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MONETARY POLICY AND INTEREST RATES

The Response of Real Output and Inflation to Monetary Policy

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This paper presents a small model of the U.S. economy for estimating the response of inflation and real output to a change in monetary policy. Measures obtained from the model's reduced-form equations provide estimates of the complete adjustment paths of inflation and real output to a monetary disturbance. By **complete** adjustment we mean that the response of each variable to a change in monetary policy continues until the variable's level and rate of change have both reached their respective long-run values. In other words, prices will continue to change until both the inflation rate and the level of real money balances reach their respective long-run values, while real output will continue to adjust until it equals the level of potential output and is growing at the rate of potential output. Most other reduced-form models focus only upon the adjustment of rates of change in prices and output to a monetary disturbance. In contrast with these, our model provides results which are consistent with the neutrality of money, which is perhaps one of the most generally accepted properties regarding economic behavior. It holds that changes in the money supply ultimately affect only nominal variables, such as prices and wages, leaving all real quantities, such as goods and services, unchanged.

Our empirical estimates provide the adjustment patterns of both real GNP and inflation to their respective long-run values implied by the neutrality property. Notably these time patterns show a relatively quick, short-run response of both inflation and real GNP to a change in monetary growth, with those responses completed within about two years' time. This contrasts with conventional model estimates which range from three to five years. Our results suggest that a monetary contraction is likely to bring inflation down faster with less adverse affect upon real economic activity than previously anticipated.

In the next section we describe the model and detail its long- and short-run properties, emphasizing the expected lag pattern between changes in monetary policy and changes in the level and rates of change of prices and real output. In the following section, we estimate the reduced-form equations of the model, utilizing an estimation technique suggested by John Scadding (see Appendix 1). Using this method, we are able to place restrictions on both the steady-state level of a variable and its rate of change. The final section provides policy implications and conclusions.

I. Model of Real Output and Inflation

The structure of the model is concerned directly with behavior in the markets for goods, money and labor. Each market is characterized by fairly standard economic relationships which are detailed in Appendix 2, and which may be combined to provide the aggregate-demand and aggregate-supply equation shown in Table 1. Each of the variables is measured in terms of its natural logarithm.

Aggregate demand, which is stated in terms of the level of output relative to its potential, $(Y/YP)^d$, is inversely related to the rate of inflation, dP_t. This occurs because — given the rate of growth in the nominal money supply, dM_t — a reduction in the rate of inflation raises

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real money balances at each level of output. Higher real balances lead to a fall in real interest rates, which in turn increases interest-sensitive spending. By the same line of reasoning, aggregate real demand declines when the inflation rate increases.

However, demand is positively related to expected changes in the rate of inflation, as indicated by the positive sign associated with the coefficient a_{12}. For example, if people anticipate an increase in the inflation rate, their demand for goods and services will increase in the current period as they take advantage of this period's relatively lower prices. The negative value associated with the coefficient a_{13} indicates that demand is inversely associated with changes in the government-budget surplus. For example, an increase in government revenues relative to spending acts to depress aggregate demand. Also, past increases in the real-GNP gap tend to reduce current real growth. This may occur, for instance, when past increases in income tend to increase current saving more than investment. Finally w_{1t} represents the random disturbance or error term in estimating aggregate demand. It captures the sometimes sizable but unsystematic effect upon aggregate demand of many factors whose total impact over time averages zero. For instance, the impact of both labor troubles and unusual weather would generally be included in this term.

The aggregate-supply equation states that deviations of output from potential are determined by unexpected changes in the current inflation rate and by past deviations of output from potential. Since these past deviations can in turn be related to past unexpected price changes, the supply equation indicates that the impact of unexpected inflation will persist for some time.[1] The equilibrium condition states that aggregate demand equals supply in any given period of time.

The specification of inflation expectations for the current and subsequent periods, dP^*_t and dP^*_{t+1}, completes the model. The hypothesis used here, following John Muth, states that expectations are informed predictions of future events, based on the available information and the relevant economic theory at the time the expectations are formed. This is the well-known "rational expectations hypothesis."[2] We will leave the detailed specification of inflation expectations for later. For now, these general comments are sufficient to begin analysis of the model under steady-state and long-run conditions.

Long-run Behavior

Certain economic conditions characterize an economy in its steady-state or long-run equilibrium. In summary, they indicate that money is neutral in the long-run with respect to the level and rate of change of output.

First, there are no surprises regarding people's expectations. Therefore, the actual money-supply growth rate and actual inflation rate are both equal to their respective anticipated rates.

Second, actual GNP is equal to potential GNP, as can be seen from the aggregate-supply equation in Table 1. In the long-run, unanticipated inflation is zero ($dP - dP^* = 0$), and therefore the logarithm of the real GNP gap, Y/YP, is zero.

Third, in the long-run, the inflation rate is determined by the excess money supply — by the growth rate of the money supply less the amount of money the public demands to finance the changing quantity of output. This result may be derived by considering the market conditions underlying aggregate demand which are stated in equations (1) through (5) in Appendix 2. In the long-run, several terms in those equations are zero, and we are left with the following specification for the inflation rate:

$$dP_t = dM_t - a_6 dYP_t$$

where a_6 represents the income elasticity of money demand.

Fourth, in the long-run, changes in the money-supply growth rate are fully reflected in changes in the inflation rate: one percentage point more (less) monetary growth means one percentage point higher (lower) inflation. This result is represented in the above equation by the one-to-one relationship between those two rates — dP_t and dM_t — and by the earlier assumption that potential GNP is exogenous with respect to the variables in our model.

Some critics argue that the long-run is an inappropriate state in which to analyze economic behavior; in other words, the economy is so frequently shocked away from its steady state that that state may never in fact be attained. However, long-run conditions may also provide useful "rules-of-thumb" regarding average economic behavior over spans of time. For instance, over long enough periods of time, the average rate of GNP will approach the economy's potential growth rate as determined by its labor-force productivity growth. Similarly, the average rate of inflation will approach the rate of excess money supply.

Table 1
Summary of the Model

Aggregate Demand

$$(Y/YP)^d_t = k_0 - a_{11}(dP_t - dM_t) + a_{12}(dP^*_{t+1} - dP^*_t) - a_{13}\, dG_t - c_1 B(L)\, d(Y/YP)_{t-1} + (Y/YP)_{t-1} + w_{1t} \quad (10)$$

Aggregate Supply

$$(Y/YP)^s_t = a_{10}(dP_t - dP^*_t) + J(L)\,(Y/YP)_{t-1} + w_{2t} \quad (11)$$

Equilibrium Condition

$$(Y/YP)^d_t = (Y/YP)^s_t \quad (12)$$

where the coefficients and weights of the polynomials are combinations of the coefficients and weights in the structural equations (1) - (7) in Appendix 2*.

The unknown variables are Y^d_t, Y^s_t, dP^*_t, dP^*_{t+1}, and dP_t.

List of Variables

- G = Federal government real high-employment surplus, measured as the ratio of revenues to expenditures,
- Y = Real GNP,
- YP = Real potential GNP,
- Y/YP = Real GNP gap,
- M = Money supply,
- P = GNP implicit price deflator,
- P* = Expected GNP implicit price deflator.

Each of the variables is measured in terms of its natural logarithm. Therefore, the change in real GNP, dY, is a measure of the rate of change in real GNP, and Y/YP is a measure of real GNP as a percent of potential. The variables G, YP, UN and M are exogenous variables.

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*$c_1 = a_7/(a_7 + a_4a_6)$
$a_{10} = a_9a_8$
$a_{11} = a_4/(a_7 + a_4a_6)$
$a_{12} = a_4 a_7/(a_7 + a_4a_6)$
$a_{13} = a_5a_7/(a_7 + a_4a_6)$
$k_0 = (-d\,YP_t - c_1B(L)\,dYP_{t-1})$

Short-run Adjustments

In the short-run, we focus on the transition period after a monetary change as the economy tends to move from one steady state to another. We emphasize the short-run adjustment paths of real GNP, inflation and real money balances which result from an increase in the money-supply growth rate. Similar reasoning may be applied to a decrease in monetary growth.

Any such monetary stimulus would lead initially to the creation of excess real money balances in the hands of the public. (This can be shown in the aggregate-demand equation by increasing the money-growth rate, dM_t, while leaving all other variables unchanged.) Individuals now will attempt to reduce those balances by increasing expenditures. Whether this increased demand will be met with greater production, higher prices, or some combination of the two depends upon aggregate supply behavior. If both employers and labor expect an increase in prices equal to the increased monetary growth, and if no contractual impediments exist, prices and wages will adjust quickly. Accordingly, there will be no change in the rate of output growth, and the increased monetary stimulus will result only in price and nominal-wage increases.

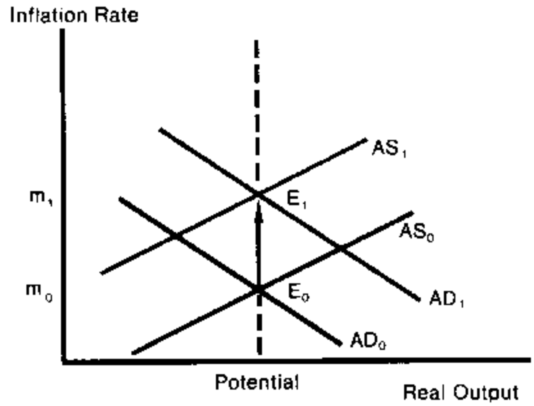
These results may be shown graphically with the use of the aggregate demand and supply equations. To simplify, we assume no change in fiscal policy and in the level of potential output, but a continued change in expected inflation. In addition, we assume zero weights for both polynomial functions, (B(L) and J(L)), in the aggregate demand and supply equations. With these assumptions, we can specify aggregate demand and supply with the following two simplified equations, each written with the inflation rate on the left-hand side.

$$dP_t = -1/a_{11}(Y/YP)^d_t + k_0/a_{11} + dM_t + 1/a_{11}(Y/YP)_{t-1} \quad \text{Aggregate Demand (10.1)}$$

$$dP_t = dP^*_t + 1/a_{10}(Y/YP)^s_t \quad \text{Aggregate Supply (11.1)}$$

Chart 1

Inflation and Output Adjustment to Increase in Money-Growth Rate: Immediate Adjustment in Price Expectations



We begin with the economy in equilibrium for some time, illustrated in Chart 1 by the intersection of aggregate demand, AD_0, and aggregate supply, AS_0, at point E_0. At that point, the level of real output is equal to potential and the rate of inflation is equal to the money-supply growth rate, "m_0", on the vertical axis. Now, let the money growth rate increase to m_1, which shifts aggregate demand upward by the full amount of that increase to AD_1, according to equation (10). If price expectations increase by the same amount as money growth, aggregate supply will shift upward by the same amount as aggregate demand, to AS_1. The intersection of AS_1 and AD_1 at point E_1 provides the new solution: the change in money growth leads only to an equal increase in the inflation rate — without any change in the quantity of output — as inflation expectations change at the same time and by the same amount as the permanent change in money growth.

However, empirical evidence suggests that prices and inflation expectations do not adjust quickly to their new long-run values.[3] Working in the face of uncertainty, individuals appear to rely heavily on observations of past behavior and other relevant information in forming expectations of the future. These expectations, although rational in the sense of being well-informed and based upon the relevant information, nevertheless provide imperfect predictions of the future at any given time. As a consequence, each increase in the money-growth rate may be followed by an increase in both real GNP and inflation.

Graphically, in Chart 2, the initial equilibrium position is again marked as point E_0. Let the rate of money growth increase to M_1. The aggregate-demand function will again shift upward to AD_1, and aggregate supply probably will also shift. But since we allow for inflation expectations which do not adjust immediately and completely to their new long-run value (m_1 on the vertical axis), the supply shift is smaller than the demand shift. For purposes of illustration, assume that aggregate supply shifts upward to AS_1. The new, short-run equilibrium is then at point B, which indicates an initial increase of both real output and inflation in response to the money-growth increase.

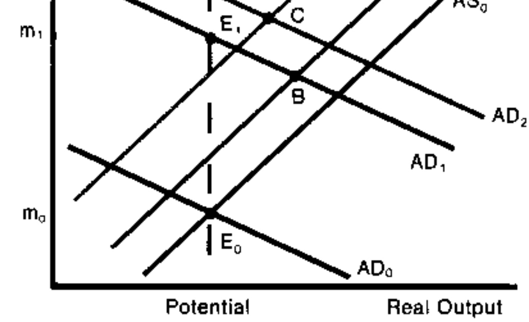
At the beginning of the next period, aggregate demand will shift upward again, according to equation 10, as past real income increases from the level associated with point E_0 to that of point B. Also, aggregate supply will shift upward again as market participants reevaluate price expectations in light of information not previously available. The intersection of aggregate demand and supply at point C illustrates the new short-run equilibrium position.

It is important to note that inflation increases from the level m_0 to a rate close to its new equilibrium rate, m_1, and then overshoots that value as continued adjustments in aggregate demand and supply lead to a new, short-run equilibrium at point C. Changes in price expectations (in light of revised forecasts) influence the shifts in supply, while past income and real money balances produce adjustments in aggregate demand. This characteristic overshooting property — "the key element in monetary theories of cyclical fluctuations," according to Milton Friedman — is a widely noted economic phenomenon.[4]

The inflation pattern is associated with a particular adjustment in real GNP, as shown in Chart 2. After the initial increase in monetary growth, real GNP increases above its potential level, and continues to increase as long as the inflation rate is below its new equilibrium rate, m_1. This occurs because as long as inflation increases more slowly than money growth (m_1), real money balances will increase and, accordingly, stimulate aggregate demand. Once the inflation rate starts to overshoot its new equilibrium level, real money balances will begin to decline. As a result of this contractionary force, real GNP begins to decline from its previous value.

The adjustments of real output, inflation and real money balances continue until each reaches a new, long-run value associated with the permanent increase in money-supply growth (Chart 3). First consider the response of inflation to an increase in the money rate, from m to m_1(Chart 3A). Initially, the inflation rate increases by less than the change in

Chart 2
Inflation and Output Adjustment to Increase in Money-Growth Rate: Lagged Adjustment in Price Expectations



money growth each period, but completes its initial adjustment phase at time T_1. At that time, the inflation rate will equal the long-run rate of change in money growth, although price expectations and real income will still not have made a complete adjustment. Subsequent to T_1, inflation overshoots and then returns to its long-run rate. In our illustration, we depict inflation as gradually returning to its new equilibrium position, although alternatively, it may exhibit a damped cyclical adjustment to its long-run value.

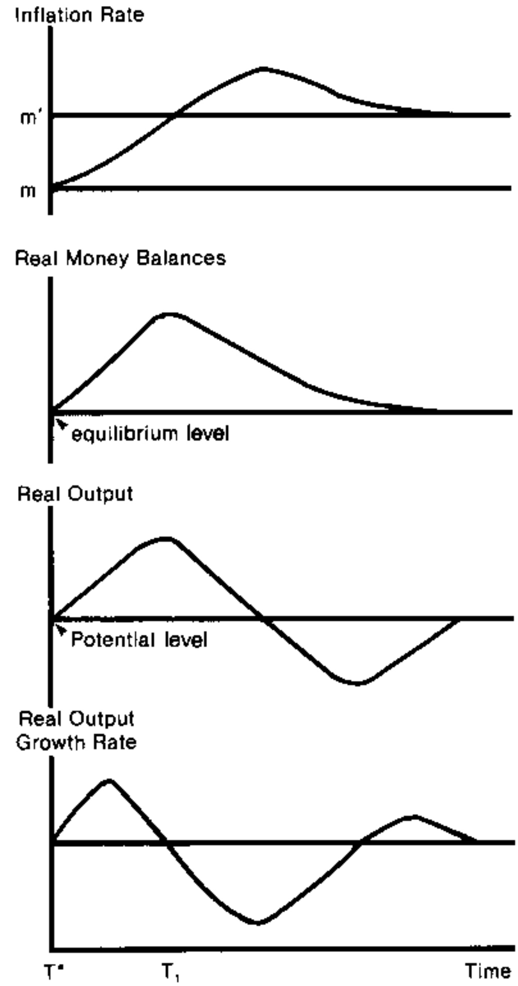
Next consider the response of real money balances to an increase in the money growth rate (Chart 3B). In the initial equilibrium, individuals hold a certain desired proportion of income in that form. For some time after money-supply growth increases, prices increase less rapidly than the nominal money supply in each period. As a result, real money balances increase and reach a maximum when the inflation rate initially reaches its long-run rate at time T_1. Subsequently, these balances steadily decline toward their new, long-run level, which we have shown to be equal to their initial level. With any sensitivity to interest rates, however, the level of real balances in the new equilibrium will be lower than initially. In the U.S., money demand generally moves inversely with interest rates. Therefore, we would expect that, after a permanent increase in monetary growth, real money balances will ultimately be slightly less than before the change.

Finally consider the response of real GNP to a change in the money growth rate (Chart 3C). In this adjustment process, the **level** of real output returns to its long-run path after rising above potential for some time. Meanwhile, the **rate of change** in real GNP increases and then declines in response to an increase in monetary growth, and finally increases again before approaching its long-run rate of change (Chart 3D). This pattern of growth is consistent with the adjustment in the level of GNP shown in Chart 3C.

In summary, a change in money-supply growth may have no significant effect on real output, at least as long as prices and price expectations adjust immediately. In other words, the entire change in money may be absorbed by a change in prices. In contrast, if there is no immediate price adjustment, monetary changes may be accompanied by changes in both inflation and real output. The key feature in the adjustment process is the cyclical response of both output and inflation After a permanent increase in money growth, inflation will increase, overshoot, and then decline towards its long-run value. The real-output growth rate will at first increase, then decline,

Chart 3

Response of Selected Variables to Permanent Increase in Money-Growth Rate



and may increase again as the level of output returns to potential.

Formation of Inflation Expectations

Before deriving the final model equations, we must first express the functional forms for expected inflation. These will then be substituted into the aggregate demand and supply equations, which in turn can be solved for the final equations for real GNP and inflation.

To obtain the specifications for expected inflation, we follow a procedure suggested by Sargent and Wallace (1975). The mathematics are shown in Appendix 3. Given the rational-expectations feature of the model, expected inflation depends upon the public's forecasts of future monetary and fiscal policies, as well as the past history of deviations of output from its potential level. (These deviations directly affect current inflation, due to the lagged response of prices to past money-supply changes.) Particularly important is the process or rule by which the public forecasts future rates of change in the money supply and in Federal-budget surpluses. We assume that people forecast these values by considering their past history, and that they update their forecasts each period as new information becomes available. Consequently, expected inflation is determined by the past history of the money supply, Federal-budget surpluses and the GNP gap.

The derived specifications for expected inflation (shown in the appendix) can next be used to obtain the final equations for inflation and real GNP. However, changes in the processes by which the public predicts future monetary and fiscal policies will lead to changes in the parameters of the estimated equations. Consequently, those equations will provide appropriate means of forecasting real GNP and inflation as long as the public does not change the rule by which it predicts future government policies.[5]

II. Solution of the Model: Reduced-form Equations

In this section, we provide the reduced-form equations for the inflation rate and real GNP which are solutions of the system (equations 10, 11, 12, plus equations 16.1 and 17.1 from Appendix 3), and which serve as the basic equations estimated in the next section.

First, we obtain the equation for real GNP, which will be stated in terms of the GNP gap, Y/YP, by substituting the specifications for inflation and expected inflation — provided by equations (13) and (14) in Appendix 3 — into the aggregate-supply equation.

$$(Y/YP)_t = \sum_{i=0}^{N} m_{1i}(dM - E_{t-1-i}\,dM)_{t-i} - \sum_{i=0}^{N} g_{1i}(dG - E_{t-1-i}\,dG)_{t-i} \quad (18)$$

where the coefficients m_{1i} and g_{1i} are combinations of the coefficients in equations (1) through (9) in Appendix 2.*

Also, we have assumed a finite length of the lags, N. The equation thus states that the cyclical movement in real GNP is determined by distributed lags on unanticipated changes in money-supply growth and in the Federal-budget surplus. (An unanticipated change is defined as the actual less the expected value of a variable.)

$^*m_{10} = a_{11}Z$
$M_{1i} = a_{11}ZJ_i,\ i = 0, 1, 2, \ldots$

(Recall that J_i are the weights of the polynomial $J(L)$ in the aggregate supply equation.)

$g_{10} = a_{13}Z$
$g_{1i} = a_{13}ZJ_i,\ i = 0, 1, 2, \ldots$

We estimate the level of GNP relative to its potential and then derive the rate of growth of real GNP from that equation. We do this because the level specification possesses the desirable property that the long-run values of both the level and rate of change will be independent of the initial conditions of the forecast.

After a monetary shock, our real GNP estimates eventually return to their long-run values. This long-run property does not hold for conventional reduced-form equations, which estimate rates of change in real GNP directly. In such cases, the long-run value of income equals the initial level of income times the subsequent rate of change in potential GNP. As a result, those equations predict a persistent gap in real output equal to the initial gap at the time of the monetary shock. By doing so, the rates-of-change specifications ignore the economic impact resulting from deviations of unemployment from its natural rate. Consequently, these specifications may produce biased estimates of the impact of monetary growth upon real GNP.

To obtain the reduced form for the inflation rate, we substitute equations (16.1) and (17.1) into equation (13) in Appendix 3, and collect terms.

$$dP_t = \sum_{i=0}^{K} m_{2i} dM_{t-i} - \sum_{i=0}^{J} g_{2i} dG_{t-i} + c' + y_1(Y/YP)_{t-1} - \sum_{i=1}^{N} y_{2i}(Y/YP)_{t-1-i} \quad (19)$$

where the coefficients are combinations of the coefficients in the equations (1) to (9) and of the money-supply and fiscal-policy processes.*

The equation states that the rate of inflation is determined by 1) a distributed lag on current and past rates of growth in the money supply, 2) a distributed lag on current and past changes in the Federal surplus, and 3) past values of the cyclical component of GNP, Y/YP.[11]

Empirical Results

Following the above specifications, we estimated equations for real GNP and the inflation rate over the sample period 1966.2-1978.1. The starting date was dictated by the availability of M-1B data and the length of the estimated lag distribution in the equations. The estimation period ends in 1978.1, to permit a relatively sizable time span outside the sample period for assessing the model's performance. The estimation results are shown in Box 1.

In estimating these equations, we applied a method suggested by John Scadding and detailed in Appendix 1. The method enables us to place restrictions on both the steady-state level of a variable and its range of change. For example, we can impose the restriction of the long-run neutrality of money, with respect to both the level and growth rate of real GNP. Similarly, in the inflation equation the restrictions may be imposed that the elasticity of the inflation rate with respect to the growth rate of money is unity, and that the level of prices is consistent with the level of money, given the demand for money.

In the real-GNP equation, we have added a lagged GNP-gap term to the reduced-form specification, equation 18. The equation then may be interpreted as estimating a rational distributed lag between the dependent variable and the $\overline{M}$ and $\overline{E}$ variables. In addition, in the inflation equation we have added two variables, D_1 and D_2, to capture the possible impact of Nixon-era wage and price controls. These same variables, although significant in the inflation equation, had no significant impact upon real GNP and are therefore not included in that equation.

$$^*\sum_{i=0}^{K} m_{2i} dM_{t-i} = [w_1H_1(L) + w_2H_3(L) + w_4]\, dM_t$$

$$\sum_{i=0}^{J} g_{2i}\, dG_{t-i} = [w_1H_2(L) + w_2H_4(H) + w_5]\, dG_t$$

$$\sum_{i=0}^{N} y_{2i} = q_i[1 + (w_1 + w_2)k_3/(1 - w_1)$$

$$c^1 = k_1k_3(w_1 + w_2) + c_0$$

$$k_3 = \sum_{j=0}^{\infty} v_1^j$$

$$y_1 = k_3v_3(w_1 + w_2) + w_3$$

Both the real GNP and inflation regressions account for over 80 percent of the variation in the dependent variables. The adjusted coefficient of determination, R^2, is .89 in the case of real GNP and .83 for inflation. The Durbin-Watson statistic (D.W.) indicates support for the hypothesis of no serial correlation in the error terms. Thus, on the basis of the relevant T-statistics, the explanatory variables in each equation have a statistically significant impact upon the determination of the dependent variables.

According to the estimated real-GNP equation, money is neutral in the long run with respect to the level of real GNP and its rate of change. These findings follow from several characteristics of the equation. First, in the long run, both unanticipated money and unanticipated federal expenditures equal zero. Second, the constant term does not significantly differ from zero, and the coefficient on the lagged gap term is less than unity, assuring the stable long-run result of a zero value for the log of Y/YP.

According to the Scadding method, the coefficient on the current change in unanticipated money, $\overline{M}_t$, in the real GNP equation provides an estimate of the total short-run effect of such a change. The estimate of .187 indicates that a one-percentage-point increase (decrease) in unanticipated money growth leads to a small transitory gain (loss) in real GNP. Since economic theory does not indicate an expected value of the transitory impact, we have left unconstrained the coefficient on the current value of unanticipated monetary growth.

The estimates also indicate that unanticipated increases in Federal expenditures, $\overline{E}$, will at first lead to increases in real GNP, but after 14 quarters will have no significant impact upon either the level or rate of growth of real GNP. This result follows from the estimated sum of −.068, which is not significantly different from zero.

With regard to the inflation equation, we constrained the coefficient of dM_t to be unity, after testing for the appropriateness of that constraint. Consequently, in the long run, changes in money growth are fully reflected in changes in the inflation rate. The coefficient on the second difference of money, according to our estimation procedure, is equivalent to the long-run elasticity of the price level with respect to a change in the money-supply growth rate. In other words, the coefficient is equivalent to the long-run elasticity of real money balances with respect to money (with the signs reversed).[6]

Our estimate indicates that real money balances, in the long-run, will decline by .21 percentage point when money growth increases by one percentage point each quarter. This value appears consistent with estimates for the long-run elasticity of money demand for our sample period.[7] Accordingly, we did not constrain the estimate to take on any particular value. Together the two coefficients indicate that the inflation equation is consistent with the long-run neutrality of money. In the first instance, changes in the money growth rate are fully reflected in prices; and in the second, the price level is consistent with the level of money, given the demand for money in the steady state.

Only unanticipated changes in Federal spending significantly affected the inflation rate. There was no effect in the long run, because unanticipated changes are zero in that case. The results, however, indicate a short-run or transitory impact from spending changes which are initially unanticipated.

Finally, the sum of coefficients on the lagged gap measure was not significantly different from zero, so that we constrained their sum to that value. The estimated coefficients on the first difference of the gap indicate that a positive widening of the gap in period t-1 will lead to a small negative effect upon the current quarter's inflation rate. Thereafter, the positive impact of the lagged-gap values offset the initial negative effect upon inflation.

Next, consider the response patterns of real GNP and inflation to changes in the money-supply growth rate. These patterns may be obtained from the empirical estimates by "unscrambling" the estimated coefficients, according to the method outlined in Appendix

Box 1
REAL GNP EQUATION

$$(Y/YP)_t = \underset{(-.6)}{-.0006} + \underset{(.9)}{.187\overline{M}_t} + \sum_{i=0}^{14} b_i dM_{t-i} + \sum_{i=0}^{14} e_{1i} E_{t-i} + \underset{(15.0)}{.869(Y/YP)_{t-1}}$$

Lag	b_i	t-statistic	e_{1i}	t-statistic
0	.207	3.7	.0042	1.2
1	.387	3.7	.0074	1.1
2	.542	3.7	.0096	1.1
3	.670	3.8	.0107	1.0
4	.773	3.8	.0109	0.9
5	.850	3.8	.0100	0.8
6	.900	3.9	.0080	0.6
7	.925	3.9	.0050	0.4
8	.924	3.9	.0011	0.08
9	.896	4.0	-.0039	-.32
10	.843	4.0	-.0100	-.82
11	.764	4.0	-.0171	-1.34
12	.659	3.9	-.0252	-1.77
13	.527	3.1	-.0343	-2.03
14	.370	2.1	-.0444	-2.25
			Sum -.068	(-.479)

The distributed lags are 2nd-degree polynomials, with a near-end constraint.

Adjusted r^2 = .89 D.W. = 2.01 Standard Error = .00749

INFLATION EQUATION

$$dP_t = \underset{(-.7)}{-.0004} + dM_t + \underset{(1.8)}{.21d^2M_t} + \sum_{i=0}^{20} h_i d^3M_{t-i} + \sum_{i=0}^{12} e_{2i} E_{t-1-i} - \sum_{i=0}^{16} y_i d(Y/YP)_{t-1-i}$$

$$\underset{(-1.4)}{-.002D_1} + \underset{(3.7)}{.009D_2}$$

Lag	h_i	t-statistic	e_{2i}	t-statistic	y_i	t-statistic
0	-1.23	(-10.0)	-.001	(-.7)	-.083	(-2.70)
1	-2.23	(-10.1)	.000	(.01)	-.081	(-3.03)
2	-3.00	(-10.1)	.006	(.9)	-.078	(-3.35)
3	-3.57	(-10.0)	.011	(1.7)	-.075	(-3.61)
4	-3.97	(-9.9)	.015	(2.2)	-.072	(-3.75)
5	-4.20	(-9.8)	.018	(2.5)	-.069	(-3.75)
6	-4.29	(-9.6)	.020	(2.6)	-.065	(-3.65)
7	-4.25	(-9.2)	.020	(2.7)	-.061	(-3.47)
8	-4.09	(-8.8)	.020	(2.7)	-.056	(-3.26)
9	-3.85	(-8.3)	.018	(2.7)	-.046	(-2.84)
10	-3.53	(-7.6)	.018	(2.7)	-.051	(-3.04)
11	-3.15	(-6.9)	.011	(2.6)	-.040	(-2.65)
12	-2.74	(-6.1)	.006	(2.6)	-.034	(-2.48)
13	-2.29	(-5.3)			-.028	(-2.34)
14	-1.87	(-4.5)			-.021	(-2.21)
15	-1.43	(-3.7)			-.015	(-2.09)
16	-1.04	(-2.9)			-.007	(-1.99)
17	- .70	(-2.2)				
18	- .41	(-1.7)				
19	- .22	(-1.2)				
20	- .13	(-1.2)				
			Sum .170	(2.14)		

Adjusted r^2 with unitary constraint on dM_t = .83 D.W. = 1.97 Standard Error = .0033

The distributed lags on expenditures and income are polynomial distributed lags, 2nd degree, with far-end constraints. On money a third-degree polynomial was used with a near-end constraint.

Definition of Variables

YP_t = An estimate of potential GNP. We estimate this variable as a simple trend over the period 1964.1 - 1980.1

$$\ln GNP_t = 6.8121 + .007877\,\text{Time}$$
$$(1200) \qquad (51)$$

This provides an estimate of 3.2 percent as the average annual rate of growth of real GNP.

$\overline{M}_t$ = The monetary variable is M1B, here represented by the difference between the rate of change in M1B in the current quarter and its average rate of change over the past two years, at a quarterly rate. The two-year average change is our estimate of the expected growth of the money supply, so that this variable may be interpreted as the unexpected (or unanticipated) rate of growth in M1B in period t.

$\overline{E}_t$ = The fiscal variable is represented by Federal government expenditures, E, which is the sum of Federal purchases of goods and services, Federal aid to state and local governments, and subsidies less current surplus in the National Income and Product Accounts. Each of these is regarded as independent of the stage of the cycle, and therefore may be considered as an appropriate exogenous policy variable. The fiscal variable is measured similarly to the monetary variable; it is the difference between the rate of change in Federal expenditures in the current quarter and its average rate of change over the past three years. The three-year average change is our estimate of the expected rate of change in expenditures.

dM_t = Rate of change in M1B in the current quarter, at quarterly rates.

D_1 = Wage and Price Control Dummy, equals 1 during 1971.3 - 1974.1 and 0 elsewhere.

D_2 = Wage and Price Decontrol Dummy, equals 1 during 1974.2 - 1974.4 and 0 elsewhere.

Y_t = GNP in constant 1972 dollars.

P_t = GNP Implicit Deflator.

1. This "unscrambling" may be done either directly, through algebraic manipulations of the estimated coefficients — or by obtaining estimated lag patterns from dynamic multiplier simulations of the equations.

These computer simulations essentially solve the estimated equations for a given rate of money growth, and then repeat the solution with another rate of money growth exactly one percentage point higher in each and every period. The difference between the results in each period provides the estimate of the response of the dependent variable (such as real GNP or the deflator) to the specified increase in money growth. In these dynamic simulations, the lagged dependent variables are solutions of the model after the initial period.

Real GNP Estimates

Our model has been able to capture the complete adjustment paths of both the level and rate of change in real GNP to the change in monetary policy. This can be seen from the similarity between the empirical multiplier estimates (Charts 4A and 4B), and the expected estimates from the theoretical model (Charts 3C and 3D).

If we begin, say, with an initial equilibrium situation in which the level of real GNP is equal to potential (Chart 4A), an increase of one percentage point in the money growth rate will lead to a small initial response in the level of real GNP (Table 2).

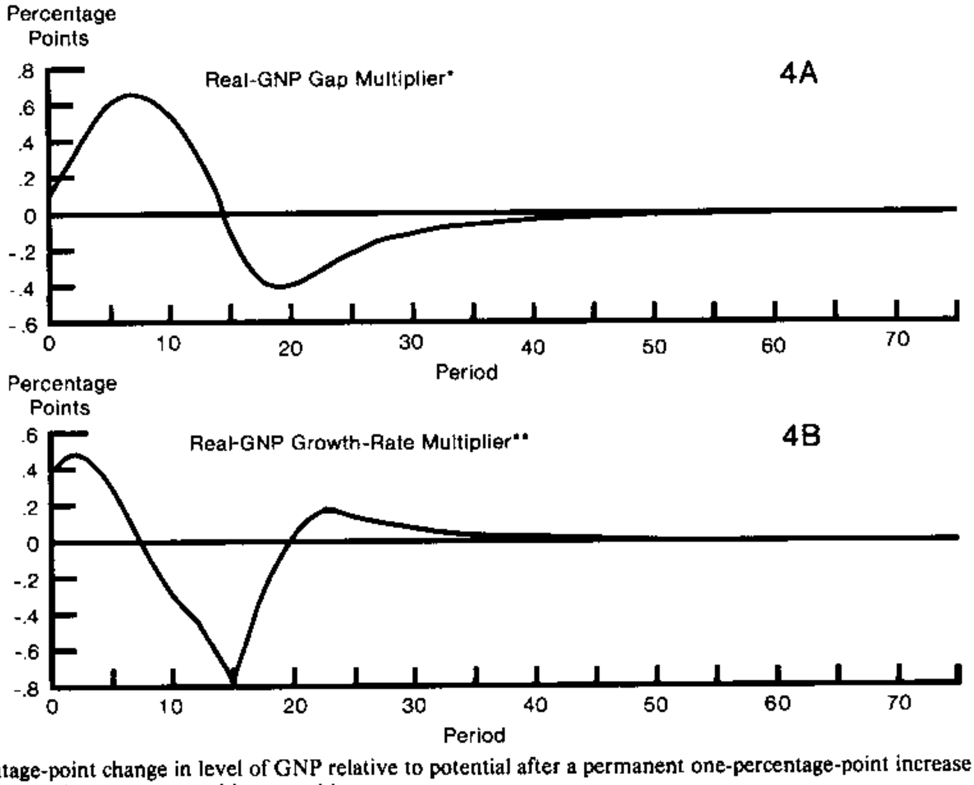
After a rise of seven quarters, ouput reaches a peak .66 percentage point higher than potential, but it then returns to its potential level around the 14th quarter after the money change. The stimulus to real GNP thus appears to end within about two years, although the final adjustment does not end until about ten years after the initial change.

The growth rate of real GNP follows a cyclical pattern over time in response to a permanent increase in money growth. For instance, from an initial position of equilibrium with real GNP rising at a steady 3.2 percent rate each quarter, a one-percentage point rise in money growth would raise GNP growth to 3.7 percent within a half-year of the initial monetary change. The growth rate then begins to decline, and reaches its initial rate after the end of two years. Following a further decline, the growth rate increases again in the final phase of adjustment, settling at its long-run rate of 3.2 percent in about 10 years' time.

With this adjustment pattern, the initial stimulus to real GNP is almost matched by an equivalent contraction of real GNP in the final phases of economic adjustment. As a result, the monetary change leads to a small but transitory gain in real output. However, in the long run, these gains disappear as real GNP returns to its potential path regardless of the rate of money growth. Moreover, most of the

Chart 4

Response of Selected Variables to Permanent Increase in Money-Growth Rate



*Percentage-point change in level of GNP relative to potential after a permanent one-percentage-point increase in the money growth rate, measured in natural logs.

**Percentage-point change in growth rate of real GNP after a permanent one-percentage-point increase in money growth rate, measured in natural logs.

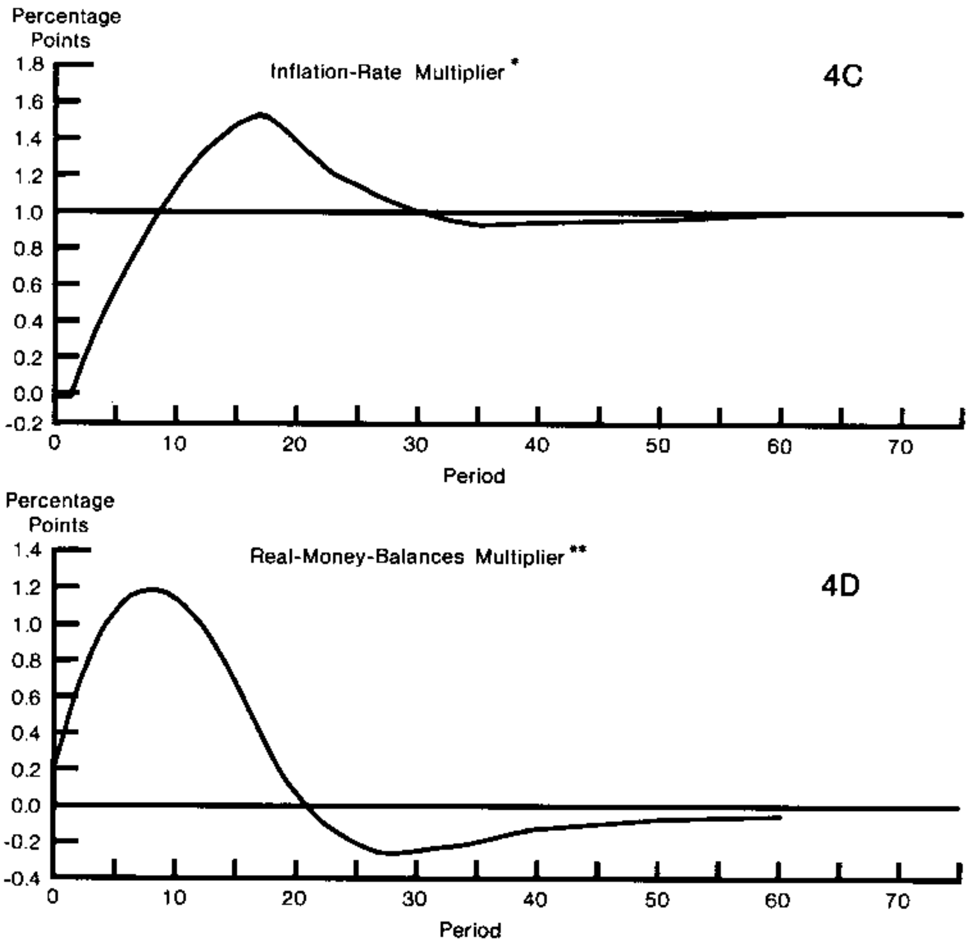
stimulative effect of an increase in monetary growth appears to end within two years of the initial stimulus.

Our estimates capture the systematic changes in real GNP, although not the sharp saw-toothed variations characteristic of this series (Chart 5). The standard error of the quarterly estimates of GNP, at annual rates, is 2.92 percent within the estimation period (1962.2-1978.1) and 3.30 percent outside that period (1978.2-1980.4).[8]

Inflation Estimates

Our model, again, permits us to estimate the complete adjustment of the level of prices and the rate of inflation to a change in money growth. This can be seen from the conformity of the estimated lag patterns in Charts 4C and 4D to those anticipated in Charts 3A and 3B.

In the long-run, the change in the rate of inflation equals the change in the rate of money growth. By the end of the first year, the inflation rate exhibits about 50 percent of its



*Percentage-point change in inflation rate after a permanent one-percentage-point increase in money growth rate, measured in natural logs.

**Percentage-point change in level of real money balances after a permanent one-percentage-point increase in money supply, measured in natural logs.

Table 2

Response of Real GNP to Permanent Percentage-Point Increase in Money-Supply Annual Growth Rate

Percentage-Point Change in Level of Real GNP

Lag	Estimate	Lag	Estimate of m_N
0	.09	25	−.22
1	.20	26	−.19
2	.32	27	−.16
3	.43	28	−.14
4	.53	29	−.12
5	.60	30	−.11
6	.64	31	−.09
7	.66	32	−.08
8	.63	33	−.07
9	.59	34	−.06
10	.52	35	−.05
11	.43	40	−.03
12	.32	50	.00
13	.21		
14	.08		
15	−.11		
16	−.24		
17	−.33		
18	−.38		
19	−.40		
20	−.39		
21	−.37		
22	−.33		
23	−.29		
24	−.25		

Percentage-Point Change in Rate of Growth of Real GNP

Lag	Estimate	Lag	Estimate
0	.38	25	.13
1	.46	26	.12
2	.48	27	.10
3	.46	28	.09
4	.39	29	.08
5	.30	30	.07
6	.18	31	.06
7	.05	32	.05
8	−.09	33	.04
9	−.20	34	.04
10	−.29	35	.03
11	−.37	40	.02
12	−.45	50	.00
13	−.48		
14	−.54		
15	−.78		
16	−.56		
17	−.37		
18	−.21		
19	−.08		
20	.03		
21	.10		
22	.15		
23	.18		
24	.15		

Table 3

Response of Inflation Rate and Real Money Balances to Permanent Percentage-Point Increase in Money Supply Annual Growth Rate

Percentage Change in Inflation Rate

Lag	Estimate	Lag	Estimate
0	−.02	16	1.52
1	−.02	17	1.54
2	.16	18	1.49
3	.32	19	1.46
4	.47	20	1.34
5	.60	21	1.37
6	.72	22	1.23
7	.84	23	1.20
8	.95	24	1.17
9	1.06	25	1.14
10	1.16	26	1.10
11	1.24	27	1.07
12	1.33	28	1.04
13	1.39	29	1.01
14	1.46	30	.99
15	1.49	35	.93
		40	.95
		50	.99
		60	1.00

Percentage Change in Real Balances

Lag	Estimate	Lag	Estimate
0	.24	16	.53
1	.48	17	.39
2	.69	18	.27
3	.85	19	.16
4	.98	20	.07
5	1.07	21	−.02
6	1.14	22	−.08
7	1.18	23	−.13
8	1.19	24	−.17
9	1.17	25	−.20
10	1.13	26	−.23
11	1.07	27	−.25
12	.99	28	−.26
13	.89	29	−.26
14	.77	30	−.26
15	.66	35	−.19
		40	−.12
		50	−.07
		60	−.05

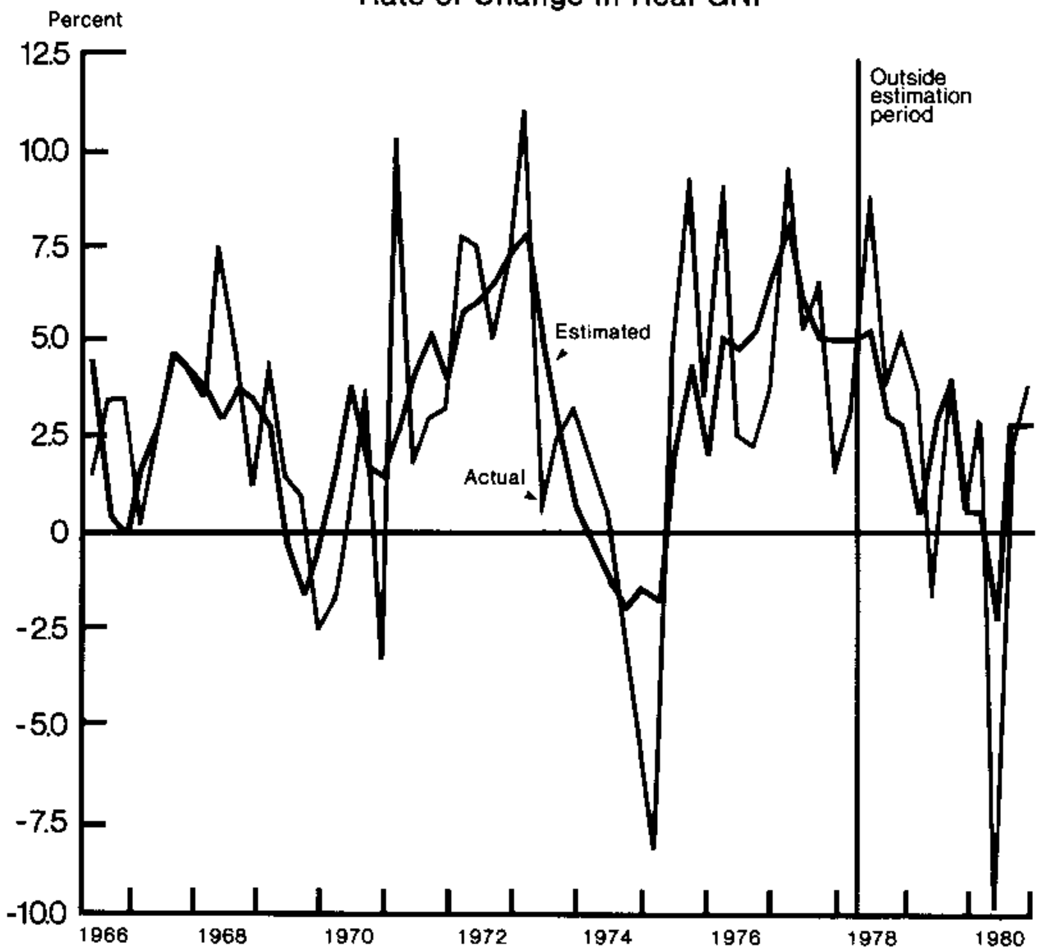
ultimate change, and within two years exhibits a 100-percent change (Table 3). But over the next two years, it overshoots its ultimate change by about 50 percent. For instance, with a one-percentage-point increase in money growth associated with an inflation-rate increase from 7.0 percent to ultimately 8.0 percent, we expect to see inflation rise from 7.0 to 8.0 percent within two years, rise further to 8.5 percent within four years, and then slowly descend to 8.0 percent by the end of 10 years.[9]

Real money balances at first increase steadily in response to the monetary change, reaching a maximum at the end of two years (Chart 4D). Thereafter, they generally decline towards their new long-run value — about .05 percentage-point lower after a one-percentage-point annual rate of increase in money growth.

Within the estimation period, 1966.2-1978.1, our equation follows the general movements in the inflation rate fairly closely (Chart 6). The model makes no adjustment for supply shocks, other than for the episodes of price control and decontrol, which sharply affected short-run inflation estimates. Within the sample period, the standard error is 1.24 percentage points, at an annual rate, or 22 percent of the mean inflation rate of 5.6 percent. Outside that period, from 1978.2-1980.4, the standard error is 1.8 percentage points, at an

Chart 5

Rate of Change in Real GNP



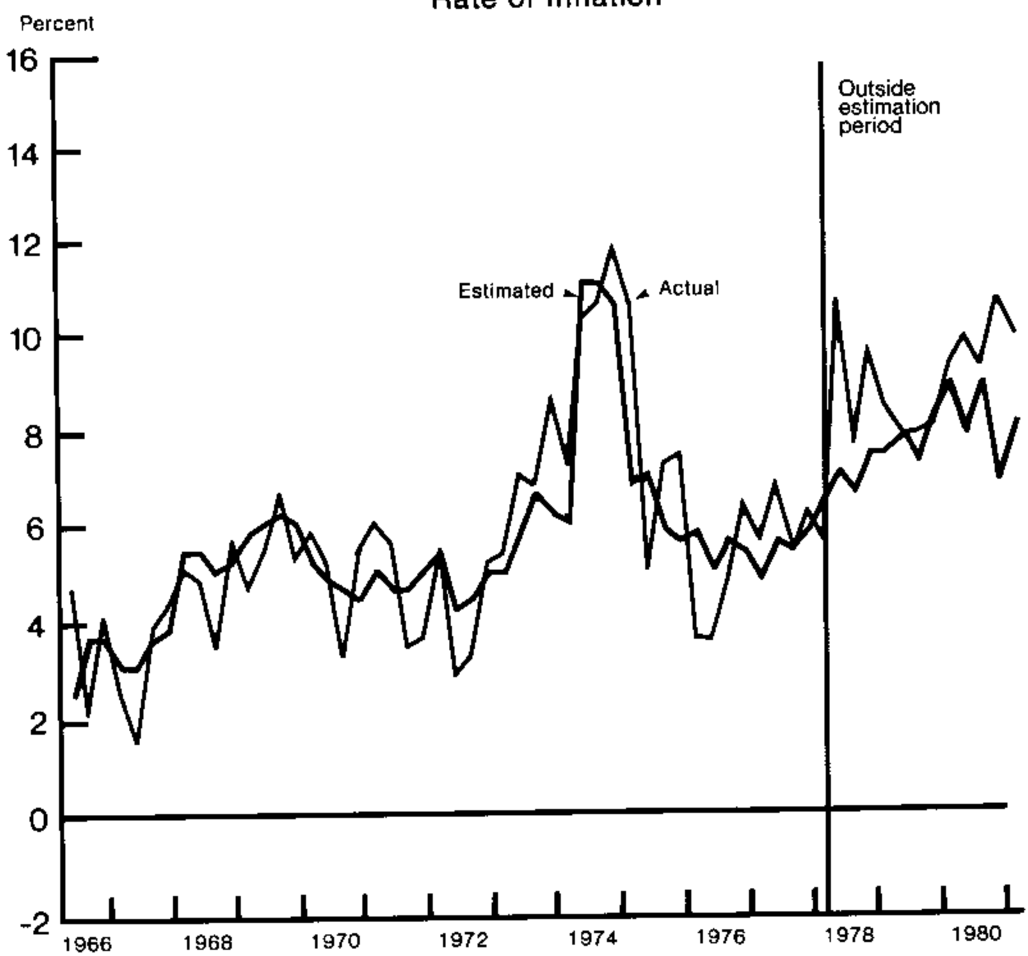
annual rate, or 20 percent of the mean inflation rate of 9.0 percent.

Consistency of the Estimates

Our estimates of real GNP and inflation appear generally consistent. As long as real money balances increase, the level of real GNP also increases. Once the inflation rate overshoots its long-run value and real money balances begin to decline — that is, around the eighth quarter after the initial monetary change — the level of output begins to decline, falling below its potential around the fifteenth quarter. The rate of inflation then declines around the seventeenth quarter. Finally, both real GNP and inflation virtually complete their adjustments around 10 years after the initial monetary shock.

Chart 6

Rate of Inflation

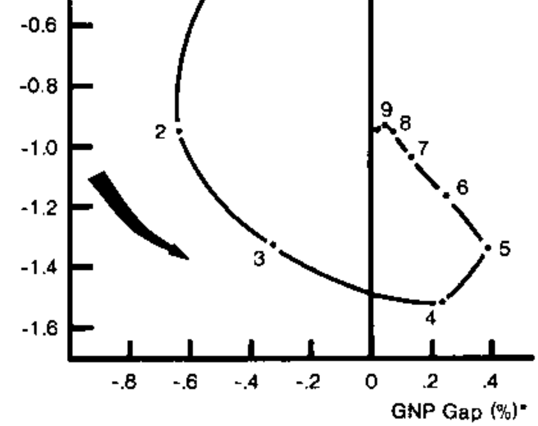


III. Policy Implications

Monetary policy makers often have tried to counteract the business cycle, with either stimulative or deflationary policies. In this paper, we have concentrated upon the effects of a stimulative policy. For a deflationary policy, the response patterns for real GNP and inflation will be the same, but with the signs reversed. Let us consider such a policy — especially one where the decrease in money growth is at first unanticipated.

Within the first year, inflation declines by about half of the decrease in money growth, and by the end of two years, by 100 percent of that change. Some overshooting then occurs, followed by a gradual and slow adjustment, so that inflation returns to its long-run value within 7 to 10 years. Thus, in the long-run, the permanent decrease in inflation matches the permanent decrease in the money growth rate.

Chart 7
Response of Inflation Rate and Real GNP to a Permanent Decrease of 1 Percent in Money-Growth Rate



*(GNP-Potential)/Potential

Our results show a more substantial and rapid change in inflation than we are accustomed to expect from standard inflation models. According to conventional models, the initial inflation adjustment to a change in monetary growth takes about five instead of two years to complete. However, these standard models do not estimate the complete adjustment of prices. As a result, they fail to capture a period of overshooting, which is essential if the model is to capture the long-run effect of money on both the rate of inflation and the level of prices.

Recognizing this shortcoming, several authors have recently devised models which incorporate the long-run neutrality of money (as we have done), but with methods which are far more complicated than the Scadding method.[10] Nonetheless, in each case, the intitial adjustment period for inflation is much shorter than estimated from the standard model — from six to eight quarters for the U.S. (Giddings), and about five quarters on average for eleven developing countries (Khan). These results suggest that the inflation response has been relatively faster than what has been captured in standard-type estimates.

Our results also indicate that the major contraction in real GNP will be completed within two years following an unanticipated decrease in monetary growth. Within another year or two it will have returned to its initial level. Thereafter, the level of output will overshoot and, tracing a cyclical pattern, will return to its permanent level within about 10 years of the initial monetary change.

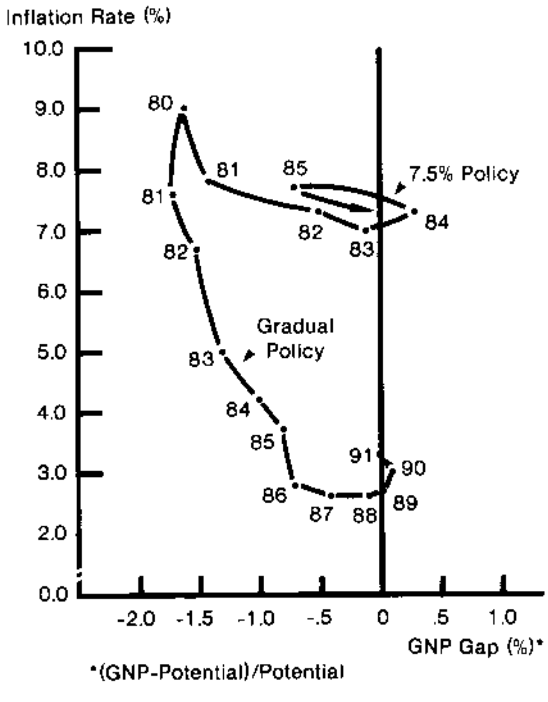
Point A in Chart 7 represents the initial position of the economy, where output equals potential and where the inflation rate is stable. In response to a one-percentage point decrease in monetary growth, which is initially not anticipated, in one year's time inflation decreases by about .5 percentage points and real GNP falls .5 percent below what it otherwise would have been (point 1). By the end of the second year, inflation declines 1.0 percentage points and real GNP declines a maximum of about .7 percentage points below potential. Thereafter inflation overshoots its long-run value, while real GNP begins to approach potential after temporarily overshooting that level. Between the third and fourth years, real output has returned to its initial level. Between the seventh and tenth years, both the inflation rate and real GNP virtually attain their permanent values.

The cost of a deflationary policy in terms of lost output occurs within the first four years of the policy change. After that, real GNP overshoots potential and some gain occurs. On balance, we estimate that the short-run loss of real GNP exceeds the cumulative gain — specifically by a net $16 billion when real GNP equals $1540 billion (potential in 1981.1) at point A, in 1972 prices.

George Perry (Brookings Institution) argues that real GNP would have to decline by $33 billion annually for three years to bring inflation down by one percentage point, again in 1972 dollars. Similarly, our results indicate a $27-billion annual loss during the first 15 quarters after the initial shock, but the loss is partially offset and is reduced to $16 billion (as indicated) by the end of the adjustment period.

Chart 8

Comparison of Inflation and GNP Gap: Steady Growth vs. Gradual Reduction in Monetary Growth



*(GNP-Potential)/Potential

Separately, we can compare the results of two different policy assumptions — holding money growth constant at 7.5 percent a year — the average of the past three years — or reducing money growth gradually from 6.0 to 3.5 percent over the next half-decade — essentially what the Reagan Administration assumes. The results, shown in Chart 8, portray the consequences of the monetary assumptions but of no other outside shocks.

The policy of constant 7.5-percent money growth should raise real GNP from 1.5 percentage points below potential in 1980 until it equals potential GNP by the end of 1983. Thereafter some cyclical response occurs, producing some over-and undershooting, but not enough to lead to any recession. The inflation rate meanwhile should drop to around 7.0 percent by the end of 1982 and thereafter stay close to its long-run value of 7.3 percent.

In contrast, the policy of gradual reduction of money growth should hold real GNP below potential until mid-1988, and after some overshooting, should help the economy reach (and maintain) potential by 1991. With this approach, inflation should decline from around 6.5 percent by the end of 1982 to around 3.5 percent in 1985, and after some overshooting, should reach its permanent level of 3.5 percent in 1991.

The gradual policy could lead to $115 billion less output than the stable money-growth policy, in 1972 prices — or approximately 6 percent of the average level of potential over the 1980-91 period. Nonetheless, we may be able to obtain a gradual reduction in inflation without having to incur a recession in the process. But this gradual reduction in inflation generally would be associated with only moderate rates of real growth — averaging 3.9 percent in 1984 but thereafter remaining close to its potential rate of 3.2 percent.

IV. Summary and Conclusions

In this paper, we consider the output and price effects of a permanent increase in the money-supply growth rate which is at first unanticipated. Initially, inflation steadily increases but by less than the permanent increase in money. During this period, real money balances expand, providing the stimulus for increases in real demand and real GNP.

Yet in the long-run, an increase in money growth apparently does not affect the level or the rate of growth of real GNP, at least to a first approximation. Consequently, the level of real money balances also should remain generally unchanged, since these are held in desired proportion to income. Interest rates could also be a determining factor, however, at least in this country. The desired level of real balances could be somewhat less in the new situation than before the monetary increases occurred, because the rate of interest can be expected to increase with the higher rate of inflation.

Our model of economic behavior is consistent with the behavior of real GNP and inflation just discussed. We have estimated reduced-form equations of the model for both the rate of inflation and the real GNP gap, defined as the level of output as a percent of its potential. For both variables, and also for the rates of change in real GNP and real money balances, our results indicate a cyclical response to a permanent change in money growth. Notably, both inflation and real GNP respond quickly to a change in monetary policy, with the major stimulative or deflationary phase occurring within two years of the initial change. Our findings thus conflict with most of the published literature, which suggests that output and prices require about five years to respond to a change in money growth.

Some analysts suggest that it will take a long time to bring down the inflation rate, and that we risk an economic recession in the process. Our results offer an alternative viewpoint. Changes in monetary growth, at least since the mid-1960's, apparently have acted fairly rapidly upon inflation — and hence upon aggregate demand as well. Thus, since a monetary contraction is likely to bring inflation down faster than previously anticipated, less of the brunt of that contraction need be borne by real GNP, so that a major decline or loss of real income need not result when we adopt a policy which gradually reduces monetary growth.

Appendix I

John Scadding*

Simple Technique for Imposing Restrictions on Sums of PDL coefficients

In estimating polynomial distributed lags, researchers typically are not so much interested in the individual coefficients as they are in certain sums of the coefficients. In many problems, for example, considerable importance attaches to whether the total sum of coefficients is unity or not. This appendix is designed to illustrate a simple method for estimating directly the sum of coefficients, or alternately for imposing on it any point restriction. The method illustrated can also be used to impose or estimate more complicated restrictions, and an illustration is given in the example below.

Suppose we take the familiar PDL relationship between money growth (measured as first differences in the log of money) and inflation (measured by first differences in the log of prices):

$$\Delta \log P_t = \sum_{j=0}^{N} a_j \Delta \log M_{t-j} \qquad , a_j = 0, \text{for } j \geq N. \qquad (1)$$

Interest usually focuses on how inflation adjusts to a permanent change in the rate of monetary growth. The answer to that question is given by the sequence of coefficients

$$w_0 = a_0$$
$$w_1 = a_0 + a_1$$
$$\cdots\cdots$$
$$w_i = a_0 + a_1 + \ldots + a_i$$
$$w_N = \sum_{j=0}^{N} a_j$$

Thus w_0 gives the contemporaneous response of inflation to a one-percentage-point increase in money growth, w_1 measures how much higher inflation will be in the next period (compared to the rate before monetary expansion increased), and so on. The last coefficient, w_N, measures the steady-state response of inflation to an increase in the rate of monetary growth. It is usual to inquire whether this long-run response is unity — i.e., whether ultimately changes in the rate of monetary expansion are fully reflected in the rate of inflation. The usual way to answer this question is to estimate (1) and sum the estimated a's. An alternative is to rearrange (1) in such a way that w_N can be estimated directly. To do that, we intergrate (1) by parts to obtain

$$\Delta \log P_t = \sum_{j=0}^{N-1} w_j \, \Delta^2 \log M_{t-j} + w_N \Delta \log M_{t-N}, \qquad (2)$$

where Δ^2 denotes second differences. Adding and subtracting w_N from each of the terms in the first summation yields

$$\log P_t = \sum_{j=0}^{N-1} (w_j - w_N) \, \Delta^2 \log M_{t-j}$$
$$+ w_N \Delta \log M_{t-N} + w_N \sum_{j=0}^{N-1} \Delta^2 \log M_{t-j} \qquad (3)$$

$$\Delta \log P_t = \sum_{j=0}^{N-1} (w_j - w_N) \, \Delta^2 \log M_{t-j} + w_N \Delta \log M_t$$
$$= \sum_{j=0}^{N-1} \tilde{w}_j \Delta^2 \log M_{t-j} + w_N \Delta \log M_t$$
$$\tilde{w} \equiv w_j - w_N$$

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Thus it is possible to rewrite the distributed lag as another distributed lag in second differences of log M, plus a term in the contemporaneous growth rate of money whose coefficient is the sum of distributed-lag coefficients. Hence w_N can be directly estimated; alternatively any restriction on w_N can be imposed simply by taking the last term in (3) over to the left-hand side of the question.

To illustrate how the method can be used to impose more complicated restrictions, consider the question of whether a change in the rate of monetary expansion permanently affects the level of real money balances. Sufficient conditions that the level of real balances be unchanged in the new steady state (i.e. that the long-run elasticity of the level of prices with respect to money is unity) are:

$$w_N = 1 \tag{4a}$$

$$\sum_{j=0}^{N} \tilde{w}_j = 0 \tag{4b}$$

Define the following coefficients:

$$v_0 = \tilde{w}_0$$

$$v_1 = \tilde{w}_0 + \tilde{w}_1$$

$$\cdots$$

$$v_i = \tilde{w}_0 + \tilde{w}_1 + \ldots + \tilde{w}_i$$

$$v_N = \sum_{j=0}^{N} \tilde{w}_j$$

Next, integrate (3) by parts again to obtain

$$\Delta \log P_t = \sum_{j=0}^{N-2} (v_j - v_N)\, \Delta^3 \log M_{t-j} + w_N \Delta \log M_t$$

$$= \sum_{j=0}^{N-2} \tilde{v}_j \Delta^3 \log M_{t-j} + w_N \Delta \log M_t$$

$$+ v_N \Delta^2 \log M_t \,;\; \tilde{v}_j \equiv v_j - v_N \tag{5}$$

Hence it is possible to rewrite the distributed lag as yet another distributed lag, this time in third differences of log M and two other terms — a term in contemporaneous money growth, and a term in first differences of money growth. The coefficients on these two variables provide estimates respectively of the long-run elasticity of the inflation rate (w_N) and of the price level (v_N) with respect to money. Alternatively, equation (5) allows assumptions about either or both of these elasticities to be easily imposed.

Appendix 2
Model of Real Output and Inflation

The structure of the model is concerned directly with behavior in the goods, money and labor markets. Each market is characterized by simplified and highly aggregative relationships, and labor is the only factor market directly considered. The terms long-run, steady-state and potential are used interchangeably.

List of Variables

- S = Total real savings in the national-income accounts;
- I = Total real investment in the national-income accounts;
- G = Federal government real high-employment surplus, measured as the ratio of revenues to expenditures;
- Y = Real GNP,
- YP = Real potential GNP;
- Y/YP = Real GNP gap;
- R = Nominal rate of interest;
- M = Money supply;
- P = GNP implicit price deflator;
- U = Unemployment rate;
- N = Natural rate of unemployment;
- U/UN = Unemployment gap;
- P* = Expected GNP implicit price deflator.

With the exception of R, the nominal rate of interest, each of the variables is measured in terms of its natural logarithm. Therefore, the change in real GNP, dY, is a measure of the rate of change in real GNP, and dP is a measure of the inflation rate. The variables G, YP, UN and M are exogenous variables.

Goods Market

$$dS_t = F(L)\,dY_t + a_2\,dG_t \qquad (1)$$

$$dI_t = a_3(dR_t - (dP^*_{t+1} - dP^*_t)) \qquad (2)$$

$$dS_t = dI_t \qquad (3)$$

Equation (1) indicates that the change in total savings depends upon a distributed lag in current and past changes in real income, $F(L)\,dY_t$, and the change in the high-employment Federal-government budget surplus, dG_t. The expression $F(L)$ is a polynomial so that $F(L)dy_t$ represents a polynomial distributed lag of the variable dY, and $F(L)dY_t = f_0dY_t + f_1dY_{t-1} + f_2dY_{t-2} + \ldots +$. Equation (2) relates changes in real investment expenditures to changes in the real rate of interest, which in turn is represented by the change in the nominal rate of interest, R, minus the change in the expected rate of inflation. Equation (3) expresses equilibrium in the goods market.

From equations (1) - (3) we derive the IS function which expresses the equilibrium conditions in the goods market.

$$dY_t = -B(L)\,dY_{t-1} - a_4(dR_t - (dP^*_{t+1} - dP^*_t)) - a_5dG_t \qquad \text{IS function (3a)}$$

where the weights of the polynomial, $B(L)$, and the coefficients, a_4 and a_5, are combinations of the coefficients in the structural equations (1) - (3).

Money Market

$$dM^d_t = dP_t + a_6\,dY_t - a_7\,dR_t \qquad (4)$$

$$dM^d_t = dM^s_t \qquad (5)$$

Equation (4) states that the demand for real money balances $(dM^d_t - dP_t)$ is positively related to current income and declines with increases in nominal interest rates. Equation (5) states the equilibrium conditions in the money market, that nominal money demanded equals nominal money supplied in any given period. From equations (4) and (5), we derive the LM function, which expresses the equilibrium conditions in this market.

$$dM_t = dP_t + a_6\,dY_t - a_7\,dR_t \qquad \text{LM function (5a)}$$

Factor Market

$$(U/UN)_t = a_8(dP_t - dP^*_t) + J(L)(U/UN)_{t-1} \quad (6)$$

$$(Y/YP)_t = a_9(U/UN)_t \quad (7)$$

Equation (6) states that deviations in unemployment from its natural level are determined by unexpected changes in the current inflation rate and by past deviations of unemployment from its natural rate, $J(L)(U/UN)_{t-1}$. Since these past deviations of unemployment can in turn be related to past unexpected price changes, equation (6) indicates that the economic impact of unexpected inflation will persist for some time into the future. The reasoning here is that, in labor markets, suppliers of labor and of goods bargain with respect to the expected real price of the product they supply. Suppliers of goods initially interpret unexpected increases in inflation as an increase in the relative price of the product which is being supplied. This increases the derived demand for labor, and places upward pressure on nominal wages. Labor interprets the unexpected demand for its services and the increase in its nominal wage as an increase in real wages. This occurs initially because labor receives price information about labor-supplied items faster than information about labor-purchased items.

Unemployment is linked to levels of output via the production function, which is represented in the model as a type of Okun's Law equation, equation (7).

The specification of inflation expectations completes the model. Our hypothesis, following John Muth's proposal, states that expectations are informed predictions of future events, based on the available information and the relevant economic theory at the time the expectations are formed. This implies that expected inflation can be represented by the conditional mathematical expectation of inflation, dP_t, based on the economic model and all the information assumed to be available as of the end of period $(t-1)$. Inflation expectations may then be represented with the following equations:

$$dP^*_t = E_{t-1}\, dP_t \quad (8)$$

$$dP^*_{t+1} = E_{t-1}\, dP_{t+1} \quad (9)$$

where the E operator signifies expectations conditional on information available as of the end of period $(t-1)$. This hypothesis regarding the formation of expectations has become popularly known as "rational expectations."

Appendix 3
Formation of Inflation Expectations

Before the final equations for the inflation rate and real GNP can be derived from the model, we must express the functional forms for expected inflation, $E_{t-1}dP_t$ and $E_{t-1}dP_{t+1}$, equations (8) and (9) of the model. These will then be substituted into the aggregate demand and supply equations, which in turn can be solved for the final equations for the inflation rate and real GNP.

To obtain the specifications for expected inflation, $E_{t-1}dP_t$ and $E_{t-1}dP_{t+1}$, we follow a procedure suggested by Sargent and Wallace (1975). The first step is to obtain the solution of the system of equations (8)-(12) for the inflation rate, dP_t. The expectations operator, E_{t-1}, is then applied recursively to that equation to yield the equations for inflation expectations.

This procedure results in the following equations. First, the solution for the inflation rate is,

$$dP_t = w_1E_{t-1}dP_t + w_2E_{t-1}dP_{t+1} + w_3(Y/YP)_{t-1} + \sum_{i=2}^{N} q_i(Y/YP)_{t-i} + w_4dM_t - w_5dG_t + c_0$$

where the coefficients are functions of the parameters of the equations (1)-(9):*

Applying the E operator to equation 13, we obtain,

$$E_{t-1}P_t = w_1E_{t-1}dP_t + w_2E_{t-1}dP_{t+1} + w_3(Y/YP)_{t-1} - \sum_{i=2}^{N} q_i(Y/YP)_{t-i} + w_4E_{t-1}dM_t - w_5E_{t-1}dG_t + c_0$$

$$E_{t-1}P_{t+1} = w_1E_{t-1}dP_{t+1} + w_2E_{t-1}dP_{t+2} + w_3(Y/YP)_{t-1} - \sum_{i=1}^{N} q_i(Y/YP)_{t-1-i} + w_4E_{t-1}dM_{t+1} \quad w_5E_{t-1}dG_{t+1} + c_0$$

Applying equations (14) and (15) recursively and then gathering terms yields the solution for expected inflation,

$$E_{t-1}dP_t = v_2\sum_{i=0}^{\infty} v_1^i E_{t-1}dM_{t+i} + v_3\sum_{i=0}^{\infty} v_1^i(Y/YP)_{t-1} - \sum_{i=0}^{\infty} v_1^i \sum_{i=2}^{N} (q_i/(1-w_1))\,(Y/YP)_{t-i} - v_5\sum_{i=0}^{\infty} v_1^i\, dG_{t+i} + \sum_{i=0}^{\infty} v_1^i k_1$$

$$E_{t-1}dP_{t+1} = v_2\sum_{i=0}^{\infty} v_1^i E_{t-1}dM_{t+i+1} + v_3\sum_{i=0}^{\infty} v_1^i(Y/YP)_{t-1} - \sum_{i=0}^{\infty} v_1^i \sum_{i=2}^{N} (q_i/(1-w_1))\,(Y/YP)_{t-i} - v_5\sum_{i=0}^{\infty} v_1^i\, dG_{t+1+i} + \sum_{i=0}^{\infty} v_1^i k_1$$

where the coefficients are combinations of the coefficients in equation (13).**

In equations (16) and (17), expected inflation for time t and time t+1 formed at time t−1 will depend upon the predictions formed at time t−1 for the exogenous variables, dM and dG, for all future periods. Particularly relevant, then, is the process or rule by which the public forms expectations of these variables. There are a number of alternatives for specifying such rules or processes. For instance, we may postulate a model which relates expected future monetary growth and fiscal policy to a set of predetermined variables relative to the model, or we may choose to postulate an ARIMA process for each variable. In the latter case, only past values of a variable are used to predict future values of the same variable. For now, we assume that the public forecasts future values of changes in M and G by considering the past history of these variables.

In addition, we assume that expectations regarding future policy variables are updated each period, as new information becomes available, in accordance with the theorem of optimal least-squares learning and expectations formation stated by Benjamin Friedman. This updating process is fully optimal in the sense of meeting the information-exploitation assumptions of Muth's rational-expectations hypothesis. We may write the expectation of future variables as a polynomial distributed lag

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*$w_1 = Z - (a_{12}/a_{10})$

$w_2 = (a_{12}Z)/a_{10}$

$w_3 = (1 - (Z/a_{10})(j_0 + c_1b_0))$

$w_4 = (a_{11}Z/a_{10})$

$w_5 = (a_{13}Z)/a_{10}$

$Z = (a_7 + a_4a_6) / ((a_7 + a_4a_6) + a_3)$

$q_i = (Z/a_{10})(j_i + c_1(b_0 - b_1)),\ i = 1,2,\ldots$

$c_0 = (r_o Z)/a_{10}$

**$v = w_2/(1 - w_1)$
$v_2 = w_4/(1 - w_1)$
$v_3 = w_3/(1 - w_1)$
$v_5 = w_5/(1 - w_1)$
$k_1 = c_0/(1 - w_1)$

of past values of that variable. Expected future money growth is then specified as follows:

$$E_{t-1}dM_t = F_0(L)\, dM_t$$
$$E_{t-1}dM_{t+1} = F_1(L)\, dM_{t+1}$$

or, in general

$$E_{t-1}dM_{t+j} = F_j(L)\, dM_{t+j}$$

where $F_j(L)$ indicates a polynomial in the lag operator, and $j = 0, 1, 2, \ldots$. Similar specifications can be written for the fiscal variable, dG.

These specifications for expected monetary growth and fiscal policy may next be substituted into the expected inflation equations (16) and (17). Expected inflation thus depends upon linear combinations of past monetary growth and past fiscal-budget changes. We may gather terms and write the expected inflation equations as polynomials in past exogenous variables and the lagged endogenous variables, as is done in the following equations.

$$\begin{aligned} E_{t-1}P_t = {} & H_1(L)\, dM_t - H_2(L)\, dG_t \\ & + v_3 \sum_{i=0}^{\infty} v_1^i\, (Y/YP)_{t-1} \\ & - \sum_{i=0}^{\infty} v_1^i \sum_{i=2}^{N} (q_i/(1-w_1))\, (Y/YP)_{t-2} \\ & + v^i k_1 \end{aligned} \tag{16.1}$$

$$\begin{aligned} E_{t-1}P_{t+1} = {} & H_3(L)\, dM_t - H_4(L)\, dG_t \\ & + v_3 \sum_{i=0}^{\infty} v_1^i\, (Y/YP)_{t-1} \\ & - \sum_{i=0}^{\infty} v_1^i \sum_{i=2}^{N} (q_i/(1-w_1))\, (Y/YP)_{t-i} \\ & + v_1^i k_1 \end{aligned} \tag{17.1}$$

Equations (16.1) and (17.1) represent the final equations for expected inflation. The model now may be represented by the equations shown in Table 1 of the text and quotations 16.1 and 17.1.

FOOTNOTES

1. For similar models, see Thomas J. Sargent and Neil Wallace and Roque B. Fernandez.

2. For a discussion of rational expectations, see Benjamin Friedman.

3. For discussion of alternative models of the inflation-adjustment process, see Robert J. Gordon (1980), George Perry, David H. Resler and Keith M. Carlson.

4. See Milton Friedman (1969), pages 1 - 14.

5. See Sargent and Wallace for an early discussion of this point.

6. This is shown in Appendix 1.

7. From 1960.4 - 1980.1, the money-demand equation estimated in first-difference form by Adrian Throop, (Federal Reserve Bank of San Francisco) is:

$$\begin{aligned} (\ln M1B_t/P_t) = {} & .00133 + \underset{(6.29)}{.585} \ln (M1B_{t-1}/P_t) \\ & \quad (-1.60) \\ & - \underset{(-1.73)}{.00965} \ln RTB_t \\ & - \underset{(-1.03)}{.00654} \ln RCBPASS_t \\ & + \underset{(3.72)}{.264} \ln (Y_t/P_t) \end{aligned}$$

$\bar{R}^2 = .576$ D.W. = 2.00 S.E. = .00490

The variables are defined as:

RTB = rate on three-month Treasury bills, annual effective yield;

RCBPASS = rate on commercial-bank passbook deposits, annual effective yield;

Y = nominal GNP;

P = GNP price deflator

The long-run elasticity of money demand with respect to the treasury-bill rate is −.0232. According to that estimate, and assuming that nominal interest rates in the long-run fully reflect a change in the money growth rate, the expected change in real money balances with respect to a change in the level of interest rates would be .34. Our estimate is .21, which is not significantly different from .34 at the 10-percent level of significance. On the other hand, Throop's estimate appears lower than Goldfeld's estimate of −.048 for the long-run elasticity of money demand with respect to the commercial-paper rate. This implies a .8 elasticity of real money balances with respect to a change in the level of short-term rates, and indicates that our estimate may be on the low side, given Goldfeld's estimate.

8. These estimates were obtained from dynamic simulations in each sample period, in which only the initial lagged-dependent variable is the actual value, and actual money and Federal expenditures appear as the right-hand side of the equation.

9. Our estimates of the inflation multipliers shown in Table 3 and Chart 4C reflect what I shall call the "full effect" of a permanent change of 1.0 percentage point in the money growth rate upon the inflation rate. By "full effect" I mean that two effects are considered in deriving those multipliers: (1) the "direct" effect of

changes in money on the inflation rate, and (2), the indirect effects of money as they work their way through changes in the GNP gap (since lagged gap values also appear in the inflation equation). We derived these multipliers from dynamic simulation of both the GNP and price equations. If we do not take into consideration the indirect effects, we may easily obtain estimates for the inflation multiplier and the standard errors associated with those multipliers, as shown below. The reader will notice that the multipliers are slightly different from those reported in Table 3 in the text. For example, the initial adjustment period is moved up by one-to-two quarters to the seventh quarter following the initial change in monetary growth when the indirect effect is not considered. Also, in the latter instance, the total length of the lag is 22 quarters rather than about 10 years. When the indirect effects are considered, then, they add a considerable period of time in which slow adjustment continues. Importantly, the small size of the standard errors indicates that the estimated multipliers are statistically significant, and that the overshooting property of inflation is highly significant.

Period	Inflation Multiplier	Standard Error
0	-.019	.098
1	.008	.098
2	.224	.078
3	.424	.062
4	.605	.050
5	.768	.043
6	.914	.041
7	1.042	.042
8	1.152	.044
9	1.244	.046
10	1.319	.048
11	1.376	.048
12	1.415	.048
13	1.437	.046
14	1.440	.045
15	1.426	.045
16	1.394	.048
17	1.345	.055
18	1.277	.066
19	1.192	.082
20	1.089	.102
21	1.127	.105

10. See Gittings and Khan.

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