Discussion of Comin and Mulani (2006)

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Discussion outline

- Empirical relationships.
- Calibrated model predictions.
  - Social planner solution.

Firm size and productivity volatility

- COMPUSTAT: Increase in firm size and productivity volatility.
- LBD: Overall decrease in firm size and productivity volatility (Davis et al. (2006)).
  - Publicly held: increase in volatility.
  - Privately held: decrease in volatility.
  - Overall firm population trend dominated by privately held firms.
- Comin and Mulani (2006) model is not a model of publicly held firms, only.
- Aggregate growth and volatility measures include production from privately held firms.
  - Could consider producing aggregate measures on data from publicly held firms, only.

Sectoral productivity growth and R&D expenditure

- Authors find positive relationship between two-digit sectoral R&D intensity and within sector firm volatility. Adopt causal interpretation.
- What causes cross-sector R&D intensity variation?
  - Endogeneity bias?
  - Even a casual “indicative evidence” usage of this regression is probably too strong.

- Authors' model calibration is,

	1950	2000
δ_h	1.011	1.011
δ_q	1.125	1.125
λ^h	2.070	1.036
λ^q	0.020	0.050
γ_y	0.025	0.017

- Growth implication for 2000 probably a bit low.
- Mapping into model parameters? Existence?
  - Production function parameters α, β.
  - Mass of followers relative to leaders, m.
  - R&D cost and arrival process parameter, $\lambda^q = \bar{\lambda} n^q / (1 - s)$.
  - GI cost and arrival process parameters, $\lambda^h = \bar{\lambda}^h \left(n^h\right)^{\rho_h}$.

- Directly form Comin and Mulani (2006):
- Optimal GI innovation condition,

$$\frac{1}{\rho_h}\left(\bar{\lambda}^h\right)^{\frac{-1}{\rho_h}}\left(\lambda_t^h\right)^{\frac{1-\rho_h}{\rho_h}} = \frac{(1-s_t)(\delta_h - 1)}{\bar{\lambda}\delta_q} \tag{1}$$

- No arbitrage condition for R&D innovation

$$(1-s_t) = \bar{\lambda}\delta_q \frac{(1-\alpha)\chi^l - c(\lambda^h)}{r + \lambda_t^q - \lambda_t^h(\delta_h - 1)}, \tag{2}$$

where

$$\chi^l = \left(\frac{(\beta\alpha^\alpha)^{\frac{1}{1-\alpha}}}{(\beta\alpha^\alpha)^{\frac{1}{1-\alpha}} + (1-\beta)^{\frac{1}{1-\alpha}}}\right).$$

- From footnote 30, sales of leaders are 70% higher than sales of followers,

$$m = 1.7\left(\frac{1-\beta}{\beta\alpha^\alpha}\right)^{\frac{1}{1-\alpha}} \Rightarrow \chi^l = \frac{1.7}{1.7+m}. \tag{3}$$

- R&D subsidies, s_t, driving process. Given the GI innovation cost specification, there exists a solution only if,

$$\frac{1-s_t}{1-s_{t+1}} = \frac{r+\lambda^q_{t+1} - (1-\rho_h)\lambda^h_{t+1}(\delta_h - 1)}{r+\lambda^q_t - (1-\rho_h)\lambda^h_t(\delta_h - 1)} \tag{4}$$

- Assume $s_{1950} = 0$. This implies $s_{2000} = 0.3612$.
- The GI innovation cost curvature is given by,

$$\rho_h = \left[1 + \frac{\ln(1-s_t) - \ln(1-s_{t+1})}{\ln\lambda_{h,t} - \ln\lambda_{h,t+1}}\right]^{-1} = 0.6070. \tag{5}$$

- By $\alpha \in (0,1)$ it follows that $(1-\alpha)\chi^l \in (0,1)$. This establishes a lower bound on $\bar{\lambda}^h > 5.1$.

- Make the identifying assumption that $\alpha = .5$. In this case, I obtain

	$m = 2$	$m = 10$	$m = 100$	$m = 10,000$
$\bar{\lambda}^h$	12.500	25.000	92.700	1502.100
$\bar{\lambda}$	0.239	0.749	6.485	637.883

- Will use in social planner analysis.

Multiple products in a two-digit sector?

- is m large?
- Taken literally, if a U.S. two-digit sector has only one leader, m on the order of $40,000$.
- Seems like a non-starter when concerned with explaining the great diversity in size, productivity, and dynamics at the firm level in a two-digit sector.
- Rather, consider multiple products, $J = 40,000/(m+1)$. Each product has its own R&D process independent of the other products.
- In this case, variance of productivity growth within sector is,

$$V(\gamma_{y_s}) = \frac{\lambda_s^q}{J} \ln(\delta_q)^2 + \lambda^h \ln(\delta_h)^2.$$

- If J is large, all of sector volatility due to GI innovation process $\Rightarrow$ perfect co-movement. Cannot explain lower co-movement through increases in λ_s^q.

- Hamiltonian,

$$
\begin{aligned}
H = & \ln\left(1 - n^q - n^h\right) + \ln q_t + \ln h_t + \frac{1}{\alpha}\ln\left[\beta\left(x_l\right)^\alpha + (1-\beta)\left(mx_f\right)^\alpha\right] \\
& + \omega_1\left[L - x_l - mx_f\right] \\
& + \omega_2\bar{\lambda}n^q \ln\left(\delta_q\right) \\
& + \omega_3\left(1+m\right)\bar{\lambda}^h\left(\frac{n^h}{1+m}\right)^\rho \ln\left(\delta_h\right)
\end{aligned}
$$

- Given calibration, corner solution where $n^q = 0$. Optimal n^h given by,

$$\rho\bar{\lambda}^h\left(n^h\right)^{\rho-1}\ln\left(\delta_h\right) = \frac{r}{1-\left(m+1\right)n^h}.$$

- Optimal growth rates,

	$m=0$	$m=2$	$m=10$	$m=100$	$m=10,000$
γ_y	0.024	0.165	0.653	6.171	613.453

Final remarks

- Extreme planner results partly a feature of $c'(0) = 0$.
- Relationship between R&D and firm volatility less obvious in multiproduct firm models like Klette and Kortum (2004) and Lentz and Mortensen (2006).