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Computing early-warning pattern-information in the dawn of crisis: a captive monkey system and a banking system.

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[Abstract:]
Consider a system in its stable phase that is well approximated by a network that embraces a flow-based hierarchy built upon all its nodes, called power structure. In this system, all nodes can use the flow-based hierarchy to instantly access the same global collection of local information, but each node has only heterogeneous and limited computing capabilities to handle such a large amount of information. We argue that, under diverse and large enough stresses, such a system would be driven through a series of unstable phases, and commonly ending in a sudden collapse of its power structure. The manifestation of this collapse involves the loss of all long and cross-layers connectivity, while only keeping short and discrete, unconnected branches because all individuals now primarily focus on their illusive immediate survival. We further point out that such a sudden whole-scale shut-down of connectivity is by no means a collective phenomenon, and its seeming synchrony is primary due to simultaneous response to the same constraint of instant information availability.

To capture the coming of such a sudden structural change on a single network can be rather difficult. However this power structure collapsing phenomenon motivates us to couple such a network with another behavioral network, and then perform joint modeling on the pair of behavioral networks. We then extract early-warning pattern-information from observed changes in inter-behavioral associations. The fundamental concept used here is: when a system is undergoing its power structure collapsing process, joint-modeling can effectively capture the induced trend of inter-behavioral independence, which is characterized by a loss of inter-behavioral subtlety in regulation and coordination across all nodes in the dawn of a crisis.

[Section 1: Introduction].

Technological advances in phone and internet communications now permit the constant and rapid transfer of information across the globe on a sub-second basis. Thus, the availability of global information has increased so dramatically in recent years that we now live in a drastically different society than we did just 20 years ago. One of the most significant aspects that differentiates our current generation from the previous one is the instant availability of a global collection of local (GCL) information, and this information is available to almost everyone. The highly integrated and globalized nature of the banking system of today is dependent upon, and structured around, this availability of GCL (as are many other aspects of our society). Therefore, when we try to compute the intrinsic structure of the banking system and develop potential early-warning pattern-information in the dawn of a financial crisis, it is unthinkable that this aspect would be overlooked. And yet that is exactly what we see: in the literature on systemic risks to banking systems, the availability of GCL is not addressed. Most likely this is due to a lack of methodology of handling its implied global effects. In this paper we discuss one such global effect, and then develop a direction for computing early-warning pattern-information.

First, we need to look at the underlying setting of our banking system and the way in which it is modeled. For instance, electric grid networks [Amin and Schewe(2007)], banking ecosystems [Haldane and May (2010, 2011), May, et al (2008), May and Arinaminpathy (2010)], and flow networks are among many modeling approaches developed after the most recent financial crisis. The theme behind these modeling efforts is to treat the banking network as a complex dynamic system. This modeling approach has been promoted by the joint study on New Direction for Understanding Systemic Risk (National Research Council of National Academies 2007), and several conferences for this study were cosponsored by the Federal Reserve Bank of New York and the National Academy of Sciences.

All of these modeling approaches share one common structure: each node takes in only information from local interactions and local connections, and similarly delivers its reaction to only its local vicinity. This is the primary way of shock propagation and risk contagion under consideration in the entire literature. This state of research seems to reflect the definition of "systemic risk" by the Bank for International Settlement as" the risk that the failure of a participant to meet its contractual obligations may in turn cause other participants to default with a chain reaction leading to broader financial difficulties".

However, in sharp contrast, this structural assumption sounds unrealistic, or even contradictory to the real world setting where the banking system is indeed subject to the aforementioned GCL information perspective. Hence the potential implications possibly derived from the availability of global collections of local information are left unmentioned. Intuitively these potential consequences cannot only have greater impact, but can also percolate at a much faster pace than effects brewed via a slow chain reaction mechanism could possibly deliver.

Not only do current models ignore global collections of local information, but also system dynamics. One implicit assumption of the ecological, power-grid and flow-theory modeling approaches is that the network remains constant and static during the course of shock propagation [Gai and Kapadia (2010)]. This assumption almost completely ignores the "mutuality" between a node and the banking system. Under the setting of GCL, both large and small nodes directly and closely interact with the system, producing mutually reinforcing feedback loops. These nonlinear feedback loops can collectively change the network wiring construct very quickly, and such changes can occur continuously over time. Thus this is a potential underlying mechanism of the marked feature of diminishing availability of interbank loans, known as "liquidity-hoarding" or "funding liquidity shock". From our point of view, this sort of shock should not be used as the initial shock in studies of contagion of systemic risks, as in Haldane and May (2011), because this shock itself is actually one manifestation of systemic risk under study.

This feedback mechanism and the resultant changes in the network wiring all point to one reality facing researchers studying systemic risk on a banking system: current attempts at modeling the whole banking system have been rather unrealistic for this age of information, at least from the GCL information perspective. Further it might be beneficial to recognize that the endeavors on finding the early-warning pattern-information based on a model system could also be unrealistic, if not dangerously misleading.

Recent research indicates that identification and detection of early-warning pattern-information is still in its infancy, see Scheffer, et al. (2009) for a review. The shared common focus is on detecting the "tipping-points" in a closed dynamic system. It then becomes essential to detect when a system is approaching the threshold of, but has not yet transitioned into, another state, a phenomenon known as "critical slowing", in order to extract early-warning patterns. However, identifying the phenomenon of "critical slowing" is also quite difficult within a dynamic system given the fact that dynamic models used may not be able to realistically represent the banking system. Instead, it may be more beneficial to look beyond dynamic modeling for a different approach to extract realistic early-warning pattern information in the dawn of a banking crisis.

[Section 2: A banking system and a captive monkey system]
To illustrate the modeling-free approach, we first compare two systems here: one banking system and one captive monkey system.

A banking system: as a banker watches the constantly running sovereign and corporative bond's yield rates in the world-wide market on one computer screen, and prices and indexes in world-wide stock market on the other computer screen, and political and weather news channels on the others. This is the GCL information shared by everyone in every bank around the globe.

A banker constantly monitors changes from multiple sources of global collections of local information, such as bond yield rates in the world market, prices and indexes in the world stock market, political news, and weather news. Such information is available to everyone in every bank around the world. Suppose every individual banker is equipped with a device for computing the current world-wide "capital flows" at any time point by taking in GCL information. So its screen shows the flows going into "safe-havens", and contrastingly going out of financially hot spots. A banker then tinkers or changes the bank's portfolio according to this computed information and implied one from GCL information. It is important to note that, though the computing device could have heterogeneous powers depending on the size of the bank, all similar devices are made based on the same currently available banking theory, such as arbitrage pricing theory (APT).

A captive monkey system: Like the banker, a monkey is constantly watching the social interactions of its group members, such as who grooms, helps, or harasses whom. Even very large groups of 200 monkeys, individuals can easily monitor the social interactions of others and act accordingly in their own future interactions. This is also a global collection of local information.

All of this GCL information is noted, remembered, and incorporated into the monkey's perception of the current situation in its social group. Grooming behavior communicates whether kin relationships are still strong, which pairs of monkeys are in a sexual consortship, or whether a new coalition may form. Aggressive and submissive interactions suggest whether the alpha male is still well respected by the group, whether the stability of the hierarchy has changed, and if there might be opportunities to rise in rank.

Many networks in a banking system:
When looking at the interior structure of a global banking system, we see a collection of bank nodes, such as the World Bank, the International bank settlement, central banks, mega international banks, and regional banks. The network relationships among these bank nodes are defined by various mutual and contractual relations, such as Inter-bank lending, trading in Federal Funds Market, shared- holding on common assets and various other financial products and banking behaviors. The key concept here is that each type of inter-bank relationship constitutes one banking network. Lending networks describe inter-bank lending relationships, for example. Each banking network can be used to approximate a single dynamic aspect of the banking system as a whole. That is, there are versatile relationships being represented by versatile networks among all banks. These versatile banking networks as a whole constitute the multifaceted dynamics of this banking system from the behavior perspective.

Many networks in a captive monkey system:
Monkey society is also a complex system consisting of many layers of inter-behavioral relationships. Monkeys groom each other, fight with each other, offer coalitionary support during some of these fights, and exchange status signals that communicate social power and dominance. Each type of behavior can be used to construct the network of a single aspect of monkey society. A grooming network describes the grooming relationships among group members, for example. So the four corresponding behavior-specific networks together provide a very good approximation of behavioral dynamics of the society as a whole.

System crisis:
A banking crisis could be best described by a sudden, unintended system-wide loss of banking behaviors or inter-bank interactions. Monkey societies may suffer from an analogous crisis, known as cage wars or social collapse, in which serious fighting erupts because group members no longer agree on the dominance hierarchy. Typically in these societal collapses, lower-ranking monkeys attack and kill the highest ranking family, which completely disrupts the dominance hierarchy. These tragic events, though relatively infrequent, are extremely financially costly and create many management problems, as the entire group must be disbanded and relocated elsewhere.

Summarized corresponding characteristics between the banking and monkey systems:

1) Node-wise: monkeys cannot emigrate from captive confinement, while a bank is captured by its global banking network; that is a bank typically would cease to exist outside of the banking network.
2) Dyadic Behavioral relationships: both the banking and the captive monkey system is defined by many behavioral networks of interest;
3) Accessing GCL information: each monkey as well as bank can access GCL information and then make individual decisions.
4) Crisis: A social collapse is to a group of captive monkeys as a banking crisis is to a banking system.

To better see the structural similarity between these two systems, we further suggest the following potential contrasting behaviors:

1) Grooming vs. Bank loans
2) Aggression vs. derivatives
3) Alliance vs. corporate bond
4) Status vs. interbank lending (overnight) (Federal fund trading)

After seeing the structural similarity between these two systems, in the next section, we attempt to establish the common “tipping-point” structure shared by these two dynamics as “the collapse of power structure” under GCL information perspective. And then we propose that the “key idea” for computing of the pattern-information of early-warning signals for a social collapse, and suggest a similar fundamental idea for computing early-warning signals for a bank crisis.

[Section 3: Collapse of power structure as the tipping-point in the dawn of a crisis]

Each aforementioned behavior in the two systems commonly gives rise to a weighted directed graph (or network). The distinct network “flow” characteristic is our focus. A flow between two nodes is a path of successive same direction links leading from one starting node to the ending one. We show that this collection of flows collectively constitute a computable pattern, called a “power structure”. This power structure indeed functionally summarizes the behavior-specific aspect of system dynamics. It should be noted here that some behaviors are more suitable and informative than others to serve as the bases for constructing such a power structure. The choice is a judgment call based on subject matter knowledge.

The structural tipping-point may be identified, aka the “critical slowing” phenomenon, by comparing two power structures derived from networks observed at two different time points (1) when the system is stable, and (2) when it is unstable (preferably right before the crisis occurs). Using the GCL perspective, we argue that the tipping-point is most likely to be a collapse in the power structure. Furthermore, the phenomenon of critical slowing suggests that we should be able to compute, or even observe, bi-network independence in which the pattern of relationships in one network become independent of the other behavioral networks. This statistical concept can be much more easily quantified than the critical slowing for deriving the early-warning pattern-information in the next section.

[Subsection: Monkey's Power structure.]

In monkey societies, the four behaviors of grooming, aggression, alliance, and status all play an integral part in the system dynamics as a whole. However, our knowledge and understanding of monkey society tells us that status interactions are the backbone of the society, the relationships upon which the other behaviors are based. Thus, the status signaling network is the fundamental basis for the power structure underlying monkey society. Status interactions are governed by dominance – they are signals given by a subordinate animal to a dominant animal. Most important is the "silent-bared-teeth" (SBT) display, which is a peaceful communication of subordination. Because the SBT is unidirectional (always given by subordinate to dominant), primatologists often use the SBT to determine rank orders.

Further the behavioral logic goes as follows. SBTs express true dominance relationships. Once dominance is understood, dominance governs aggression, grooming, and alliances. Aggression is mostly from dominant to subordinate. Grooming is often from subordinate to dominant, unless dominants initiate grooming to reconcile after a fight. And alliances are most often made between kin, which rank near each other. That's how the status network is the base for all other behaviors. For this reason, we choose status behavior to be the base for our power structure construction.

A considerable amount of computation is required to reveal the power structure of a monkey society, just as an overall hierarchy must be carefully calculated from a collection of reliable dyadic rankings. We discuss the computation below. However, we note that the resultant pattern-information is crucial. Once a flow-based construct of power structure has been determined, this structure immediately suggests a concept of "flow topology": the flow of information through this power structure highlights topological features such as which nodes are linked in hierarchical order and separate "streams" of flow are visible. This concept indeed opens a new window for understanding a directed network. Its application is especially evident when comparing a series of two or more flow-topologies among the [more or less] same population of nodes. Pairwise comparison easily reveals changes between these two networks, and serial comparison allows further discovery of the changing dynamics.

Below we briefly describe the key steps for building such a power structure on a directed network using flow topology. A detailed description of this method can be found in a separate report by Fujii, et al. (2012, UCD manuscript). The key ingredient is to compute the conductance between each pair of nodes, regardless of whether there is an empirically observed link. Conductance, a term borrowed from electrical circuitry, describes the strength or current flowing through a potentially missing link. In an electrical network, conductance is the effective current flowing from one point to another. In a behavioral network, conductance is the strength or current that flows between any two nodes, regardless of empirically observed interaction. Thus, conductance is computed by accumulating the strength along all possible same-direction pathways leading from one node to the target node. In a directed network, conductance strength is imputed from indirect pathways between nodes, and these computed values update the conductance strength beyond the observed network. This concept says that if power information is perceived to transmit through an edge as if through an electrical circuit network, then the signal should transmit not only via direct (empirically observed) edges, but also between any two nodes indirectly.

Two approaches can be employed for computing the dyadic conductance: A) Tricking-down percolation, i.e. via Markov random walk flowing among node-media; B) Transitivity, i.e. via exhaustively computing all flows of all orders.

[A. Trickling-down Percolation].
The underlying mechanism is information transitivity that was proposed in Fushing et al. [2011a & b]. This percolation tends to go through the node-media following a downward direction. From this directional aspect, this percolation is rather unique and quite distinct from unidirectional percolation on a network.

Let the empirical (status) relational data matrix $C = [c_{ij}]$ and the Beta random field {Beta(ac_{ij}+b, ac_{ji}+b)} built onto it are the foundation of this percolation. The algorithm of this trickling down percolation is given as following steps:

- [P-0:] Consider a potential dominance action initiated by the i-th subject toward a randomly selected immediate neighbor, say j-th subject, that is, $c_{ij} > 0$. The probability of this action being successful is s_{ij} which is random simulated strengths from Beta(ac_{ij}+b, ac_{ji}+b).
- [P-1:] Generate a Bernoulli random variable $B(1, s_{ij})$ with probability $q^{(0)}(i,j)$ for the outcome "success (=1)". If it turns out to be a "failure (=0)", this trickling down process stops.
- [P-2:] Repeat the [P-0:]- [P-1:] cycle until it stops. Then record the trickling down path in a progressive fashion into a matrix format as:
  1) Let the trickling-down path be $< i -(i_1, ...i_k)-i_{k+1} >$ with $i = i_0$ and only the ending action from i_k to i_{k+1} being a failure;
  2) The percolation matrix is denoted by E_m=[$e_{\hbar l}$], initially been set to zeros for all its entries, and then the entries on i-th row and {i_1, ...i_k} columns are added by 1; the entries on i_1-th row and {i_2, ...i_k} are added by 1. Proceed this similar recording until the entry (i_{k-1}, i_k) being added by 1.
  3) we record the path-ending action by adding 1 to the entry (i_{k+1}, i_k), since it is failure.

- [P-3:] We repeat M times on the step [P-2] to construct an ensemble of trickling down paths and record them into the ensemble E_M=$\sum_{m=1}^{M} E_m$.
- [P-4:] We convert ensemble matrix E_M into an action transmission matrix $A_M = [a_{ij}]$ with $a_{ij} = \frac{E_{M,ij}}{E_{M,ij}+E_{M,ji}}$.

- [P-5:] Finally we perform the rescaling step:$DA_M = dig(\dots, \sum_{j=1} c_{ij,\dots\dots})[a_{ij}]$ as the final conductance matrix.

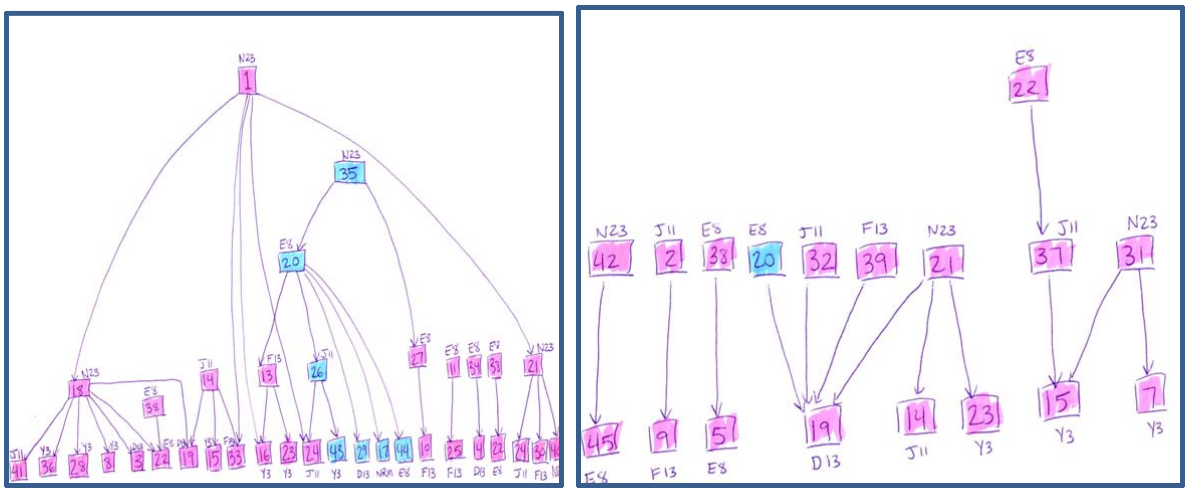
The percolation moves through the node-media via a random walk. First, the direction of the random walk is determined by randomly selecting a starting node, and next an immediate neighbor with whom to interact. Next, the length of the random walk is determined by drawing from a distribution of randomly simulated strengths (from a Beta random field), the probability of this particular interaction occurring. The random walk proceeds to this next node if the strength of the link between them is sufficiently high. These steps are repeated until the strength drawn indicates no interaction (fail), and the exact path is recorded. Importantly, the previously calculated conductance guides the random walk, making random walks more likely to occur via pathways with higher conductance strength.
The tricking-down percolation implicitly takes the series of strengths along the flow pathway into computations. The drawback is that it more often goes through "popular" and relative short paths. That is, it suffers the limitation of not finding the long and "unpopular" paths. The next approach, called transitivity, takes all paths of equal length equally. The advantage is the computing efficiency for all possible paths, but the disadvantage is ignoring the fact that some paths are more popular and should be given more weighting than others.
[B: Transitivity]

- [T-1]. For each dyad, we compute the numbers of flow paths of all different lengths.
- [T-2]. Every flow pathway contains some information about "dyadic dominance", and the longer pathway is, the less dominance is worthy. A logistic regression is performed to evaluate the net averaged dominance of each order of flow paths.
- [T-3]. The conductance is imputed by combining the predicted dominance plus the observed one.

After computing all conductance (from all flow pathways) for all possible pairs, a hierarchy flow-chart is computed which represents the power structure for the flow topology on the system, see Fujii, et al. (2012, UCD manuscript) for all details. Below we illustrate this by showing flow topologies via conductance for SBT networks from one monkey group at a stable time point (in 2009) and at an unstable time point (in 2011) four months before a cage war.

Figure 1. SBT and its power structure: (a) 2009; (b) 2011'

(a) (b)



[Subsection: Power structure comparison and its implications]
By comparing these two power structures, we see clearly, but not surprising, the following:

1) 2009 power structure seems to describe a steady Power structure among this group of monkeys during the peaceful period of time;
2) While the 2011 flow-topology strongly indicates the previous power structure has almost completely collapsed even before the cage war.

Here we make our first **<u>main conjecture</u>** of this paper:
The collapse of the power structure is chiefly due to availability of GCL information, and conversely the collapse of the power structure is the only form emerging from a crisis under GCL information perspective setting.

Our argument for this conjecture is given as follows. With available GCL information, a monkey 'computes' its perception of current societal dynamics. This is the monkey's perception of his/her position within the society as well as his/her perception of the state of the society as a whole (e.g. opportunities to rise in rank due to disintegration of others' alliances or dominance rank). This very subject-specific summarized dynamics can be very different from monkey to monkey depending on its brain size, or computing capability, personality (bold individuals may see impending collapse as an opportunity to increase rank, while fearful/gentle individuals may see the same situation as something to avoid), and its current position in the power structure (this influences which opportunities are noticed and which go unnoticed). Hence subject-specific computations of the current dynamics consist of many pieces of global and local understanding with varying degrees of correctness.

When there is a merging negative trend within the monkey society, as a merging constraint placed on all computations, every monkey would need to "simultaneously" modify its behaviors according their own computed subject-specific dynamics, which are indeed converging. The relationships going through several layers in the hierarchy of the power structure are especially prone to being terminated because more monkeys are involved, meaning having more uncertainty. For instance, a power structure contains a serial dominance (>), say A>B>C. If monkey C observes that monkey B stops showing SBT-status to A, then monkey C might run into danger of conflicting with monkey B if it indeed shows SBT to monkey A. Very importantly this kind of adapting is whole-scale under GCL setting. Hence this synchrony of acting is more of reflex to GCL information than being collectively adapting to nearest neighbors' actions.

Though this is heavily based on Gibson's (1979) idea that organisms regulate their behaviors with respect to the "affordance" of the environment (see also Barrett, et al. (2012, Phil. Trans. Royal Society B.), our emphasis here is on the aspect of synchrony.
In the face of the accumulating inertia of this negative trend, the offering from environment is depleting. Individuals synchronously behave to ensure their own stability. Hence the more subtle relationships among group members are severed almost simultaneously as each individual monkey responds to its perception of the current situation, and these subtle, indirect relationships are the ones that constituted the power structure. That is, there emerges an on-going whole-scale feedback loop

between individual decisions to change their behavior based upon the perceived environment, which augments the perception that the social environment is in upheaval, which causes more monkeys to change their behavior. This feedback loop continues until the negative inertia erupts into deleterious aggression and a cage war. Consequently a crisis occurs on top of the collapse of the power structure.

[Subsection: Potential implications on Federal Fund or interbank Market.]

The concept of this power structure collapse can have far reaching implications for studies of systemic risk. The power structure collapse in the monkey society in 2011 preceded a cage war later that year, so we can confidently postulate that the process leading to the cage war (the crisis) involved a structural phase shift. Although the society might recover from a given power structure collapse before a crisis ensues, the discovery of a series of phase-shifts in the same direction (i.e. toward loss of power structure) would indicate increasing degrees of urgency that a global crisis is at hand. This dynamic perspective could have similar bearing in most systemic risk research.

As the key means of distributing liquidity throughout the financial system, Federal Fund Market in US and Interbank markets in many advanced economic countries are important components in global banking system. The overnight lending between two banks is not typically insured. Hence it is subject to risk upon the potential of default of the borrowing bank, and the risk is reflected through the interest rate.

A power structure of such a market is would be very informative flow-topology for prescribing the state of the whole banking system at any temporal time point. Further, over a long enough time span, its topological dynamics could be very relevant to systemic risk evaluation on the scale of economic region as well as on the scale of global banking system.

Several social network studies of bank markets show that research is moving in the right direction. The “social network” topology of Austrian Interbank market was constructed in Boss et. al (2004) and of Federal Fund market in Bech and Atalay (2008). However, social network analysis does not include power structure, but only averaged summarizing statistics. These statistics miss the essence of flow dynamics contained in the data networks. On the other hand, if flow-topology can be constructed across long periods of time, then we are likely to be able to detect potential footprints or critical phases in regarding to systemic risks in dynamics of the global banking system.

[Section: Maximum entropy based joint modeling approach]

In a stable monkey society, primatologists have discovered that social dynamics are governed by a set of general rules or constraints. For example, females form close alliances with kin to defend their resources and their family rank against other families in the group. This means that aggression, status, and alliance interactions have interdependencies. Dyads that form alliances tend not to fight much (and these are likely to be kin). Aggression and status both follow dominance relationships and are primarily unidirectional. We holistically term the overall interrelationships as behavioral subtlety.

On the other hand, when the monkey society is approaching its crisis, the tipping-point embraced by the group dynamics is in the form of power structure collapse. During the unstable stage, the processing of collapsing power structure should be revealed through gradually losing the behavioral subtlety in the monkey dynamics, that is, all aspects of behavioral interdependences are getting lighter and thinner, until we see that two behavioral networks are nearly independent of each other. For example, if dominance no longer governs aggressive and status interactions, then the inter-dependence between these networks will be gradually lost, and become increasingly independent.

Though interdependences between two or multiple behaviors are not directly observable, in this paper we show how to evaluate such dynamic features by coupling multiple network data. It is noted that, since one monkey can interact with only one behavior to another monkey at a time point, there is no data of multivariate format. So the classical Pearson correlation and its variants are not applicable here. The data format is in the behavior specific networks constructed across a temporal span. Thus a new methodology is needed to evaluate interdependences among different networks. We have developed one evaluating technique based on the maximum entropy principle taken from statistical mechanics [Chan, et al, 2012]. This technique should be capable of providing essential information for early-warning pattern-information.

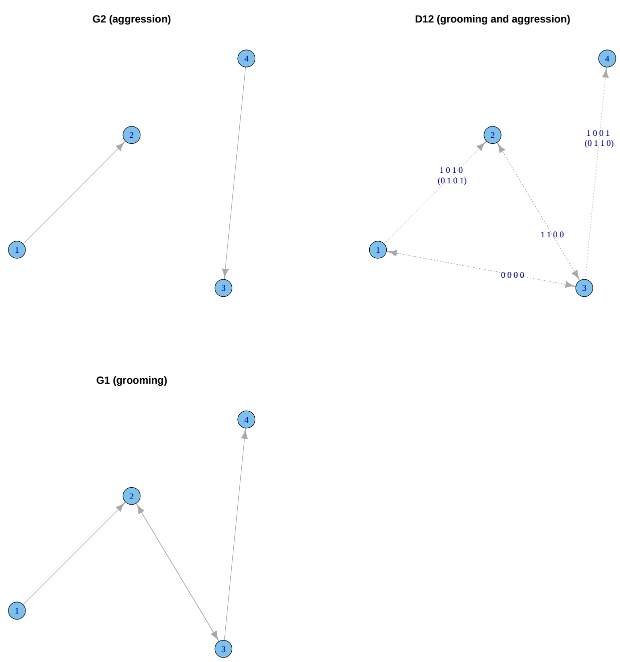
Our technique provides a new way of alleviating the well-known difficulty in capturing such a critical slowing leading to a sudden phase change in the dynamics among many hundreds or thousands of nodes. Together, the behavioral subtlety concept and the joint modeling technique have great potential for dynamic analysis in general. The concept of behavioral interdependence provides a framework by which to both characterize and evaluate the structural features of tipping-points and critical slowing phases. Therefore, given the dynamic similarity between the monkey and banking systems under the GCL setting, we believe our approach proposed here should be valuable for evaluating early-warning pattern-information in banking system.

The maximum entropy based joint modeling approach is briefly described below with derivations given in the Appendix (for full detail of about this methodology, see Chan, et al. (2012)).

As the monkey's SBT status behavior is chosen to derive the power structure, it is natural to focus on coupling the status network with grooming, aggression, and alliance networks. Here we discuss only pairwise coupling for simplicity. Our analytical technique is termed 'joint modeling' and it is based upon the maximum entropy principle. In the case of the banking system, though we have no suitable data sets for similar analysis that can be presented here, we suggest coupling the network of Federal fund or interbank markets with other banking behavioral networks, as mentioned in the previous section, for similar joint modeling analysis.

To simplify our illustration, the basic idea of jointly modeling for only two binary (un-weighted) networks, corresponding to two types of social behaviors, will be described in detail here. Our specific goal is modeling the probabilistic distribution of a link in one network being associated a link in the other network. By association, we mean that each directed link is encoded by a binary code: either 00, 01 or 11. Here the 2-dimensional binary code represents the link presence in both directions of the relationship between two nodes. Therefore, the link pair between every pair of nodes in a two-behavior network is encoded by a 4-dimensional binary code. For example, let the two behaviors be grooming and (SBT) status. A monkey dyad with mutual grooming, but no status can be represented by the 4-dimensional code vector $(1, 1, 0, 0)$ (see nodes 2, 3 in Figure 1). A pair of monkeys with opposite directional grooming and status represented by a linkage vector $(1, 0, 0, 1)$ (see nodes 3,4 in Figure 1). Thus, there are 16 possible 4-dimensional linkage vectors, although there are only 10 biologically-distinct vectors. The empirical distribution of these 10 categories of linkage vectors represents the empirical association information between these two behaviors of interest.

Figure 2.



Our maximum entropy based joint modeling is equipped with an iterative procedure to construct a distribution from a known candidate distribution by adding a new structural component at a time when approaching the good fitness of the empirical distribution. Each of the structural components, also called (inter-relational) constraint, can be taken as one learned knowledge that scientists need in order to recreate a realistic parametric probability fit. The set of components explicitly reveals all key association information embedded within the empirical distribution. The iterative steps are heuristically described as follows.

We begin with the distribution, assuming independence between the two behaviors of interest (i.e. no association among the links). First, the four empirical marginal distributions of the vector codes are calculated and then the expected probabilities (or counts) of the 10 linkage vector categories are computed by assuming that each of the links of the two networks are independent. By comparing the empirical distribution with the expected one (assuming independence), any significant discrepancy in any category indicates a missing piece of information regarding the association between the two behaviors in the null model. It should be noted, however, that this is also subject to randomness of finite sampling.

Next, to correct any significant discrepancies we need to choose a constraint function that captures the missing association, and incorporate such a chosen constraint function into the revised version of probability distribution. The latter incorporation is the work of the maximum entropy principle, which chooses the maximum entropy distribution among all distributions fitting the constraints with empirical values calculated from the data. The advantage of using maximum entropy is that no extra or artificial assumptions are taken into the modeling. We now compare this new computed maximum entropy distribution with the empirical distribution; ideally the new distribution would include the right amount of association and improved in fitting the data.

We repeat this cycle of choosing a proper constraint function to describe the discrepancy and then updating the probability distribution until the discrepancy between the overall expected and empirical counts of the 10 categories is below a critical Chi-squared percentile. We discuss our analysis results and corresponding implications in the next section.

[Section: Analysis results and conclusions]

The iterative results of joint model on status and grooming networks are reported in Table 1 and 2 on 2009 and 2011, respectively. The four constraints shown successively improve the modeling. So they can be seen as four extracted features between these two behavioral networks.

The important evidence revealed by these four features by comparing the two tables is that all four features are needed to barely improve our fitting to an acceptable level with respect to the Chi-square values on the 2009 joint model, but only the f_1 would be enough on the 2011 joint model. This clearly means that these two behaviors are complexly inter-related with each other in 2009, while this subtle complexity disappeared in 2011. Grooming and status are statistically independent in 2011, which is highly unlikely, given the dynamics of the monkey system in stable state.

If the independence phase has been gradually building for some time since 2009, then this phase can be easily detected long before the crisis event. Similar conclusions can be made by comparing the joint modeling for status and alliance in Table 3 and 4, respectively.

Table 1. Maximum entropy calculations for joint modeling of Grooming and Status networks in 2009

grooming status	Total	indep	f1	f2	f3	f4
1 0 0 0	98	153.92 (20.32)	143.27 (14.31)	140.30 (12.75)	140.12 (12.66)	130.84 (8.24)
1 1 0 0	32	5.01 (145.29)	38.86 (1.21)	38.05 (0.96)	38.01 (0.95)	35.49 (0.34)
0 0 1 0	412	400.34 (0.34)	399.28 (0.41)	462.35 (5.48)	461.77 (5.36)	444.71 (2.41)
0 0 1 1	0	33.91 (33.91)	33.82 (33.82)	5.41 (5.41)	5.41 (5.41)	5.21 (5.21)
1 0 1 0	15	13.04 (0.30)	12.14 (0.68)	14.05 (0.06)	8.41 (5.16)	13.81 (0.10)
1 0 0 1	30	13.04 (22.07)	12.14 (26.30)	14.05 (18.10)	23.41 (1.85)	38.43 (1.85)
1 1 1 0	6	0.42 (73.21)	3.29 (2.23)	3.81 (1.26)	3.81 (1.26)	6.25 (0.01)
1 0 1 1	0	1.10 (1.10)	1.03 (1.03)	0.16 (0.16)	0.16 (0.16)	0.27 (0.27)
1 1 1 1	0	0.04 (0.04)	0.28 (0.28)	0.04 (0.04)	0.04 (0.04)	0.07 (0.07)
0 0 0 0	4775	4726.30 (0.50)	4713.75 (0.80)	4616.01 (5.48)	4610.16 (5.89)	4619.64 (5.22)
total χ^2		297.0714	81.04574	49.71641	38.7586	23.72612

Table 2 Maximum entropy calculations for joint modeling of Grooming and Status networks in 2011

grooming status	Total	indep	f1	f2	f3	f4
1 0 0 0	116	149.39 (7.46)	140.81 (4.37)	140.58 (4.30)	140.42 (4.25)	137.22 (3.28)
1 1 0 0	28	4.24 (133.05)	30.22 (0.16)	30.17 (0.16)	30.14 (0.15)	29.45 (0.07)
0 0 1 0	238	240.52 (0.03)	240.11 (0.02)	247.09 (0.33)	246.81 (0.31)	242.46 (0.08)
0 0 1 1	6	11.00 (2.27)	10.98 (2.26)	5.82 (0.01)	5.82 (0.01)	5.71 (0.01)
1 0 1 0	5	6.83 (0.49)	6.44 (0.32)	6.62 (0.40)	3.29 (0.88)	4.37 (0.09)
1 0 0 1	15	6.83 (9.77)	6.44 (11.39)	6.62 (10.59)	13.29 (0.22)	17.65 (0.40)
1 1 1 0	2	0.19 (16.82)	1.38 (0.28)	1.42 (0.23)	1.42 (0.24)	1.89 (0.01)
1 0 1 1	0	0.31 (0.31)	0.29 (0.29)	0.16 (0.16)	0.16 (0.16)	0.21 (0.21)
1 1 1 1	0	0.01 (0.01)	0.06 (0.06)	0.03 (0.03)	0.03 (0.03)	0.04 (0.04)
0 0 0 0	5298	5260.61 (0.27)	5251.78 (0.41)	5242.99 (0.58)	5237.21 (0.71)	5241.20 (0.62)
total χ^2		170.4748	19.56063	16.78067	6.9537	4.810583

Table 3 Maximum entropy calculations for joint modeling of alliance and status networks in 2009

alliance status	Total	indep	f1	f2	f3	f4
1 0 0 0	87	129.10 (13.73)	121.58 (9.84)	119.06 (8.63)	118.93 (8.57)	112.50 (5.78)
1 1 0 0	26	3.49 (145.27)	28.10 (0.16)	27.51 (0.08)	27.48 (0.08)	26.00 (0.00)
0 0 1 0	423	404.68 (0.83)	403.87 (0.91)	467.66 (4.27)	467.14 (4.17)	455.09 (2.26)
0 0 1 1	0	34.28 (34.28)	34.21 (34.21)	5.48 (5.48)	5.47 (5.47)	5.33 (5.33)
1 0 1 0	25	10.94 (18.09)	10.30 (20.99)	11.93 (14.33)	20.07 (1.21)	30.57 (1.01)
1 0 0 1	12	10.94 (0.10)	10.30 (0.28)	11.93 (0.00)	7.07 (3.44)	10.77 (0.14)
1 1 1 0	3	0.30 (24.75)	2.38 (0.16)	2.76 (0.02)	2.75 (0.02)	4.19 (0.34)
1 0 1 1	0	0.93 (0.93)	0.87 (0.87)	0.14 (0.14)	0.14 (0.14)	0.21 (0.21)
1 1 1 1	0	0.03 (0.03)	0.20 (0.20)	0.03 (0.03)	0.03 (0.03)	0.05 (0.05)
0 0 0 0	4803	4777.47 (0.14)	4767.90 (0.26)	4669.04 (3.84)	4663.82 (4.15)	4670.97 (3.73)
total χ^2		238.1352	67.86995	36.82985	27.28788	18.85823

Table 4. Maximum entropy calculations for joint modeling of Alliance and Status networks in 2011

alliance status	total	indep	f1	f2	f3	f4
1 0 0 0	127	157.15 (5.79)	148.50 (3.11)	148.25 (3.05)	147.97 (2.97)	146.89 (2.69)
1 1 0 0	28	4.71 (115.20)	28.30 (0.00)	28.26 (0.00)	28.20 (0.00)	28.00 (0.00)
0 0 1 0	239	239.78 (0.00)	239.50 (0.00)	246.45 (0.23)	245.98 (0.20)	244.60 (0.13)
0 0 1 1	6	10.96 (2.25)	10.95 (2.24)	5.81 (0.01)	5.80 (0.01)	5.76 (0.01)
1 0 1 0	17	7.19 (13.41)	6.79 (15.36)	6.99 (14.35)	16.88 (0.00)	18.38 (0.10)
1 0 0 1	3	7.19 (2.44)	6.79 (2.11)	6.99 (2.27)	2.88 (0.00)	3.14 (0.01)
1 1 1 0	1	0.22 (2.86)	1.29 (0.07)	1.33 (0.08)	1.33 (0.08)	1.45 (0.14)
1 0 1 1	0	0.33 (0.33)	0.31 (0.31)	0.16 (0.16)	0.16 (0.16)	0.18 (0.18)
1 1 1 1	0	0.01 (0.01)	0.06 (0.06)	0.03 (0.03)	0.03 (0.03)	0.03 (0.03)
0 0 0 0	5276	5244.62 (0.19)	5238.33 (0.27)	5229.56 (0.41)	5219.56 (0.61)	5220.95 (0.58)
total χ^2		142.4655	23.5324	20.59748	4.070777	3.872492

Table 5 reveals that the 2009 inter-relationship between status and aggression behaviors is rather complicated. The Chi-square values are reduced step-by-step, though they never reach the critical level. In other words, these four constraint functions are not the correct features for the 2009 network data. But these constraints suitable for the same inter-relationship in 2011. Hence this comparison of joint modeling between 2009 and 2011 on status and aggression behaviors indeed again provides us the same characteristic evidence: inter-behavioral relationships are much more complex and subtle in 2009, while the subtlety is lost by 2011. Along this line of argument, we are confident to detect early-warning pattern-information in the captive monkey dynamic system. We are also confident that similar computations would potentially lead to computing for early-warning pattern-information in banking system.

Table 5. Maximum entropy calculations for joint modeling of Aggression and Status networks in 2009

aggression status	total	indep	f1	f2	f3	f4
1 0 0 0	332	488.56 (50.17)	433.97 (23.96)	424.97 (20.34)	412.90 (15.85)	384.25 (7.10)
1 1 0 0	114	59.64 (49.56)	146.38 (7.16)	143.35 (6.01)	139.27 (4.59)	129.61 (1.88)
0 0 1 0	289	339.03 (7.38)	340.93 (7.91)	394.78 (28.34)	383.56 (23.31)	356.15 (12.66)
0 0 1 1	0	28.72 (28.72)	28.88 (28.88)	4.62 (4.62)	4.49 (4.49)	4.17 (4.17)
1 0 1 0	147	41.38 (269.55)	36.76 (330.60)	42.57 (256.22)	155.97 (0.52)	193.60 (11.22)
1 0 0 1	2	41.38 (37.48)	36.76 (32.87)	42.57 (38.66)	10.97 (7.33)	13.61 (9.91)
1 1 1 0	25	5.05 (78.78)	12.40 (12.81)	14.36 (7.89)	13.95 (8.75)	17.32 (3.41)
1 0 1 1	0	3.51 (3.51)	3.11 (3.11)	0.50 (0.50)	0.48 (0.48)	0.60 (0.60)
1 1 1 1	0	0.43 (0.43)	1.05 (1.05)	0.17 (0.17)	0.16 (0.16)	0.20 (0.20)
0 0 0 0	4225	4002.40 (12.38)	4024.83 (9.96)	3941.38 (20.41)	3829.40 (40.87)	3863.96 (33.73)
total χ^2		537.9403	458.3071	383.1573	106.3561	84.88729

Table 6. Maximum entropy calculations for joint modeling of Aggression and Status networks in 2011

aggression status	total	indep	f1	f2	f3	f4
1 0 0 0	372	456.74 (15.72)	426.28 (6.91)	425.56 (6.74)	419.23 (5.32)	404.69 (2.64)
1 1 0 0	80	45.30 (26.57)	88.20 (0.76)	88.06 (0.74)	86.75 (0.52)	83.74 (0.17)
0 0 1 0	172	210.53 (7.05)	211.36 (7.33)	217.50 (9.52)	214.26 (8.34)	205.01 (5.32)
0 0 1 1	2	9.63 (6.04)	9.66 (6.08)	5.13 (1.91)	5.05 (1.84)	4.83 (1.66)
1 0 1 0	78	20.88 (156.23)	19.49 (175.66)	20.06 (167.41)	79.89 (0.04)	95.58 (3.23)
1 0 0 1	3	20.88 (15.31)	19.49 (13.95)	20.06 (14.50)	4.89 (0.73)	5.85 (1.39)
1 1 1 0	7	2.07 (11.73)	4.03 (2.18)	4.15 (1.96)	4.09 (2.07)	4.89 (0.91)
1 0 1 1	2	0.95 (1.14)	0.89 (1.38)	0.47 (4.94)	0.47 (5.06)	0.56 (3.74)
1 1 1 1	0	0.09 (0.09)	0.18 (0.18)	0.10 (0.10)	0.10 (0.10)	0.12 (0.12)
0 0 0 0	4734	4604.85 (3.62)	4622.87 (2.67)	4615.13 (3.06)	4546.47 (7.74)	4562.17 (6.47)
total χ^2		243.5242	217.1091	210.8734	31.75803	25.63412

[Discussion]

An important question about the dynamics of a banking system is: what does the tipping-point look like? Many suggestions have been proposed. Here we cite one description of the tipping point from Prof. G. Sugihara's article in 2012 Seed Magazine:

"… Indeed, with regard to risk management through diversification, it is ironic that diversification become so extreme that diversification was lost:

everyone owning part of everything creates complete homogeneity. ….".

And then comes the common saying: "homogeneity breeds disaster". So if complete portfolio homogeneity is taken as a tipping-point in banking dynamics, then we might need to measure and monitor the trajectory of the degree of homogeneity via either asset or liability sides of the portfolio of all involved banks in order to detect the presence of its critical slowing phase. From our perspective, portfolio homogeneity may not be the true underlying mechanism which leads to a crisis, but rather the collapse of the power structure.

Another natural question is what role does an individual node play in the banking system? How does an individual bank's behavior fit in with our understanding of the system? We suggest that any answer that does not incorporate GCL information (which is not a node-centric perspective) is not a mechanistic answer.

Hence we like to conclude that a banking system is more like a captive monkey system and less like an electric power grid system being studied by electrical engineers, or an ecological system by biologists. Both electrical power grids and ecological systems assume constant network structure, and this is unrealistic for a banking system which we know is quite dynamic. Although such constant-structure models could be relevant to a financial trouble in a small scale, they are certainly not equipped to handle the global one.

Further we should ask: are our proposed properties realistic enough to be transformed into realistic understanding toward such a complex banking system? Here we do not pretend to know the answer to this question. However we do believe that, as far as approaching a crisis is concerned, the two systems considered here share similar important ingredients in dynamics: the ability to access the global collection of local (GCL) information among all nodes, big or small, and the power structure collapse as their tipping-point. Hence we confidently suggest our early-warning pattern-information to researchers with interest in banking system.

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[Appendix (Chan, et al, 2012): Derivation of Maximum Entropy Procedure]

To simplify notation, we let $\boldsymbol{x}$ be $(x_1^{12}, x_2^{12}, x_3^{12}, x_4^{12})$, the four dimensional vector. We maximize the relative entropy for $\mathbb{p}_{\mathrm{m}}$ by maximizing

$$\sum \mathbb{p}_{\mathrm{m}}(\boldsymbol{x}) \log\left(\frac{\mathbb{p}_{\mathrm{m}}(\boldsymbol{x})}{\mathbb{p}_0(\boldsymbol{x})}\right)$$

summed over all probabilities where $\mathbb{p}_0$ is the null probability distribution and $\mathbb{p}_{\mathrm{m}}$ probability distribution with maximum entropy subject to the constraints of the data, and the constraint These constraints are

$$\widehat{E_1}[f_1(\boldsymbol{x})] = \sum f_1(\boldsymbol{x}) \frac{d(\boldsymbol{x})}{n_{12}}$$

where the expectation is determined by

$$\widehat{E_1}[f_1(\boldsymbol{x})] = \sum_{\boldsymbol{x}} \mathbb{p}_1(\boldsymbol{x}) f_1(\boldsymbol{x})$$

We thus have two constraints:

$$\sum_{\boldsymbol{x}} \mathbb{p}_1(\boldsymbol{x}) f_1(\boldsymbol{x}) = \widehat{E_1}[f_1(\boldsymbol{x})]$$

and the sum all probabilities is 1:

$$\sum_{\boldsymbol{x}} \mathbb{p}_1(\boldsymbol{x}) = 1$$

Therefore we can maximize the entropy using the Lagrange operation

$$L(\mathbb{p}_1, \lambda_1, \mu) = \sum \mathbb{p}_1(\boldsymbol{x}) \log\left(\frac{\mathbb{p}_1(\boldsymbol{x})}{\mathbb{p}_0(\boldsymbol{x})}\right)$$

$$-\lambda_1\left(\sum_{\boldsymbol{x}} \mathbb{p}_1(\boldsymbol{x}) f_1(\boldsymbol{x}) - \widehat{E_1}[f_1(\boldsymbol{x})]\right) - \mu\left(\sum_{\boldsymbol{x}} \mathbb{p}_1(\boldsymbol{x}) - 1\right)$$

We take the derivative of the Lagrange operation to get

$$\frac{\partial}{\partial \mathbb{p}_1} L(\mathbb{p}_1, \lambda_1, \mu) = \log\left(\frac{\mathbb{p}_1(\boldsymbol{x})}{\mathbb{p}_0(\boldsymbol{x})}\right) + 1 - \lambda_1 f_1(\boldsymbol{x}) - \mu = 0$$

By solving this equation for $\mathbb{p}_1$, we get

$$\mathbb{p}_1(\boldsymbol{x}) = \mathbb{p}_0(\boldsymbol{x})\text{exp}\big(-\lambda_1 f_1(\boldsymbol{x})\big)\text{exp}(-\mu + 1)$$

Let $Z(\lambda_1) = \sum_{\boldsymbol{x}} \mathbb{p}_0(\boldsymbol{x})\text{exp}\big(-\lambda_1 f_1(\boldsymbol{x})\big)$**,** which is called the partition function**.** Applying the constraint that all probabilities must sum to 1, we determine that

$$\text{exp}(-\mu + 1) = \frac{1}{Z(\lambda_1)}$$

Then applying the first constraint, we get

$$\sum_{\boldsymbol{x}} \frac{1}{Z(\lambda_1)} \mathbb{p}_0(\boldsymbol{x})\text{exp}\big(-\lambda_1 f_1(\boldsymbol{x})\big) f_1(\boldsymbol{x}) = \widehat{E_1}[f_1(\boldsymbol{x})]$$

which is equivalent to

$$\frac{\partial}{\partial \lambda_1} \log Z(\lambda_1) = \widehat{E_1}[f_1(\boldsymbol{x})]$$

In order to find
$\mathbb{p}_1(x_1^{12}, x_2^{12}, x_3^{12}, x_4^{12}) = \mathbb{p}_1(\boldsymbol{x}) = \frac{1}{Z(\lambda_1)} \mathbb{p}_0(\boldsymbol{x})\text{exp}\big(-\lambda_1 f_1(\boldsymbol{x})\big)$ we solve for λ_1 by the previous equation. This process can be repeated iteratively for each f_k and$\mathbb{p}_\mathrm{k}$.