Deficits and Inflation: HANK meets FTPL[*]

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Abstract

In HANK models, fiscal deficits drive aggregate demand and thus inflation because households are non-Ricardian; in the Fiscal Theory of the Price Level (FTPL), they instead do so via equilibrium selection. *Because* of this difference, the mapping from deficits to inflation in HANK is robust to active monetary policy and free of the controversies surrounding the FTPL. *Despite* this difference, a benchmark HANK model with sufficiently slow fiscal adjustment predicts just as much inflation as the FTPL. This is true even in the simplest FTPL scenario, in which deficits are financed *entirely* by inflation and debt erosion. In practice, however, unfunded deficits are likely to trigger a persistent boom in real economic activity and thus the tax base, substituting for debt erosion. In our quantitative explorations, this reduces the inflationary effects of unfunded deficits by about half relative to that simple FTPL arithmetic.

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1 Introduction

Do fiscal deficits drive inflation? And if so, how, and how much? One answer is provided by the Fiscal Theory of the Price Level (FTPL): deficits not backed by commensurate future surpluses *must* be accompanied by an increase in nominal prices and a corresponding drop in the real value of the outstanding nominal debt.[1] This theory has received much attention following the recent inflationary episode (Bianchi et al., 2023; Anderson and Leeper, 2023; Barro and Bianchi, 2024), yet it remains controversial, because of its reliance on subtle, untestable assumptions regarding equilibrium selection (Kocherlakota and Phelan, 1999; Buiter, 2002; Canzoneri et al., 2001; Niepelt, 2004; Atkeson et al., 2010). Another answer is provided by mainstream Keynesian logic: once Ricardian Equivalence fails due to liquidity constraints, finite lives, or imperfect foresight, fiscal deficits naturally stimulate aggregate demand and can thereby lead to an inflationary boom. This mechanism is absent in the textbook New Keynesian model, because households there are Ricardian, but it lies at the heart of both the old IS-LM framework and the modern Heterogeneous Agent New Keynesian (HANK) literature.

This paper builds a bridge between the FTPL and the Keynesian accounts of how fiscal deficits drive inflation. We establish an equivalence result: *despite* the different mechanisms at work, a benchmark HANK model with sufficiently delayed fiscal adjustment can feature just as much inflation as what is predicted by the FTPL. At the same time, *because* of the difference in mechanism, HANK naturally sidesteps the controversies surrounding the FTPL: the deficit-inflation mapping is now grounded on testable assumptions about consumer behavior, is consistent with a "dominant" monetary authority, and is robust to plausible equilibrium refinements.

We complement these lessons with another, more practical, takeaway. The simplest FTPL arithmetic stipulates that unfunded deficits induce an exactly offsetting increase in nominal prices. Although our HANK-FTPL equivalence result holds even in this extreme scenario, in practice unfunded deficits can partially finance themselves by triggering a boom in real economic activity and thus the tax base (Angeletos et al., 2024). In a variety of empirically disciplined quantitative exercises, this channel cuts down deficit-driven inflation by around one half relative to the simple FTPL arithmetic.

Environment. For our baseline analytical results, we consider an overlapping generations New Keynesian model, similarly to Farhi and Werning (2019), Galí (2021), Aguiar et al. (2024) and Angeletos et al. (2024). As in those papers, finite lives can be interpreted as a proxy for borrowing constraints. When households live infinitely, our model reduces to RANK—the standard, representative-agent, New Keynesian model. Otherwise, our model emulates HANK.

[1] This basic prediction holds in both the flexible-price version of the FTPL (Leeper, 1991; Sims, 1994; Woodford, 1995; Bassetto, 2002; Cochrane, 2005; Kaplan et al., 2023) and its modern, sticky-price incarnation (Bianchi and Ilut, 2017; Bianchi et al., 2023; Cochrane, 2017, 2018, 2023). In this paper we are concerned exclusively with the latter.

The two core questions that we study are *how* and by *how much* inflation responds to fiscal deficit shocks, here modeled for concreteness as unexpected, deficit-financed, lump-sum transfers to households (i.e., "stimulus checks"). We start our analysis by noting that inflation is uniquely determined by real economic activity, via the familiar New Keynesian Phillips Curve (NKPC). It follows that fiscal deficits can drive inflation *only if* they also drive consumption, employment, and output—meaning that, in both RANK and HANK, a failure of Ricardian Equivalence is *necessary* for deficits to be inflationary. This elementary observation will help explain both the tensions faced by the prevailing formalization of the FTPL and the robustness that is provided by our HANK alternative.

RANK-FTPL. In RANK, households are Ricardian as in Barro (1974): they have infinite horizons, can freely borrow and save at the same interest rate as the government, and are rational enough to understand that fiscal policy does not have wealth effects in equilibrium.[2] Nevertheless, because RANK generally admits multiple equilibria, it is possible for Ricardian Equivalence to fail, and so for deficits to drive inflation, through appropriate equilibrium selection. Standard practice (e.g., as in Galí, 2008) rules this out by assuming an "active-monetary, passive-fiscal" policy regime; this selects an equilibrium in which Ricardian Equivalence is preserved, and thus fiscal deficits have no effect on inflation. If, however, one assumes the opposite scenario of an "active-fiscal, passive-monetary" regime, then a different equilibrium—the FTPL equilibrium—is selected; in this equilibrium, Ricardian equivalence fails by the amount necessary to ensure government budget balance.

The sharpest and most familiar version of the FTPL equilibrium obtains when real rates and future tax revenue are held fixed in response to deficit shocks. In this case, which we refer to as the simple FTPL arithmetic, the entirety of a deficit shock is financed by an exactly offsetting jump in nominal prices and, thereby, the real value of public debt. For example, a deficit shock equal to one percent of GDP induces a price jump equal to the inverse of the debt-to-GDP ratio. More generally, the FTPL equilibrium allows a deficit shock to be partially financed by a reduction in the government's cost of borrowing and a boom in the tax base, reducing the need for debt erosion and hence for inflation. The mechanism, however, remains the same: even though households are Ricardian in the textbook sense of Barro (1974), Ricardian Equivalence fails through equilibrium selection, with equilibrium output and prices adjusting by exactly the amount necessary to substitute for the missing tax hikes.

The FTPL's controversies—and our own angle. The FTPL's account of *how*, and by *how much*, fiscal deficits drive inflation has been subject to controversy. The most familiar debate is whether the FTPL relies on an off-equilibrium threat to "blow up the government budget" (Kocherlakota and Phelan,

[2] This statement may appear to contradict an argument from Cochrane (2005, 2023) about how public debt enters aggregate demand in RANK-FTPL. That argument is a conjecture about how adjustment happens *off* equilibrium. We instead reason about what happens *on* equilibrium—and for *any* equilibrium. See Section 3.3 for details.

1999; Bassetto, 2002; Buiter, 2002; Cochrane, 2005; Niepelt, 2004; Atkeson et al., 2010). Building on the global games literature, Angeletos and Lian (2023) have furthermore argued that the FTPL equilibrium is not robust to small noise anchoring far-ahead beliefs. We here establish a related fragility to a plausible equilibrium refinement: the FTPL equilibrium is not robust, and deficits do not have any effect on output and inflation, if consumers expect the economy to return to steady state at an arbitrarily long but *finite* horizon, as opposed to mere *asymptotic* convergence.

Our paper shifts the focus away from these debates and towards the FTPL's predictions about how much inflation responds to an increase in deficits. As explained next, our main result is that the same predictions—i.e., the same "how much"—arise robustly in HANK, yet without any of the above controversies, precisely because in HANK the mechanism—i.e., the "how"—is very different.

HANK meets FTPL, without the controversies. In HANK, fiscal deficits influence aggregate demand, and thus output and inflation, via a classical non-Ricardian channel—by shifting the tax burden to future generations or, less literally, by relaxing liquidity constraints. In the HANK equilibrium studied in Section 4 and the rest of the paper, fiscal policy's impact on output and inflation operates *exclusively* via this classical, non-Ricardian mechanism, and not via equilibrium selection as in RANK-FTPL. Our first result is that, *despite* this difference in mechanism, HANK can actually replicate the FTPL's core prediction about how much fiscal deficits contribute to inflation.

We begin by proving this result in the special case of a monetary authority that stabilizes the real rate. For a given deficit shock, consider reducing the speed of fiscal adjustment—i.e., shift any potential tax hikes further into the future. Because households are non-Ricardian, this shift contributes to a larger and more persistent real boom, and hence also to a larger jump in prices. We show that, with short-term debt, the date-0 price jump and the corresponding debt erosion converge monotonically to their FTPL counterparts as the fiscal adjustment is delayed more and more. This equivalence holds independently of the strength of the tax base channel (i.e., the automatic increase in tax revenue that is caused by an increase in real economic activity); if this channel is muted, then the jump in prices *entirely* finances the deficit, as in the simple FTPL arithmetic. Finally, the equivalence extends to more complex monetary policies, provided that the monetary authority does not raise rates too aggressively during booms. Intuitively, if the real rate response is sufficiently muted, then it is still feasible for the fiscal authority to delay adjustment to the far-ahead future, and so HANK continues to predict the same inflation as a comparable RANK-FTPL scenario featuring the same movements in real rates.

Our second main result is that, *because* of the difference in mechanism, HANK avoids the controversies surrounding the FTPL. This robustness is most easily understood in a particular active-monetary, passive-fiscal scenario: the fiscal authority commits to repaying any accumulated debt within a sufficiently long (but still finite) horizon, and at the same date the monetary authority becomes active. This change in policy rules out any effect of deficits on output and inflation in RANK, but has only a vanishingly small effect in HANK. Intuitively, this is because non-Ricardian households in HANK respond to concurrent transfers while heavily discounting tax hikes in the distant future. The same logic explains why HANK is also robust to the refinements of far-ahead beliefs discussed above.

Further results. In addition to delivering robustness, HANK also implies that the inflationary effects of fiscal deficits are more front-loaded and short-lived than in FTPL. Intuitively, again because of discounting, non-Ricardian households spend any innovation in current disposable income *rapidly*, in line with the empirical evidence. This front-loading is further reinforced, without affecting our main results, in an extension that allows for realistic heterogeneity: if government bonds are predominantly held by households with low marginal propensities to consume (MPCs), while transfers are received by high-MPC households, then the fiscally-led inflation burst gets front-loaded even more.

In our final two extensions we allow for long-term government debt and for inertia in inflation (via a hybrid NKPC). With long-term debt, debt erosion depends on the *cumulative* inflation triggered by a deficit shock.[3] If the tax base channel is switched off, then this cumulative inflation is again the same in FTPL and HANK (with delayed fiscal adjustment). But if instead it is operative, then the inflation response is now *smaller* in HANK, precisely because the inflation burst is short-lived—which, with long-term debt, leaves less room for debt erosion relative to tax-base financing. Accommodating realistic inertia in inflation instead has the opposite effect: HANK's more transitory boom now leads to greater short-run inflation, *increasing* the scope for debt erosion relative to tax-base financing.

Quantification. Our theoretical analysis has revealed that the simple FTPL arithmetic—i.e., the prediction that prices jump by exactly enough to finance an unfunded fiscal deficit—can also emerge in HANK. Our final contribution, in Section 6, is a quantitative evaluation of this possibility.

This evaluation is based on a richer, and empirically disciplined, variant of our baseline model, featuring: intertemporal marginal propensities to consume (iMPCs) consistent with empirical evidence (taken from Fagereng et al., 2021); plausible heterogeneity in fiscal transfer incidence and nominal wealth; an estimated hybrid NKPC (taken from Barnichon and Mesters, 2020); a realistic average maturity for government debt; and a meaningful feedback from economic activity to fiscal surpluses.

Our headline finding is that, in response to a fiscal deficit shock, and even with a relatively steep Phillips curve, debt-erosion-relevant cumulative inflation responses are reduced by about half compared to the simple FTPL arithmetic. This is due to two reasons, both already mentioned above. First, deficits partially finance themselves by triggering a boom in real economic activity and, consequently, the tax base. Second, the inflation burst front-loading implied by HANK helps moderate cumulative

[3] To be precise, the relevant cumulative measure discounts future inflation at a rate that reflects the maturity structure of government debt; see Section 5.1 for details.

inflationary pressures through its interaction with long-term debt, and this effect is only partially offset by the impact of inflation inertia. We further show that these conclusions extend to a range of model variants, and finally close with a quantitative application to post-covid inflation.

Related literature. This paper builds on our earlier work on the self-financing of fiscal deficits (Angeletos et al., 2024). Our core contributions here are: to establish equivalence in the inflationary effects of deficits between HANK and RANK-FTPL; to elaborate on the difference in mechanism between the two theories; and to show that this difference explains HANK's greater robustness. The same points separate our work from Auclert et al. (2024), whose Intertemporal Keynesian Cross prism we leverage here; from Aguiar et al. (2024), who employ an overlapping generations setting like ours; and more broadly from a large and diverse literature that studies fiscal policy in New Keynesian settings with non-Ricardian features (e.g., Galí et al., 2007; Bilbiie, 2020; Kaplan et al., 2018; Eusepi and Preston, 2018; Hagedorn et al., 2019; Campos et al., 2024). To the best of our knowledge, our lessons regarding HANK-FTPL equivalence and robustness are new to this literature.[4]

Our paper also offers a new angle on a literature that structurally estimates different policy regimes within RANK (Bianchi and Ilut, 2017; Bianchi et al., 2023; Smets and Wouters, 2024). In light of our results, the empirical patterns that this literature attributes to active fiscal policy could also be rationalized by a classical failure of Ricardian Equivalence. This helps insulate that literature's applied lessons on the fiscal origins of inflation from the theoretical controversies surrounding the FTPL.

Finally, our paper adds to a topical literature on the post-covid inflationary episode. While some research has emphasized the connection to the FTPL (e.g., Bianchi et al., 2023; Anderson and Leeper, 2023; Barro and Bianchi, 2024; Bigio et al., 2024), much of the policy debate has remained anchored in conventional Keynesian logic (e.g., Blanchard, 2021; Summers, 2021; Bernanke and Blanchard, 2024). Our contribution here is threefold. First, we show that the gap between the two perspectives may be much smaller than previously thought. Second, we offer a quantitive evaluation of the inflationary effects of unfunded stimuli, based on empirically disciplined HANK models. And third, we show that the empirical patterns identified in Barro and Bianchi (2024) are broadly consistent with HANK.

Outline. Section 2 introduces our baseline model. Section 3 reviews RANK's conventional and FTPL solutions. Section 4 presents our main results: it shows how HANK can replicate FTPL's core prediction, albeit via a different and more robust mechanism. Section 5 discusses several extensions, setting the stage for the richer quantitative explorations in Section 6. Finally Section 7 concludes. Proofs and supplementary results are provided in several appendices.

[4]An additional contribution is to illustrate that, compared to the simpler model used in our theoretical analysis, accommodating realistic heterogeneity in wealth, MPCs, and transfer incidence appears to have a limited effect on the *cumulative* inflation generated by unfunded fiscal stimuli, even though it affects their *dynamic* propagation. This in turn complements Kaplan et al. (2023), who study how such heterogeneity matters in a flexible-price version of the FTPL.

2 Environment

For our main analysis, we consider a perpetual-youth, overlapping-generations (OLG) version of the New Keynesian model, where finite lives can also be interpreted as a proxy for liquidity frictions (as, e.g., in Farhi and Werning, 2019; Angeletos et al., 2024). Since the micro-foundations are standard, we delegate the detailed set-up to Appendix A.1. In the main text, we instead work with the relevant log-linearized relations. By the same token, the equilibria characterized in this paper should be interpreted as approximations of the corresponding non-linear equilibria around a steady state in which inflation is zero, real allocations are given by their flexible-price counterparts, and government debt is fixed at some arbitrary level. Finally, time is discrete, indexed by $t \in \{0, 1, ...\}$; uppercase variables denote levels; and lowercase variables denote (log-)deviations from steady state.[5]

2.1 Aggregate demand

The model environment builds on Angeletos et al. (2024). The economy is populated by a unit continuum of households. A household survives from one period to the next with probability $\omega \in (0, 1]$ and is replaced by a new one whenever it dies. Households have standard separable preferences over consumption and labor, and can save and borrow through an actuarially fair, risk-free, nominal annuity, backed by government bonds. To facilitate aggregation, we assume that all households receive the same dividend payments, pay the same taxes, face the same wage, and supply the same (union-intermediated) labor. Finally, we abstract from the steady-state implications of finite lives (or other non-Ricardian effects), and let all cohorts have the same wealth in steady state, by assuming that old households make appropriate, time-invariant, contributions to a social fund whose proceeds are distributed to newborn households.[6]

Deriving the (log-linearized) consumption function of each household, and aggregating across households, we obtain the following aggregate consumption function:

$$c_t = (1 - \beta\omega)\left(a_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k(y_{t+k} - t_{t+k})\right]\right) - \beta\left(\sigma\omega - (1 - \beta\omega)\frac{A^{ss}}{Y^{ss}}\right)\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k r_{t+k}\right], \tag{1}$$

where c_t is aggregate consumption, a_t is real private wealth, y_t is real private income (labor income plus dividends), t_t is real tax payments, r_t is the expected real rate of interest, σ is the elasticity of

[5]To accommodate the case of zero debt, all fiscal and household wealth variables will be measured in *absolute* deviations from the steady state, scaled by steady-state output; all other variables will instead be measured in log-deviations.

[6]This assumption makes sure that the flexible-price steady state is invariant to both ω and the real level of government debt—which in turn means that the point around which we log-linearize our economy remains the same as we vary either ω or the fiscal and monetary policies. A different question, outside the scope of our paper, is how non-Ricardian effects influence the steady state itself and how this in turn may interact with price-level determination when prices are flexible. See Hagedorn (2016) and Kaplan et al. (2023) for two contributions in this direction.

intertemporal substitution, A^{ss}/Y^{ss} is the steady state wealth-to-income ratio, β is the discount factor (also the reciprocal of R^{ss}, the steady-state gross real interest rate), and $\mathbb{E}_t$ is the rational-expectations operator. Equation (1) generalizes the familiar infinite-horizon Permanent Income Hypothesis (PIH): the first term in the right hand side captures financial wealth and permanent income, and the second term captures the substitution and wealth effects of real interest rates.

Connection to HANK. As we move from $\omega = 1$ to $\omega < 1$, our model incorporates two key—and empirically relevant—properties of consumption behavior: (i) households discount future income and future taxes at a rate higher than the steady-state interest rate; (ii) relative to the PIH benchmark, households exhibit a higher MPC out of current income and current wealth. As will become clear, all our conclusions regarding HANK derive from these two properties. While these properties are modeled here as a result of finite lives, they can also be framed as the outcome of liquidity constraints (as in Farhi and Werning, 2019), and they extend naturally to a broad class of HANK models (e.g., see Kaplan et al., 2018; Auclert et al., 2024). An obvious limitation is that our model abstracts from heterogeneity in wealth, marginal propensities to consume, and exposure to fiscal transfers. However, as shown in Sections 5.2 and 6, these abstractions do not cause serious loss for our purposes.

2.2 Aggregate supply

The production side of the economy is the same as in the textbook New Keynesian model: there is a unit-mass continuum of monopolistically competitive retailers, who set prices subject to the standard Calvo friction, hire labor on a spot market, produce according to a technology that is linear in labor, and then pay out all their profits as dividends back to the households. Together with our assumptions about labor supply and time-invariant tax distortions, this guarantees that the supply block of our economy reduces to the standard New Keynesian Phillips curve (NKPC):

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t [\pi_{t+1}], \tag{2}$$

for some $\kappa > 0$ that captures the degree of price flexibility. In Section 5.3, we show that the essence of our analysis remains the same if (2) is replaced with a more empirically relevant, hybrid NKPC.

Iterating (2) forward pins down the path of inflation as a function of the path of output:

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t \left[y_{t+k} \right]. \tag{3}$$

Fiscal deficits can therefore be inflationary *only if* they trigger a boom in real economic activity. Put differently, a failure of Ricardian Equivalence is *necessary* for fiscal deficits to drive inflation, irrespective of whether $\omega = 1$ (RANK) or $\omega < 1$ (HANK). This link between inflation and Ricardian Equivalence is absent in flexible-price versions of the FTPL (e.g., Sims, 1994; Bassetto, 2002; Cochrane, 2005): in those models, the nominal price level can be a "free variable," disconnected from real economic activity. By contrast, this link is at the heart of the modern, sticky-price version of the FTPL—and a focal point of the subsequent analysis.

2.3 Fiscal policy

The government issues non-contingent, short-term, nominal debt; the extension to long-term debt is provided in Section 5.1. Let B_t denote the level of nominal public debt outstanding at the beginning of period t, P_t the nominal price level, and $D_t \equiv B_t/P_t$ the real value of public debt. In levels, the government's flow budget constraint is $B_{t+1} = I_t(B_t - P_t T_t)$, where T_t is real tax revenue (also, the real primary surplus) in date t and I_t is the gross nominal rate between dates t and $t+1$. Re-writing this flow constraint in real, log-linearized terms, the real value of government debt follows

$$d_{t+1} = \underbrace{\frac{1}{\beta}(d_t - t_t) + \frac{D^{ss}}{Y^{ss}} r_t}_{\text{expected debt burden, } \mathbb{E}_t d_{t+1}} - \underbrace{\frac{D^{ss}}{Y^{ss}}(\pi_{t+1} - \mathbb{E}_t[\pi_{t+1}])}_{\text{debt erosion due to inflation surprise}}, \tag{4}$$

where $r_t = i_t - \mathbb{E}_t[\pi_{t+1}]$ is the expected real rate and D^{ss}/Y^{ss} is the steady-state debt-to-GDP ratio, which, by asset-market clearing, equals the steady state wealth-to-income ratio A^{ss}/Y^{ss}. The government has to satisfy the flow constraint (4) at each date t, along with the no-Ponzi condition $\mathbb{E}_t\left[\lim_{k\to\infty} \beta^k d_{t+k}\right] = 0$.[7] As we assume that the economy starts in steady state (and hence $x_{-1} = 0$ for any variable $x \in \{d, t, r, y, \pi\}$), we can evaluate (4) at date 0 to obtain the following initial condition:

$$d_0 = -\frac{D^{ss}}{Y^{ss}} \pi_0. \tag{5}$$

If public debt had been indexed to inflation, d_0 would have been historically predetermined; but since debt is nominal, d_0 can jump in proportion to the jump in the nominal price level.

Tax rule. We close the fiscal block of the model by assuming that the fiscal authority sets tax revenue according to the following rule, for some $\tau_d, \tau_y \in [0, 1)$:

$$t_t = \underbrace{-\varepsilon_t}_{\text{deficit shock}} + \underbrace{\tau_d(d_t + \varepsilon_t)}_{\text{fiscal adjustment}} + \underbrace{\tau_y y_t}_{\text{tax base}}. \tag{6}$$

This rule mirrors those commonly used in applied work. Its first component, ε_t, identifies the exogenous fiscal innovation—the "deficit shock." For concreteness, we interpret ε_t as an unexpected, one-off, lump-sum transfer (e.g., a surprise issuance of stimulus checks). We assume that this shock is independently distributed over time; furthermore, for technical reasons, we also assume that it

[7] When $\omega = 1$, this condition can also be derived as an equilibrium condition, by combining the representative household's transversality and no-Ponzi conditions with asset market clearing. To simplify the exposition, we impose $\mathbb{E}_t\left[\lim_{k\to\infty} \beta^k d_{t+k}\right] = 0$ as an a priori constraint on fiscal policy and refer to it as the government's no-Ponzi condition.

has bounded support and ceases to occur at some finite T.[8] The second component captures how much taxes adjust over time in response to accumulated debt, conditional on aggregate income. For simplicity, and in line with the FTPL, this adjustment is non-distortionary, i.e., it takes the form of lump-sum tax hikes. Finally, the third term indicates how much tax revenue covaries with aggregate income, arising from a time-invariant, proportional tax on total household income at rate τ_y.[9]

Similarly to Leeper (1991), τ_d parameterizes the speed of fiscal adjustment following a deficit shock: taxes adjust with greater delay as τ_d falls, and *never* adjust if $\tau_d = 0$. An important policy question, and one central to the remainder of our analysis, is which values of τ_d are consistent with the requirement that "public debt does not explode," either in the sense that d_t remains bounded, or in the weaker sense that the government satisfies its no-Ponzi condition. Finally, and similarly to Angeletos et al. (2024), τ_y parameterizes the automatic feedback from economic activity to tax revenue.

2.4 Monetary policy

We abstract from the zero lower bound and let the monetary authority set i_t, the nominal interest rate between dates t and $t+1$, according to the following Taylor-type rule:

$$i_t = \mathbb{E}_t[\pi_{t+1}] + \phi y_t,$$

for some $\phi \in \mathbb{R}$. Re-writing this in terms of the (expected) *real* rate, we have

$$r_t = \phi y_t. \tag{7}$$

Monetary policy is thus parameterized by whether it implements lower or higher real rates (respectively, $\phi < 0$ or $\phi > 0$) in response to any demand-driven boom in real economic activity (and thereby in inflation). We allow $\phi < 0$ and $\phi > 0$, to accommodate both "passive" and "active" monetary policies, though we restrict $\phi > \underline{\phi} \equiv -\frac{1}{\sigma}$, for tangential technical reasons.[10] We finally note that, although monetary policy ends up reinforcing a fiscally-induced boom when $\phi < 0$, the monetary authority does not *directly* condition its policy instrument on ε_t or d_t. This separates the FTPL and HANK mechanisms that we study from a third mechanism—that of a monetary authority that *deliberately* reduces real rates when fiscal needs are high, for given output or inflation.

[8] The sole purpose of the latter assumption is to ensure that the RANK-FTPL equilibrium characterized in Section 3 remains bounded even in the case with fixed real rates ($\phi = 0$), which induces a random walk.

[9] By assuming that the proportional tax τ_y is time-invariant and that tax hikes are lump-sum, we abstract from time-varying distortions that would otherwise appear as cost-push shocks in the Phillips curve, thus isolating the failure of Ricardian Equivalence on the demand side of the economy. That said, since our HANK-FTPL equivalence result concerns the limit where tax hikes vanish ($\tau_d = 0$), the assumption of non-distortionary tax hikes is without any loss of generality.

[10] We restrict $\phi > \underline{\phi} \equiv -\frac{1}{\sigma}$ to rule out oscillatory impulse responses—a well-known, but immaterial, nuisance. Note also that we have departed slightly from the common practice of specifying monetary policy as $i_t = \psi \pi_t$. As a result, the Taylor principle translates here to $\phi > 0$ instead of $\psi > 1$.

2.5 Equilibrium definition

A standard equilibrium definition combines (i) optimality for households and firms, (ii) market clearing, and (iii) the flow budget for the government, together with its no-Ponzi condition. With the assumed policy rules, this gives the following:

Definition 1. *An equilibrium is a stochastic path* $\{c_t, y_t, \pi_t, a_t, d_t, t_t, r_t\}_{t=0}^{\infty}$ *for consumption, output, inflation, the real values of household wealth and public debt, total tax revenue, and real interest rates that satisfies all of the following: the aggregate consumption function* (1) *and the NKPC* (2)*; market clearing* $c_t = y_t$ *and* $a_t = d_t$*; the government's flow budget* (4) *along with the initial condition* (5) *and the no-Ponzi condition* $\mathbb{E}_t\left[\lim_{k\to\infty}\beta^k d_{t+k}\right] = 0$*; and the fiscal and monetary policy rules* (6) *and* (7).

Unlike Leeper (1991), we do not a priori require that d_t be bounded; instead, we only impose the no-Ponzi condition $\mathbb{E}_t\left[\lim_{k\to\infty}\beta^k d_{t+k}\right] = 0$. This eliminates a small discrepancy between the notions of "passive" and "active" fiscal policy found in Leeper (1991) and those implicit in much of the FTPL literature. In particular, we here define a passive fiscal policy as one that guarantees that the no-Ponzi condition is satisfied regardless of the paths of output, inflation and interest rates, and an active fiscal policy as one for which this happens only for a particular combination of such paths. In RANK ($\omega = 1$), and under the policy rule (6), these definitions translate to $\tau_d > 0$ for passive fiscal policy and $\tau_d = 0$ for active fiscal policy. Our definitions thus agree with the textbook treatment of the FTPL in Cochrane (2023). Further details are made clear in the next section.

3 A review of RANK-FTPL

This section examines the predictions and mechanism of the modern, sticky-price version of the FTPL ("RANK-FTPL"), which is nested in our environment with $\omega = 1$, $\phi \le 0$ and $\tau_d = 0$. Section 3.1 begins by reviewing the different types of equilibria that obtain in RANK ($\omega = 1$) under different policy mixes (different ϕ and τ_d). Section 3.2 characterizes the FTPL equilibrium, focusing on the predicted inflationary effects of fiscal deficits—the "how much" part. Finally, Section 3.3 provides a new perspective on the mechanism supporting the FTPL equilibrium—the "how" part—, revisits some of the controversies surrounding this theory, and connects to our upcoming HANK analysis.

3.1 Equilibrium characterization

When $\omega = 1$, there is a representative, infinitely-lived household and aggregate consumption obeys the familiar Euler equation, $c_t = -\sigma r_t + \mathbb{E}_t[c_{t+1}]$. Together with goods market clearing ($y_t = c_t$) and the monetary policy rule (7), this yields the following equilibrium restriction on output, referred to as the "DIS" equation:

$$y_t = -\sigma\phi y_t + \mathbb{E}_t\left[y_{t+1}\right]. \tag{8}$$

Consider any bounded solution of this equation.[11] To translate any such solution into a full equilibrium as defined in Definition 1, it is necessary and sufficient to complete the following steps. First, use the NKPC to obtain inflation (as in (3)) and then apply the monetary policy rule to obtain the real interest rate as $r_t = \phi y_t$. Next, use the flow budget constraint (4) together with the fiscal policy rule (6) to construct the implied process for government debt d_t. Finally, verify that this process satisfies the no-Ponzi condition $\mathbb{E}_t\left[\lim_{k\to\infty}\beta^k d_{t+k}\right] = 0$. Then and only then have we satisfied all the conditions for an equilibrium. We thus have a simple two-step process for characterizing equilibria: first, study the bounded solutions to equation (8); and second, check which of these solutions and corresponding debt processes satisfy the no-Ponzi condition.

Focusing on the first step, we see that two cases are possible, depending on the value of ϕ. When $\phi > 0$ ("active monetary policy"), the unique bounded solution to equation (8) is $y_t = 0$. This corresponds to the conventional solution of the New Keynesian model (e.g., Galí, 2008), which preserves Ricardian Equivalence—fiscal deficits have no effect on output and inflation. When instead $\phi \le 0$ ("passive monetary policy"), $y_t = 0$ remains a solution, but it is not the only one. Instead, any of the following processes is also a bounded solution:

$$y_t = \varrho y_{t-1} + \eta_t, \tag{9}$$

with $\varrho \equiv 1 + \sigma\phi \in (0,1]$ and η_t a bounded but otherwise *arbitrary* innovation such that $\mathbb{E}_{t-1}\left[\eta_t\right] = 0$.[12]

Turning to the second step, we see that $\tau_d > 0$ ("passive fiscal policy") suffices for the no-Ponzi condition to be satisfied for *any* bounded path of output, inflation and interest rates. It follows that, as long as $\tau_d > 0$, any of the solutions to equation (8) automatically translates to a complete equilibrium. In particular, the conventional solution of the New Keynesian model is completed by combining $\phi > 0$ from the previous step with $\tau_d > 0$ in this step. By contrast, when $\tau_d = 0$ ("active fiscal policy"), the no-Ponzi condition is not satisfied anymore at the conventional solution, and so the latter ceases to be an equilibrium. Instead, in order to satisfy the no-Ponzi condition, we must first let $\phi \le 0$, so as to

[11] By boundedness for a variable x, we mean that there exists $M > 0$ such that $|x_t| < M$ for all t and all realizations of uncertainty. As usual, the use of log-linearized relations justifies the focus on bounded solutions. Furthermore, note that Cochrane (2011, 2023) recognizes the rationale of requiring that y_t be bounded but questions the economic justification for requiring that d_t be bounded, consistent with our treatment here.

[12] In the knife-edge case of $\phi = 0$ (equivalently, $\varrho = 1$), y_t remains bounded according to the definition in Footnote 11, provided that the innovation η_t ceases to occur at a finite date. This in turn is satisfied in the RANK-FTPL equilibrium of Proposition 1 (see below) by the assumption that the deficit shock itself ceases to occur at finite date. Finally, both this technicality and another related one—the tension that $\varrho = 1$ creates with the log-linearization of the model—can be bypassed by reinterpreting $\phi = 0$ as $\phi \to 0^-$ (i.e., ϕ negative but arbitrarily close to zero).

open the door to the additional bounded solutions to equation (8) described in equation (9), and then select the one in which η_t is a *specific* multiple of ε_t, the concurrent fiscal innovation—a multiple that is precisely such that the no-Ponzi condition is satisfied.

We thus arrive at the following result, which is our version of Leeper (1991).[13]

Proposition 1. *Suppose that* $\omega = 1$.

1. *When* $\phi \in (\underline{\phi}, 0]$ *and* $\tau_d > 0$ *(i.e., both policies are passive), there are multiple equilibria in which* y_t *is bounded, including: (a) one in which* $y_t = \pi_t = 0$*; and (b) a continuum of equilibria in which* y_t *follows* (9) *and* $\pi_t = \frac{\kappa}{1-\beta\varrho} y_t$*, where* $\varrho \equiv 1 + \sigma\phi \in (0, 1]$ *and where* η_t *is an arbitrary innovation such that* $\mathbb{E}_{t-1}[\eta_t] = 0$.

2. *When* $\phi > 0$ *and* $\tau_d > 0$ *(i.e., active monetary policy and passive fiscal policy), there exists a unique equilibrium in which* y_t *is bounded. This equilibrium has* $y_t = \pi_t = 0$*, as in case (a) above.*

3. *When* $\tau_d = 0$ *and* $\phi \in (\underline{\phi}, 0]$ *(i.e., active fiscal policy and passive monetary policy), there exists a unique equilibrium in which* y_t *is bounded, referred to as the FTPL equilibrium. This corresponds to case (b) above, with the output innovation pinned down to*

$$\eta_t = \frac{1 - \beta\left(1 + \sigma\phi\right)}{\tau_y + \left(\kappa - \beta\phi\right)\frac{D^{ss}}{Y^{ss}}} \varepsilon_t. \tag{10}$$

The corresponding inflation surprise—i.e., the price jump causing debt erosion—is

$$\pi_t - \mathbb{E}_{t-1}[\pi_t] = \pi^{FTPL}_{\varepsilon,0} \cdot \varepsilon_t \qquad \textit{with} \qquad \pi^{FTPL}_{\varepsilon,0} \equiv \frac{\kappa}{\tau_y + \left(\kappa - \beta\phi\right)\frac{D^{ss}}{Y^{ss}}}. \tag{11}$$

The first part of Proposition 1 highlights that the New Keynesian model admits multiple equilibria when both policies are passive ($\phi \le 0$ and $\tau_d > 0$). In this case, one can rationalize not only a failure of Ricardian Equivalence, but also an *arbitrary* relation between deficits and inflation. The second part presents the conventional equilibrium: an active monetary authority ($\phi > 0$) ensures that Ricardian Equivalence is preserved and deficits have no effect on output or inflation. The third part presents the FTPL alternative: by committing *not* to adjust taxes ($\tau_d = 0$), an active fiscal authority selects a different equilibrium, in which Ricardian Equivalence fails and the deficit shock triggers an inflationary boom precisely as large as necessary to finance the deficit without any fiscal adjustment. In the next subsection we dig deeper into how precisely this equilibrium and the corresponding output and price jumps are determined—i.e., the "how much" part of the deficit-inflation nexus in RANK-FTPL.[14]

[13] There are two inessential differences from Leeper (1991): the re-parameterization of monetary policy and the accommodation of unbounded d_t. If we replace (7) with $i_t = \psi\pi_t$, Proposition 1 holds with active monetary policy redefined to $\psi > 1$; and if we require that d_t be bounded, the result holds with active fiscal policy redefined to $\tau_d \in [0, 1-\beta)$.

[14] A fourth scenario arises when both policies are active ($\phi > 0$ and $\tau_d = 0$). In this case, no equilibrium exists in which y_t is bounded and the implied process for d_t satisfies the no-Ponzi condition.

3.2 How much?

We begin our analysis of the "how much" question—i.e., the size of the price jump predicted by the FTPL equilibrium ($\tau_d = 0$)—with the special case of $\phi = \tau_y = 0$, i.e., constant real rates and no feedback from output to taxes. We refer to this case as the "simple FTPL arithmetic" because it eliminates every margin of adjustment other than debt erosion, thus necessitating a jump in prices to absorb the entirety of a deficit shock. Indeed, in this case, the government's no-Ponzi condition is satisfied if and only if

$$\frac{D^{ss}}{Y^{ss}}\left(\pi_t - \mathbb{E}_{t-1}[\pi_t]\right) = \varepsilon_t. \tag{12}$$

In words, the real value of the outstanding public debt must drop by exactly the same amount as the increase in the fiscal deficit. Equivalently, the price jump per unit of deficit must equal the reciprocal of the debt-to-GDP ratio—this is captured in (11) with $\pi_{\varepsilon,0}^{FTPL} = (D^{ss}/Y^{ss})^{-1}$ when $\phi = \tau_y = 0$.

To see how this price jump is supported in equilibrium, note that, with $\phi = 0$, the solutions seen in (9) reduce to $y_t = y_{t-1} + \eta_t$. From the NKPC (2), we then also have that $\pi_t = \pi_{t-1} + \frac{\kappa}{1-\beta}\eta_t$. Combining this with (12), we conclude that

$$\eta_t = \frac{1-\beta}{\kappa \frac{D^{ss}}{Y^{ss}}}\varepsilon_t. \tag{13}$$

In short, the output innovation—which would have been a free variable under passive fiscal policy—is now selected to support the price jump required to finance the deficit shock.

This logic readily extends to $\phi < 0$ and $\tau_y > 0$. In this more general case, a deficit shock may now be financed not only by debt erosion, but also by a reduction in interest rate costs (when $\phi < 0$) and by an increase in tax revenue (when $\tau_y > 0$). This reduces the requisite price jump and rescales the relation between η_t and ε_t—equations (12) and (13) generalize to equations (10) and (11)—but does not change any of the essence. In all cases, the FTPL equilibrium corresponds to the only solution of (8) that satisfies the equilibrium requirement for the government to meet its no-Ponzi condition *despite* fiscal policy being active ($\tau_d = 0$).[15]

[15] In fact, this basic logic extends even beyond the class of fiscal rules assumed here. To illustrate, consider the following example inspired by Cochrane (2023) and Smets and Wouters (2024): following any deficit shock ε_t, the fiscal authority adjusts the discounted present value of future surpluses (inclusive of interest payments) by a fraction λ of ε_t, for some $\lambda \in [0,1)$. This again selects an equilibrium in which output and prices jump in response to ε_t, now by the amount necessary for the resulting debt erosion to cover $(1-\lambda)\varepsilon_t$, the "unfunded" portion of the deficit. Setting $\phi = \tau_y = 0$ in our setting corresponds to setting $\lambda = 0$ in this example; conversely, letting $\tau_y > 0$ and $\phi < 0$ in our setting corresponds to letting $\lambda > 0$, with the equivalent λ being an increasing function of τ_y and a decreasing function of ϕ. Finally, models such as Bianchi and Ilut (2017) and Bianchi et al. (2023) can be understood as involving a time-varying, and possibly shock-specific, λ. In all cases, the equilibrium is selected to substitute for the missing fiscal adjustment.

3.3 How?

We now turn to the "how" question—i.e., the precise mechanism behind the RANK-FTPL predictions reviewed above. In Section 3.1, we characterized the equilibria of the New Keynesian model in the "traditional" way, using the representative agent's Euler equation. We here offer a different prism, based on a suitable version of the "Intertemporal Keynesian Cross" (building on Auclert et al., 2024). While mathematically equivalent, this perspective offers two advantages. First, it provides deeper insight into the mechanism through which the FTPL breaks Ricardian Equivalence and allows deficits to drive output and inflation. Second, it facilitates the transition to our upcoming HANK analysis.

The IKC in RANK. When $\omega = 1$, the aggregate consumption function (1) reduces to

$$c_t = (1-\beta) z_t + (1-\beta) \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t [y_{t+k}] - \sigma\beta \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t [r_{t+k}], \tag{14}$$

where

$$z_t = a_t - \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t \left[t_{t+k} - \beta \tfrac{D^{ss}}{Y^{ss}} r_{t+k} \right]$$

is private financial wealth net of tax obligations and inclusive of any revaluation effects. In *any* equilibrium, $a_t = d_t$ and $A^{ss} = D^{ss}$ (by asset market clearing). Furthermore, if we iterate the government's flow budget (4) forward and use the no-Ponzi condition, we arrive at the following key equation, commonly referred to as the government's intertemporal budget constraint:[16]

$$d_t = \mathbb{E}_t \left[\sum_{k=0}^{\infty} \beta^k \left(t_{t+k} - \beta \frac{D^{ss}}{Y^{ss}} r_{t+k} \right) \right]. \tag{15}$$

It follows that $z_t = 0$ in *any* equilibrium—and since households have rational expectations, they *themselves* also understand this fact, as in Barro (1974). Using this to eliminate z_t from (14), and replacing c_t with y_t (by goods market clearing) and r_{t+k} with ϕy_{t+k} (by the monetary policy rule), we conclude that real output must solve the following fixed-point relation, which is the applicable version of the Intertemporal Keynesian Cross (IKC):

$$y_t = (1 - \beta - \sigma\beta\phi) \left(y_t + \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t [y_{t+k}] \right). \tag{16}$$

Because the IKC (16) can be rewritten recursively as the DIS (8), the fixed points of the former coincide with the solutions to the latter. Therefore, the approach taken here is mathematically equivalent to the traditional approach used previously. The benefit, however, is that we can now see more clearly the economics behind the earlier discussion. Just as in Barro (1974), permanent-income consumer behavior together with rational expectations guarantee that "government bonds are not net wealth"

[16] In line with Footnote 7, Cochrane (2005) and Cochrane (2023) have argued that equation (15) should be interpreted as an equilibrium valuation equation instead of a constraint on government behavior. Both interpretations are consistent with our paper: our formal results only use the fact that (15) must hold in any equilibrium.

(i.e., $z_t = 0$). Consumers therefore optimally equate their spending to $(1-\beta)$ times their permanent income, adjusted for any movements in real rates. Along with the fact that income and real rates are themselves pinned down by aggregate spending, this implies (i) that y_t has to solve (16) in *any* equilibrium and (ii) that fiscal policy does not enter this fixed-point relation.

Does Ricardian Equivalence hold? Our IKC prism clarifies the precise sense in which "fiscal policy does not enter aggregate demand" when $\omega = 1$: in any equilibrium, $z_t = 0$ and fiscal policy drops out of equation (16).[17] Fiscal policy does not enter aggregate supply either, in the sense of not entering the NKPC (2). One may thus reasonably guess that Ricardian Equivalence holds and that fiscal policy does not affect either output or inflation. Consistent with this logic, $y_t = 0$ (i.e., no response to fiscal shocks) is always a solution to the IKC, and this solution translates to $\pi_t = 0$ via the NKPC.

There are, however, two important caveats to this logic, opening the door for a failure of Ricardian equivalence. First, the IKC (16) admits multiple fixed points—including those where Ricardian households optimally consume more in response to deficit shock because they rationally expect other households to do the same, which in turn boosts their own income. Second, to complete the construction of an equilibrium, we must not only find a solution to the IKC (16) but also verify that equation (15)—equivalently, the government's no-Ponzi condition—is satisfied along this solution. Together, these two caveats raise the logical possibility that a solution other than $y_t = 0$ may need to be selected as a full equilibrium, and that this solution could indeed violate Ricardian equivalence. While standard practice rules out this possibility, the FTPL instead *leverages* it: with $\tau_d = 0$, a failure of Ricardian Equivalence is required to avoid a violation of the government's no-Ponzi condition, and this happens *even though* households are Ricardian in the classical sense of Barro (1974).[18]

The FTPL's controversies and our way forward. Our IKC perspective also helps connect to the controversies surrounding the FTPL. Early debates focused on whether an active fiscal policy amounts to an untestable, off-equilibrium threat to "blow up the government's budget" or to induce the nonexistence of a continuation equilibrium (e.g., Kocherlakota and Phelan, 1999; Bassetto, 2002; Buiter, 2002; Atkeson et al., 2010). More recently, Angeletos and Lian (2023) have argued that all solutions of the New Keynesian model, except for the conventional one, hinge on a form of self-fulfilling feedback and can be ruled out by adding appropriate noise, as in the global-games literature. We will echo these points in Section 4.4, where we show that the FTPL equilibrium is fragile with respect to both (i) a difficult-to-test change in the policy mix and (ii) a plausible refinement of equilibrium beliefs.

[17] Cochrane (2005, 2023) has pushed forward a different narrative, based on the idea that government bonds have wealth effects *off*-equilibrium. However, standard equilibrium concepts are not suitable for discussing off-equilibrium adjustment (Bassetto, 2005), so this paper avoids any conjecture about it. Instead, we study exclusively what happens *on* equilibrium. We then make clear that, as long as $\omega = 1$ and consumers are rational, government bonds *cannot* have a wealth effect *on* equilibrium and fiscal policy does not enter the IKC (16).

[18] Our discussion here echoes Woodford (1995), who defines a "non-Ricardian" regime as $\tau_d = 0$ within RANK.

We will further see that both of these fragilities stem from an elementary observation made in this section: with Ricardian (i.e., rational, infinite-horizon) households, fiscal policy drops out of the IKC and hence aggregate demand, in the precise sense articulated in this section, and can therefore only matter through equilibrium selection.

That said, our ultimate focus in this paper is not on these theoretical debates about equilibrium selection; rather, it is on the more tangible question of how much deficits can drive inflation. In the next section, we will let deficits drive output and inflation through a different mechanism—a classical failure of Ricardian Equivalence, as captured in our setting by $\omega < 1$. We will show that this natural and empirically grounded alternative reproduces FTPL's core predictions on how much deficits can drive inflation. At the same time, because this alternative ensures that fiscal policy no longer drops out of the IKC, the aforementioned fragilities will disappear. In a nutshell, we will insulate the FTPL's core predictions about inflation and debt erosion from the controversies reviewed here.

4 HANK meets FTPL

This section studies the HANK version of our model ($\omega < 1$). We begin in Section 4.1 by delineating the mechanism at work from its FTPL counterpart. We then show that, despite the difference in mechanism, HANK predicts the exact same price jump as FTPL as long as tax adjustment is sufficiently slow. Section 4.2 establishes this result for the instructive case of fixed real rates ($\phi = 0$), while Section 4.3 extends it to more general monetary policies ($\phi \neq 0$). Finally, Section 4.4 explains why the difference in the underlying mechanism means that HANK avoids the fragilities of its FTPL counterpart.

4.1 Classical non-Ricardian effects in HANK

As we move form $\omega = 1$ to $\omega < 1$, the only—but crucial—change in the economics is that fiscal policy now enters the IKC. The aggregate consumption function is the same as the RANK counterpart in (14), modulo the replacement of β with $\beta\omega$ and the corresponding adjustment in the definition of z_t, i.e.,

$$z_t \equiv a_t - \sum_{k=0}^{\infty} (\beta\omega)^k \mathbb{E}_t \left[t_{t+k} - \beta \frac{D^{ss}}{Y^{ss}} r_{t+k} \right].$$

This object represents the private financial wealth of currently living households, net of tax obligations and inclusive of revaluation effects. In any equilibrium, we still have $a_t = d_t$ and that d_t has to satisfy equation (15). However, because $\omega < 1$, these facts now do not anymore translate to $z_t = 0$. Instead,

$$z_t = \mathbb{E}_t \left[\sum_{k=0}^{\infty} \beta^k \tilde{t}_{t+k} - \sum_{k=0}^{\infty} \left(\beta\omega\right)^k \tilde{t}_{t+k} \right],$$

where $\tilde{t}_t \equiv t_t - \beta \frac{D^{ss}}{Y^{ss}} r_t$ is the government surplus net of interest payments (i.e., the effective total transfer from households to the government). By the same token, the IKC becomes

$$y_t = \underbrace{(1-\beta\omega)\, z_t}_{\text{non-Ricardian effect}} + \underbrace{(1-\beta\omega-\beta\sigma\phi\omega)\left\{y_t + \sum_{k=1}^{\infty}(\beta\omega)^k \mathbb{E}_t\left[y_{t+k}\right]\right\}}_{\text{permament income and intertemporal substitution}}. \tag{17}$$

For a given z_t, the IKC (17) embeds the same general equilibrium feedback between individual and aggregate spending as its RANK counterpart. The key novelty is that fiscal policy enters this relation *directly* via z_t. Intuitively, deficits can stimulate individual spending, and thereby equilibrium output, by shifting the tax burden to future generations (in our model's literal interpretation) or by relaxing borrowing constraints (in the liquidity constraint re-interpretation). This means that deficits can drive output and thereby inflation *without* the equilibrium selection mechanism articulated in the previous section, and will be key for why HANK sidesteps the FTPL's controversies.

Notwithstanding this point, RANK's equilibrium indeterminacy issue extends to HANK as well: the IKC (17) may once again admit multiple fixed points. This now raises the possibility that, when $\omega < 1$, fiscal deficits can in principle drive equilibrium output not only via the classical non-Ricardian effect captured by z_t, but *also* via equilibrium selection of a similar type as that articulated in the previous section. In Section 4.4, we will verify that this is *not* the case for the particular HANK equilibrium characterized below—its predictions are driven *exclusively* by the classical non-Ricardian effect.

4.2 HANK meets FTPL, with fixed real rates

In this subsection we restrict attention to the special case of $\phi = 0$; i.e., we let monetary policy stabilize the (expected) real rate. This case is a focal point in the HANK literature because it distills how fiscal stimuli propagate via the Keynesian general-equilibrium feedback between spending and income, abstracting from any additional feedback via real interest rates. Additionally, this case also connects to the simple FTPL arithmetic reviewed above, which similarly abstracts from any effect of deficits on real rates. Finally, as we will show later, the fixed-rate assumption is actually without serious loss of generality—our HANK-FTPL equivalence result readily extends to more general monetary policies.

Equilibrium characterization. We begin by establishing that our HANK economy admits a unique bounded equilibrium when $\phi = 0$, and by characterizing its structure.[19]

[19]As standard in the literature, a unique bounded equilibrium in the linearized economy translates to a locally determinate equilibrium in the original non-linear economy and presumes that shocks are small enough for the economy to remain in a neighborhood of the steady state around which the economy has been linearized. Whether global determinacy can be achieved with the help appropriate escape clauses for monetary policy (as, e.g., in Atkeson et al., 2010), or by other means, is outside the scope of our paper.

Proposition 2. *Suppose that $\omega < 1$, $\tau_y > 0$, $\tau_d \in [0,1)$, and $\phi = 0$. There exists a unique bounded equilibrium, henceforth referred to as the HANK equilibrium, and it is such that*

$$y_t = \chi\,(d_t + \varepsilon_t) \qquad \textit{and} \qquad \mathbb{E}_t\,[d_{t+1}] = \rho_d\,(d_t + \varepsilon_t)\,, \tag{18}$$

for some scalars $\chi > 0$ and $\rho_d \in (0,1)$ that are continuous functions of $(\beta, \omega, \tau_y, \tau_d)$. In this equilibrium, the inflation surprise due to a deficit shock—or the price jump causing debt erosion—is

$$\pi_t - \mathbb{E}_{t-1}[\pi_t] = \pi^{HANK}_{\varepsilon,0} \cdot \varepsilon_t \qquad \textit{with} \qquad \pi^{HANK}_{\varepsilon,0} \equiv \frac{\kappa\chi}{1 - \beta\rho_d + \kappa\chi\frac{D^{ss}}{Y^{ss}}}. \tag{19}$$

At the heart of this HANK equilibrium is a two-way feedback between aggregate demand and fiscal conditions. Because households are non-Ricardian ($\omega < 1$), deficits naturally increase aggregate consumption, thereby boosting output and inflation ($\chi > 0$), which in turn helps stabilize debt ($\rho_d < 1$) through both higher tax revenue and debt erosion. Equation (18) provides the structure of this equilibrium. The finding that y_t is proportional to $d_t + \varepsilon_t$ reflects the aforementioned non-Ricardian channel: $d_t + \varepsilon_t$ is, in effect, a sufficient statistic for z_t, and thereby also for y_t. By direct implication, χ decreases with ω : a larger departure from the permanent-income benchmark implies a larger response of equilibrium output, and hence also of inflation, to any innovation in private wealth and in fiscal transfers. Equation (19) then zeroes in on the main object of interest: the price jump triggered by a deficit shock along the HANK equilibrium. We will soon relate this price jump in HANK to its FTPL counterpart.

This two-way feedback also explains why moving to HANK (i.e., $\omega < 1$) changes the equilibrium determinacy properties of the model. Recall that RANK admits *multiple* equilibria when $\phi = 0$ and $\tau_d > 0$. Under the exact same conditions, we now instead obtain a *unique* bounded equilibrium; we will later verify that this equilibrium is also the common limit of $\phi \to 0^+$ and $\phi \to 0^-$. Relatedly, recall that RANK allowed d_t to grow without bound, yet slowly enough to satisfy the no-Ponzi condition, whenever $\tau_d \in (0, 1-\beta)$. Here, instead, debt necessarily converges back to steady state, as emphasized in our prior work (Angeletos et al., 2024), thanks again to the aforementioned two-way feedback. Finally, while the equilibrium set was (right-)discontinuous at $\tau_d = 0$ in RANK, here it is now continuous: there is no more a material difference between "adjusting taxes very slowly" ($\tau_d > 0$ but small) and "never adjusting taxes" ($\tau_d = 0$). All these properties are manifestations of the different mechanism at work: in our HANK equilibrium, both ϕ and τ_d operate only via conventional partial-equilibrium and general-equilibrium demand channels, as encapsulated in the IKC—and not via equilibrium selection (whether of the Taylor principle or FTPL type). We will return to these points in Section 4.4.

HANK meets FTPL. Despite the difference in the underlying economic mechanism, we find that our HANK equilibrium can—if fiscal adjustment is sufficiently delayed—replicate the core prediction of the FTPL regarding inflation and debt erosion.

Proposition 3. *Let $\omega < 1$, $\tau_y > 0$, and $\phi = 0$ and consider $\pi^{HANK}_{\varepsilon,0}$, the initial price jump in response to a deficit shock obtained in the HANK equilibrium. This jump decreases continuously in the speed of fiscal adjustment, $\tau_d \in [0,1)$, and becomes exactly the same as the FTPL counterpart when $\tau_d = 0$:*

$$\lim_{\tau_d \to 0^+} \pi^{HANK}_{\varepsilon,0} = \pi^{HANK}_{\varepsilon,0}\Big|_{\tau_d=0} = \frac{\kappa}{\tau_y + \frac{D^{ss}}{Y^{ss}}\kappa} = \pi^{FTPL}_{\varepsilon,0}. \tag{20}$$

This equivalence result holds independently of the strength of the tax-base channel (τ_y). As evident from equation (20), the common price jump in HANK and FTPL decreases with the relative strength of this channel (i.e., it increases with τ_y and decreases with κ). However, if this channel is absent ($\tau_y \to 0$), or if prices are very flexible ($\kappa \to \infty$), then the jump in prices *entirely* finances the deficit shock, just as in the simple FTPL arithmetic.

Corollary 1. *Let $\omega < 1$, $\phi = 0$ and $\tau_d = 0$. If either $\kappa \to \infty$ or $\tau_y \to 0$, then the price jump in response to a deficit shock in the HANK economy converges to that predicted by the simple FTPL arithmetic:*

$$\pi^{HANK}_{\varepsilon,0} \to \left(\frac{D^{ss}}{Y^{ss}}\right)^{-1} \tag{21}$$

Figure 1 provides a visual illustration of Proposition 3 and Corollary 1.[20] We see that, the weaker the fiscal adjustment (in the sense of smaller τ_d), the larger the impact inflation response to a deficit shock, converging to the FTPL limit as $\tau_d \to 0$. If $\tau_y \to 0$, then this limit is the textbook FTPL arithmetic of prices jumping by exactly enough to fully finance the deficit; if $\tau_y > 0$, then the price jump is strictly smaller. As we have stressed throughout, the intuition for these results is rooted in the classical non-Ricardian effects of fiscal stimulus: if fiscal adjustment is sufficiently weak and delayed, then, since households are non-Ricardian, an initial fiscal deficit will increase demand.[21] If the tax base channel is absent ($\kappa \to \infty$ or $\tau_y \to 0$), then the erosion of private wealth—equivalently, the erosion of government debt—must be large enough to negate the initial transfer; i.e., the endogenous reduction in d_t must fully offset the exogenous ε_t, and the price jump that does so is $\left(\frac{D^{ss}}{Y^{ss}}\right)^{-1}$. If instead the tax base channel is operative, then some flow of funds back towards the government occurs via the tax base increase, dampening the price jump required to achieve convergence in general equilibrium.

To further connect our HANK results here to the FTPL discussion in Section 3, let us examine how the government's intertemporal budget constraint (15) is satisfied when $\tau_d = 0$. With fixed real rates, (15) reduces to $d_0 = \mathbb{E}_0\left[\sum_{k=0}^{\infty} \beta^k t_k\right]$. Substituting t_k from the policy rule (6) and setting $\tau_d = 0$, we

[20]For this visual illustration, we set $\omega = 0.8$ and $\kappa = 0.1$, representing a meaningful failure of Ricardian equivalence and a steep NKPC. For our later quantitative analysis, we will consider empirically disciplined variants of our model.

[21]In this paper, we associate delayed fiscal adjustment with smaller values of $\tau_d > 0$. A natural alternative assumes no tax adjustment for a finite number of periods, followed by a tax hike sufficient to bring government debt back to steady state immediately. The equivalence of these two notions of delayed fiscal adjustment is shown in Angeletos et al. (2024).

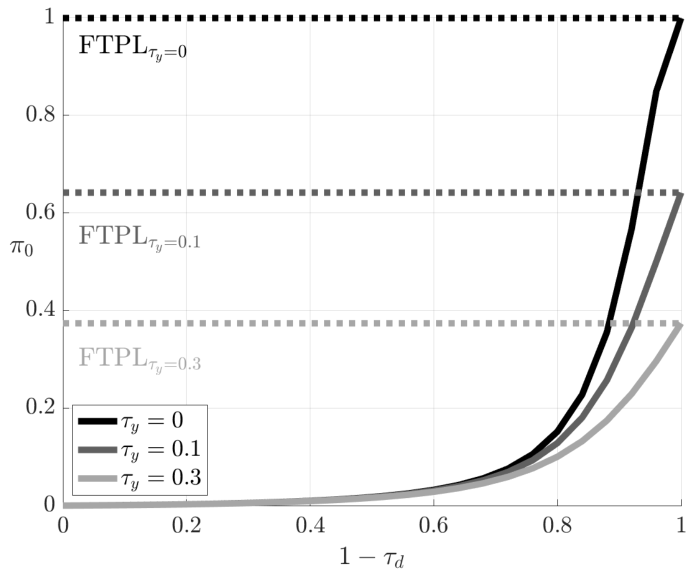

Figure 1: Date-0 inflation response to a fiscal deficit shock in HANK (solid), for different τ_d and τ_y. The dashed lines show the corresponding inflation response in the FTPL equilibrium. The size of the shock is normalized to give a date-0 FTPL inflation response of 1 per cent for $\tau_y = 0$.

obtain $d_0 = -\varepsilon_0 + \tau_y \mathbb{E}_0 \left[\sum_{k=0}^{\infty} \beta^k y_k\right]$. Finally, combining this with the initial condition (5), we obtain

$$\underbrace{\varepsilon_0}_{\text{deficit shock}} = \underbrace{\frac{D^{ss}}{Y^{ss}}\pi_0}_{\text{debt erosion}} + \underbrace{\tau_y \mathbb{E}_0 \left[\sum_{k=0}^{\infty} \beta^k y_k\right]}_{\text{tax base expansion}}.$$

In words, since fiscal adjustment has been ruled out (i.e., $\tau_d = 0$), the initial fiscal deficit must be financed by inflation and its induced debt erosion, by an expansion in the tax base, or by a mixture of both. Furthermore, this equation must hold in *both* our HANK economy and its RANK-FTPL counterpart. Therefore, the *sum* of the two terms on the right-hand side of this equation must be the same in both economies. Finally, because inflation follows the NKPC and hence (3), the *ratio* of these two terms is also the same and is given by

$$\frac{\frac{D^{ss}}{Y^{ss}}\pi_0}{\tau_y \mathbb{E}_0 \left[\sum_{k=0}^{\infty} \beta^k y_k\right]} = \frac{D^{ss}}{Y^{ss}} \frac{\kappa}{\tau_y}.$$

If both the sum and the ratio of these two terms are the same, then each term itself must also be the same, and so RANK-FTPL and our limit HANK economy must deliver the exact same debt erosion, and hence the same initial price jump. This argument is simple: it merely leverages government budget arithmetic along with the NKPC. However, this argument would have been vacuous if our HANK equilibrium did not exist for $\tau_d = 0$. It would also have been of limited use if the inflation response were dramatically different for $\tau_d \to 0^+$, or if the HANK equilibrium were supported by the same mechanism as the FTPL equilibrium. The value-added of our equivalence result, therefore, rests not only on the above argument but also on two key qualities of the HANK equilibrium: (i) its existence and continuity for all $\tau_d \in [0,1)$, as stated in Proposition 3; and (ii) its reliance on classical non-Ricardian effects, rather than equilibrium selection, as further substantiated in Section 4.4 below.

Finally, we note that our equivalence result, like the underlying continuity of the equilibrium with respect to τ_d, holds for *any* $\omega < 1$, including values arbitrarily close to (but below) 1. However, for any given $\tau_d > 0$, a higher ω implies a smaller boom and less inflation, increasing the distance between $\pi_{\varepsilon,0}^{FTPL}$ and $\pi_{\varepsilon,0}^{HANK}$. Together with the monotonicity of $\pi_{\varepsilon,0}^{HANK}$ with respect to τ_d, this implies that the value of τ_d required for $\pi_{\varepsilon,0}^{HANK}$ to stay close to $\pi_{\varepsilon,0}^{FTPL}$ becomes smaller as ω gets closer to 1. It follows that, although the theoretical result is valid for any departure from the PIH benchmark, its practical relevance depends on this departure being non-trivial: a *moderate* delay in fiscal adjustment produces quantitatively similar inflation in HANK as in the FTPL if and only if households discount future taxes and spend any transfer or other transitory income *sufficiently* fast.

Front-loading. While identical in their predictions regarding the initial price jump and the associated debt erosion, FTPL and HANK do differ in their predictions on the timing and persistence of the induced inflation. To see this, recall that when $\phi = 0$, RANK-FTPL produces a random walk for output and, consequently, for inflation. By contrast, HANK produces a mean-reverting process for both variables. To operationalize this point, we consider the following measure of the "front-loadedness" of the inflation response:

$$\pi^{\dagger} \equiv \frac{\pi_{\varepsilon,0}}{\sum_{k=0}^{\infty} \beta^k \pi_{\varepsilon,k}}, \tag{22}$$

where $\pi_{\varepsilon,k} \equiv \frac{d\mathbb{E}_t[\pi_{t+k}]}{d\varepsilon_t}$ is the response of inflation to the deficit shock k periods earlier; i.e., $\pi^{\dagger}$ is the initial impact relative to the cumulative inflation response. This will prove to be a useful statistic not only here, but also when we allow $\phi \neq 0$ (shortly) or consider other extensions (in Section 5).

Proposition 4. *Let $\omega < 1$, $\tau_y > 0$, and $\phi = 0$. The inflation impulse response to a deficit shock in the HANK economy is more front-loaded when households are less Ricardian, i.e., $\pi^{\dagger}$ increases when ω is lower. Furthermore, $\pi^{\dagger}$ is bounded from below by its FTPL counterpart,*

$$\pi^{\dagger,HANK} > \pi^{\dagger,FTPL} = 1 - \beta, \tag{23}$$

with the distance between the two vanishing when $\tau_d = 0$ and $\omega \to 1^{-}$.

The intuition underlying this finding is straightforward, and yet again rooted in short household horizons. Because of those short horizons, the demand boom—and thus the inflationary pressure that it causes—in HANK is necessarily short-lived. More precisely, in the case of fixed real rates considered here, output and thus also inflation in RANK-FTPL follow a random walk, while in HANK the demand boom is transitory, i.e., $\rho_d \in [0,1)$.

4.3 HANK meets FTPL, with interest rate feedback

We next relax the restriction $\phi = 0$; that is, we allow fiscal deficits to now trigger a change in (expected) real rates via the monetary authority's response to output (or inflation). We first clarify the conditions under which the HANK equilibrium characterized in Proposition 2 continues to exist for $\phi \neq 0$, before then extending our HANK-FPTL equivalence result to this more general case.

The HANK equilibrium with $\phi \neq 0$. We continue to assume that $\omega < 1$, but now let $\phi \neq 0$ and ask the following question: what are the values of ϕ such that an equilibrium of the same form—and same economics—as that in Proposition 2 continues to exist for *all* values of τ_d, including $\tau_d = 0$?

Proposition 5. *Suppose that $\omega < 1$ and $\tau_y > 0$. There exists a threshold $\bar{\phi} > 0$ such that: if $\phi \in \left(\underline{\phi}, \bar{\phi}\right)$, then for all $\tau_d \in [0,1)$, a bounded equilibrium of the form (18) exists and is unique. The equilibrium coefficients χ and ρ_d, and the resulting inflation impulse responses, are all continuous in $\left(\beta, \omega, \tau_y, \tau_d, \phi\right)$.*

Intuitively, if the monetary authority raises real interest rates sufficiently aggressively in response to the fiscal deficit-led boom (namely, if $\phi > \bar{\phi}$), then it both arrests the boom and raises the government's cost of borrowing. Fiscal adjustment must then be fast enough (i.e., τ_d must be sufficiently higher than 0), or else public debt will not be stabilized. It follows that no bounded equilibrium exists for $\tau_d = 0$ if $\phi > \bar{\phi}$. If instead the rate hikes are modest (i.e., if $0 < \phi < \bar{\phi}$), then they only partially offset the aforementioned two-way feedback between fiscal conditions and real economic activity, making it possible to sustain $\tau_d = 0$, similarly to the case of $\phi = 0$.[22] Finally, if monetary policy lets real rates *fall* in response to the deficit-led boom (i.e., if $\phi < 0$), then this only further speeds up the boom—and so public debt is again stabilized and $\tau_d = 0$ is supported.[23] It follows that, as stated in Proposition 5, a bounded equilibrium with $\tau_d = 0$ exists in our HANK economy on both sides of $\phi = 0$, and this equilibrium is furthermore continuous in ϕ.

HANK meets FTPL, again. Pick any $\phi \in \left(\underline{\phi}, \bar{\phi}\right)$ and consider the HANK equilibrium obtained for $\tau_d = 0$. We ask whether this equilibrium predicts the same price jump and debt erosion as a properly defined FTPL counterpart. In defining such a counterpart, we must deal with two challenges. First, while our HANK equilibrium exists for both $\phi > 0$ and $\phi < 0$, the FTPL equilibrium ceases to exist for $\phi > 0$. Second, even if we restrict to $\phi < 0$, HANK and FTPL are not directly comparable because the

[22] Consistent with this intuition, $\bar{\phi}$ increases with both τ_y and $1-\omega$: if the feedback is strong, then even very aggressive monetary reactions are consistent with stable public debt in the absence of fiscal adjustment ($\tau_d = 0$).

[23] The lower bound $\underline{\phi} \equiv -\frac{1}{\sigma}$, which is inherited from RANK, has the following property in HANK: as monetary policy becomes more and more accommodative ($\phi \to \underline{\phi}^+$), the deficit-induced boom becomes so large and so front-loaded that debt is stabilized immediately ($\rho_d \to 0^+$).

front-loading property discussed above translates the same ϕ to *different* paths for real rates, which in turn affects both aggregate demand and the government budget. We address these challenges and provide the natural "apples-to-apples" comparison as follows: for any $\phi \in (\underline{\phi}, \bar{\phi})$, we first take the HANK equilibrium that occurs for $\tau_d = 0$; we then identify the FTPL equilibrium that occurs in RANK under a *modified* monetary policy, which induces the same path of (expected) real interest rates as in our HANK equilibrium; and finally, we compare the inflation predictions of these two equilibria. Proposition 6 summarizes the results of this exercise.[24]

Proposition 6. *Suppose that $\omega < 1$, $\tau_y > 0$, and $\phi \in (\underline{\phi}, \bar{\phi})$, and consider the HANK equilibrium that obtains when $\tau_d = 0$. Select any realization of the initial fiscal shock ε_0, abstract from any future shocks, and let $\{r_t^{HANK}\}_{t=0}^{\infty}$ be the equilibrium path of the (expected) real rate obtained in this equilibrium. Now consider an analogous RANK-FTPL economy in which $\omega = 1$, fiscal policy follows the same rule as in our HANK economy (with $\tau_d = 0$), and monetary policy follows the passive rule $r_t = r_t^{HANK}$. Then, the comparisons established in Propositions 3 and 4 continue to hold, i.e.,*

$$\pi_{\varepsilon,0}^{HANK} = \pi_{\varepsilon,0}^{FTPL} \qquad \text{and} \qquad \pi^{\dagger,HANK} > \pi^{\dagger,FTPL}.$$

Proposition 6 establishes that both conclusions of Section 4.2—i.e., the HANK-FTPL equivalence and the front-loading—directly extend to more general monetary policies. Intuitively, once we equate the response of interest rates to deficit shocks in the two economies, we also equate the government's interest rate costs of servicing its outstanding debt. This ensures that, even though the sum of debt erosion and the tax base expansion now differs from the $\phi = 0$ benchmark, reflecting the increase or decrease in the aforementioned interest rate costs, this sum must still be equated between the two economies. Finally, the ratio of these two components also necessarily remains the same across the two economies, as it is directly determined by the slope of the Phillips curve and the strength of the tax base channel. The HANK-FTPL equivalence then follows from the same argument as before: if both sum and ratio are the same, then each of these components must also be the same; and since debt erosion is determined by the initial price jump, the latter must also be the same. Finally, to understand the result regarding front-loading, note that $\phi < 0$ will naturally make the fiscally-led boom more front-loaded in *both* HANK and RANK-FTPL, by effectively adding a monetary stimulus atop the fiscal stimulus. However, as long as the two economies receive the same monetary stimulus (i.e., the same path for r_t), the response of output and inflation in HANK must be *more* front-loaded than its FTPL counterpart, once again for the same reason as before: discounting due to finite household horizons.

[24] In the interest of parsimony, Proposition 6 focuses on $\tau_d = 0$ and does not repeat the continuity and monotonicity of $\pi_{\varepsilon,0}^{HANK}$ in $\tau_d \in [0, 1)$, although these properties continue to hold. See the proof of Proposition 5 for details.

4.4 The robustness of HANK

In Section 3.3 we revisited some controversies surrounding the modern, sticky-price version of the FTPL, all rooted in the particular mechanism at work—breaking Ricardian equivalence by force of equilibrium selection. In this section, we verify that, unlike its FTPL counterpart, the deficit-inflation mapping in HANK is robust to, first, a change in the policy mix that makes fiscal policy unambiguously passive and, second, an equilibrium refinement that anchors long-run beliefs. Together, these results underscore that the HANK alternative presented in this paper sidesteps those controversies, precisely because of the difference in underlying mechanism.

A change in far-ahead policies. We consider the following policy experiment. Up to some finite but far-ahead date H, the monetary and fiscal authorities follow our baseline policy rules (6) and (7). After that date, however, the fiscal authority becomes completely passive, adjusting taxes to make sure that government debt returns to its steady-state value immediately and regardless of the economy's history before date H; the monetary authority at the same time turns active, aggressively leaning against any booms past that date. As shown next, this change in far-ahead policy eliminates any fiscally-led fluctuations in output and inflation in RANK, while having only a negligible effect in HANK, precisely due to the difference in the underlying mechanism.

Proposition 7. *Fix any $\tau_y > 0$, $\tau_d \in [0,1)$, and $\phi \in (\underline{\phi},\bar{\phi})$. Let H be any finite date; and suppose that the fiscal and monetary authorities follow the rules (6) and (7) for $t < H$ but switch to, respectively,*

$$t_t = d_t + \beta\frac{D^{ss}}{Y^{ss}}r_t \quad \text{and} \quad r_t = \phi' y_t \quad \text{for } t \geq H, \tag{24}$$

for arbitrary $\phi' > 0$. Then, there exists a unique equilibrium in which y_t is bounded. Furthermore:

1. *In RANK ($\omega = 1$), this equilibrium has $y_t = \pi_t = 0$ for all t and all realizations of uncertainty.*
2. *In HANK ($\omega < 1$), this equilibrium has $y_t = \pi_t = 0$ for all $t \geq H$ but not for $t < H$. Instead, this equilibrium is "near" that of Proposition 5 in the following sense: for any $T > 0$, as $H \to \infty$, $\{y_t, \pi_t\}_{t=0}^{T}$ converges to its counterpart in Proposition 5, for all realizations of uncertainty.*

To understand this result, we begin by studying what happens at $t \geq H$. The first part of the policy mix (24) means that taxes adjust to ensure that $d_t = 0$ for all $t \geq H+1$ and for all realizations of uncertainty. The no-Ponzi condition is therefore satisfied regardless of the economy's history, fiscal policy is unambiguously passive, and there is no threat to "blow up the government budget." This also ensures that the non-Ricardian channel is fully inactive after H: for all $t \geq H$, the IKC (17) holds with $z_t = 0$.[25]

[25] To see this note that, when z_t is identically zero, HANK's IKC (17) reduces to RANK's IKC (16), modulo the replacement of β with $\beta\omega$. Next, recall that RANK's IKC (16) is itself equivalent to the DIS equation (8), which is invariant to β (and hence invariant to the replacement of β with $\beta\omega$). We conclude that, once z_t is identically zero, as it is after date H, HANK is described by the same DIS equation as RANK—a property that echoes Werning (2015).

By further assuming that monetary policy turns active ($\phi' > 0$) for $t \geq H$, we finally guarantee that, in RANK and HANK alike, $y_t = 0$ for all $t \geq H$.

In RANK, it is furthermore immediate that we must have $y_t = 0$ also for all $t < H$. To see this, recall that the IKC for RANK is equivalent to the DIS relation (8), which we repeat here for convenience:

$$y_t = -\sigma r_t + \mathbb{E}_t\left[y_{t+1}\right] = -\sigma\phi y_t + \mathbb{E}_t\left[y_{t+1}\right].$$

Starting with $y_H = 0$ and then solving backwards, we immediately get that $y_t = 0$ for all $t < H$. This completes the proof of the first part of Proposition 7 and illustrates the first point mentioned above—the critical reliance of the FTPL equilibrium on the hard-to-test assumption that taxes *never adjust enough* to finance the initial deficit increase, e.g., as discussed in Kocherlakota and Phelan (1999). In particular, we see the importance of a point previewed in Section 3.3: fiscal policy does enter aggregate demand, in the precise sense that it drop out of the IKC, and so fiscal deficits become irrelevant for output and inflation as soon as the equilibrium selection margin is switched off. The left panel of Figure 2 illustrates: for any finite H, a date-0 deficit shock has no effect on inflation; it is only when "$H = \infty$" that real output and prices jump in response to the shock.

Turning to HANK, we observe that the path of y_t prior to date H is similarly uniquely pinned down, but now departs from steady state because—and *only* because—$z_t \neq 0$ for $t < H$. Along with the fact that the equilibrium has been selected by imposing the Taylor principle after date H, this verifies that, in the equilibrium obtained here, fiscal policy operates exclusively through the classical non-Ricardian channel—and not via the FTPL mechanism articulated in 3.3. Finally, and most importantly, the second part of the proposition establishes that this equilibrium converges to our original HANK equilibrium as $H \to \infty$. This convergence is, once again, due to the households' extra discounting of the future: as $H \to \infty$, this extra discounting ensures that the promised tax hike at date H, as well as any recession triggered by it around that date, has a negligible effect in the short run (i.e., for $t \ll H$). Households instead just spend the initial transfer in the short run, and so we converge to the outcomes characterized in Proposition 5. The right panel of Figure 2 illustrates: as H increases, fiscal deficits become progressively more expansionary, and thus more inflationary, with no discontinuity between large but finite H and $H = \infty$.

The role of far-ahead beliefs. While the above discussion was phrased in terms of assumptions about far-ahead policy, we note that there is a more general principle at work: both parts of Proposition 7 follow *directly* from the property that the economy is expected to return to the steady state at a finite date. The policy mix assumed after H merely *rationalizes* this expectation.

Corollary 2. *Replace (24) with the requirement that the economy returns to the steady state at date H (i.e., $y_t = \pi_t = 0$ for $t \geq H$), where H is finite but arbitrarily large. Then, both parts of Proposition 7*

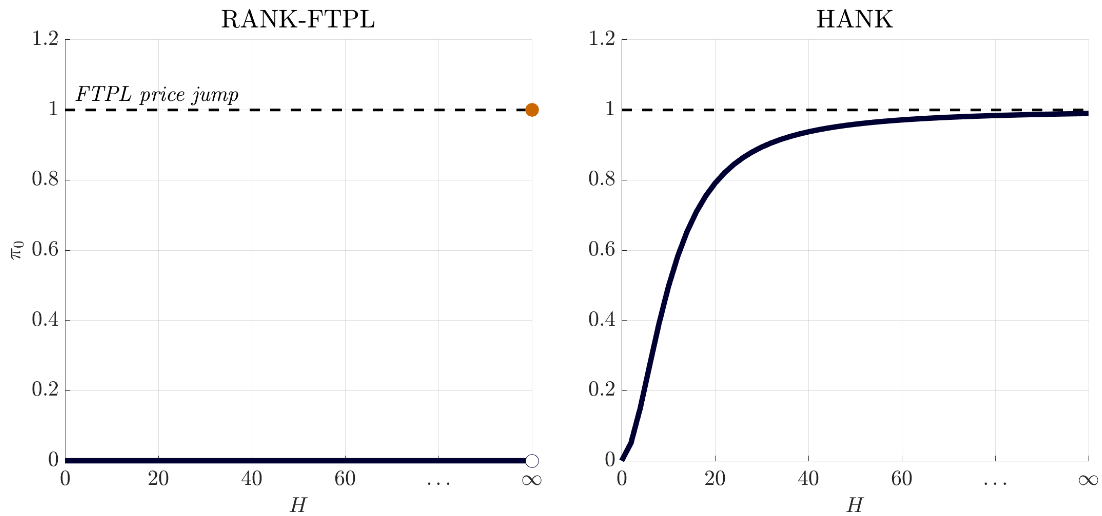

Figure 2: Date-0 inflation response to a deficit shock in RANK (left panel) and HANK (right panel) for different H. The size of the shock is normalized so that the FTPL price jump is 1 per cent.

continue to hold.

Because H can be arbitrarily large, this requirement can be interpreted as a refinement of equilibrium beliefs: it requires consumer beliefs at far-ahead horizons to be anchored to the steady state. At first glance, the fragility of RANK-FTPL to this refinement may appear surprising. In RANK, the economy converges back to steady state asymptotically, in both the conventional scenario ($\phi > 0$ and $\tau_d > 0$) as well as in the FTPL scenario ($\phi < 0$ and $\tau_d = 0$). By analogy to turnpike theorems, one might thus have conjectured only a small change in equilibrium outcomes if such convergence occurs at a finite but sufficiently far-ahead horizon. This conjecture holds true for RANK's conventional solution, as well as for our HANK equilibrium, but not for the FTPL equilibrium.

The logic for this fragility of RANK-FTPL echoes Angeletos and Lian (2023). In RANK, because consumers understand that fiscal policy has no wealth effect on equilibrium (as argued in Section 3.3), they are willing to condition their spending on fiscal deficits only if they expect other consumers to do the same in perpetuity. If, instead, consumers expect the effect of fiscal policy on aggregate spending to cease at some far-ahead but finite date, they can infer—essentially through the same backward-induction argument used in the proof of the first part of Proposition 7—that the shock cannot have any effect before that date either. Consequently, fiscal policy must be passive; otherwise, government debt would violate the no-Ponzi condition, contradicting equilibrium.[26]

[26] In Angeletos and Lian (2023), far-ahead beliefs were anchored by introducing small but appropriate noise in information, in the spirit of the global games literature (Morris and Shin, 1998). Here, the same essence is captured by assuming that the economy is expected to return to the steady state at an arbitrarily long but finite horizon. The key in both cases is that the indeterminacy problem of the New Keynesian model, and thus the freedom to select an equilibrium, is eliminated by appropriately anchoring far-ahead beliefs.

Our HANK equilibrium instead does not suffer from this fragility, for essentially the same reason that RANK's conventional solution does not suffer from it either. To see this, return to RANK, but now allow the economy to be subject to discount rate shocks. The DIS equation (8) becomes

$$y_t = -\sigma\phi y_t + \mathbb{E}_t\left[y_{t+1}\right] + \xi_t, \tag{25}$$

where ξ_t captures the discount rate shocks. If we anchor $\mathbb{E}_{H-1}\left[y_H\right]$ to 0 and then solve the above equation backwards, we get a unique solution for $\{y_t\}_{t=0}^{H-1}$, which naturally varies with $\{\xi_t\}_{t=0}^{H-1}$, simply because the ξ_t's show up directly in (25). The same logic applies to our HANK equilibrium: because households are non-Ricardian, the term z_t drives y_t via the IKC (17) in the same way that ξ_t drives y_t via the above DIS equation in RANK. In contrast, in FTPL, fiscal variables do not show up *directly* in the DIS equation (or, equivalently, in the IKC)—and this absence is what lies at the heart of the lack of robustness discussed above.

Bottom line. Combining Proposition 7 and Corollary 2 with our HANK-FTPL equivalence results, we can conclude that, while the prevailing *formalization* of the FTPL depends on subtle and controversial assumptions regarding equilibrium selection and far-ahead beliefs, the *essence* of the FTPL does not hinge on these assumptions: its core prediction obtains naturally when households are non-Ricardian, fiscal adjustment is sufficiently slow, and the monetary policy reaction is sufficiently timid.

5 Extensions

The preceding analysis focused on two tasks: (i) to establish that HANK can produce FTPL-like outcomes; and (ii) to clarify the crucial difference in the underlying mechanisms and the implications of this difference in terms of robustness (and, relatedly, front-loading). To accomplish these tasks as transparently as possible, we used a highly tractable model. We now discuss how our results on the deficit-inflation mapping in FTPL and in HANK extend to three important model extensions of practical relevance: long-term government debt in Section 5.1; heterogeneous household bond holdings as well as transfer receipts in Section 5.2; and a hybrid NKPC in Section 5.3. All of these extensions will feature prominently in our quantitative analysis in Section 6.

5.1 Long-term government debt

We allow for government debt to be long-term. In keeping with much of the FTPL literature, we consider the analytically tractable case of a geometric maturity structure (e.g., Cochrane, 2001). For simplicity we furthermore here restrict attention to the special case of fixed (expected) real rates, with the extension to interest rate feedback relegated to Appendix A.2.

Environment. Nominal public debt is long-term, with its maturity parameterized by $\delta \in [0,1]$; the baseline case of short-term debt is nested as $\delta = 0$. The government flow budget now becomes

$$d_{t+1} = \underbrace{\frac{1}{\beta}(d_t - t_t) + \frac{D^{ss}}{Y^{ss}} r_t}_{\mathbb{E}_t[d_{t+1}]} - \underbrace{\frac{D^{ss}}{Y^{ss}}\left(\pi^{\delta}_{t+1} - \mathbb{E}_t\left[\pi^{\delta}_{t+1}\right]\right)}_{\text{debt erosion due to inflation surprise}} - \underbrace{\frac{D^{ss}}{Y^{ss}}\left(r^{\delta}_{t+1} - \mathbb{E}_{t-1}\left[r^{\delta}_{t+1}\right]\right)}_{\text{debt revaluation due to real rate surprises}} \tag{26}$$

where

$$\pi^{\delta}_t \equiv \mathbb{E}_t\left[\sum_{k=0}^{\infty} (\beta\delta)^k \pi_{t+k}\right] \quad \text{and} \quad r^{\delta}_t \equiv \mathbb{E}_t\left[\sum_{k=0}^{\infty} (\beta\delta)^{k+1} r_{t+k}\right] \tag{27}$$

A derivation of (26) is provided in Appendix A.2, but the logic is straightforward: $d_{t+1} - \mathbb{E}_t[d_{t+1}]$, the innovation in the real market value of the government debt, is proportional to the innovations in cumulative inflation as well as real rates over the duration of public debt. The remainder of the model is exactly as in Section 2. The essential structure of the HANK equilibrium furthermore remains exactly the same, in the sense that (18) in Proposition 2 continues to hold with the same values of χ and ρ_d. The only relevant change is the equilibrium size of the deficit-led boom, and how this boom translates to deficit-relevant inflation and real rate surprises, as explained next.

HANK meets FTPL. The comparison of inflation in HANK and FTPL now concerns the maturity-adjusted *cumulative* inflation response $\pi^{\delta}_{\varepsilon} \equiv \frac{d\pi^{\delta}_t}{d\varepsilon_t}$—i.e., the summary statistic of the impact of the inflation surprise on the government budget in (26). This object has received much attention in the FTPL literature (e.g., see Barro and Bianchi, 2024), precisely because it is the object for which the FTPL makes the starkest prediction. We also note that, for the empirically relevant case of δ close to 1, it is very similar to the full cumulative inflation impulse response, an object customarily studied in the monetary economics literature (e.g., see Alvarez et al., 2016). For this object, we now find a weaker form of equivalence, as summarized in Proposition 8: still exact equivalence if the tax base channel is absent, but otherwise, RANK-FTPL serves as an *upper bound.*

Proposition 8. *Let $\omega < 1$, $\tau_y > 0$, $\tau_d = 0$, $\delta > 0$, and $\phi = 0$.*[27] *There exists a unique bounded equilibrium in the HANK economy. The quantity $\pi^{\delta}_{\varepsilon}$, which measures the degree of debt erosion or, equivalently, the maturity-adjusted cumulative inflation response to a deficit shock, is strictly lower in the HANK economy than its FTPL counterpart:*

$$\pi^{\delta,HANK}_{\varepsilon} = \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}(1 - \beta\delta\rho_d)} < \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}(1 - \beta\delta)} = \pi^{\delta,FTPL}_{\varepsilon}, \tag{28}$$

with the distance between the two vanishing when $\tau_y \to 0$ or $\kappa \to \infty$ (no tax-base self-financing channel) or when $\delta \to 0$ (short term debt).

[27] As in Proposition 6, Proposition 8 focuses on $\tau_d = 0$ and does not repeat the continuity and monotonicity of $\pi^{\delta,HANK}_{\varepsilon}$ in $\tau_d \in [0,1)$, although these properties continue to hold. See the proof of Proposition 8 for details.

The key takeaway is that the (cumulative) inflation response in HANK is *smaller* than in RANK-FTPL. The intuition is simple, reflecting the interaction of long-term debt with the inflation front-loading implied by HANK. Since inflation is at all dates proportional to the present discounted value of future output responses, making any given output boom more front-loaded (while holding its present value fixed) will leave the impact inflation unchanged, but lower the subsequent inflation responses. This then reduces the scope for debt erosion—and thus also the cumulative inflation response, discounted by δ—in HANK relative to RANK-FTPL. The interaction of front-loading and long-term debt is evident in equation (28), with ρ_d and δ entering $\pi_\varepsilon^{\delta,HANK}$ only via the product $\delta\rho_d$.

Though lower in magnitude, $\pi_\varepsilon^{\delta,HANK}$ shares the comparative statics of its RANK-FTPL counterpart with respect to both $\frac{D^{ss}}{Y^{ss}}$ and δ: as shown in the proof of Proposition 8, the cumulative inflation response in HANK decreases with the level of government debt and increases with its maturity. This connects directly to the empirical findings of Barro and Bianchi (2024). That paper provides cross-country evidence that a country's debt-to-GDP ratio and average debt maturity help predict how much that country's inflation co-varied with its government spending during the post-covid period. Our analysis reveals that this exact empirical pattern is also consistent with HANK.[28]

5.2 Heterogeneous distributional incidence

While the simple OLG model studied thus far captures the key feature of richer HANK models that is essential for our purposes—namely the classical failure of Ricardian Equivalence—it abstracts from various forms of heterogeneity and, consequently, from the distributional effects of inflation. Specifically, by eroding the real value of government bonds (or other nominal assets), fiscally-induced inflation will necessarily redistribute real wealth from households with large savings in such assets to households with small savings (or with debt). In complementary work, Kaplan et al. (2023) emphasize this channel in a flexible-price heterogeneous-agent model. Here, we ask whether and how this channel matters for the propagation of fiscal deficit shocks in the New Keynesian framework and in particular how it affects our HANK-FTPL equivalence. To address this question, we consider a tractable extension of our baseline model, featuring two types of non-Ricardian households—rich, low-MPC households and poor, high-MPC households.

Environment. We study a hybrid model that combines our baseline OLG block with a margin of hand-to-mouth spenders, with $\mu \in (0,1)$ denoting the share of spenders. From Auclert et al. (2024) and Wolf (2024), we know that such models can fit relatively well both the available microeconomic evi-

[28] Furthermore, if the tax base channel is weak (because either $\tau_y \to 0$ or $\kappa \to \infty$), then we again converge to equality in (28)—i.e., the maturity-adjusted cumulative inflation in HANK also converges to the inverse of the debt-to-GDP ratio, as in the simple RANK-FTPL arithmetic, which is indeed the theoretical benchmark in Barro and Bianchi (2024).

dence on consumer responses to transfers and the entire profile of iMPCs generated by fully-fledged quantitative HANK models. Furthermore, since spenders do not hold any assets, such a model can also capture—albeit in a crude way—the redistributional effects mentioned above.

In this extension, the aggregate consumption function (1) generalizes to

$$c_t = \left(1-\beta\omega\right) a_t + \left(\mu + (1-\mu)(1-\beta\omega)\right)\left(\left(y_t - t_t\right) + \frac{(1-\mu)(1-\beta\omega)}{\mu + (1-\mu)(1-\beta\omega)}\mathbb{E}_t\left[\sum_{k=1}^{\infty}\left(\beta\omega\right)^k\left(y_{t+k} - t_{t+k}\right)\right]\right), \tag{29}$$

where we have for simplicity already imposed the assumption of a neutral monetary policy, i.e. $\phi = 0$ in (7). The remainder of the model is as in Section 2, except that we will allow for long-term debt.

HANK meets FTPL. Even in this generalized model variant we continue to obtain similar comparison results between HANK and FTPL; there is exact equivalence when $\delta = 0$, and it takes the form of an upper bound when $\delta > 0$, mirroring Propositions 3 and 8.

Proposition 9. *Let* $\omega < 1$, $\tau_y > 0$, $\tau_d = 0$, $\phi = 0$, *and* $\mu \in (0,1)$. *There exists a unique bounded equilibrium, and it has the following properties.*

1. *If* $\delta = 0$, *the initial price jump in response to a deficit shock is exactly the same as its counterparts in our baseline HANK economy and in FTPL:*

$$\pi_{\varepsilon,0} = \pi_{\varepsilon,0}^{HANK} = \pi_{\varepsilon,0}^{FTPL}.$$

2. *If* $\delta > 0$, *the maturity-adjusted cumulative inflation response to a deficit shock is bounded from above by its analogue in our baseline HANK economy and hence also by FTPL:*

$$\pi_{\varepsilon}^{\delta} < \pi_{\varepsilon}^{\delta,HANK} < \pi_{\varepsilon}^{\delta,FTPL}. \tag{30}$$

By triggering inflation and eroding the real value of the government bonds held by rich, low-MPC households, fiscal deficit redistribute from these households to poor, high-MPC households (i.e., the hand-to-mouth households). This additional impetus to demand front-loads the fiscally-led boom even more. If government debt is short-term ($\delta = 0$), then this additional front-loading is immaterial for the initial price jump and so the overall debt erosion obtained when $\tau_d = 0$; if instead debt is long-term ($\delta > 0$), then the additional front-loading further dampens inflationary pressures, by exactly the same reasoning as that behind Proposition 8.[29]

Even more general aggregate demand. While formally proved only for our benchmark OLG setting and the two-type extension of this section, our equivalence and robustness results are materially more general, and in particular extend to richer, numerically solved HANK-type environments. As

[29] A complementary analysis is provided in Diamond et al. (2022), focusing on the role of mortgage debt.

emphasized throughout, the two properties of consumer behavior driving our conclusions are (i) that households discount the future at a higher rate than the interest rate on government bonds, and (ii) that they spend any additional income faster than in the permanent-income benchmark, leading to a transitory boom.[30] Provided this holds, any initial deficit will pass through the IKC and the NKPC to boost output and prices by an amount that increases with the delay in fiscal adjustment; as this delay grows larger, the price jump approaches its FTPL counterpart, by the same logic as that discussed in Section 4, and thus with the same robustness properties. Our quantitative analysis in Section 6—which contains a fully-fledged HANK model, as well as several other demand structures—will further illustrate this discussion.

5.3 Hybrid NKPC

Our analysis thus far has featured the textbook NKPC. We now ask how our results change with generalized Phillips curves, such as those implied by price indexation (e.g., Christiano et al., 2005), menu costs (e.g., Auclert et al., 2023), or bounded rationality (e.g., Angeletos and Huo, 2021). All these cases boil down to replacing (2) with a more flexible mapping from the path y_t to the path of π_t.

We begin with some preliminary observations. First, the main conceptual point of Section 3 clearly extends to *any* such mapping: equilibrium selection still remains the only channel through which fiscal deficits can drive output and inflation in RANK. Second, and similarly, the core of our HANK analysis in Section 4 does not depend on the specific form of the Phillips curve: fiscal policy still influences aggregate demand through the term z_t in the IKC (17). However, things can change *quantitatively*, as the specification of the Phillips curve will affect the relative contributions of debt erosion and tax-base expansion in deficit financing. Replacing (2) with generalized Phillips curves could, therefore, affect the precise HANK-FTPL equivalence, unless the tax-base channel is shut down entirely.

The remainder of this section elaborates on this last observation. We focus on the empirically relevant case of a Hybrid NKPC that allows price-setting to be partially backward-looking, thus capturing the sluggishness of inflation observed in the data.

Environment. Following the above discussion, we replace (2) with the Hybrid NKPC:[31]

$$\pi_t = \kappa y_t + \xi \beta \pi_{t-1} + (1-\xi)\beta \mathbb{E}_t[\pi_{t+1}], \tag{31}$$

[30] In sequence-space terms (Auclert et al., 2021), such discounting translates to the off-diagonal elements of the intertemporal MPC matrix decaying to zero sufficiently quickly. See Section 5.2 and Appendix E.1. of Angeletos et al. (2024).

[31] The conventional micro-foundation of (31) is price indexation (Christiano et al., 2005), while an empirically plausible alternative is incomplete information or bounded rationality (Angeletos and Huo, 2021). In either case, the appeal of (31) lies in its ability to better account for the inflation dynamics observed in the data.

where $\xi \in [0,1]$ parameterizes the degree of backward-lookingness in price-setting. The remainder of the model is exactly as in Section 2; in particular, we restrict attention to the case of short-term government debt, for reasons that will become clear shortly.

HANK meets FTPL. With a hybrid NKPC, the exact equivalence between HANK and FTPL continues to obtain in the absence of the tax base channel. If this channel is present, however, then the short-run inflationary pressures are *larger* in HANK than in FTPL—exactly the opposite of the case with long-term government debt discussed earlier.

Proposition 10. *Let $\omega < 1$, $\tau_y > 0$, $\tau_d = 0$, $\delta = 0$, and $\phi = 0$, and let inflation now follow the hybrid NKPC (31) with any $\xi \in (0,1]$. There exists a unique bounded equilibrium in the HANK economy, of the same form as in Proposition 2. The initial price jump in response to a deficit shock is strictly higher than the FTPL counterpart with the same hybrid NKPC:*

$$\pi^{HANK}_{\varepsilon,0} > \pi^{FTPL}_{\varepsilon,0},$$

with the distance between the two vanishing when $\tau_y \to 0$ or $\kappa \to \infty$.

The intuition is as follows. Compared to the textbook NKPC, its hybrid generalization (31) is less forward-looking, so current inflation depends more heavily on output in the near future. Since the output boom itself is more front-loaded in HANK than in RANK-FTPL, this means that the initial inflation increase in the former is *larger* than in the latter.

Note that this result assumes short-term government debt to isolate how the front-loading of the output response, due to finite household horizons, interacts with inflation inertia, due to the hybrid NKPC. However, we have already shown that the same front-loading in output, when combined with long-term debt, moves inflation in the opposite direction. It follows that the precise relationship between HANK and FTPL becomes ambiguous in the general case, which features both long-term debt and inflation inertia. Our quantitative analysis in the next section will combine all of the model ingredients considered here—and more—to shed light on the empirically relevant scenario.

6 Quantitative analysis

We now complement our theoretical results with a quantitative analysis of the deficit-inflation nexus in HANK. Our results so far suggest that even the predictions of the textbook extreme version of the FTPL—in which current deficits are financed *entirely* through a commensurate jump in prices—can emerge in HANK economies. The main takeaway of this section, however, is that, in practice, deficits are likely to be much less inflationary than predicted by the simple FTPL arithmetic.

To this end, we study the mapping from deficits to inflation in a version of our HANK model that is disciplined through direct evidence on the key ingredients of our theory. Section 6.1 describes model and calibration, Sections 6.2 and 6.3 contain the main results, and Section 6.4 closes with an application to post-covid inflation dynamics.

6.1 Extended HANK model and calibration

We consider a variant of the model environment in Section 2, with three additions, following our discussion in Section 5. First, government debt is now long-term. Second, we allow for moderate household heterogeneity, with three types of households i, indexed by heterogeneous survival probabilities ω_i. This extension will allow the model to be simultaneously consistent with empirical evidence on (i) intertemporal marginal propensities to consume (e.g., Auclert et al., 2024) as well as (ii) household wealth holdings and transfer receipts. Third, we consider a hybrid NKPC, yielding more realistic inflation dynamics. The remainder of this section presents calibration details for all model blocks, with a summary provided in Table 1. We will study several further model variants—including a full-fledged HANK model—in Section 6.3, with details for those model variants provided in Appendix B.1.

Throughout this section, and as in Sections 2 - 5, the policy experiment that we consider is a one-off, surprise fiscal deficit increase at date 0 (i.e., a tax cut), equal to one per cent of steady-state GDP.

Consumers. We extend the consumer block of Section 2.1 to allow for three types of households i, with respective population shares χ_i. Households differ in their death probabilities ω_i—or, less literally, in their probably of being subject to a binding borrowing constraint—, steady-state wealth shares A_i^{ss}/A^{ss}, and exposure to fiscal deficit shocks (i.e., transfer receipts). We choose population shares and death probabilities to match empirical evidence on average intertemporal marginal propensities to consume, from Fagereng et al. (2021). Wealth shares are set to roughly replicate the skewness of the U.S. wealth distribution (e.g., see Kaplan et al., 2018), with the bottom 15 per cent holding no wealth, and the top quantile holding 60 per cent of all wealth. Finally, consistent with U.S. policy practice, transfer receipts are somewhat more concentrated at the bottom. We also set $\sigma = 1$ (giving log preferences), and back out β to hit a steady-state real rate of interest of one per cent (annual). Our model variants in Section 6.3 will consider several alternative assumptions on the departure from Ricardian equivalence, wealth shares, and transfer receipts, including a full HANK model.

Nominal rigidities. We assume a hybrid NKPC, as discussed in Section 5.3. For the slope κ we consider two headline values: the shallow slope estimated by Hazell et al. (2022); and a three-times steepening of that NKPC, as estimated in Cerrato and Gitti (2022) for the post-covid inflationary period. Finally, for the backward-forward split (ξ vs. $1-\xi$), we take the headline point estimates reported in

Parameter	Description	Value	Target
Demand Block			
χ_i	Population shares	$\{0.218, 0.629, 0.153\}$	Fagereng et al.
ω_i	Survival rates	$\{0.972, 0.833, 0\}$	Fagereng et al.
A_i^{ss}/A^{ss}	Wealth shares	$\{0.6, 0.4, 0\}$	See text
ε_i	Transfer receipt	$\{0.122, 0.706, 0.172\} \times \varepsilon$	See text
σ	EIS	1	Standard
β	Discount factor	0.998	Annual real rate
Supply Block			
κ	Slope of Hybrid NKPC	$\{0.006, 0.019\}$	Hazell et al.; Cerrato and Gitti
ξ	Backward-lookingness	0.288	Barnichon and Mesters
Policy			
τ_y	Tax rate	0.33	Average Labor Tax
D^{ss}/Y^{ss}	Gov't debt level	1.79	See text
δ	Gov't debt maturity	0.95	Av'g debt maturity
τ_d	Tax feedback	0	Anderson and Leeper
ϕ	Inflation feedback	0	See text

Table 1: Quantitative model, calibration.

Barnichon and Mesters (2020).

In our main quantitative analysis we will furthermore report results for an entire (and wide) range of κ's. The alternative model variants studied in Section 6.3 will also feature alternative assumptions on the backward-forward split in the NKPC.

Policy. We set $\tau_y = 0.33$, implying meaningful—and empirically realistic—feedback from economic activity to primary surpluses. Government debt, D^{ss}, is set to match the total amount of domestically, privately held U.S. government debt, and $\delta = 0.95$ gives an average debt maturity of five years.[32] Consistent with legislative evidence on the post-covid fiscal stimulus (e.g., see the detailed discussion Anderson and Leeper, 2023), we consider an "unbacked" fiscal expansion, so $\tau_d = 0$. Finally, as in our main analysis, we set $\phi = 0$, corresponding again to a fixed real-rate rule. We do so for two reasons. First, in that case, our simulations will be informative about the pure effect of the deficit, without any direct monetary offset or accommodation. Second, as discussed in Angeletos et al. (2024), this case is

[32]We target government debt—rather than household liquid wealth, as in Kaplan et al. (2018)—since the government debt-to-GDP ratio is what matters for the FTPL arithmetic. With a share of around 42 per cent of U.S. government debt being held by private, domestic entities (Department of the Treasury, 2024, p.50), the pre-covid (2020:Q1) quarterly debt-to-GDP ratio of 4.28 gives $D^{ss}/Y^{ss} = 1.79$.

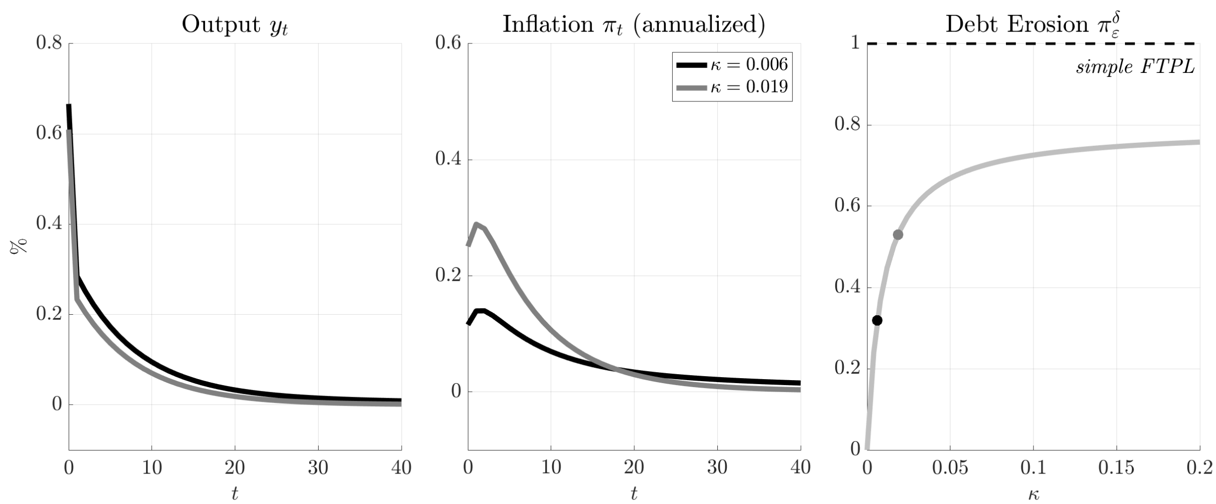

Figure 3: Output and inflation impulse responses to a date-0 deficit shock of size D^{ss}/Y^{ss} for different values of κ (left and middle), and π_ε^δ as a function of κ (right).

actually a quite reasonable approximation to many past fiscal stimulus episodes.

For our alternative model variants and the quantitative post-covid application we will pay particular attention to what happens under alternative assumptions on fiscal adjustment (τ_d) and on the monetary policy reaction (ϕ).

6.2 Benchmark specification

We study how, in our quantitative model, fiscal deficits transmit to inflation. Figure 3 shows impulse responses of aggregate output and inflation to a deficit shock that, according to the simple FTPL arithmetic, would move cumulative (maturity-adjusted) inflation by 1 per cent (left and middle panel), for our two headline values of κ (shades of grey). The right panel then displays the cumulative (maturity-adjusted) inflation response π_ε^δ as a function of κ, over a large range.

The main takeaway from the figure is that the inflationary pressures associated with the unfunded fiscal deficit shock are—while material—quite substantially weaker than predicted by the simplest textbook FTPL arithmetic. The key panel is the right one, which shows the cumulative inflation response as a function of κ, relative to the simple FTPL arithmetic prediction (dashed line). We see that, even for an NKPC three-times as steep as the pre-covid estimates of Hazell et al. (2022), the cumulative inflation response is actually only around half of the simple FTPL prediction. The left panel provides the answer for why: output booms with a cumulative multiplier around 1.37 - 1.94 (for our two headline values of κ), generating meaningful tax revenue through the feedback from economic activity to primary surpluses (with $\tau_y = 0.33$). Such a tax base expansion substitutes for the cumulative inflation response (and its induced debt erosion) to finance the deficit shock. Finally, the middle panel shows the time profile of the inflation response: consistent with our theoretical results, the inflation that does occur is front-loaded and relatively short-lived, with around a quarter of the entire inflation response already occurring over the first year. Given that government debt is long-term, this front-loading—which is further reinforced by the fiscal shock's distributional incidence—is also part of the dampening of the overall cumulative inflation response visible in the right panel.

The remainder of this section extends our analysis in two ways. First, in Section 6.3, we go beyond the benchmark model parameterization and explore the effects of various possible model alterations. Second, in Section 6.4, we discuss implications of our results for the post-covid inflationary episode.

6.3 Model variants

We now study the deficit-inflation mapping in several alternative variants of our quantitative model, allowing us to shed light both on the broader relevance of our conclusions as well as on the role played by the various model ingredients. Details for all variants are provided in Appendix B.

- *Consumers.* For a first set of experiments, we alter our empirically disciplined consumer block to feature no heterogeneity in bond holdings and transfer receipts ("iMPC"), heterogeneity only in bond holdings ("Het. B"), and heterogeneity only in transfer receipts ("Target"). Second, we consider what happens if households are behavioral, with a sticky information friction as in Auclert et al. (2020) ("Behavioral"). Third, we replace our consumer block by the one-type OLG structure of Section 2 ("OLG") and by a full-blown HANK structure ("HANK").

- *Nominal rigidities.* Our analysis in Section 6.2 already shed light on the role of NKPC slope κ. We here additionally consider what happens if our empirically disciplined hybrid NKPC is replaced by a simple textbook forward-looking one ("f-NKPC").

- *Policy.* To further illustrate our theoretical "robustness" discussion, we also investigate what happens with gradual fiscal adjustment ("Fiscal Adjustment", $\tau_d = 0.02$) and with active monetary policy ("Active MP", $\phi = 0.25$, together with fiscal adjustment, $\tau_d = 0.02$). We also consider a model variant in which the average government debt maturity is halved ("Half Mat.", $\delta = 0.9$).

Our results are reported in Figures 4 and 5. Figure 4 shows the cumulative (maturity-adjusted) inflation response π_ε^δ (in the x-axis) and the short-run inflation share (defined as the share of inflation in the first year relative to the first five years, in the y-axis), under various model specifications. The simple FTPL arithmetic is in the bottom right ("simple FTPL"), with the cumulative inflation response normalized to 1. Starting from this reference point and then adding tax-base self-financing ("FTPL w/ τ_y") does not affect the persistence of the inflation burst, but dampens its magnitude. Moving

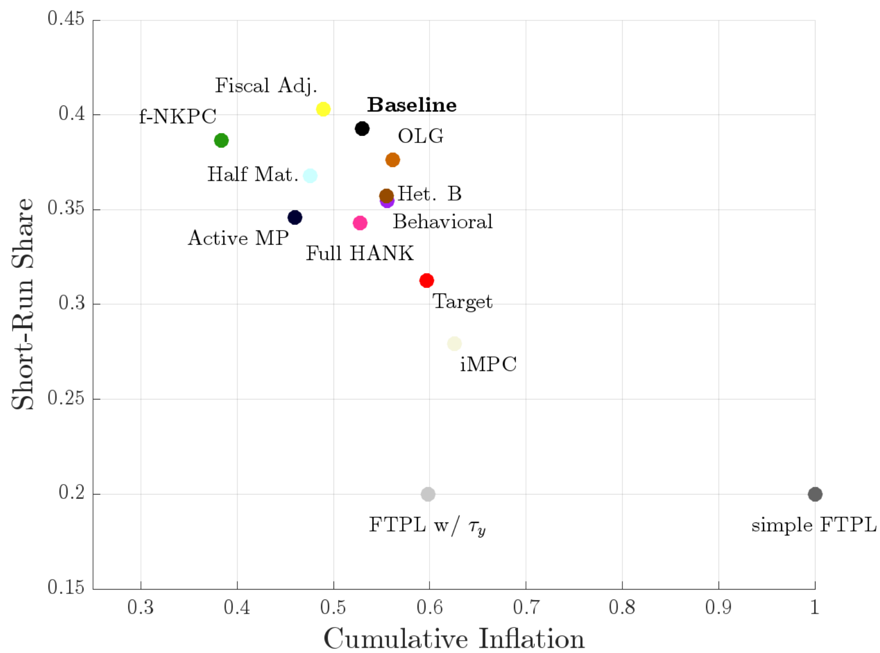

Figure 4: Cumulative inflation response and short-run response share to a date-0 deficit shock of size D^{ss}/Y^{ss}, for different model variants, indicated by dots.

to our HANK model ("baseline") reduces cumulative inflation a bit more while materially increasing the short-run inflation share. This increase simply reflects the front-loading property, while the reduction in cumulative inflation is governed by the interaction of front-loading with long-term debt and the hybrid NKPC. Long-term debt significantly dampens the inflation response, with the hybrid NKPC partially offsetting this effect (cf. the "Baseline" and "f-NKPC" dots). Finally, *all* other HANK variants (all other dots) remain in the top left of the figure: while changing model parameterization details affects the precise magnitudes, it does not alter the core finding that inflation responses are substantially smaller and more front-loaded than in the simple FTPL benchmark.

Figure 5 shows full impulse responses for selected model variants, allowing us to dig deeper into the role played by the various model alterations. First, with a more aggressive monetary policy, the inflation response is—as expected—dampened, but of course remains present, illustrating our theoretical results on the robustness of the deficits-inflation mapping in HANK-type models. Second, in the less forward-looking behavioral model, the intertemporal Keynesian cross underlying the deficit-inflation mechanism in HANK plays out more slowly, and so the inflation burst is slightly more persistent. And third, moving to a full-blown HANK model has very limited effect on our results, consistent with prior work establishing that analytical models of the sort provided here provide an excellent approximation to aggregate output and inflation dynamics in HANK.

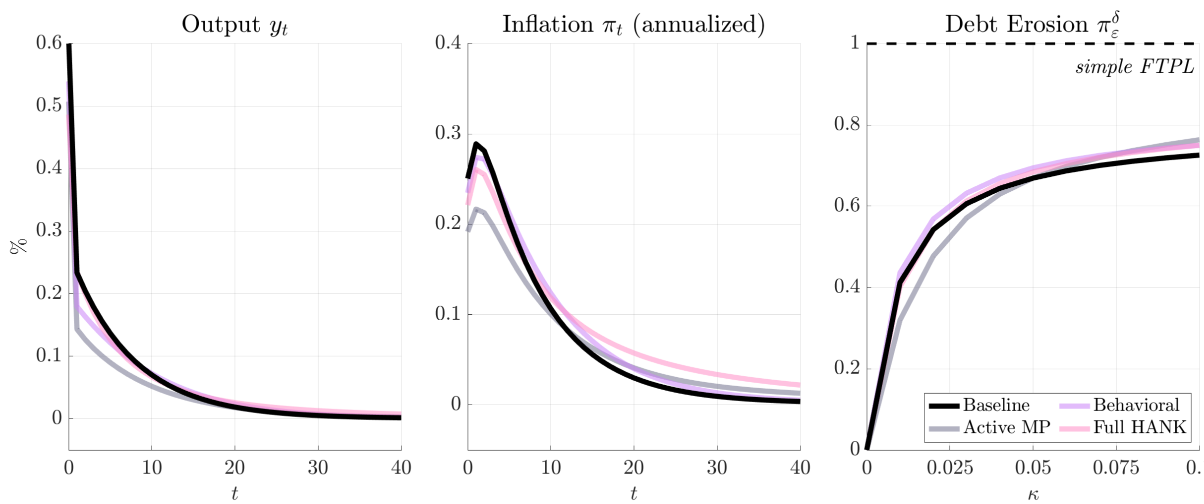

Figure 5: Output and inflation impulse responses to a date-0 deficit shock of size D^{ss}/Y^{ss} (left and middle) and π_ε^δ as a function of κ (right), for different model variants.

Finally, we also note that, while the results in Figure 4 assume a fixed real rate (except, of course, for the model variant with an active monetary rule), our results do not hinge on that assumption. Specifically, Appendix B.2 repeats our analysis for a fiscal stimulus accompanied by monetary accommodation (lower real rates). In that case, the standard FTPL also features a front-loaded inflation response, since the real rate cut encourages households to front-load consumption. Crucially, however, in our HANK model variants, and for the same real rate path, the inflation response is *even more* front-loaded (again because of discounting), thus overall delivering the same picture as in Figure 4.

6.4 Application to post-covid inflation dynamics

Finally, we use our quantitative model for an application to post-covid inflation dynamics. Results are reported in Figure 6, which shows output and inflation impulse responses as well as the discounted cumulative inflation response under different assumptions on policy.

Policy experiments. We consider a two-step fiscal deficit shock: first, at $t = 0$, there is a shock equal to \$0.795tr (payments to households as part of the CARES Act), and second, at $t = 3$, there is a shock equal to \$0.844tr (payments to households as part of the ARP Act). We restrict attention to payments to households because our theoretical analysis only directly speaks to the propagation of this kind of fiscal deficit increase. We then furthermore assume that there is no fiscal adjustment (i.e., we set $\tau_d = 0$), consistent with actual legislation so far (e.g., see the review in Anderson and Leeper, 2023).[33]

[33] We assume that the two stimulus packages are surprises. We obtain very similar results under the opposite extreme of perfect foresight, see Appendix B.3. The precise numbers for the payments to households in our policy experiment are taken from Committee for a Responsible Federal Budget (2024).

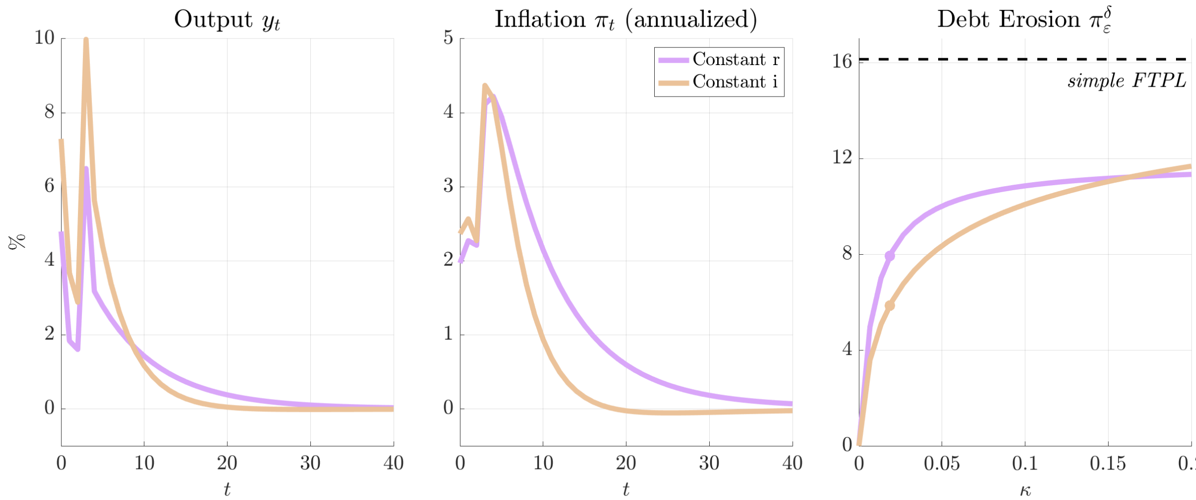

Figure 6: Output and inflation impulse responses (left and middle) to the post-covid fiscal deficit shock (see text) and π_ε^δ (right) as a function of κ, under two different assumptions on the monetary policy reaction: fixed real rates (purple) and fixed nominal rates (orange).

We study impulse responses to this fiscal deficit shock under two different assumptions on the monetary policy reaction. First, we keep real rates fixed. The resulting impulse responses will identify the causal effect of the fiscal expansion *in isolation*; i.e., what is the incremental impetus to inflation, keeping the monetary policy stance—in terms of real rates—exactly as observed in the data? Second, we keep nominal rates fixed. This counterfactual keeps the monetary stance in policy instrument space as in the data, and thus—since the fiscal deficit will be inflationary—embeds the effects of additional monetary accommodation, i.e., a decline in real interest rates.

Results. The results from our policy experiments are reported as the purple and orange lines in Figure 6. Consider first the overall magnitudes. Given the size of the deficit shock, the simple textbook FTPL accounting would predict a cumulative discounted inflation response of around 16%. We see that both policy experiments in our setting predict material dampening relative to that upper bound, consistent with our results in Sections 6.2 - 6.3. The burst in inflation is furthermore, in both cases, concentrated in the first couple of years after the fiscal deficit shock.

We next investigate further the role of the monetary policy response by contrasting the two sets of impulse responses. The counterfactual of nominal interest rates kept as in the data corresponds to additional monetary accommodation, and thus leads to a larger and more front-loaded demand boom, together with a reduction in government borrowing costs. Taken together, stronger front-loading as well as reduced borrowing costs (by the flip-side of the classical "stepping-on-a-rake" effect, as studied in Sims, 2011) *lower* the overall cumulative inflation response.

7 Conclusion

How, and by how much, do fiscal deficits drive inflation? We addressed these questions in the New Keynesian framework, comparing and contrasting FTPL and HANK. These two theories diverge sharply on the "how": in FTPL, deficits influence economic activity and inflation through equilibrium selection; in HANK, they operate via classical non-Ricardian effects. *Despite* this difference, the theories can align—not only qualitatively but also quantitatively—on the "how much": provided that fiscal adjustment and monetary policy reactions are sufficiently slow, HANK can produce the same inflation responses and debt erosion as the FTPL. However, *because* of the difference in mechanisms, this common prediction is, in HANK, robust to an "active" monetary authority and a "passive" fiscal authority, as well as to plausible refinements about far-ahead beliefs. Together, these findings shift the research focus away from the untestable debate on equilibrium selection and toward the more tangible question of how quickly fiscal adjustment and monetary policy reactions take effect.

Our contribution concluded with a quantitative evaluation of just how inflationary fiscal deficits are likely to be in practice. To this end, we disciplined the theory's core components—i.e., intertemporal MPCs, the slope of the Phillips curve, the maturity structure of government debt, and the automatic feedback from real economic activity to the tax base—with relevant empirical evidence. We then benchmarked our results against the simple FTPL arithmetic, which assumes no such feedback and posits that prices rise enough to fully finance any fiscal deficit shock. The key takeaway from this part of the paper was that, while fiscal deficits are undoubtedly inflationary, their impact is only about half of what the simple FTPL arithmetic would predict.

Our analysis suggests at least three avenues for future research. First, our quantitative findings were model-based, with empirical discipline applied indirectly through evidence on individual model components; it would be valuable to confront the theory with more direct evidence on the deficit-inflation relationship (e.g., as done in Hazell and Hobler, 2024). Second, our analysis assumed rational expectations, abstracting from the possibility that private agents may perceive a different deficit-inflation relationship than the actual one (e.g., as in Bigio et al., 2024); extending the analysis to account for this is another open question. Finally, while HANK shares FTPL's *positive* predictions, it likely has different *normative* implications due to the difference in underlying mechanisms.

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Appendices for:
Deficits and Inflation: HANK meets FTPL

This Appendix contains further material for the article "Deficits and Inflation: HANK meets FTPL". We provide: (i) supplementary details for our baseline model environment (Section 2) and its various extensions (Section 5); (ii) supplementary model details, additional analysis, and alternative results for our quantitative investigations in Section 6; and (iii) all proofs.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded by "A."—"C." refer to the main article.

A Supplementary theoretical details and extensions

Appendix A.1 provides further details for the headline model environment of Section 2, while Appendix A.2 does the same for the extended model with long-term debt (see Section 5.1), including a version of our HANK-FTPL equivalence result for general monetary policy.

Throughout, we will use uppercase variables to indicate levels; unless indicated otherwise, lowercase variables denote log-deviations from the economy's deterministic steady state. We log-linearize around a steady state in which inflation is zero ($\Pi^{ss} = 1$), real allocations are given by their flexible-price counterparts, and the real debt burden is constant at some level $D^{ss} \geq 0$. As discussed below, our assumptions on annuities and the social fund ensure that $R^{ss} = \frac{1}{\beta} > 1$, and steady-state taxes satisfy $T^{ss} = (1-\beta)D^{ss}$. While we will throughout focus on on the empirically relevant scenario with $D^{ss} > 0$, we do wish to accommodate $D^{ss} = 0$, and so we let $d_t \equiv (D_t - D^{ss})/Y^{ss}$, $t_t \equiv (T_t - T^{ss})/Y^{ss}$, and $a_{i,t} = \left(A_{i,t} - A^{ss}\right)/Y^{ss}$—i.e., we measure fiscal variables (and so also household wealth) in terms of absolute deviations (rather than log-deviations) from steady state, scaled by steady-state output.

A.1 Environment

Aggregate demand. The household block is the same as in Angeletos et al. (2024), which is restated here for completeness. The economy is populated by a unit continuum of households. A household survives from one period to the next with probability $\omega \in (0,1]$ and is replaced by a new one whenever it dies. Households have standard separable preferences regarding consumption and labor, and do not consider the utility of future households that replace them. The expected utility of any (alive) household i in period $t \in \{0,1,\dots\}$ is hence

$$\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\left[u(C_{i,t+k}) - v(L_{i,t+k})\right]\right], \tag{A.1}$$

where $C_{i,t+k}$ and $L_{i,t+k}$ denote household i's consumption and labor supply in period $t+k$ (conditional on survival), $u(C) \equiv \frac{C^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}$, $v(L) = \iota\frac{L^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$.

Households can save and borrow through an actuarially fair, risk-free, nominal annuity, backed by government bonds. Conditional on survival, households reserve a nominal return I_t/ω, where I_t is the nominal return on government bonds. Households furthermore receive labor income and dividend income $W_t L_{i,t}$ and $Q_{i,t}$ (both in real terms), and pay taxes. The real tax payment $T_{i,t}$ depends on both the individual's income and aggregate fiscal conditions:[34]

$$T_{i,t} = \tau_y Y_{i,t} + \bar{T} - \mathscr{E}_t + \tau_d\left(D_t - D^{ss} + \mathscr{E}_t\right), \tag{A.2}$$

[34] After (log-)linearization and aggregation, (A.2) becomes the tax rule (6) in the main text, where $\varepsilon_t \equiv \mathscr{E}_t/Y^{ss}$.

where $Y_{i,t} \equiv W_t L_{i,t} + Q_{i,t}$ is the household's total real income, $\tau_y \in [0,1)$ captures the rate of a proportional tax $\tau_y \in [0,1)$ on household heir total income, $\bar{T} = T^{ss} - \tau_y Y^{ss}$ is set to guarantee budget balance at steady state, $\mathscr{E}_t$ is a mean-zero and i.i.d. deficit shock (e.g., issuance of stimulus checks), and $\tau_d \in [0,1)$ is a scalar that parameterizes the speed of fiscal adjustment.

Finally, old households make contributions to a "social fund" whose proceeds are distributed to the newborn households. We use $S_{i,t}$ to denote the transfer from or contribution to the fund, with $S_{i,t} = S^{\text{new}} = D^{ss} > 0$ for newborns and $S_{i,t} = S^{\text{old}} = -\frac{1-\omega}{\omega} D^{ss} < 0$ for old households. This guarantees $(1-\omega)S^{\text{new}} + \omega S^{\text{old}} = 0$, ensuring that the fund is balanced. The fund thus ensures that all cohorts, regardless of their age, enjoy the same wealth and hence consumption in steady state. This simplifies aggregation and implies that the steady state of our model is the same as its RANK counterpart. In particular, the fund guarantees—together with the annuities, which offset mortality risk—that the steady-state rate of interest is β^{-1} (thus "$r > g$").

Together, the date-t budget constraint of household i is given as

$$P_{t+1} A_{i,t+1} = \underbrace{\frac{I_t}{\omega}}_{\text{annuity}} P_t \cdot \big(A_{i,t} + \underbrace{W_t L_{i,t} + Q_{i,t}}_{Y_{i,t}} - C_{i,t} - T_{i,t} + S_{i,t}\big), \tag{A.3}$$

where $A_{i,t}$ denotes i's real saving at the beginning of date t and P_t is the date-t price level.

In terms of household income, we assume that all households receive identical shares of dividends. Moreover, we abstract from heterogeneity in labor supply. We assume that labor supply is intermediated by labor unions,[35] which demand identical hours worked from all households $L_{i,t} = L_t$ and bargain on behalf of those households, equalizing the (post-tax) real wage and the average marginal rate of substitution between consumption and labor supply; i.e., we have that

$$(1-\tau_y) W_t = \frac{\iota L_t^{\frac{1}{\varphi}}}{\int_0^1 C_{i,t}^{-\frac{1}{\sigma}} di}. \tag{A.4}$$

Together, all households receive the same income and face the same taxes, $Y_{i,t} = Y_t$ and $T_{i,t} = T_t$.

Aggregate supply. Log-linearizing (A.4),

$$\frac{1}{\varphi} \ell_t = w_t - \frac{1}{\sigma} c_t. \tag{A.5}$$

Together with market clearing ($c_t = y_t$) and technology ($y_t = \ell_t$), this pins down the real wage as $w_t = \left(\frac{1}{\varphi} + \frac{1}{\sigma}\right) y_t$. Firm optimality pins down the optimal reset price as a function of current and expected future real marginal costs (wages), and thus also inflation. Together, the aggregate supply of the economy can be summarized by the familiar NKPC (2), where $\kappa = \frac{(1-\theta)(1-\beta\theta)\left(\frac{1}{\varphi}+\frac{1}{\sigma}\right)}{\theta} \geq 0$ and $1-\theta$ is

[35] This assumption simplifies the analysis by avoiding deficit-driven heterogeneity in the labor supply and income of different generations, without changing the essence of our self-financing results.

the Calvo reset probability.

A.2 Long-term government debt and interest rate feedback

We first provide the missing details for our extended environment with long-term government debt. We then discuss how our "HANK-meets-FTPL" results here extend to the case with interest rate feedback (i.e., $\phi \neq 0$).

Details about the environment with long-term bonds. The fiscal authority issues nominal government bonds, whose maturity is parameterized by $\delta \in [0,1]$. Each unit of government debt outstanding at t pays \$1 at t, and \$$\delta^k$ at $t+k$ for all $k>1$. We use J_t to denote the units of government debt outstanding at the start of period t. We use Q_t to denote the post-coupon dollar price at the end of period t for a government debt that pays \$1 at $t+1$ and \$$\delta^{k+1}$ at $t+k+1$. As a result, $B_t = J_t(1+\delta Q_t)$ captures the nominal value of government debt outstanding at the beginning of period t. The government budget constraint (in levels) can then be written as

$$J_{t+1} = \left(\frac{J_t - P_t T_t}{Q_t} + \delta J_t\right), \tag{A.6}$$

where P_t is the price level at t and T_t is total tax revenue at t. Rewriting (A.6) in terms of the nominal value of government debt B_t, we have

$$B_{t+1} = \left(\frac{1+\delta Q_{t+1}}{Q_t}\right)(B_t - P_t T_t), \tag{A.7}$$

where $I^g_{t+1} = \left(\frac{1+\delta Q_{t+1}}{Q_t}\right)$ is the realized nominal rate of return on government bonds between dates t and $t+1$. The monetary authority sets the date-t expected nominal rate of return on government debt as $I_t = \mathbb{E}_t\left[I^g_{t+1}\right]$. The government must satisfy the flow budget constraint (A.7) at all dates and states of nature, as well as the standard no-Ponzi condition (in the limit as $t \to \infty$).

We use $R_t \equiv \mathbb{E}_t\left[\frac{1+\delta Q_{t+1}}{Q_t \Pi_{t+1}}\right]$ to denote the expected real return on government bonds, $\Pi_{t+1} = P_{t+1}/P_t$ to denote the inflation from t to $t+1$, and $D_t \equiv B_t/P_t$ to denote the real value of total public debt that is outstanding at the beginning of period t, which by market-clearing equals total real household saving A_t. Re-writing (A.7) in real terms, log-linearizing, iterating forward, and imposing the household transversality condition, we have that the nominal price of the long-term bond is given by the negative of the present value of nominal short-term rates (or equivalently inflation plus real short-term rates), discounted by $\beta\delta$:

$$q_t = -\mathbb{E}_t\left[\sum_{k=0}^{\infty} (\beta\delta)^k (\pi_{t+k+1} + r_{t+k})\right]. \tag{A.8}$$

Next, re-writing (A.7) in real terms and linearizing, we obtain

$$d_{t+1} = \underbrace{\frac{1}{\beta}(d_t - t_t) + \frac{D^{ss}}{Y^{ss}} r_t}_{\text{expected debt burden tomorrow}} \underbrace{- \frac{D^{ss}}{Y^{ss}} \left(\pi_{t+1} - \mathbb{E}_t[\pi_{t+1}] - \beta\delta\left(q_{t+1} - \mathbb{E}_t[q_{t+1}]\right)\right)}_{\text{debt erosion due to inflation and bond price surprise}} \tag{A.9}$$

together with

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0 + \beta\delta\frac{D^{ss}}{Y^{ss}} q_0. \tag{A.10}$$

Finally, using (A.8) in (A.9) and (A.10), we arrive at (26) and

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0^{\delta} - \frac{D^{ss}}{Y^{ss}} r_0^{\delta}, \tag{A.11}$$

where $\{\pi_t^{\delta}, r_t^{\delta}\}_{t=0}^{\infty}$ is defined in (27).

HANK meets FTPL with interest rate feedback. Our "HANK-meets-FTPL" result extends naturally to the case with interest rate feedback, as in our baseline analysis with short-term government debt. Proposition A.1 states the formal result, with the proof provided in Appendix C.

Proposition A.1. *Suppose that $\omega < 1$, $\tau_y > 0$, $\delta > 0$, and $\phi \in (\underline{\phi}, \bar{\phi})$, and consider the HANK equilibrium that obtains when $\tau_d = 0$. Select any realization of the initial fiscal shock ε_0, abstract from any future shocks, and let $\{r_t^{HANK}\}_{t=0}^{\infty}$ be the equilibrium path of the (expected) real rate obtained in this equilibrium. Finally, consider an analogous RANK-FTPL economy in which $\omega = 1$, fiscal policy follows the same rule as in our HANK economy (with $\tau_d = 0$), and monetary policy follows the passive rule $r_t = r_t^{HANK}$. Then, the comparison established in Proposition 8 continues to hold, i.e.,*

$$\pi_{\varepsilon}^{\delta,HANK} < \pi_{\varepsilon}^{\delta,FTPL}. \tag{A.12}$$

B Additional results for quantitative analysis

In Appendix B.1 we provide supplementary details for the alternative model variants analyzed in Section 6.3. In Appendices B.2 and B.3 we then report results from two further sets of experiments, supplementing our main analysis in Section 6.

B.1 Further model details

The model variants discussed in Section B.1 alter the baseline environment along three margins: consumers, nominal rigidities, and policy. Our alterations along the pricing and policy margins were already described in detail in the main text, so we here just provide the missing details for the consumer block of the model.

The model variants with no cross-sectional heterogeneity in bond holdings or transfer receipts (or both) are self-explanatory: we set $A_i^{SS} = A^{SS}$ and $\varepsilon_i = \varepsilon$ for all groups i. For the behavioral model variant, we add a sticky information friction, modeled as in Angeletos et al. (2024, Appendix B.2), the behavioral coefficient set to $\theta = 0.95$. For the single-type OLG model variant, we set $\omega = 0.8/\beta$. Finally, for the HANK variant, we consider the exact same heterogeneous-agent block as in Angeletos et al. (2024, Appendix E.6.1), but with one important change in the model calibration: we set total liquid household wealth to $A^{SS} = D^{SS} = 1.79$, exactly as in our benchmark model. Even with this slightly elevated liquid wealth level we still obtain a large quarterly MPC of around 0.24.

B.2 Deficits and inflation with real rate response

To construct Figure B.1 we assumed a fixed real rate path, i.e., $\phi = 0$. We now ask what happens if instead (expected) real interest rates follow the exogenous path

$$r_t = \rho^t r_0,$$

i.e., the fiscal stimulus is accompanied by a particular movement in real rates. Specifically, we consider a one per cent fiscal deficit shock, and then set $r_0 = -0.15$ and $\rho = 0.6$—a meaningful and persistent monetary easing. In particular, relative to our baseline exercise, this almost doubles the size of the initial fiscal boom, and because of intertemporal substitution makes it front-loaded also in FTPL. Results are reported in Figure B.1.

The main takeaway from the figure is that our headline results are unchanged relative to Figure B.1. A real rate cut now makes the inflation burst front-loaded also in FTPL, but it remains *more* front-loaded in our HANK model variants.

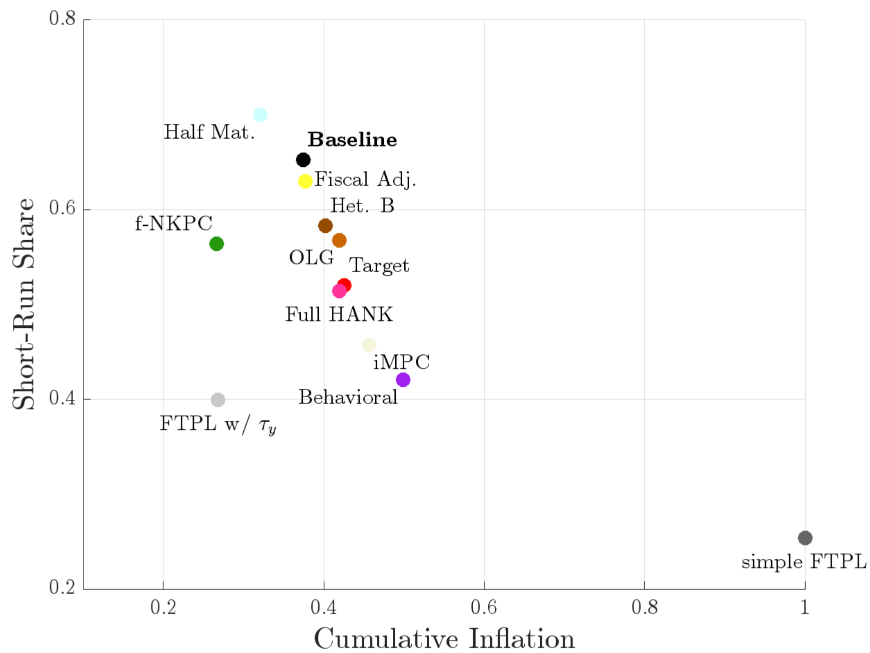

Figure B.1: Cumulative inflation response and short-run response share to a date-0 deficit shock of size D^{ss}/Y^{ss} accompanied by a transitory real rate cut, for different model variants, indicated by dots.

B.3 Perfectly anticipated covid stimulus

For our baseline exercise in Figure 6 we assumed that the two parts of the fiscal stimulus—reflecting the CARES and ARP Acts, respectively—arrived as surprises. Here we asks what happens if instead the ARP Act was perfectly anticipated at the time of CARES Act.

Results are displayed in Figure B.2. We say that our conclusions are qualitatively and quantitatively robust to alternative assumptions on household expectations.

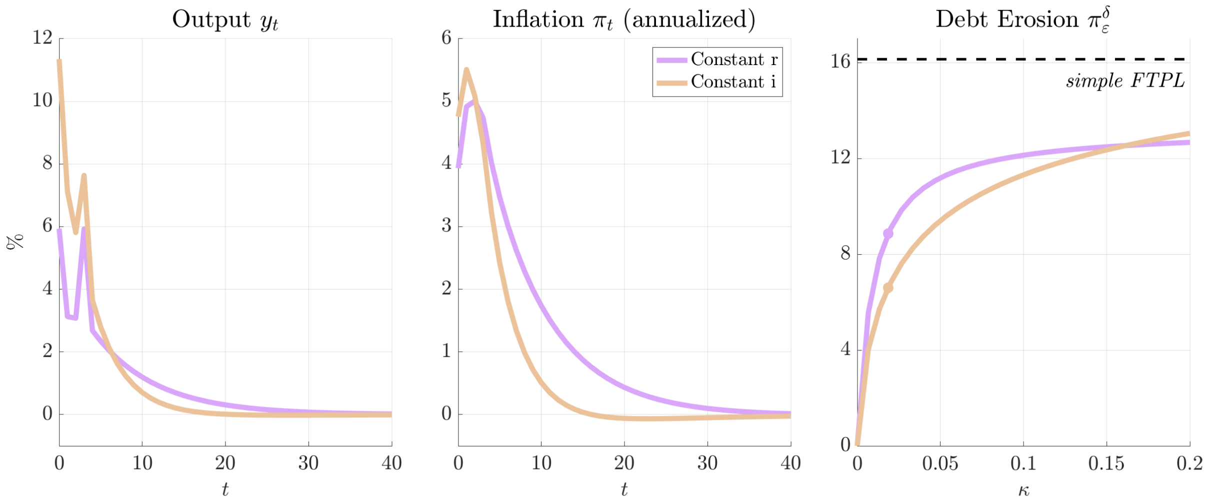

Figure B.2: Output and inflation impulse responses (left and middle) to the post-covid fiscal deficit shock (see text) and π^δ_ε (right) as a function of κ, with perfect foresight and under two different assumptions on the monetary policy reaction: fixed real rates (purple) and fixed nominal rates (orange).

C Proofs

C.1 Proof of Proposition 1

1. For any process $\{y_t\}_{t=0}^{\infty}$ of (9) (which is bounded and satisfies household optimality (1) and (8)), we can find $\{\pi_t\}_{t=0}^{\infty}$ from the NKPC (3), $\{d_t, r_t, t_t\}_{t=0}^{\infty}$ from the policy block (4) – (7), and $\{c_t, a_t\}_{t=0}^{\infty}$ from goods and asset market clearing. To prove $\{c_t, y_t, \pi_t, a_t, d_t, t_t, r_t\}_{t=0}^{\infty}$ is an equilibrium according to Definition 1, we only need to prove that $\mathbb{E}_t\left[\lim_{k\to\infty}\beta^k d_{t+k}\right]=0$. From (4) – (7), we know that, for $k\geq 1$,

$$\begin{aligned}\mathbb{E}_t\left[\beta^k d_{t+k}\right] &= (1-\tau_d)^k(d_t+\varepsilon_t)-\left(\tau_y-\beta\frac{D^{ss}}{Y^{ss}}\phi\right)\sum_{l=0}^{k-1}\beta^l(1-\tau_d)^{k-1-l}\mathbb{E}_t\left[y_{t+l}\right]\\ &= (1-\tau_d)^k(d_t+\varepsilon_t)-\left(\tau_y-\beta\frac{D^{ss}}{Y^{ss}}\phi\right)\sum_{l=0}^{k-1}\beta^l(1-\tau_d)^{k-1-l}\varrho^l y_t\\ &= (1-\tau_d)^k(d_t+\varepsilon_t)-\left(\tau_y-\beta\frac{D^{ss}}{Y^{ss}}\phi\right)\frac{(1-\tau_d)^k-\beta^k\varrho^k}{1-\tau_d-\beta\varrho}y_t. \end{aligned}\tag{C.1}$$

Because $\tau_d\in(0,1)$, $\varrho\in(0,1]$, and $\beta\in(0,1)$, we know that $\mathbb{E}_t\left[\lim_{k\to\infty}\beta^k d_{t+k}\right]=0$. This proves part 1.

2. From the main text, we know that there is a unique bounded process $\{y_t\}_{t=0}^{\infty}$ satisfying household optimality (1) and (8), that is, $y_t=0$ for all t. We can find $\{\pi_t\}_{t=0}^{\infty}$ from the NKPC (3), $\{d_t, r_t, t_t\}_{t=0}^{\infty}$ from the policy block (4) – (7), $\{c_t, a_t\}_{t=0}^{\infty}$ from goods and asset market clearing, and construct $\{c_t, y_t, \pi_t, a_t, d_t, t_t, r_t\}_{t=0}^{\infty}$ uniquely. To prove that $\{c_t, y_t, \pi_t, a_t, d_t, t_t, r_t\}_{t=0}^{\infty}$ is an equilibrium according to Definition 1, we only need to prove that $\mathbb{E}_t\left[\lim_{k\to\infty}\beta^k d_{t+k}\right]=0$. From (4) – (7), we know that, for $k\geq 1$,

$$\mathbb{E}_t\left[\beta^k d_{t+k}\right]=(1-\tau_d)^k(d_t+\varepsilon_t)$$

Because $\tau_d\in(0,1)$, we know that $\mathbb{E}_t\left[\lim_{k\to\infty}\beta^k d_{t+k}\right]=0$. This proves part 2.

3. From the main text, we know that, any equilibrium in which $\{y_t\}_{t=0}^{\infty}$ is bounded must satisfy (9), which derives from the household optimality (1) and (8). From (4) – (7) and similar to (C.1), we know that

$$\mathbb{E}_t\left[\beta^k d_{t+k}\right]=d_t+\varepsilon_t-\left(\tau_y-\beta\frac{D^{ss}}{Y^{ss}}\phi\right)\frac{1-\beta^k\varrho^k}{1-\beta\varrho}y_t.$$

Because $\varrho\in(0,1]$ and $\beta\in(0,1)$, we know that the no-Ponzi condition $\mathbb{E}_t\left[\lim_{k\to\infty}\beta^k d_{t+k}\right]=0$ implies $d_t+\varepsilon_t=\frac{\tau_y-\beta\frac{D^{ss}}{Y^{ss}}\phi}{1-\beta\varrho}y_t$. As a result,

$$d_t-\mathbb{E}_{t-1}\left[d_t\right]+\varepsilon_t=\frac{\tau_y-\beta\frac{D^{ss}}{Y^{ss}}\phi}{1-\beta\varrho}\left(y_t-\mathbb{E}_{t-1}\left[y_t\right]\right).\tag{C.2}$$

From (3), (4), (5), and (9), we know

$$d_t - \mathbb{E}_{t-1}[d_t] = -\frac{D^{ss}}{Y^{ss}} \frac{\kappa}{1-\beta\varrho} \eta_t \quad \text{and} \quad y_t - \mathbb{E}_{t-1}[y_t] = \eta_t.$$

Together with (C.2), we know that any equilibrium in which $\{y_t\}_{t=0}^{\infty}$ is bounded must satisfy

$$\eta_t = \frac{1-\beta(1+\sigma\phi)}{\tau_y + (\kappa - \beta\phi)\frac{D^{ss}}{Y^{ss}}} \varepsilon_t$$

which is (10). From (3),

$$\pi_t - \mathbb{E}_{t-1}[\pi_t] = \pi_{\varepsilon,0}^{FTPL} \cdot \varepsilon_t \qquad \text{with} \qquad \pi_{\varepsilon,0}^{FTPL} \equiv \frac{\kappa}{\tau_y + (\kappa - \beta\phi)\frac{D^{ss}}{Y^{ss}}},$$

which is (11). One can construct the rest of the equilibrium $\{c_t, y_t, \pi_t, a_t, d_t, t_t, r_t\}_{t=0}^{\infty}$ uniquely: $\{d_t, r_t, t_t\}_{t=0}^{\infty}$ from the policy block (4) – (7) and $\{c_t, a_t\}_{t=0}^{\infty}$ from goods and asset market clearing. This proves part 3.

C.2 Proof of Proposition 2

The characterization of the equilibrium follows from Proposition 1 in Angeletos et al. (2024), which is restated here for completeness. Note that we restrict that $\omega \in (0,1)$, $\tau_y \in (0,1)$, and $\tau_d \in [0,1)$. Imposing $r_t = 0$ (fixed real rates), $y_t = c_t$ (goods market clearing), and $a_t = d_t$ (asset market clearing) in (1), we have, for all $t \geq 0$,

$$y_t = (1-\beta\omega)\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty} (\beta\omega)^k (y_{t+k} - t_{t+k})\right]\right).$$

We now write it recursively using the government's flow budget (4). For all $t \geq 0$,

$$\begin{aligned} y_t &= (1-\beta\omega)(y_t + d_t - t_t) + \beta\omega\mathbb{E}_t[y_{t+1} - (1-\beta\omega)\cdot d_{t+1}] \\ &= (1-\beta\omega)(y_t + d_t - t_t) + \beta\omega\mathbb{E}_t\left[y_{t+1} - \frac{1-\beta\omega}{\beta}(d_t - t_t)\right] \\ &= \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(d_t - t_t) + \mathbb{E}_t[y_{t+1}]. \end{aligned}$$

Applying the fiscal rule (6), we have, for all $t \geq 0$,

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1 + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}(d_t + \varepsilon_t) + \frac{1}{1 + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\mathbb{E}_t[y_{t+1}].$$

Applying period-t expectations $\mathbb{E}_t[\cdot]$ to (4), we have, for all $t \geq 0$,

$$\begin{pmatrix} \mathbb{E}_t[d_{t+1}] \\ \mathbb{E}_t[y_{t+1}] \end{pmatrix} = \begin{pmatrix} \frac{1-\tau_d}{\beta} & -\frac{\tau_y}{\beta} \\ -\frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{\beta\omega} & 1 + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y \end{pmatrix} \begin{pmatrix} d_t + \varepsilon_t \\ y_t \end{pmatrix} \tag{C.3}$$

The two eigenvalues of the system are given by the solutions of

$$\lambda^2 - \lambda\left(\frac{1}{\beta}(1-\tau_d) + 1 + \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) + \frac{1}{\beta}(1-\tau_d) = 0,$$

with

$$\begin{aligned}
\lambda_1 &= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)+\sqrt{\left(1+\frac{1}{\beta}(1-\tau_d)+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2-4\frac{1}{\beta}(1-\tau_d)}}{2}\\
&= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)+\sqrt{\left(1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2+4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \qquad \text{(C.4)}\\
&> \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)+\left|1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right|}{2} \geq 1
\end{aligned}$$

and

$$\begin{aligned}
\lambda_2 &= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)-\sqrt{\left(1+\frac{1}{\beta}(1-\tau_d)+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2-4\frac{1}{\beta}(1-\tau_d)}}{2}\\
&= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)-\sqrt{\left(1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2+4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \qquad \text{(C.5)}\\
&< \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)-\left|\frac{1}{\beta}(1-\tau_d)+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)-1\right|}{2} \leq 1,
\end{aligned}$$

with $\lambda_2 > 0$ too since $\lambda_1\lambda_2 = \frac{1}{\beta}(1-\tau_d) > 0$. Let $(1,\chi_2)'$ denote the eigenvector associated with λ_2, where

$$\lambda_2 = \frac{1}{\beta}\left(1-\tau_d-\tau_y\chi_2\right) \quad \text{and} \quad \chi_2 = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y-\lambda_2} > 0. \qquad \text{(C.6)}$$

This means that any bounded path of $\{d_t, y_t\}_{t=0}^{+\infty}$ that satisfies (C.3) takes the form of

$$y_t = \chi(d_t+\varepsilon_t) \quad \text{and} \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t+\varepsilon_t),$$

where χ and ρ_d are uniquely given by

$$\chi = \chi_2 > 0 \quad \text{and} \quad \rho_d = \lambda_2 \in (0,1), \qquad \text{(C.7)}$$

and are continuous functions of $(\beta,\omega,\tau_y,\tau_d)$. In other words, any bounded equilibrium must take the form of (18).

From (5) and (3) , we can find d_0 as a function of the deficit shock ε_0 :

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0 = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\mathbb{E}_0[y_k] = -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_0+\varepsilon_0) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\varepsilon_0. \qquad \text{(C.8)}$$

Similarly, for $t \geq 1$, from (4) and (3),

$$d_t - \mathbb{E}_{t-1}[d_t] = -\frac{D^{ss}}{Y^{ss}}(\pi_t - \mathbb{E}_{t-1}[\pi_t]) = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\left(\mathbb{E}_t[y_{t+k}] - \mathbb{E}_{t-1}[y_{t+k}]\right)$$
$$= -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_t - \mathbb{E}_{t-1}[d_t] + \varepsilon_t) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\varepsilon_t. \tag{C.9}$$

Together with (3) and (18), we find a bounded equilibrium path of $\{\pi_t, d_t, y_t\}_{t=0}^{+\infty}$. In particular,

$$\pi_t - \mathbb{E}_{t-1}[\pi_t] = \pi_{\varepsilon,0}^{HANK}\cdot\varepsilon_t \qquad \text{with} \qquad \pi_{\varepsilon,0}^{HANK} \equiv \frac{\kappa\chi}{1-\beta\rho_d+\kappa\chi\frac{D^{ss}}{Y^{ss}}}.$$

We can then find $c_t = y_t$, $a_t = d_t$, and t_t from the fiscal rule (6), and the entire equilibrium path $\{c_t, y_t, \pi_t, a_t, d_t, t_t\}_{t=0}^{\infty}$ satisfying Definition 1. The uniqueness comes from the fact that χ and ρ_d are uniquely pinned down by (C.7). Finally, from (3) and (18), for all $k \geq 0$,

$$\pi_{\varepsilon,k}^{HANK} \equiv \frac{d\mathbb{E}_t[\pi_{t+k}]}{d\varepsilon_t} = \rho_d^k\pi_{\varepsilon,0}^{HANK}. \tag{C.10}$$

C.3 Proof of Proposition 3

From (C.5), (C.6), and (C.7), we know that τ_d and χ are continuous in $\tau_d \in [0,1)$ and

$$\lim_{\tau_d\to 0^+}\pi_{\varepsilon,0}^{HANK} = \pi_{\varepsilon,0}^{HANK}\Big|_{\tau_d=0}.$$

From the second part of (18), we know that

$$\frac{\chi}{1-\beta\rho_d} = \frac{\chi}{\tau_d+\tau_y\chi}. \tag{C.11}$$

From (C.5) and (C.7), we know

$$\rho_d = \lambda_2 = f(a,b) \equiv \frac{a+b+1-\sqrt{(a+b-1)^2+4b}}{2} \tag{C.12}$$

where $a = \frac{1}{\beta}(1-\tau_d) > 0$ and $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega) > 0$. Since $\frac{\partial f}{\partial a} = \frac{1}{2} - \frac{(a+b-1)}{2\sqrt{(a+b-1)^2+4b}} > 0$, we know that ρ_d decreases with τ_d. From (C.6) and (C.7), we then know $\chi = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y-\rho_d}$ also decreases in τ_d. From (19), we know $\pi_{\varepsilon,0}^{HANK}$ decreases in τ_d.

When $\tau_d = 0$, again using (C.6), and (C.7), we have

$$\frac{\chi}{1-\beta\rho_d}\bigg|_{\tau_d=0} = \frac{1}{\tau_y} \quad \text{and} \quad \pi_{\varepsilon,0}^{HANK}\Big|_{\tau_d=0} = \frac{\kappa}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}} = \pi_{\varepsilon,0}^{\text{FTPL}}.$$

C.4 Proof of Corollary 1

This follows directly from (20).

C.5 Proof of Proposition 4

From the proof of Part 3 of Proposition 1, we know that (3) and (9) imply

$$\pi_{\varepsilon,k}^{FTPL} \equiv \frac{d\mathbb{E}_t\left[\pi_{t+k}^{FTPL}\right]}{d\varepsilon_t} = \varrho^k \pi_{\varepsilon,0}^{FTPL} = \pi_{\varepsilon,0}^{FTPL},$$

because $\varrho = 1$ when $\phi = 0$. As a result, $\pi^{\dagger,FTPL} = 1-\beta$.

From (C.10), $\pi^{\dagger,HANK} = 1-\beta\rho_d$. From (C.12), we know

$$\rho_d = \frac{a+b+1-\sqrt{(a+b+1)^2-4a}}{2} = \frac{2a}{a+b+1+\sqrt{(a+b+1)^2-4a}},$$

where $a = \frac{1-\tau_d}{\beta} > 0$ and $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega) > 0$. From the second part of the equation, we know that ρ_d decreases in $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)$ and increases in ω. As a result, $\pi^{\dagger,HANK}$ decreases in ω. Moreover, when

$$\lim_{\omega\to 1^-} \rho_d = \frac{a+1-|a-1|}{2} \leq 1,$$

with the equality obtained when $\tau_d = 0$. As a result, $\pi^{\dagger,HANK} > \pi^{\dagger,FTPL}$ and the distance between the two vanishes when $\tau_d = 0$ and $\omega \to 1^-$.

C.6 Proof of Proposition 5

The characterization of the equilibrium with interest rate feedback follows from Theorem 2 in Angeletos et al. (2024), which is restated here for completeness. We restrict $\phi \in \left(\underline{\phi}, \bar{\phi}\right)$, where the thresholds are given by

$$\underline{\phi} \equiv -\frac{1}{\sigma} \quad \text{and} \quad \bar{\phi} \equiv \frac{\frac{(1-\beta\omega)(1-\omega)}{\omega}\tau_y}{\sigma\left(1-\beta\right) + \frac{(1-\beta\omega)(1-\omega)}{\omega}\beta\frac{D^{ss}}{Y^{ss}}} < \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}}, \tag{C.13}$$

Aggregating the individual demand relation (1), together with the government budget (4), and goods and asset market clearing, leads to the following recursive aggregate demand relation for all $t \geq 0$:

$$\begin{aligned} y_t &= \left(1-\beta\omega\right)\left(\mathbb{E}_t\left[\sum_{k=0}^{\infty}\left(\beta\omega\right)^k y_{t+k}\right]\right) - \beta\sigma\omega\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k r_{t+k}\right] \\ &+ \left(1-\beta\omega\right)\left(\mathbb{E}_t\left[\sum_{k=0}^{\infty}\beta^k\left(1-\omega^k\right)\left(t_{t+k}-\beta\frac{D^{ss}}{Y^{ss}}r_{t+k}\right)\right]\right) \\ &= -\sigma r_t + \frac{\left(1-\beta\omega\right)(1-\omega)}{\beta\omega}\left(d_t - t_t + \beta\frac{D^{ss}}{Y^{ss}}r_t\right) + \mathbb{E}_t\left[y_{t+1}\right] \end{aligned} \tag{C.14}$$

Together with the baseline fiscal policy (6) and monetary policy (7). we arrive at the following aggregate demand relation for for all $t \geq 0$:

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}(d_t+\varepsilon_t)+\frac{1}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}\mathbb{E}_t\left[y_{t+1}\right].$$

Applying the period-t expectation operator $\mathbb{E}_t[\cdot]$ to (4), we have, for all $t \geq 0$,

$$\begin{pmatrix} \mathbb{E}_t[d_{t+1}] \\ \mathbb{E}_t\left[y_{t+1}\right] \end{pmatrix} = \begin{pmatrix} \frac{1-\tau_d}{\beta} & -\frac{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}{\beta} \\ -\frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{\beta\omega} & 1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}{\beta\omega} \end{pmatrix}\begin{pmatrix} d_t+\varepsilon_t \\ y_t \end{pmatrix} \tag{C.15}$$

The two eigenvalues are given by the solutions of

$$\lambda^2-\lambda\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+(1+\sigma\phi)\frac{1-\tau_d}{\beta}=0. \tag{C.16}$$

Because $\phi \in \left(-\frac{1}{\sigma}, \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}}\right)$ and $\tau_d \in [0,1)$, we know that $\lambda_1+\lambda_2 \geq 0$ and $\lambda_1\lambda_2 \geq 0$, so $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$. Moreover,

$$\begin{aligned}\lambda_1 &= \frac{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\sqrt{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}}{2} \\ &= \frac{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\sqrt{\left(1+\sigma\phi-\frac{1-\tau_d}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2+4(1+\sigma\phi)\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}}{2},\end{aligned} \tag{C.17}$$

and

$$\begin{aligned}\lambda_2 &= \frac{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\sqrt{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}}{2} \\ &= \frac{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\sqrt{\left(1+\sigma\phi-\frac{1-\tau_d}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2+4(1+\sigma\phi)\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}}{2}.\end{aligned} \tag{C.18}$$

Moreover, for $\phi \in \left(-\frac{1}{\sigma}, \bar{\phi}\right)$,

$$\lambda_1 \geq \frac{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\left|\frac{1-\tau_d}{\beta}+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)-1-\sigma\phi\right|}{2} > \frac{1-\tau_d}{\beta} \tag{C.19}$$

When $\phi \in \left(-\frac{1}{\sigma}, 0\right)$, from (C.17) and (C.18),

$$\lambda_2 \leq \frac{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\left|1+\sigma\phi-\frac{1-\tau_d}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right|}{2} \leq 1+\sigma\phi < 1$$

When $\phi \in [0, \bar{\phi})$, from (C.13), we have

$$\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right) > \sigma\phi\left(\frac{1}{\beta}-1\right). \tag{C.20}$$

Hence

$$\lambda_2 = \frac{2\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}{\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)+\sqrt{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}} \tag{C.21}$$

$$< \frac{2\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}{\frac{1-\tau_d}{\beta}+1+\frac{\sigma\phi}{\beta}+\sqrt{\left(\frac{1-\tau_d}{\beta}+1+\frac{\sigma\phi}{\beta}\right)^2-4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}}$$

$$= \frac{\frac{1-\tau_d}{\beta}+1+\frac{\sigma\phi}{\beta}-\sqrt{\left(\frac{1-\tau_d}{\beta}+1+\frac{\sigma\phi}{\beta}\right)^2-4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}}{2} \le 1.$$

The last step is from the fact that

$$\frac{\frac{1-\tau_d}{\beta}+1+\frac{\sigma\phi}{\beta}-\sqrt{\left(\frac{1-\tau_d}{\beta}+1+\frac{\sigma\phi}{\beta}\right)^2-4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}}{2} \le 1 \Longleftrightarrow \frac{1-\beta-\tau_d+\sigma\phi}{\beta} \le \sqrt{\left(\frac{1-\tau_d+\sigma\phi+\beta}{\beta}\right)^2-4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}$$

$$\Longleftrightarrow 4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta} \le \left(\frac{1-\tau_d+\sigma\phi+\beta}{\beta}\right)^2-\left(\frac{1-\beta-\tau_d+\sigma\phi}{\beta}\right)^2$$

$$\Longleftrightarrow (1+\sigma\phi)(1-\tau_d) \le (1-\tau_d+\sigma\phi) \Longleftrightarrow 0 \le \phi.$$

Let $(1,\chi_1)'$ and $(1,\chi_2)'$ denote the eigenvector associated with λ_1 and λ_2, where $\chi_1 = \frac{1-\tau_d-\beta\lambda_1}{\tau_y} < 0$

$$\lambda_2 = \frac{1}{\beta}(1-\tau_d-\tau_y\chi_2) \quad \text{and} \quad \chi_2 = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)-\lambda_2} > 0. \tag{C.22}$$

Consider any bounded equilibrium that takes the form of (18) for some scalars $\chi > 0$ and $\rho_d \in (0,1)$. Because $\chi_1 < 0$, we know that χ and ρ_d are uniquely given by

$$\chi = \chi_2 > 0 \quad \text{and} \quad \rho_d = \lambda_2 \in (0,1), \tag{C.23}$$

which are continuous in $(\beta, \omega, \tau_y, \tau_d, \phi)$ and, in particular, in $\tau_d \in [0,1)$. Furthermore, from (C.18), we know

$$\rho_d = \lambda_2 = f(a,b) \equiv \frac{a+b+1+\sigma\phi-\sqrt{(a+b-1-\sigma\phi)^2+4b(1+\sigma\phi)}}{2} \tag{C.24}$$

where $a = \frac{1}{\beta}(1-\tau_d) > 0$ and $b = \frac{1-\beta\omega}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)(1-\omega) > 0$. Since $\frac{\partial f}{\partial a} = \frac{1}{2} - \frac{(a+b-1-\sigma\phi)}{2\sqrt{(a+b-1-\sigma\phi)^2+4b(1+\sigma\phi)}} > 0$, we know that ρ_d decreases with τ_d. From (C.22), we then know χ also decreases in τ_d.

Note that (C.8) and (C.9) remain to be true. We can then find a bounded equilibrium path of $\{\pi_t, d_t, y_t\}_{t=0}^{+\infty}$ where

$$\pi_t - \mathbb{E}_{t-1}[\pi_t] = \pi_{\varepsilon,0}^{HANK} \cdot \varepsilon_t \qquad \text{with} \qquad \pi_{\varepsilon,0}^{HANK} \equiv \frac{\kappa\chi}{1-\beta\rho_d + \kappa\chi\frac{D^{ss}}{Y^{ss}}},$$

where $\pi_{\varepsilon,0}^{HANK}$ is continuous in $(\beta, \omega, \tau_y, \tau_d, \phi)$ and, in particular, in $\tau_d \in [0,1)$. Moreover, $\pi_{\varepsilon,0}^{HANK}$ decreases in $\tau_d \in [0,1)$.

We can then find $c_t = y_t$, $a_t = d_t$, and t_t from the fiscal rule (6), and the entire equilibrium path $\{c_t, y_t, \pi_t, a_t, d_t, t_t, r_t\}_{t=0}^{\infty}$ satisfying Definition 1. The uniqueness comes from the fact that χ and ρ_d are uniquely pinned down by (C.7). Finally, from (3) and (18), for all $k \geq 0$,

$$\pi_{\varepsilon,k}^{HANK} \equiv \frac{d\mathbb{E}_t[\pi_{t+k}]}{d\varepsilon_t} = \rho_d^k \pi_{\varepsilon,0}^{HANK}, \tag{C.25}$$

continuous in $(\beta, \omega, \tau_y, \tau_d, \phi)$ and, in particular, in $\tau_d \in [0,1)$.

C.7 Proof of Proposition 6

In this proof, objects without superscripts, such as $\{\pi_t, d_t, y_t\}_{t=0}^{+\infty}$ and (ρ_d, χ) capture relevant objects in the HANK economy characterized in Proposition 5. Objects with the superscript FTPL, such as $\{\pi_t^{FTPL}, d_t^{FTPL}, y_t^{FTPL}\}_{t=0}^{+\infty}$ and $(\rho_d^{FTPL}, \chi^{FTPL})$ capture the corresponding objects in the RANK-FTPL economy which shares the same path of (expected) real interest rates as the HANK economy.

Consider the HANK economy with $\tau_d = 0$. From the flow budget (4) and the government's no-Ponzi condition, the government's intertemporal budget is:

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{t=0}^{+\infty} \beta^{t+1} r_t = \frac{D^{ss}}{Y^{ss}} \pi_0 + \tau_y \sum_{t=0}^{+\infty} \beta^t y_t, \tag{C.26}$$

where we drop the expectation operator because we abstract from any future shocks after the initial shock ε_0.

Now we feed the equilibrium path of the (expected) real rate obtained in the HANK equilibrium $\{r_t\}_{t=0}^{\infty}$ into the RANK-FTPL economy in which $\omega = 1$, fiscal policy follows the same rule as in our HANK economy (with $\tau_d = 0$), and monetary policy follows the passive rule $r_t^{FTPL} = r_t$. The government's intertemporal budget also holds for the FTPL equilibrium,

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{t=0}^{+\infty} \beta^{t+1} r_t = \frac{D^{ss}}{Y^{ss}} \pi_0^{FTPL} + \tau_y \sum_{t=0}^{+\infty} \beta^t y_t^{FTPL}. \tag{C.27}$$

From (3), we know that

$$\pi_0 = \pi_0^{FTPL} = \frac{\kappa}{\tau_y + \frac{D^{ss}}{Y^{ss}}\kappa} \left(\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{t=0}^{+\infty} \beta^{t+1} r_t \right).$$

As a result, $\pi_{\varepsilon,0}^{HANK} = \pi_{\varepsilon,0}^{FTPL}$.

To prove $\pi^{\dagger,HANK} > \pi^{\dagger,FTPL}$, we first need to establish some additional property of the HANK economy characterized in Proposition 5. From (C.25), we know that

$$\pi^{\dagger,HANK} = 1 - \beta\rho_d.$$

From (7) and (18), we know that, for all $t \geq 0$,

$$r_t = \rho_d^t r_0 = \phi\rho_d^t y_0. \tag{C.28}$$

From the recursive demand relation (C.14) and the government budget (4), for $t \geq 0$,

$$y_t = -\sigma r_t + \frac{\left(1-\beta\omega\right)(1-\omega)}{\omega}\rho_d\left(d_t + \varepsilon_t\right) + y_{t+1},$$

where $\varepsilon_t = 0$ for all $t \neq 0$. Because $\rho_d \in (0,1)$ so $\lim_{t\to\infty} y_t = 0$ in the HANK equilibrium. We have, for $t \geq 0$,

$$y_t = -\frac{\sigma}{1-\rho_d} r_t + \frac{\left(1-\beta\omega\right)(1-\omega)}{\omega\left(1-\rho_d\right)}\rho_d\left(d_t+\varepsilon_t\right)$$

$$\sum_{t=0}^{+\infty}\beta^t y_t = -\frac{\sigma}{\left(1-\rho_d\right)\left(1-\beta\rho_d\right)} r_0 + \frac{\left(1-\beta\omega\right)(1-\omega)}{\omega\left(1-\rho_d\right)\left(1-\beta\rho_d\right)}\rho_d\left(d_0+\varepsilon_0\right). \tag{C.29}$$

where we use (18) for the second equation. Putting them into (C.26) and using (3), we have, for $k \geq 0$,

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}}\left(\frac{\beta}{1-\beta\rho_d}\right) r_0 + \frac{\sigma\left(\kappa\frac{D^{ss}}{Y^{ss}} + \tau_y\right)}{\left(1-\beta\rho_d\right)\left(1-\rho_d\right)} r_0 = \frac{\kappa\frac{D^{ss}}{Y^{ss}} + \tau_y}{\left(1-\beta\rho_d\right)\left(1-\rho_d\right)}\frac{\left(1-\beta\omega\right)(1-\omega)}{\omega}\rho_d\left(d_0+\varepsilon_0\right) \tag{C.30}$$

Now we turn to the RANK-FTPL economy sharing the same path of $\{r_t = r_t^{HANK}\}_{t=0}^{\infty}$. Similar to (8), the equilibrium path of $\{y_t^{FTPL}\}_{t=0}^{\infty}$ can be characterized by the familiar DIS equation, for $t \geq 0$,

$$y_t^{FTPL} = -\sigma r_t + y_{t+1}^{FTPL}. \tag{C.31}$$

Similar to (C.29) but without imposing $y_\infty^{FTPL} \equiv \lim_{t\to\infty} y_t^{FTPL} = 0$,

$$y_t^{FTPL} = -\frac{\sigma}{1-\rho_d} r_t + y_\infty^{FTPL} \tag{C.32}$$

$$\sum_{t=0}^{+\infty}\beta^t y_t^{FTPL} = -\frac{\sigma}{\left(1-\rho_d\right)\left(1-\beta\rho_d\right)} r_0 + \frac{1}{1-\beta} y_\infty^{FTPL}$$

Putting them into (C.27) and using (3), we have, for $k \geq 0$,

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}}\left(\frac{\beta}{1-\beta\rho_d}\right) r_0 + \frac{\sigma\left(\kappa\frac{D^{ss}}{Y^{ss}} + \tau_y\right)}{\left(1-\beta\rho_d\right)\left(1-\rho_d\right)} r_0 = \frac{\kappa\frac{D^{ss}}{Y^{ss}} + \tau_y}{1-\beta} y_\infty^{\text{FTPL}}.$$

Compared with (C.30), we know that $y_\infty^{\text{FTPL}} = \frac{(1-\beta)(1-\beta\omega)(1-\omega)}{(1-\beta\rho_d)(1-\rho_d)\omega}\rho_d\left(d_0+\varepsilon_0\right)$. From (C.8), we know that y_∞^{FTPL} has the same sign as ε_0.

From this point on, we will use the positive fiscal deficit shock $\varepsilon_0 > 0$ as an example, which means $y_\infty^{\text{FTPL}} > 0$; the proof with $\varepsilon_0 < 0$ is symmetric. With $\varepsilon_0 > 0$, from (18) and (C.8), we know that, in

HANK, $\pi_t > 0$ and $y_t > 0$ because $\chi > 0$ and $\rho_d \in (0,1)$. When $\phi \in [0,\bar{\phi}), r_t \geq 0$ for all $t \geq 0$. From (C.31), $y_0^{FTPL} \leq y_1^{FTPL} \leq y_2^{FTPL} \leq \cdots$. From (3), we have

$$0 < \pi_0 = \pi_0^{FTPL} \leq \pi_1^{FTPL} \leq \pi_2^{FTPL} \leq \cdots.$$

We hence know that $\pi^{\dagger,FTPL} \leq 1-\beta < 1-\beta\rho_d = \pi^{\dagger,HANK}$. When $\phi \in \left(-\frac{1}{\sigma},0\right)$, $r_t = \rho_d^t r_0 < 0$ for all $t \geq 0$. From $y_\infty^{\text{FTPL}} > 0$ and (C.32), we know that $y_{t+1}^{FTPL} > \rho_d y_t^{FTPL} > 0$ for all $t \geq 0$. From (3), we know that $\pi_{t+1}^{FTPL} > \rho_d \pi_t^{FTPL} > 0$ for all $t \geq 0$. We hence know that $\pi^{\dagger,FTPL} < 1-\beta\rho_d = \pi^{\dagger,HANK}$.

C.8 Proof of Proposition 7

1. Given (24) and similar to (8), we know that, for $t \geq H$,

$$y_t = -\sigma\phi' y_t + \mathbb{E}_t\left[y_{t+1}\right]. \tag{C.33}$$

   Similar to Part 2 of Proposition 3, there exists a unique equilibrium in which y_t is bounded. In this equilibrium, $y_t = \pi_t = 0$ for $t \geq H$. Using (8) for $t < H$ and (3), we know that $y_t = \pi_t = 0$ for all t and all realizations of uncertainty.

2. Imposing $y_t = c_t$ (goods market clearing), $a_t = d_t$ (asset market clearing), and using the government's flow budget (4), we can write aggregate demand (1) recursively

$$y_t = -\sigma r_t + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(d_t - t_t + \beta\frac{D^{ss}}{Y^{ss}} r_t\right) + \mathbb{E}_t\left[y_{t+1}\right]. \tag{C.34}$$

   Given (24), we know that, for $t \geq H$,(C.33) also holds under HANK. As a result, there exists a unique equilibrium in which y_t is bounded. In this equilibrium, $y_t = \pi_t = 0$ for $t \geq H$. We find the equilibrium path of $\{y_t, \pi_t, d_t\}_{t=0}^{H-1}$ through backward induction starting from

$$y_H = \chi_0 d_H \quad \text{with} \quad \chi_0 = 0. \tag{C.35}$$

   Applying the fiscal and monetary rules (6) and (7) in (C.34), we know that, for $t \leq H-1$,

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)}(d_t+\varepsilon_t) + \frac{1}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)}\mathbb{E}_t\left[y_{t+1}\right]. \tag{C.36}$$

   As a result, for $t \leq H-1$,

$$y_t = \chi_{H-t}(d_t+\varepsilon_t) \quad \text{with} \quad \chi_{H-t} = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)} + \frac{\frac{1}{\beta}\left(1-\tau_d-\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\chi_{H-t}\right)}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)}\chi_{H-t-1} \tag{C.37}$$

Rearranging terms, we find the following recursive formula for the χs:

$$\chi_{H-t} = \frac{\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi_{H-t-1}}{\beta}\right)(1-\tau_d)}{1+\sigma\phi+\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi_{H-t-1}}{\beta}\right)\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)} \equiv g\left(\chi_{H-t-1}\right), \tag{C.38}$$

where $\tau_y - \beta\phi\frac{B^{ss}}{Y^{ss}} > 0$ and $1+\sigma\phi > 0$ because $\phi \in (\underline{\phi}, \bar{\phi})$ and

$$g'\left(\chi\right) = \frac{1-\tau_d}{\beta}\frac{1+\sigma\phi}{\left(1+\sigma\phi+\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}+\frac{\chi}{\beta}\right)\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2} \geq 0 \quad \forall \chi \geq 0.$$

We thus know that

$$\chi_k \in \left(0, \frac{1-\tau_d}{\tau_y - \beta\phi\frac{B^{ss}}{Y^{ss}}}\right) \quad \forall k \geq 1 \quad \text{and} \quad \chi_k \text{ increases in } k. \tag{C.39}$$

Now let's find the fixed point of g such that $g\left(\chi\right) = \chi$, where

$$\omega\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\chi^2 + \left(\beta\omega\left(1+\sigma\phi\right) - \omega\left(1-\tau_d\right) + \left(1-\beta\omega\right)\left(1-\omega\right)\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)\chi - \left(1-\beta\omega\right)\left(1-\omega\right)\left(1-\tau_d\right) = 0.$$

We know that there is only one of such fix point such that $\chi > 0$ because $-\left(1-\beta\omega\right)(1-\omega)(1-\tau_d) < 0$ and $\omega\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right) > 0$. From (C.39), we that $\lim_{k\to+\infty}\chi_k = \chi$. Moreover, because (C.36) also holds under the HANK equilibrium we characterized in Proposition 5, so the fixed point $\chi > 0$ here corresponds to the χ in the equilibrium (18) in Proposition 5.

From (4), (5), and (3), we can construct the equilibrium path of $\left\{y_t, \pi_t, d_t\right\}_{t=0}^{H-1}$ based on $\left\{\chi_k\right\}_{k=0}^{H}$. In particular, for $t \leq H-1$,

$$\mathbb{E}_0\left[d_t\right] = \frac{1}{\beta^t}\Pi_{j=0}^{t-1}\left(1-\tau_d - \left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\chi_{H-j}\right)\left(d_0+\varepsilon_0\right), \tag{C.40}$$

where

$$\frac{1-\tau_d-\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\chi_k}{\beta} \to \frac{1-\tau_d-\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\chi}{\beta} = \rho_d \in (0,1),$$

where ρ_d is the one in (18) in Proposition (5). Together with $\lim_{k\to+\infty}\chi_k = \chi$, we know that, for any $T > 0$, as $H \to \infty$, $\{y_t, \pi_t\}_{t=0}^{T}$ converges to its counterpart in Propositions 5, for all realizations of uncertainty.

C.9 Proof of Corollary 2

In the proof of Proposition 7, the only role played by (24) to make sure that there exists a unique equilibrium in which y_t is bounded, and in this equilibrium, the economy returns to the steady state at date H (i.e., $y_t = \pi_t = 0$ for $t \geq H$). As a result, the proof of Proposition 7 continues to hold if we directly require that the economy returns to the steady state at date H, where H is finite but arbitrarily large.

C.10 Proof of Proposition 8

We first characterize the HANK equilibrium with $\omega < 1$, $\tau_y > 0$, $\tau_d \in [0,1)$, $\delta > 0$, and $\phi = 0$. Applying period-t expectation to (26) leads to the same $\mathbb{E}_t[d_{t+1}]$ as applying period-t expectation to (4). As a result, (C.3) in Proposition 2 for the $\delta = 0$ case characterizing the evolution from $\left(d_t + \varepsilon_t, y_t\right)'$ to $\left(\mathbb{E}_t[d_{t+1}], \mathbb{E}_t[y_{t+1}]\right)'$ is exactly the same under $\delta > 0$ case. This means that any bounded path of $\{d_t, y_t\}_{t=0}^{+\infty}$ still takes the form of

$$y_t = \chi(d_t + \varepsilon_t) \quad \text{and} \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t + \varepsilon_t),$$

where χ and ρ_d are uniquely given by the same (C.7) in Proposition 2 for the $\delta = 0$ case, continuous in $(\beta, \omega, \tau_y, \tau_d)$. As a result,

$$\pi^{\dagger,HANK} = 1 - \beta\rho_d > 1 - \beta. \tag{C.41}$$

The maturity of government debt $\delta > 0$, however, matters for the mapping from ε_t to $d_t - \mathbb{E}_{t-1}[d_t]$ in (C.8) and (C.9). From (3) and (A.11), we can find d_0 as a function of the deficit shock ε_0 :

$$\begin{aligned} d_0 &= -\frac{D^{ss}}{Y^{ss}}\pi_0^{\delta} = -\frac{D^{ss}}{Y^{ss}}\frac{\kappa}{1-\beta\rho_d}\sum_{k=0}^{+\infty}(\beta\delta)^k\mathbb{E}_0[y_k] \\ &= -\frac{\frac{D^{ss}}{Y^{ss}}}{1-\beta\delta\rho_d}\frac{\kappa}{1-\beta\rho_d}\chi(d_0+\varepsilon_0) = -\frac{\frac{\kappa\frac{D^{ss}}{Y^{ss}}\chi}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}}{\frac{\kappa\frac{D^{ss}}{Y^{ss}}\chi}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}+1}\varepsilon_0. \end{aligned} \tag{C.42}$$

Similarly, for $t \geq 1$, from (3) and (A.9),

$$d_t - \mathbb{E}_{t-1}[d_t] = -\frac{\frac{\kappa\frac{D^{ss}}{Y^{ss}}\chi}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}}{\frac{\kappa\frac{D^{ss}}{Y^{ss}}\chi}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}+1}\varepsilon_t. \tag{C.43}$$

Together with (3) and (18), we find the unique bounded equilibrium path of $\{\pi_t, d_t, y_t\}_{t=0}^{+\infty}$. In particular, for all $t \geq 0$,

$$\pi_t - \mathbb{E}_{t-1}[\pi_t] = \pi_{\varepsilon,0}^{HANK}\cdot\varepsilon_t, \quad \pi_{\varepsilon,0}^{HANK} \equiv \frac{\frac{\kappa\chi}{1-\beta\rho_d}}{\frac{\kappa\frac{D^{ss}}{Y^{ss}}\chi}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}+1}, \quad \text{and} \quad \pi_{\varepsilon,k}^{HANK} \equiv \frac{d\mathbb{E}_t[\pi_{t+k}]}{d\varepsilon_t} = \rho_d^k\pi_{\varepsilon,0}^{HANK}.$$

As a result,

$$\pi_{\varepsilon}^{\delta,HANK} = \sum_{k=0}^{\infty}(\beta\delta)^k\pi_{\varepsilon,k}^{HANK} = \frac{\frac{\kappa\chi}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}}{\frac{\kappa\frac{D^{ss}}{Y^{ss}}\chi}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}+1}, \tag{C.44}$$

From the proof of Proposition 3, we know that ρ_d and χ are continuous and decreasing in $\tau_d \in [0,1)$. As a result, $\pi_{\varepsilon}^{\delta,HANK}$ are continuous and decreasing in $\tau_d \in [0,1)$. Also from the proof of Proposition 3, we know that τ_d and χ are independent of $\frac{D^{ss}}{Y^{ss}}$ and δ. As a result, $\pi_{\varepsilon}^{\delta,HANK}$ decreases with $\frac{D^{ss}}{Y^{ss}}$ and increases with δ.

Now we focus on the case of $\tau_d = 0$, focused in Proposition 8. In that case, from (26), we know that, $1-\beta\rho_d = \tau_y \chi$. As a result,

$$\pi_\varepsilon^{\delta,HANK} = \sum_{k=0}^{\infty} (\beta\delta)^k \pi_{\varepsilon,k}^{HANK} = \frac{\frac{\kappa}{\tau_y(1-\beta\delta\rho_d)}}{\frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y(1-\beta\delta\rho_d)} + 1} = \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}\left(1-\beta\delta\rho_d\right)} < \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}\left(1-\beta\delta\right)}. \tag{C.45}$$

We now characterize the RANK-FTPL equilibrium with $\omega = 1$, $\tau_d = 0$, $\delta > 0$, and $\phi = 0$. Household optimality (1) and (8) remain to hold no matter δ. As a result, as in Section 3 for the $\delta = 0$ case, any equilibrium in which $\{y_t^{FTPL}\}_{t=0}^{\infty}$ is bounded must satisfy (9), with $\varrho = 1$. Apply period-0 expectation to (A.11) and (26), and use the no-Ponzi condition,

$$\frac{D^{ss}}{Y^{ss}}\pi_0^{\delta,FTPL} + \tau_y \sum_{k=0}^{+\infty} \beta^k \mathbb{E}_0\left[y_k^{FTPL}\right] = \varepsilon_0.$$

Together with (3) and (9), we know that $\sum_{k=0}^{+\infty}\beta^k \mathbb{E}_0\left[y_k^{FTPL}\right] = \frac{1-\beta\delta}{\kappa}\pi_0^{\delta,FTPL}$. As a result,

$$\pi_0^{\delta,FTPL} = \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}\left(1-\beta\delta\right)}\varepsilon_0.$$

Similarly, $\pi_t^{\delta,FTPL} - \mathbb{E}_{t-1}\left[\pi_t^{\delta,FTPL}\right] = \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}(1-\beta\delta)}\varepsilon_t$. As a result,

$$\pi_\varepsilon^{\delta,FTPL} = \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}\left(1-\beta\delta\right)}.$$

Together with (C.45), we know that $\pi_\varepsilon^{\delta,HANK} < \pi_\varepsilon^{\delta,FTPL}$. Moreover, the distance between the two vanishing when $\tau_y \to 0$, $\kappa \to \infty$, or $\delta \to 0$. This proves Proposition 8.

Finally, from (9) (with $\varrho = 1$) and (C.41),

$$\pi^{\dagger,HANK} > \pi^{\dagger,FTPL} = 1-\beta.$$

C.11 Proof of Proposition 9

Let $\omega < 1$, $\tau_y > 0$, $\tau_d = 0$, $\phi = 0$, and $\mu \in (0,1)$. We work with the flow government budget (26) allowing $\delta \in [0,1)$, nesting the short-term debt case in (4). Imposing $y_t = c_t$ (goods market clearing) and $a_t = d_t$

(asset market clearing) in (29),

$$
\begin{aligned}
y_t &= (1-\beta\omega)\, d_t + \left(\mu + (1-\mu)(1-\beta\omega)\right)\left((y_t - t_t) + \frac{(1-\mu)(1-\beta\omega)}{\mu+(1-\mu)(1-\beta\omega)}\mathbb{E}_t\left[\sum_{k=1}^{\infty}(\beta\omega)^k\left(y_{t+k}-t_{t+k}\right)\right]\right)\\
&= \frac{1-\beta\omega}{(1-\mu)\,\beta\omega}d_t - \frac{\mu+(1-\mu)(1-\beta\omega)}{(1-\mu)\,\beta\omega}t_t + \frac{1-\beta\omega}{\beta\omega}\mathbb{E}_t\left[\sum_{k=1}^{+\infty}(\beta\omega)^k\left(y_{t+k}-t_{t+k}\right)\right]\\
&= \frac{1-\beta\omega}{(1-\mu)\,\beta\omega}d_t - \frac{\mu+(1-\mu)(1-\beta\omega)}{(1-\mu)\,\beta\omega}t_t + \mathbb{E}_t\left[y_{t+1}\right] + \mathbb{E}_t\left[-\frac{1-\beta\omega}{1-\mu}d_{t+1} + \frac{\mu}{1-\mu}t_{t+1}\right]\\
&= \frac{1-\beta\omega}{(1-\mu)\,\beta\omega}d_t - \frac{\mu+(1-\mu)(1-\beta\omega)}{(1-\mu)\,\beta\omega}t_t + \mathbb{E}_t\left[y_{t+1}\right] + \mathbb{E}_t\left[-\frac{1-\beta\omega}{\beta(1-\mu)}(d_t - t_t) + \frac{\mu}{1-\mu}t_{t+1}\right]\\
&= \frac{(1-\beta\omega)(1-\omega)}{\beta\omega(1-\mu)}d_t - \left(\frac{\mu}{1-\mu} + \frac{(1-\omega)(1-\beta\omega)}{\beta\omega(1-\mu)}\right)t_t + \mathbb{E}_t\left[y_{t+1}\right] + \frac{\mu}{1-\mu}\mathbb{E}_t\left[t_{t+1}\right].
\end{aligned}
$$

Applying the fiscal rule (6), we have, for all $t \geq 0$,

$$
y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega(1-\mu)}}{1+\left(\frac{\mu}{1-\mu}+\frac{(1-\omega)(1-\beta\omega)}{\beta\omega(1-\mu)}\right)\tau_y}d_t + \frac{1+\frac{\mu}{1-\mu}\tau_y}{1+\left(\frac{\mu}{1-\mu}+\frac{(1-\omega)(1-\beta\omega)}{\beta\omega(1-\mu)}\right)\tau_y}\mathbb{E}_t\left[y_{t+1}\right] + \frac{\left(\frac{\mu}{1-\mu}+\frac{(1-\omega)(1-\beta\omega)}{\beta\omega(1-\mu)}\right)}{1+\left(\frac{\mu}{1-\mu}+\frac{(1-\omega)(1-\beta\omega)}{\beta\omega(1-\mu)}\right)\tau_y}\varepsilon_t.
\tag{C.46}
$$

Applying period-t expectations $\mathbb{E}_t[\cdot]$ to (26), we have, for all $t \geq 0$,

$$
\begin{pmatrix} \mathbb{E}_t\left[d_{t+1}\right] \\ \mathbb{E}_t\left[y_{t+1}\right] \end{pmatrix} = \begin{pmatrix} \frac{1}{\beta} & -\frac{\tau_y}{\beta} \\ -\frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega(1-\mu)}}{1+\frac{\mu}{1-\mu}\tau_y} & \left(1+\frac{\frac{(1-\omega)(1-\beta\omega)}{\beta\omega(1-\mu)}\tau_y}{1+\frac{\mu}{1-\mu}\tau_y}\right) \end{pmatrix}\begin{pmatrix} d_t \\ y_t \end{pmatrix} + \begin{pmatrix} \frac{1}{\beta} \\ -\frac{\frac{\mu}{1-\mu}+\frac{(1-\omega)(1-\beta\omega)}{\beta\omega(1-\mu)}}{1+\frac{\mu}{1-\mu}\tau_y} \end{pmatrix}\varepsilon_t.
$$

The two eigenvalues of the system ($\lambda_1 > \lambda_2$) are given by the solutions of

$$
f(\lambda) \equiv \lambda^2 - \lambda\left(\frac{1}{\beta} + 1 + \frac{(1-\omega)(1-\beta\omega)\tau_y}{\beta\omega\left(1-\left(1-\tau_y\right)\mu\right)}\right) + \frac{1}{\beta} = 0.
$$

Because $f(0) > 0$ and $f(1) < 0$, we know that $\lambda_1 > 1 > \lambda_2 > 0$. Moreover,

$$
\begin{aligned}
\lambda_2 &= \frac{\frac{1}{\beta}+1+\frac{(1-\omega)(1-\beta\omega)\tau_y}{\beta\omega(1-(1-\tau_y)\mu)} - \sqrt{\left(\frac{1}{\beta}+1+\frac{(1-\omega)(1-\beta\omega)\tau_y}{\beta\omega(1-(1-\tau_y)\mu)}\right)^2 - \frac{4}{\beta}}}{2}\\
&= \frac{\frac{2}{\beta}}{\left(\frac{1}{\beta}+1+\frac{(1-\omega)(1-\beta\omega)\tau_y}{\beta\omega(1-(1-\tau_y)\mu)} + \sqrt{\left(\frac{1}{\beta}+1+\frac{(1-\omega)(1-\beta\omega)\tau_y}{\beta\omega(1-(1-\tau_y)\mu)}\right)^2 - \frac{4}{\beta}}\right)},
\end{aligned}
$$

which decreases in $\mu \in [0,1)$.

Similar to Proposition 2, there is a unique bounded equilibrium where

$$
y_t = \chi_d d_t + \chi_\varepsilon \varepsilon_t \quad \text{and} \quad \mathbb{E}_t\left[d_{t+1}\right] = \rho_d d_t + \rho_\varepsilon \varepsilon_t,
\tag{C.47}
$$

where

$$
\chi_d = \frac{1-\beta\rho_d}{\tau_y} > 0, \quad \rho_d = \lambda_2, \quad \text{and} \quad \chi_\varepsilon = \frac{1-\beta\rho_\varepsilon}{\tau_y} > \chi_d.
\tag{C.48}
$$

Because λ_2 decreases in $\mu \in [0,1)$ and the baseline HANK case in Proposition 8 corresponds to $\mu = 0$. We know that $\rho_d < \rho_d^{HANK}$.

From (3) and (A.11), we can find π_0^δ as a function of the deficit shock ε_0 :

$$
\begin{aligned}
\pi_0^\delta &= \pi_0 + \sum_{k=0}^{+\infty} (\beta\delta)^{k+1} \mathbb{E}_0 [\pi_{k+1}] \\
&= -\frac{1}{1-\beta\delta\rho_d} \frac{\kappa\chi_d}{1-\beta\rho_d} \frac{D^{ss}}{Y^{ss}} \pi_0^\delta + \kappa \left(\chi_\varepsilon + \frac{\chi_d \beta \rho_\varepsilon}{1-\beta\rho_d} \right) \varepsilon_0 + \frac{\beta\delta}{1-\beta\delta\rho_d} \frac{\kappa\chi_d\rho_\varepsilon}{1-\beta\rho_d} \varepsilon_0 \\
&= \frac{\kappa\chi_\varepsilon + \frac{\kappa\chi_d\beta\rho_\varepsilon}{1-\beta\rho_d} \left(1 + \frac{\delta}{1-\beta\delta\rho_d}\right)}{1 + \frac{1}{1-\beta\delta\rho_d} \frac{\kappa\chi_d}{1-\beta\rho_d} \frac{D^{ss}}{Y^{ss}}} \varepsilon_0 \\
&= \frac{\frac{\kappa}{\tau_y} \frac{1}{1-\beta\delta\rho_d} \left(1 + \beta\delta \left(\rho_\varepsilon - \rho_d\right)\right)}{1 + \frac{1}{1-\beta\delta\rho_d} \frac{\kappa}{\tau_y} \frac{D^{ss}}{Y^{ss}}} \varepsilon_0.
\end{aligned}
$$

As a result,

$$
\pi_\varepsilon^\delta = \frac{1 + \beta\delta \left(\rho_\varepsilon - \rho_d\right)}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa} \left(1 - \beta\delta\rho_d\right)}. \tag{C.49}
$$

When $\delta = 0$, together with Proposition 3,

$$
\pi_{\varepsilon,0} = \pi_\varepsilon^\delta = \frac{\kappa}{\tau_y + \frac{D^{ss}}{Y^{ss}} \kappa} = \pi_\varepsilon^{HANK} = \pi_\varepsilon^{FTPL}.
$$

When $\delta > 0$, from (C.48), we know that $\rho_\varepsilon < \rho_d$. Moreover,

$$
\pi_\varepsilon^\delta < \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa} \left(1 - \beta\delta\rho_d\right)} < \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa} \left(1 - \beta\delta\rho_d^{HANK}\right)}.
$$

Together with (C.45), we know that

$$
\pi_\varepsilon^\delta < \pi_\varepsilon^{\delta,HANK} < \pi_\varepsilon^{\delta,FTPL}.
$$

C.12 Proof of Proposition 10

We first derive some properties under the hybrid NKPC (31) shared by both HANK and RANK-FTPL. From the hybrid NKPC (31), for all $t \geq 0$,

$$
(1-\xi) \mathbb{E}_t [\pi_{t+1}] - \frac{1}{\beta} \pi_t + \xi \pi_{t-1} = -\frac{\kappa}{\beta} y_t. \tag{C.50}
$$

Consider two roots of

$$
(1-\xi) \lambda^2 - \frac{1}{\beta} \lambda + \xi = 0,
$$

given by

$$\Lambda_1 = \frac{1-\sqrt{1-4\beta^2\xi(1-\xi)}}{2\beta(1-\xi)} = \frac{2\xi\beta}{1+\sqrt{1-4\beta^2\xi(1-\xi)}} \le \frac{2\xi\beta}{1+|2\xi-1|} < 1,$$

$$\Lambda_2 = \frac{2\xi\beta}{1-\sqrt{1-4\beta^2\xi(1-\xi)}} = \frac{1+\sqrt{1-4\beta^2\xi(1-\xi)}}{2\beta(1-\xi)} > \frac{1+|1-2\xi|}{2\beta(1-\xi)} > \frac{1}{\beta} > 1.$$

We can rewrite (C.50) as

$$\pi_t - \Lambda_1\pi_{t-1} = \Lambda_2^{-1}\left(\frac{\kappa}{\beta(1-\xi)}y_t + \mathbb{E}_t[\pi_{t+1}] - \Lambda_1\pi_t\right).$$

Iterating forward and use $\pi_{-1} = 0$, we have

$$\pi_0 = \frac{\kappa}{\beta(1-\xi)}\sum_{k=0}^{+\infty}\Lambda_2^{-k-1}\mathbb{E}_0\left[y_k\right] \quad \text{and} \quad \pi_t - \Lambda_1\pi_{t-1} = \frac{\kappa}{\beta(1-\xi)}\sum_{k=0}^{+\infty}\Lambda_2^{-k-1}\mathbb{E}_t\left[y_{t+k}\right]. \tag{C.51}$$

We now characterize the HANK equilibrium with $\omega < 1$, $\tau_d = 0$, $\delta = 0$, $\phi = 0$, and $\xi \in (0,1]$. Note that the evolution from $\left(d_t + \varepsilon_t, y_t\right)'$ to $\left(\mathbb{E}_t[d_{t+1}], \mathbb{E}_t\left[y_{t+1}\right]\right)'$ is exactly the same as (C.3) in Proposition 2 for the $\xi = 0$ case characterizing. This means that any bounded path of $\left\{d_t, y_t\right\}_{t=0}^{+\infty}$ still takes the form of

$$y_t = \chi(d_t + \varepsilon_t) \quad \text{and} \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t + \varepsilon_t),$$

where χ and ρ_d are uniquely given by the same (C.7) in Proposition 2 for the $\xi = 0$ case. The hybrid NKPC with $\xi > 0$, however, matters for the mapping from ε_t to $d_t - \mathbb{E}_{t-1}[d_t]$ in (C.8) and (C.9). From (C.51) and the fact that $\mathbb{E}_t\left[y_{t+k}\right] = \rho_d^k y_t$, we have

$$\pi_0 = \frac{\kappa}{\beta(1-\xi)}\sum_{k=0}^{+\infty}\Lambda_2^{-k-1}\mathbb{E}_0\left[y_k\right] = \frac{\kappa}{\beta(1-\xi)}\frac{1}{\Lambda_2 - \rho_d}y_0$$

From (5) and (3) , we can find d_0 as a function of the deficit shock ε_0 :

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0 = -\frac{D^{ss}}{Y^{ss}}\frac{\kappa}{\beta(1-\xi)}\frac{1}{\Lambda_2-\rho_d}y_0 = -\frac{D^{ss}}{Y^{ss}}\frac{\kappa}{\beta(1-\xi)}\frac{\chi}{\Lambda_2-\rho_d}(d_0+\varepsilon_0) \tag{C.52}$$

As a result,

$$d_0 = -\frac{\frac{D^{ss}}{Y^{ss}}\frac{\kappa\chi}{\beta(1-\xi)(\Lambda_2-\rho_d)}}{\frac{D^{ss}}{Y^{ss}}\frac{\kappa\chi}{\beta(1-\xi)(\Lambda_2-\rho_d)}+1}\varepsilon_0 \quad \text{and} \quad \pi_0 = \frac{\frac{\kappa\chi}{\beta(1-\xi)(\Lambda_2-\rho_d)}}{\frac{D^{ss}}{Y^{ss}}\frac{\kappa\chi}{\beta(1-\xi)(\Lambda_2-\rho_d)}+1}\varepsilon_0.$$

As a result,

$$\pi_{\varepsilon,0}^{HANK} = \frac{\frac{\kappa\chi}{\beta(1-\xi)(\Lambda_2-\rho_d)}}{\frac{D^{ss}}{Y^{ss}}\frac{\kappa\chi}{\beta(1-\xi)(\Lambda_2-\rho_d)}+1}.$$

When $\tau_d = 0$, from (4), we know that, $\chi = \frac{1-\beta\rho_d}{\tau_y}$. As a result,

$$\pi_{\varepsilon,0}^{HANK} = \frac{\frac{\kappa(1-\beta\rho_d)}{\beta\tau_y(1-\xi)(\Lambda_2-\rho_d)}}{\frac{D^{ss}}{Y^{ss}}\frac{\kappa(1-\beta\rho_d)}{\beta\tau_y(1-\xi)(\Lambda_2-\rho_d)}+1}. \tag{C.53}$$

We now turn to the RANK-FTPL equilibrium with $\omega = 1$, $\tau_d = 0$, $\delta = 0$, $\phi = 0$, and $\xi \in (0,1]$. Household optimality (1) and (8) remain to hold no matter ξ. As a result, as in Section 3 for the $\xi = 0$ case, any equilibrium in which $\{y_t^{FTPL}\}_{t=0}^{\infty}$ is bounded must satisfy (9), with $\varrho = 1$. In particular, $\mathbb{E}_t\left[y_{t+k}^{FTPL}\right] = y_t^{FTPL}$ for all $t, k \geq 0$. Following similar step as above (simply replace ρ_d with $\varrho = 1$), we have

$$\pi_0^{FTPL} = \frac{\kappa}{\beta(1-\xi)} \frac{1}{\Lambda_2 - 1} y_0^{FTPL}.$$

Apply period-0 expectation to (4) and (5), and use the no-Ponzi condition,

$$\frac{D^{ss}}{Y^{ss}} \pi_0^{FTPL} + \tau_y \sum_{k=0}^{+\infty} \beta^k \mathbb{E}_0\left[y_k^{FTPL}\right] = \varepsilon_0.$$

Together, we have

$$\pi_0^{FTPL} = \frac{\frac{\kappa(1-\beta)}{\beta\tau_y(1-\xi)(\Lambda_2-1)}}{\frac{D^{ss}}{Y^{ss}} \frac{\kappa(1-\beta)}{\beta\tau_y(1-\xi)(\Lambda_2-1)} + 1} \varepsilon_0 \quad \text{and} \quad \pi_{\varepsilon,0}^{FTPL} = \frac{\frac{\kappa(1-\beta)}{\beta\tau_y(1-\xi)(\Lambda_2-1)}}{\frac{D^{ss}}{Y^{ss}} \frac{\kappa(1-\beta)}{\beta\tau_y(1-\xi)(\Lambda_2-1)} + 1}.$$

Because $\Lambda_2 > \frac{1}{\beta}$ and $\rho_d \in (0,1)$,

$$\frac{1-\beta\rho_d}{\Lambda_2 - \rho_d} > \frac{1-\beta}{\Lambda_2 - 1}.$$

Together with (C.53), we know that

$$\pi_{\varepsilon,0}^{HANK} > \pi_{\varepsilon,0}^{FTPL},$$

with the distance between the two vanishing when $\tau_y \to 0$ or $\kappa \to \infty$.

C.13 Proof of Proposition A.1

In this proof, objects without superscripts (such as $\{\pi_t, d_t, y_t\}_{t=0}^{+\infty}$ and (ρ_d, χ)) capture relevant objects in the HANK economy characterized in Proposition 5. Objects with the superscript FTPL (such as $\{\pi_t^{FTPL}, d_t^{FTPL}, y_t^{FTPL}\}_{t=0}^{+\infty}$ and $(\rho_d^{FTPL}, \chi^{FTPL})$) capture the corresponding objects in the RANK-FTPL economy which shares the same path of (expected) real interest rates as the HANK economy.

We first characterize the HANK equilibrium with $\omega < 1$, $\tau_y > 0$, $\tau_d = 0$, $\delta > 0$, and $\phi \in (\underline{\phi}, \bar{\phi})$. Applying period-$t$ expectation to (26) leads to

$$\mathbb{E}_t[d_{t+1}] = \frac{1}{\beta}(d_t - t_t) + \frac{D^{ss}}{Y^{ss}} r_t,$$

similar to applying period-t expectation to (4). As a result, (C.15) in Proposition 5 for the $\delta = 0$ case characterizing the evolution from $(d_t + \varepsilon_t, y_t)'$ to $(\mathbb{E}_t[d_{t+1}], \mathbb{E}_t[y_{t+1}])'$ is exactly the same under $\delta > 0$ case. Moreover, when $\tau_d = 0$, from (C.19), we know that $\lambda_1 > 1$. As a result, any bounded equilibrium path of $\{d_t, y_t\}_{t=0}^{+\infty}$ takes the form of

$$y_t = \chi(d_t + \varepsilon_t) \quad \text{and} \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t + \varepsilon_t), \tag{C.54}$$

where χ and ρ_d are uniquely given by the same (C.23) in Proposition 5. The maturity of government debt $\delta > 0$, however, matters for the mapping from ε_t to $d_t - \mathbb{E}_{t-1}[d_t]$ in (C.8) and (C.9). In particular, from (3) and (A.11), we can find d_0 as a function of the deficit shock ε_0 :

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\left(\pi_0^\delta + r_0^\delta\right) = -\frac{D^{ss}}{Y^{ss}}\left(\frac{\kappa}{1-\beta\rho_d} + \beta\delta\phi\right)\sum_{t=0}^{+\infty}(\beta\delta)^t\mathbb{E}_0[y_t]$$

$$= -\frac{\frac{D^{ss}}{Y^{ss}}}{1-\beta\delta\rho_d}\left(\frac{\kappa}{1-\beta\rho_d} + \beta\delta\phi\right)\chi(d_0+\varepsilon_0) = -\frac{\frac{\frac{D^{ss}}{Y^{ss}}}{1-\beta\delta\rho_d}\left(\frac{\kappa}{1-\beta\rho_d} + \beta\delta\phi\right)\chi}{\frac{\frac{D^{ss}}{Y^{ss}}}{1-\beta\delta\rho_d}\left(\frac{\kappa}{1-\beta\rho_d} + \beta\delta\phi\right)\chi + 1}\varepsilon_0. \tag{C.55}$$

Now consider any realization of the initial fiscal shock ε_0, abstract from any future shocks. When $\tau_d = 0$, from the government budget (26) and (A.11) and the government's no-Ponzi condition, the government's intertemporal budget is:

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}}\sum_{t=0}^{+\infty}\left(\beta^{t+1} - (\beta\delta)^{t+1}\right)r_t = \frac{D^{ss}}{Y^{ss}}\pi_0^\delta + \tau_y\sum_{t=0}^{+\infty}\beta^t y_t = \frac{D^{ss}}{Y^{ss}}\pi_0^\delta + \frac{\tau_y}{\kappa}\pi_0, \tag{C.56}$$

where we use (3) for the second equality. Together with (C.54) and (C.55), we know that

$$\pi_0^\delta = \frac{\kappa}{\tau_y\left(1-\beta\delta\rho_d\right) + \frac{D^{ss}}{Y^{ss}}\kappa}\left(\varepsilon_0 + \frac{D^{ss}}{Y^{ss}}\sum_{t=0}^{+\infty}\left(\beta^{t+1} - (\beta\delta)^{t+1}\right)r_t\right) = \frac{\frac{\kappa}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}\chi}{\frac{\frac{D^{ss}}{Y^{ss}}}{1-\beta\delta\rho_d}\left(\frac{\kappa}{1-\beta\rho_d} + \beta\delta\phi\right)\chi + 1}\varepsilon_0. \tag{C.57}$$

From the government budget (26) and the recursive AD (C.14), for $t \geq 0$,

$$y_t = -\sigma r_t + \frac{\left(1-\beta\omega\right)(1-\omega)}{\omega}\rho_d(d_t+\varepsilon_t) + y_{t+1},$$

where $\varepsilon_t = 0$ for all $t \neq 0$. Same as (C.28), we still have

$$r_t = \rho_d^t r_0 = \phi\rho_d^t y_0. \tag{C.58}$$

Because $\rho_d \in (0,1)$ so $\lim_{t\to\infty} y_t = 0$ in the HANK equilibrium, we have, for $t \geq 0$,

$$y_t = -\frac{\sigma}{1-\rho_d}r_t + \frac{\left(1-\beta\omega\right)(1-\omega)}{\omega\left(1-\rho_d\right)}\rho_d(d_t+\varepsilon_t)$$

$$\sum_{t=0}^{+\infty}\beta^t y_t = -\frac{\sigma}{\left(1-\rho_d\right)\left(1-\beta\rho_d\right)}r_0 + \frac{\left(1-\beta\omega\right)(1-\omega)}{\omega\left(1-\rho_d\right)\left(1-\beta\rho_d\right)}\rho_d(d_0+\varepsilon_0). \tag{C.59}$$

where we use (18) for the second equation. Putting them into (C.56) and using (3), we have, for $k \geq 0$,

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}}\left(\frac{\beta}{1-\beta\rho_d} - \frac{\beta\delta}{1-\beta\delta\rho_d}\right)r_0 + \frac{\sigma\left(\kappa\frac{D^{ss}}{Y^{ss}}\frac{1}{(1-\beta\rho_d\delta)} + \tau_y\right)}{\left(1-\beta\rho_d\right)\left(1-\rho_d\right)}r_0 = \frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{1}{(1-\beta\rho_d\delta)} + \tau_y}{\left(1-\beta\rho_d\right)\left(1-\rho_d\right)}\frac{\left(1-\beta\omega\right)(1-\omega)}{\omega}\rho_d(d_0+\varepsilon_0). \tag{C.60}$$

Together with (C.54), (C.55), and (C.57), we know that π_0, π_0^δ, $d_0 + \varepsilon_0$, and y_0 have the same sign as ε_0. For example, with $\varepsilon_0 > 0$,we have $\pi_0 > 0$, $\pi_0^\delta > 0$, $d_0 + \varepsilon_0 > 0$, and $y_0 > 0$.

We now feed the equilibrium path of the (expected) real rate obtained in the HANK equilibrium $\{r_t = r_t^{HANK}\}_{t=0}^{\infty}$ into the RANK-FTPL economy in which $\omega = 1$, fiscal policy follows the same rule as in our HANK economy (with $\tau_d = 0$), monetary policy follows the passive rule $r_t^{FTPL} = r_t$, and shares the same maturity of the HANK economy (with $\delta > 0$). The government's intertemporal budget also holds for the FTPL equilibrium,

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{t=0}^{+\infty} \left(\beta^{t+1} - (\beta\delta)^{t+1}\right) r_t = \frac{D^{ss}}{Y^{ss}} \pi_0^{\delta,FTPL} + \tau_y \sum_{t=0}^{+\infty} \beta^t y_t^{FTPL} = \frac{D^{ss}}{Y^{ss}} \pi_0^{\delta,FTPL} + \frac{\tau_y}{\kappa} \pi_0^{FTPL}, \tag{C.61}$$

where we use (3) for the second equality. Similar to (8), the equilibrium path of $\{y_t^{FTPL}\}_{t=0}^{\infty}$ can be characterized by the familiar DIS equation, for $t \geq 0$,

$$y_t^{FTPL} = -\sigma r_t + y_{t+1}^{FTPL}. \tag{C.62}$$

Similar to (C.59) but without imposing $y_\infty^{FTPL} \equiv \lim_{t\to\infty} y_t^{FTPL} = 0$,

$$y_t^{FTPL} = -\frac{\sigma}{1-\rho_d} r_t + y_\infty^{FTPL} \tag{C.63}$$

$$\sum_{t=0}^{+\infty} \beta^t y_t^{FTPL} = -\frac{\sigma}{(1-\rho_d)(1-\beta\rho_d)} r_0 + \frac{1}{1-\beta} y_\infty^{FTPL}$$

Putting them into (C.61) and using (3), we have,

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \left(\frac{\beta}{1-\beta\rho_d} - \frac{\beta\delta}{1-\beta\delta\rho_d}\right) r_0 + \frac{\sigma\left(\kappa \frac{D^{ss}}{Y^{ss}} \frac{1}{(1-\beta\rho_d\delta)} + \tau_y\right)}{(1-\beta\rho_d)(1-\rho_d)} r_0 = \frac{\kappa \frac{D^{ss}}{Y^{ss}} \frac{1}{1-\beta\delta} + \tau_y}{1-\beta} y_\infty^{\text{FTPL}}. \tag{C.64}$$

Compared with (C.60), we know that y_∞^{FTPL} has the same sign as $d_0 + \varepsilon_0$ and ε_0. From this point on, we will use the positive fiscal deficit shock $\varepsilon_0 > 0$ as an example, which means $y_\infty^{\text{FTPL}} > 0$. The proof with $\varepsilon_0 < 0$ is symmetric. With $\varepsilon_0 > 0$, from (C.54), we know that, in HANK, $\pi_t > 0$ and $y_t > 0$ because $\chi > 0$, $\rho_d \in (0,1)$, and $d_0 + \varepsilon_0 > 0$. When $\phi \in [0, \bar{\phi})$, $r_t \geq 0$ for all $t \geq 0$. From (C.62), $y_0^{FTPL} \leq y_1^{FTPL} \leq y_2^{FTPL} \leq \cdots$. From (3), we have

$$\pi_0^{FTPL} \leq \pi_1^{FTPL} \leq \pi_2^{FTPL} \leq \cdots.$$

We hence know that $\pi_0^{FTPL} \leq \frac{1}{1-\beta\delta} \pi_0^{\delta,FTPL}$. When $\phi \in \left(-\frac{1}{\sigma}, 0\right)$, $r_t = \rho_d^t r_0 < 0$ for all $t \geq 0$. From $y_\infty^{\text{FTPL}} > 0$ and (C.63), we know that $y_{t+1}^{FTPL} > \rho_d y_t^{FTPL} > 0$ for all $t \geq 0$. From (3), we know that $\pi_{t+1}^{FTPL} > \rho_d \pi_t^{FTPL} > 0$ for all $t \geq 0$. We hence know that $\pi_0^{FTPL} < \frac{1}{1-\beta\delta\rho_d} \pi_0^{\delta,FTPL}$. Together with (C.61), we know that

$$\pi_0^{\delta,FTPL} > \frac{\kappa}{\tau_y(1-\beta\delta\rho_d) + \frac{D^{ss}}{Y^{ss}}\kappa} \left(\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{t=0}^{+\infty} \left(\beta^{t+1} - (\beta\delta)^{t+1}\right) r_t\right).$$

Together with (C.57), we have $\pi_\varepsilon^{\delta,FTPL} > \pi_\varepsilon^{\delta,HANK} > 0$.