Estimating Global Bank Network Connectedness

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Financial and Macroeconomic Connectedness

- Market Risk, Portfolio Concentration Risk
  (return connectedness)
- Credit Risk
  (default connectedness)
- Counterparty Risk, Gridlock Risk
  (bilateral and multilateral contractual connectedness)
- Systemic Risk
  (system-wide connectedness)
- Business Cycle Risk
  (local or global real output connectedness)

A Natural Financial/Economic Connectedness Question:

What fraction of the H-step-ahead prediction-error variance of variable i is due to shocks in variable j, $j \neq i$?

Non-own elements of the variance decomposition: $d_{ij}^{H}, \; j \neq i$

Reading and Web Materials

Two Papers:

Diebold, F.X. and Yilmaz, K. (2014), "On the Network Topology of Variance Decompositions: Measuring the Connectedness of Financial Firms," *Journal of Econometrics*, 182, 119-134.

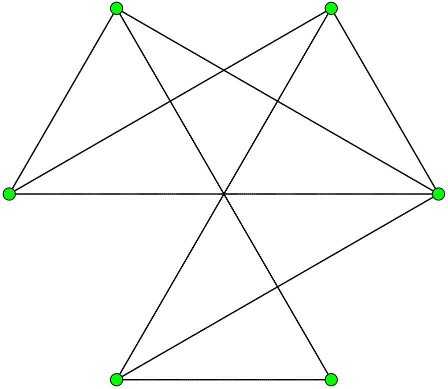
Demirer, M., Diebold, F.X. and Yilmaz, K. (2014), "Estimating Global Bank Network Connectedness," Manuscript, in progress.

Book in Press:

Diebold, F.X. and Yilmaz, K. (2015), *Financial and Macroeconomic Connectedness: A Network Approach to Measurement and Monitoring*, Oxford University Press, in press. With K. Yilmaz.

`www.FinancialConnectedness.org`

Network Representation: Graph and Matrix



$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Symmetric adjacency matrix A
$A_{ij} = 1$ if nodes i, j linked
$A_{ij} = 0$ otherwise

Network Connectedness: The Degree Distribution

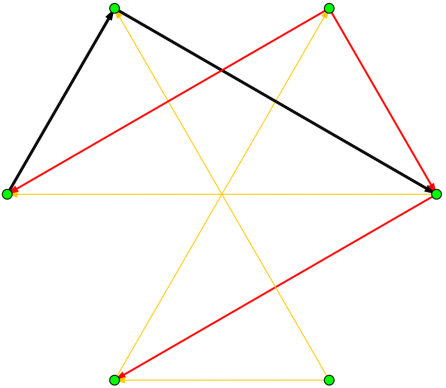
Degree of node i, d_i:

$$d_i = \sum_{j=1}^{N} A_{ij}$$

Discrete *degree distribution* on 0, ..., $N - 1$

Mean degree, $E(d)$, is the key connectedness measure

Network Representation II (Weighted, Directed)



$$A = \begin{pmatrix} 0 & .5 & .7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .3 & 0 \\ 0 & 0 & 0 & .7 & 0 & .3 \\ .3 & .5 & 0 & 0 & 0 & 0 \\ .5 & 0 & 0 & 0 & 0 & .3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

"to i, from j"

Network Connectedness II: The Degree Distribution(s)

$A_{ij} \in [0, 1]$ depending on connection strength

Two degrees:

$$d_i^{from} = \sum_{j=1}^{N} A_{ij}$$

$$d_j^{to} = \sum_{i=1}^{N} A_{ij}$$

"from-degree" and "to-degree" distributions on $[0, N-1]$

Mean degree remains the key connectedness measure

Variance Decompositions as Weighted, Directed Networks

Variance Decomposition / Connectedness Table

	x_1	x_2	...	x_N	From Others
x_1	d_{11}^H	d_{12}^H	$\cdots$	d_{1N}^H	$\sum_{j\neq 1} d_{1j}^H$
x_2	d_{21}^H	d_{22}^H	$\cdots$	d_{2N}^H	$\sum_{j\neq 2} d_{2j}^H$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_N	d_{N1}^H	d_{N2}^H	$\cdots$	d_{NN}^H	$\sum_{j\neq N} d_{Nj}^H$
To Others	$\sum_{i\neq 1} d_{i1}^H$	$\sum_{i\neq 2} d_{i2}^H$	$\cdots$	$\sum_{i\neq N} d_{iN}^H$	$\sum_{i\neq j} d_{ij}^H$

Total directional connect. "from," $C_{i\leftarrow\bullet}^H = \sum_{\substack{j=1\\j\neq i}}^N d_{ij}^H$: "from-degrees"

Total directional connect. "to," $C_{\bullet\leftarrow j}^H = \sum_{\substack{i=1\\i\neq j}}^N d_{ij}^H$: "to-degrees"

Systemwide connect., $C^H = \frac{1}{N}\sum_{\substack{i,j=1\\i\neq j}}^N d_{ij}^H$: mean degree

Relationship to *MES*

$$MES^{j|mkt} = E(r_j \,|\, \mathbb{C}(r_{mkt}))$$

- Sensitivity of firm j's return to extreme market event $\mathbb{C}$
- Market-based "stress test" of firm j's fragility

"Total directional connectedness *from*" (from-degrees)

"From others to j"

Relationship to *CoVaR*

$$VaR^{p}: \; p = P\left(r < -VaR^{p}\right)$$

$$CoVaR^{p,j|i}: \; p = P\left(r_j < -CoVaR^{p,j|i} \mid \mathbb{C}\left(r_i\right)\right)$$

$$CoVaR^{p,mkt|i}: \; p = P\left(r_{mkt} < -CoVaR^{p,mkt|i} \mid \mathbb{C}\left(r_i\right)\right)$$

- Measures tail-event linkages
- Leading choice of $\mathbb{C}\left(r_i\right)$ is a VaR breach

"Total directional connectedness *to*" (to-degrees)

"From i to others"

Estimating Global Bank Network Connectedness

- Daily range-based equity return volatilities
- Top 150 banks globally, by assets, 9/12/2003 - 2/7/2014
  - 96 banks publicly traded throughout the sample
  - 80 from 23 developed economies
  - 14 from 6 emerging economies
- Market-based approach:
  - Balance sheet data are hard to get and rarely timely
  - Balance sheet connections are just one part of the story
  - Hard to know more than the market

Many Interesting Issues / Choices

- Approximating model: **VAR**? Structural DSGE?
- Identification of variance decompositions: Cholesky? **Generalized**? SVAR? DSGE?
- Time-varying connectedness: **Rolling estimation**? Smooth TVP's? Regime switching?
- Estimation: Classical? Bayesian? **Hybrid**?
  - Selection: Information criteria? Stepwise? **Lasso**?
  - Shrinkage: BVAR? Ridge? **Lasso**?

Selection and Shrinkage via Penalized Estimation of High-Dimensional Approximating Models

$$\hat{\beta} = argmin_{\beta} \sum_{t=1}^{T} \left(y_t - \sum_{i} \beta_i x_{it} \right)^2 \quad s.t. \quad \sum_{i=1}^{K} |\beta_i|^q \leq c$$

$$\hat{\beta} = argmin_{\beta} \left(\sum_{t=1}^{T} \left(y_t - \sum_{i} \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} |\beta_i|^q \right)$$

Concave penalty functions non-differentiable at the origin produce selection. Smooth convex penalties produce shrinkage. $q \to 0$ produces selection, $q = 2$ produces ridge, $q = 1$ produces lasso.

Lasso

$$\hat{\beta}_{Lasso} = argmin_{\beta} \left(\sum_{t=1}^{T} \left(y_t - \sum_{i} \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} |\beta_i| \right)$$

$$\hat{\beta}_{ALasso} = argmin_{\beta} \left(\sum_{t=1}^{T} \left(y_t - \sum_{i} \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} w_i |\beta_i| \right)$$

$$\hat{\beta}_{Enet} = argmin_{\beta} \left(\sum_{t=1}^{T} \left(y_t - \sum_{i} \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} \left(\alpha |\beta_i| + (1-\alpha)\beta_i^2 \right) \right)$$

$$\hat{\beta}_{AEnet} = argmin_{\beta} \left(\sum_{t=1}^{T} \left(y_t - \sum_{i} \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} \left(\alpha w_i |\beta_i| + (1-\alpha)\beta_i^2 \right) \right)$$

where $w_i = 1/\hat{\beta}_i^{\nu}$, $\hat{\beta}_i$ is OLS or ridge, and $\nu > 0$.

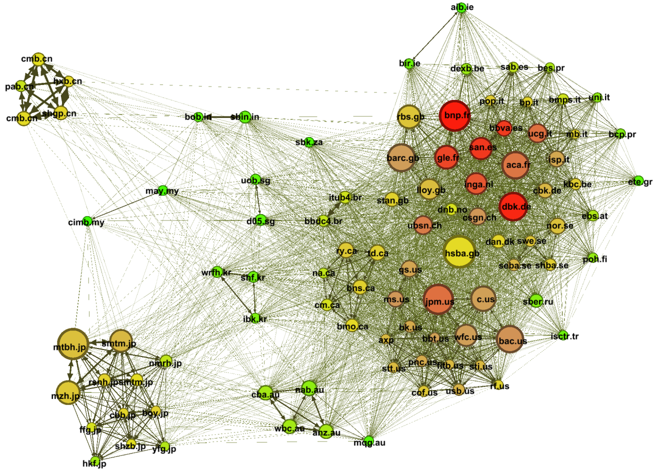
Still More Choices (Within Lasso)

- Adaptive elastic net
- $\alpha = 0.5$ (equal weight to L_1 and L_2)
- OLS regression to obtain the weights w_i
- $\nu = 1$
- 10-fold cross validation to determine λ
- Separate cross validation for each VAR equation.

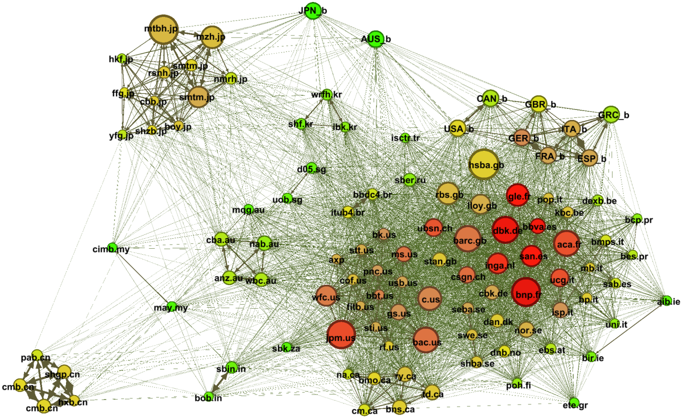
A Final Choice: Graphical Display via "Spring Graphs"

- Node size: Asset size
- Node color: Total directional connectedness "to others"
- Node location: Average pairwise directional connectedness (Equilibrium of repelling and attracting forces, where (1) nodes repel each other, but (2) edges attract the nodes they connect according to average pairwise directional connectedness "to" and "from.")
- Edge thickness: Average pairwise directional connectedness
- Edge arrow sizes: Pairwise directional connectedness "to" and "from"

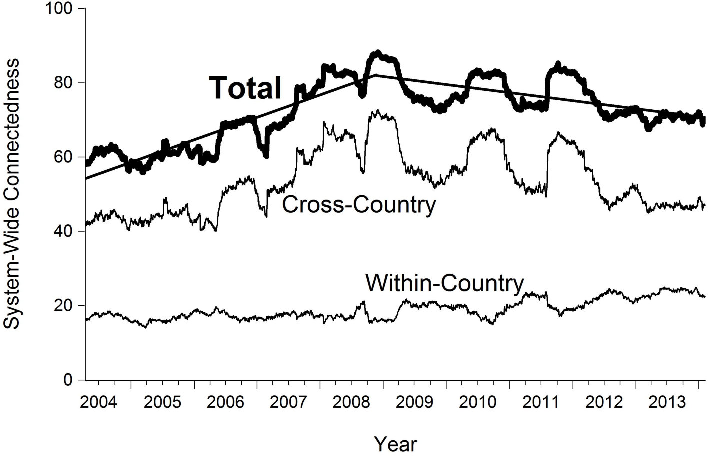
Individual Bank Network, 2003-2014



Individual Bank / Sovereign Bond Network, 2003-2014



Dynamic System-Wide Connectedness



Conclusions: Connectedness Framework and Results

- Flexible approximating models, static or dynamic estimation
- Firmly grounded in network theory
- Directional, from highly granular to highly aggregated (Pairwise "to" or "from"; total directional "to or "from"; systemwide)
- Results
- *MES* and *CoVaR* perspectives are effectively special cases
- Substantive results for global bank equity volatility connectedness