then averaged across pairs to produce the plot.

Figure A.1: Contours of the Normal, t and Skewed t Copulas.
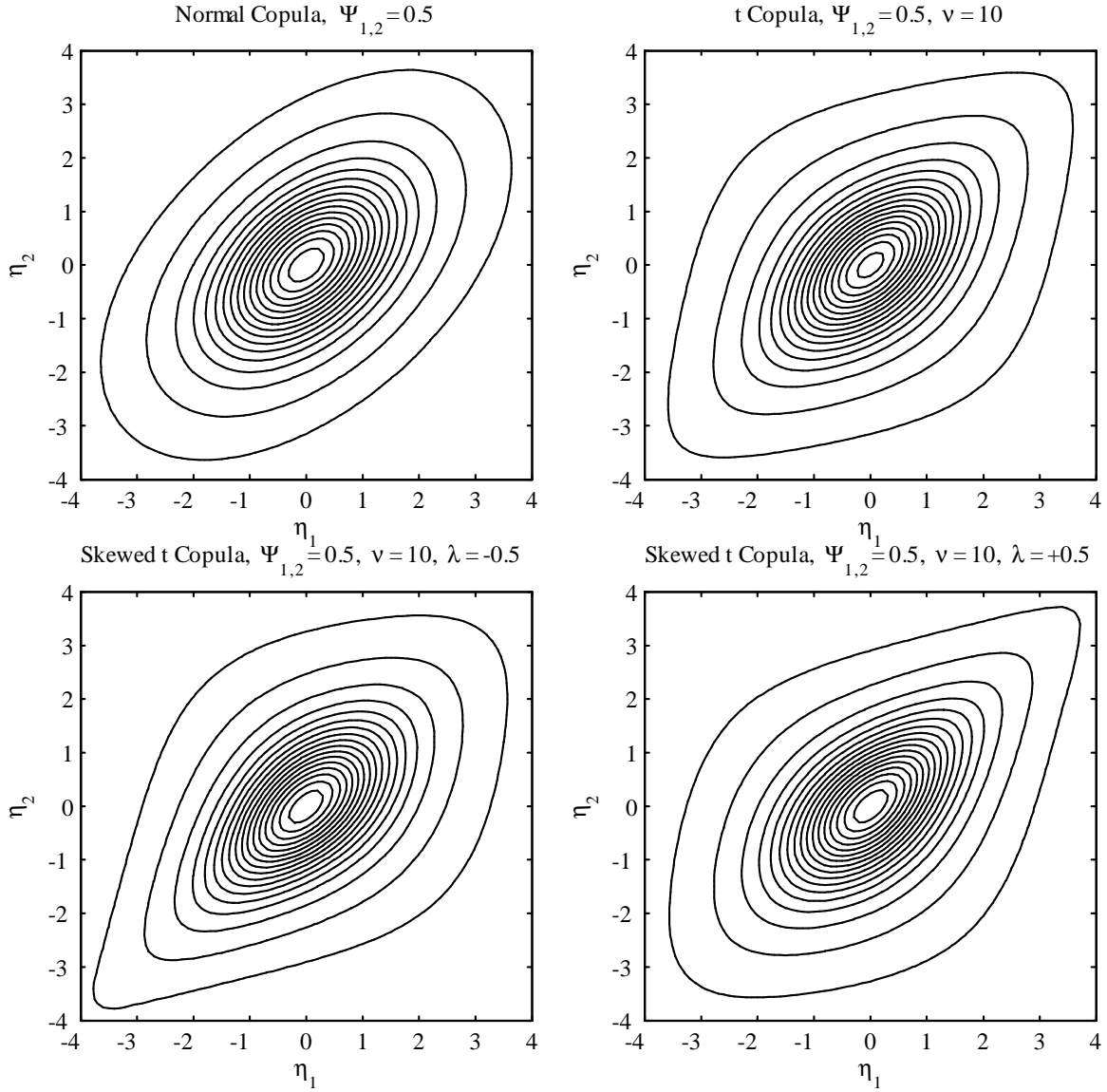
Notes to Figure: We plot the probability contours of four bivariate copula models. The top left panel shows the normal copula with a copula correlation, $\Psi_{12} = 0.5$. The top right panel shows the symmetric t copula with $\Psi_{12} = 0.5$ and degree of freedom, $v = 10$. The bottom two panels show the new skewed t copula where we keep $\Psi_{12} = 0.5$ and $v = 10$. The bottom left panel has an asymmetry parameter, $\lambda = -0.5$ and the bottom right panel has an asymmetry parameter, $\lambda = +0.5$. The probability levels for each contour are the kept the same across all four copula models.

Table 1: Descriptive Statistics for Weekly Returns on 16 DM and 13 EM (IFCG).
January 1989 to July 2008.

	Annual Mean (%)	Annual Standard Deviation (%)	Skewness	Excess Kurtosis	1st Order Auto-correlation	LB(20) P-Value on Returns	LB(20 P-value on Absolute Returns
Developed Markets							
Australia	13.21	17.13	-0.301	1.94	-0.010	0.2044	0.0000
Austria	14.50	19.63	-0.009	3.69	0.058	0.1364	0.0000
Belgium	10.93	16.55	-0.215	2.00	0.011	0.8316	0.0000
Canada	13.24	16.30	-0.489	2.35	-0.001	0.1611	0.0000
Denmark	14.55	17.83	-0.308	2.11	-0.045	0.2535	0.0000
France	12.75	17.54	-0.120	1.04	0.019	0.0632	0.0000
Germany	11.88	18.66	-0.231	1.65	0.018	0.7596	0.0000
Hong Kong	16.71	24.41	-0.355	3.11	0.030	0.0046	0.0000
Ireland	13.30	18.89	-0.286	1.74	0.011	0.2749	0.0000
Italy	9.18	20.99	0.189	3.32	0.037	0.1494	0.0000
Japan	1.45	22.28	0.339	1.56	-0.068	0.0307	0.0000
Netherlands	13.26	16.62	-0.300	3.00	0.006	0.3920	0.0000
Singapore	11.16	20.66	-0.255	5.15	-0.009	0.0010	0.0000
Switzerland	13.70	16.42	0.002	2.05	-0.012	0.5579	0.0001
United Kingdom	11.37	15.54	0.060	1.80	-0.027	0.6799	0.0019
United States	11.71	15.15	-0.434	3.46	-0.102	0.0005	0.0000
Average	12.06	18.41	-0.170	2.50	-0.005	0.2813	0.0001
Emerging Markets							
Argentina	29.44	51.18	0.804	12.22	-0.009	0.0001	0.0000
Brazil	29.85	46.23	-0.246	2.22	0.045	0.3060	0.0000
Chile	19.43	21.18	0.008	1.30	0.167	0.0000	0.0000
Colombia	22.66	26.51	0.339	6.29	0.132	0.0000	0.0000
India	15.24	27.89	-0.027	2.05	0.078	0.0494	0.0000
Jordan	15.86	18.21	0.195	5.42	0.083	0.0071	0.0000
Korea	10.18	36.43	0.045	8.54	-0.089	0.0001	0.0000
Malaysia	10.16	28.64	1.394	24.02	0.013	0.0000	0.0000
Mexico	20.99	28.02	-0.387	3.72	0.108	0.0018	0.0000
Philippines	7.21	28.57	-0.393	4.97	0.085	0.0002	0.0000
Taiwan	7.15	32.82	0.318	3.96	-0.007	0.0232	0.0000
Thailand	10.65	35.90	0.250	3.98	0.019	0.0000	0.0000
Turkey	31.02	55.59	0.178	5.27	-0.018	0.1912	0.0000
Average	17.68	33.63	0.191	6.46	0.047	0.0445	0.0000

Notes to Table: We report the first four sample moments and the first order autocorrelation of the 16 DM and 13 EM (IFCG) returns. We also report the p-value from a Ljung-Box test that the first 20 autocorrelations are zero for returns and absolute returns. The sample period is from January 20, 1989 to July 25, 2008.

Table 2: Parameter Estimates from NGARCH(1,1) on 16 DM and 13 EM (IFCG).
January 1989 to July 2008.

	α	β	θ	Variance Persistence	LB(20) P-Value on Residuals	LB(20) P-Value on Absolute Residuals	Residual Skewness	Residual Excess Kurtosis
Developed Markets								
Australia	0.055	0.804	0.785	0.893	0.633	0.702	-0.419	1.31
Austria	0.078	0.898	-0.166	0.978	0.442	0.321	-0.201	0.95
Belgium	0.136	0.594	0.737	0.804	0.818	0.200	-0.391	0.97
Canada	0.101	0.847	0.323	0.958	0.638	0.825	-0.396	0.92
Denmark	0.049	0.941	0.027	0.990	0.342	0.477	-0.352	1.52
France	0.071	0.697	1.175	0.866	0.124	0.374	-0.138	0.28
Germany	0.119	0.748	0.557	0.903	0.814	0.101	-0.298	0.74
Hong Kong	0.113	0.810	0.530	0.955	0.231	0.537	-0.230	1.20
Ireland	0.043	0.854	1.167	0.955	0.508	0.560	-0.292	1.07
Italy	0.106	0.804	0.452	0.931	0.404	0.076	-0.054	0.90
Japan	0.085	0.837	0.532	0.946	0.418	0.410	0.150	0.97
Netherlands	0.134	0.725	0.554	0.899	0.817	0.671	-0.466	0.62
Singapore	0.095	0.836	0.630	0.968	0.478	0.950	-0.359	1.79
Switzerland	0.091	0.651	0.846	0.807	0.405	0.782	-0.141	0.80
United Kingdom	0.037	0.557	2.648	0.851	0.799	0.278	-0.155	0.55
United States	0.090	0.839	0.658	0.969	0.411	0.496	-0.494	1.33
Average	0.088	0.778	0.716	0.917	0.518	0.485	-0.265	0.994
Emerging Markets								
Argentina	0.168	0.807	0.182	0.981	0.844	0.280	0.109	1.03
Brazil	0.084	0.887	0.360	0.982	0.907	0.167	-0.395	1.14
Chile	0.084	0.868	0.043	0.952	0.544	0.917	0.065	1.25
Colombia	0.238	0.599	0.044	0.838	0.625	0.633	0.129	2.13
India	0.122	0.814	0.208	0.941	0.181	0.144	-0.121	0.89
Jordan	0.009	0.879	-3.380	0.992	0.425	0.018	0.243	4.04
Korea	0.110	0.857	0.372	0.982	0.633	0.573	0.041	0.76
Malaysia	0.103	0.886	0.160	0.992	0.937	0.684	-0.241	2.47
Mexico	0.109	0.723	0.903	0.922	0.924	0.524	-0.333	1.22
Philippines	0.076	0.889	0.282	0.971	0.883	0.419	-0.420	3.71
Taiwan	0.143	0.810	0.299	0.967	0.492	0.709	0.082	1.25
Thailand	0.100	0.860	0.336	0.971	0.986	0.209	-0.027	1.48
Turkey	0.080	0.876	-0.166	0.958	0.400	0.232	0.104	2.97
Average	0.110	0.827	-0.027	0.957	0.675	0.424	-0.059	1.872

Notes to Table: We report parameter estimates and residual diagnostics for the NGARCH(1,1) models. The sample period for 16 DM and 13 EM (IFCG) weekly returns is from January 20, 1989 to July 25, 2008. The conditional mean is modeled by an AR(2) model. The coefficients from the AR models are not shown. The constant term in the GARCH model is fixed by variance targeting.

Table 3: Parameter Estimates for DECO and DCC Models. Developed, Emerging and All Markets. January 1989 to July 2008.

	DECO				DCC			
	α_Γ	β_Γ	Persistence	Composite Likelihood	α_Γ	β_Γ	Persistence	Composite Likelihood
A: Weekly Returns, January 26, 1973 to June 12, 2009								
16 Developed Markets	0.041	0.959	1.000	8693.21	0.018	0.981	1.000	8727.97
No Dynamics	0	0	0	8651.87	0	0	0	8675.94
B: Weekly IFCG Returns, January 20, 1989 to July 25, 2008								
16 Developed Markets	0.045	0.954	0.999	4882.95	0.023	0.976	0.999	4910.43
No Dynamics	0	0	0	4865.67	0	0	0	4887.19
13 Emerging Markets	0.043	0.957	1.000	3759.14	0.014	0.984	0.998	3766.54
No Dynamics	0	0	0	3751.65	0	0	0	3758.05
All 29 Markets	0.053	0.947	1.000	4345.11	0.018	0.981	0.999	4367.83
No Dynamics	0	0	0	4333.14	0	0	0	4353.17
C: Weekly IFCI Returns, July 21, 1995 to June 12, 2009								
16 Developed Markets	0.054	0.946	1.000	3407.04	0.030	0.967	0.997	3429.36
No Dynamics	0	0	0	3390.00	0	0	0	3408.39
17 Emerging Markets	0.050	0.941	0.991	2610.30	0.022	0.964	0.986	2612.97
No Dynamics	0	0	0	2602.52	0	0	0	2606.60
All 33 Markets	0.060	0.940	1.000	2975.17	0.024	0.969	0.994	2987.03
No Dynamics	0	0	0	2963.22	0	0	0	2974.97

Notes to Table: We report parameter estimates for the DCC and DECO models for the 13 emerging markets (IFCG), 17 emerging markets (IFCI), 16 developed markets, and all markets. The composite likelihood is the average of the quasi-likelihoods (correlation log likelihood + all marginal volatility log likelihood) of all unique pairs of assets. We also report the special case of no dynamics.

Table 4: Parameter Estimates for DECO and DCC Copula Models. Developed, Emerging and All Markets. January 1989 to July 2008.

	A: t DCC Copula					B: Skewed t DCC Copula					
	α_Γ	β_Γ	Persistence	ν	Composite Likelihood	α_Γ	β_Γ	Persistence	λ	ν	Composite Likelihood
16 Developed Markets	0.019	0.976	0.996	11.862	4910.46	0.017	0.979	0.996	-0.348	11.210	4912.20
No Dynamics	0	0	0	8.834	4890.44	0	0	0	-0.329	8.537	4892.48
13 Emerging Markets	0.013	0.981	0.994	17.151	3767.74	0.012	0.985	0.996	-0.417	17.801	3768.46
No Dynamics	0	0	0	14.269	3760.22	0	0	0	-0.446	14.003	3761.33
All 29 Markets	0.016	0.981	0.997	15.428	4368.34	0.014	0.983	0.997	-0.484	15.040	4369.52
No Dynamics	0	0	0	11.998	4355.50	0	0	0	-0.389	12.134	4356.96

	C: t DECO Copula					D: Skewed t DECO Copula					
	α_Γ	β_Γ	Persistence	ν	Composite Likelihood	α_Γ	β_Γ	Persistence	λ	ν	Composite Likelihood
16 Developed Markets	0.039	0.952	0.990	8.517	4886.23	0.033	0.956	0.989	-0.261	7.833	4887.67
No Dynamics	0	0	0	7.315	4872.07	0	0	0	-0.219	6.756	4873.84
13 Emerging Markets	0.041	0.953	0.994	13.973	3760.58	0.038	0.955	0.993	-0.295	12.682	3761.33
No Dynamics	0	0	0	12.667	3753.95	0	0	0	-0.332	11.853	3755.18
All 29 Markets	0.045	0.948	0.993	10.526	4347.44	0.043	0.950	0.993	-0.241	10.682	4348.36
No Dynamics	0	0	0	9.305	4337.40	0	0	0	-0.287	9.451	4338.75

	E: t DECO Copula with Dynamic Degree of Freedom					F: Skewed t DECO Copula with Dynamic Degree of Freedom					
	α_Γ	β_Γ	Persistence	Average ν	Composite Likelihood	α_Γ	β_Γ	Persistence	λ	Average ν	Composite Likelihood
16 Developed Markets	0.039	0.949	0.988	9.580	4886.84	0.039	0.950	0.988	-0.261	10.250	4888.32
No Dynamics	0	0	0	8.439	4875.26	0	0	0	-0.219	13.878	4875.32
13 Emerging Markets	0.041	0.949	0.990	18.073	3761.24	0.041	0.949	0.991	-0.284	17.573	3761.84
No Dynamics	0	0	0	15.304	3755.16	0	0	0	-0.332	21.165	3757.41
All 29 Markets	0.044	0.947	0.992	11.954	4347.97	0.046	0.946	0.992	-0.248	12.953	4348.96
No Dynamics	0	0	0	10.613	4339.23	0	0	0	-0.287	19.057	4340.86

Notes to Table: We report parameter estimates for the DECO and DCC t-copula and skewed t-copula models for the 13 emerging markets (IFCG), 16 developed markets, and all markets. The bottom panel presents the results with dynamic degree of freedom. The composite likelihood is the average of the quasi-likelihoods (copula log likelihood + all marginal QML log likelihoods) of all pairs of assets. We also report the special case of each model with no copula correlation dynamics.

Time Trend	Vol DMs	Vol EMs	Vol All	Vol(i)	R^2

Panel A: Regressand: Average Weekly Emerging Market DCC Correlation With All Other Emerging Markets

Time Trend	Vol DMs	Vol EMs	Vol All	Vol(i)	R^2
0.1055 **		0.9719 **			0.3726
(0.0013)		(0.0724)			
0.0965 **				0.1143	0.3532
(0.0012)				(0.0742)	

Panel B: Regressand: Average Weekly Emerging Market DCC Correlation With All Developed Markets

Time Trend	Vol DMs	Vol EMs	Vol All	Vol(i)	R^2
0.0755 **	0.4543 **				0.2395
(0.0016)	(0.1384)				
0.0763 **				0.0388	0.2366
(0.0016)				(0.1203)	

Panel C: Regressand: Average Weekly Emerging Market DCC Correlation With All Other Markets

Time Trend	Vol DMs	Vol EMs	Vol All	Vol(i)	R^2
0.0858 **			0.7021 **		0.2982
(0.0015)			(0.1017)		
0.0833 **				0.0459	0.2898
(0.0014)				(0.0958)	

Panel D: Regressand: Average Weekly Developed Market DCC Correlation With All Emerging Markets

Time Trend	Vol DMs	Vol EMs	Vol All	Vol(i)	R^2
0.0793 **		0.6609 **			0.5540
(0.0005)		(0.0487)			
0.0720 **				0.3446 **	0.5297
(0.0005)				(0.0889)	

Panel E: Regressand: Average Weekly Developed Market DCC Correlation With All Other Developed Markets

Time Trend	Vol DMs	Vol EMs	Vol All	Vol(i)	R^2
0.0269 **	0.4641 **				0.4573
(0.0003)	(0.0345)				
0.0269 **				0.2197 **	0.3976
(0.0003)				(0.0328)	

Panel F: Regressand: Average Weekly Developed Market DCC Correlation With All Other Markets

Time Trend	Vol DMs	Vol EMs	Vol All	Vol(i)	R^2
0.0426 **			0.3392 **		0.4505
(0.0004)			(0.0552)		
0.0410 **				0.1948 **	0.4180
(0.0004)				(0.0608)	

Notes to Table: We estimate panel regressions for the 17 emerging markets in the IFCI index and 16 developed markets, using weekly data. All samples are 1989-2008, except for Panel E, where we have 1973-2009 data available. Country fixed effects are included in each specification, and White standard errors adjusted for country cluster correlations are provided in parentheses. Vol DMs, Vol EMs, and Vol All are the equally-weighted averages of the logs of monthly volatilities across all DMs, all EMs, and all markets respectively. Vol(i) is the market specific log of in monthly volatility. The estimate of the time trend is annualized. * indicates significance at the 5% level, and ** indicates significance at the 1% level.

Table 6: Correlations and Measures of Market Openness. August 1995 to December 2006.

Time Trend	MCR	EMI	Vol DMs	Vol EMs	Vol All	Vol(i)	R^2
Panel A: Regressand: Average Monthly Emerging Market DCC Correlation With All Other Emerging Markets							
0.0236 **	0.5513 **						0.1437
(0.0017)	(0.1700)						
0.0338 **		0.1133					0.1270
(0.0022)		(0.1873)					
0.0316 **	0.4830 *	0.0088		0.2220 **			0.1691
(0.0018)	(0.1876)	(0.1636)		(0.0416)			
0.0269 **	0.5268 *	0.0298				0.1054	0.1412
(0.0018)	(0.1887)	(0.1661)				(0.0532)	
Panel B: Regressand: Average Monthly Emerging Market DCC Correlation With All Developed Markets							
0.0395 **	0.4076 *						0.1946
(0.0018)	(0.1440)						
0.0470 **		0.1068					0.1697
(0.0021)		(0.1946)					
0.0384 **	0.4147 *	0.1053	0.0490				0.2050
(0.0019)	(0.1447)	(0.2070)	(0.0563)				
0.0393 **	0.4037 *	0.0847				0.0041	0.2026
(0.0020)	(0.1511)	(0.2012)				(0.0576)	
Panel C: Regressand: Average Monthly Emerging Market DCC Correlation With All Other Markets							
0.0314 **	0.4682 **						0.1844
(0.0017)	(0.1457)						
0.0401 **		0.1046					0.1617
(0.0020)		(0.1847)					
0.0338 **	0.4404 *	0.0432			0.0929 *		0.1950
(0.0018)	(0.1545)	(0.1793)			(0.0433)		
0.0332 **	0.4526 *	0.0489				0.0612	0.1829
(0.0018)	(0.1580)	(0.1767)				(0.0519)	

Notes to Table: We estimate panel regressions for the 17 emerging markets in the IFCI index, using monthly data from August 1995 to December 2006. Country fixed effects are included in each specification, and White standard errors adjusted for country cluster correlations are provided in parentheses. "MCR" denotes the ratio of market capitalizations of the S&P/IFC investable index to the S&P/IFC global index. "EMI" denotes the integration measure implied by the Errunza and Losq (1985) model. Vol DMs, Vol EMs, and Vol All are the logs of the equally-weighted averages of volatilities across all DMs, all EMs, and all markets respectively. Vol(i) is the market specific log of monthly volatility. The estimate of the time trend is annualized. * indicates significance at the 5% level, and ** indicates significance at the 1% level.

Table 7: Parameter Estimates for Spline DECO Models. January 1989 to July 2008.
Emerging Markets, Developed Markets, and All Markets.

	α_Γ	β_Γ	Stochastic Persistence	c	w_0	w_1	Composite Likelihood
16 Developed Markets	0.0522	0.880	0.932	0.8694	-5.72E-04	1.57E-06	4884.65
No Stochastics	0	0	0	0.8407	-6.12E-04	1.59E-06	4882.25
13 Emerging Markets	0.0416	0.852	0.893	-0.0163	8.44E-04	8.57E-08	3761.11
No Stochastics	0	0	0	0.0209	7.44E-04	1.14E-07	3760.46
All 29 Markets	0.0622	0.814	0.876	0.2906	1.72E-04	8.49E-07	4346.78
No Stochastics	0	0	0	0.3027	1.30E-04	8.39E-07	4345.69

Notes to Table: We report parameter estimates for the spline DECO models for the 13 emerging markets (IFCG), 16 developed markets, and all 29 markets. The composite likelihood is the average of the quasi-likelihoods (correlation log likelihood + all marginal volatility log likelihoods) of all unique pairs of assets. We also report the special case of no stochastics ($\alpha_\Gamma = \beta_\Gamma = 0$) where the spline captures all the dynamics in the correlations.