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Gap Management: Managing Interest Rate Risk in Banks and Thrifts

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Bankers use many different asset/liability management models. Each focuses on the types and amounts of assets and liabilities needed to attain a particular goal. Gap models are concerned with the exposure of net interest income—interest income less interest expense—to changes in interest rates. These models are currently popular because recent interest rate variability has increased the uncertainty of net interest income, which currently constitutes 60 percent to 80 percent of total bank earnings.[1] As an example, net interest income in a recent survey of larger commercial banks had a quarter to quarter average variation of 5.5 percent for 1977 and 1978. From 1979 through 1980, the average variation was three times higher.

Existing gap models, however, have serious shortcomings, and several of these are revealed for the first time in this paper. One shortcoming is that these models impede banks and thrifts (henceforth, banks) from hedging the interest rate risk of their earnings by unnecessarily constraining the bank's choice of assets and liabilities that create the hedge. This inflexibility also reduces the bank's ability to accommodate customer demands for bank services. The increased competition created by financial deregulation makes customer loss particularly threatening to banks.[2] Indeed, the banks that survive in the new financial environment will be those that learn to reduce their risks while meeting customer demands for financial services.

Section I presents the fundamentals of traditional gap management models and discusses the assumptions that underlie them. Section II develops a new interest rate risk management model. This model shares a common goal with extant gap models, that of monitoring and managing the rate risk exposure of current bank earnings, while offering several advantages over current gap models. First, it provides a measure of interest rate risk that can be expressed as a single index number. Current gap models provide only "scenario modelling" through elaborate computer simulation. Second, the new model removes unnecessary restrictions imposed by extant gap models on the bank's choices of assets and liabilities to hedge bank earnings. This added flexibility makes interest rate risk management less cumbersome as it more completely accommodates bank customers. Finally, while not depending on their use, the new model can straightforwardly incorporate financial futures.

Sections I and II treat interest-rate-risk management models as useful only for hedging bank earnings against changes in interest rates. In Section III, these models are extended to consider how a bank can structure its balance sheet to adopt a prespecified level of interest rate risk in bank earnings. We also show that the new model is capable of hedging the market value of bank equity. This interest rate hedge can be constructed simultaneously with or independently from a hedged position of bank earnings.

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I. The Gap Management Model[3]

As an introduction to the reader, a basic gap model is described first below. After the description, a "state of the art" gap model is presented along with the assumptions that underlie gap models in general.

The Basic Gap Model

The gap model derives its name from the dollar gap (Gap$) that is the difference between the dollar amounts of rate-sensitive assets and rate-sensitive liabilities.

$$\text{Gap\$} = \text{RSA\$} - \text{RSL\$} \tag{1}$$

To use the model, a bank must supply four pieces of information. First, the bank must select the length of time over which net interest income is to be managed.[4] One year is usually chosen for this "gapping period." Second, the bank must decide whether to preserve the currently expected net interest income (NII) for the gapping period or to attempt to better it. For the former, the gap model is used to hedge NII against changes in interest rates; for the latter, an "active" (speculative) strategy is adopted. Third, if the bank adopts an active strategy, an interest rate forecast for the gapping period is required. Finally, the bank must determine the dollar amounts of the rate-sensitive assets and the rate-sensitive liabilities.

Rate-sensitive assets (RSA) are those that can experience contractual changes in interest rates during the gapping period. All financial assets that mature within the gapping period are rate-sensitive. Variable rate assets "repriced" during the gapping period are also rate-sensitive regardless of their maturity dates. Interest income and the periodic return of principal, as on a mortgage, are also rate-sensitive if these flows are invested in new instruments during the period. Rate-sensitive liabilities (RSL) are similarly defined.[5] CD's maturing during the gapping period, Fed Funds borrowed, Super-NOW and money market accounts are all rate-sensitive. Because Regulation Q ceiling interest rates are currently binding, regular checking and time deposits are not considered to be rate-sensitive.[6]

If the bank wishes to hedge NII against changes in interest rates, then the basic gap model recommends setting Gap$ = 0. It is argued that the Gap$ causes a rate change to influence interest income and interest expense equally.

Those banks wishing to be more aggressive may actively place NII at risk. As one Citibank official noted, "if we don't gap we can't make enough money."[7] An active gap strategy requires the formation of a mismatch between RSA$ and RSL$. The direction of this desired mismatch depends on the interest rate forecast. If rates are expected to rise, NII can be enhanced (should the rate forecast come to pass) by setting Gap$ greater than zero. In this case, more assets than liabilities shift into higher earning accounts during the gapping period. As a result, the NII realized exceeds the NII that would have been earned had either rates not increased or Gap$ been set at zero.[8] These recommendations, and similar ones for when rates decline, are consistent with the following formula:

$$\begin{aligned} E(\Delta \text{NII}) &= \text{RSA\$} \cdot E(\Delta r) - \text{RSL\$} \cdot E(\Delta r) \\ &= \text{Gap\$} \cdot E(\Delta r) \end{aligned} \tag{2}$$

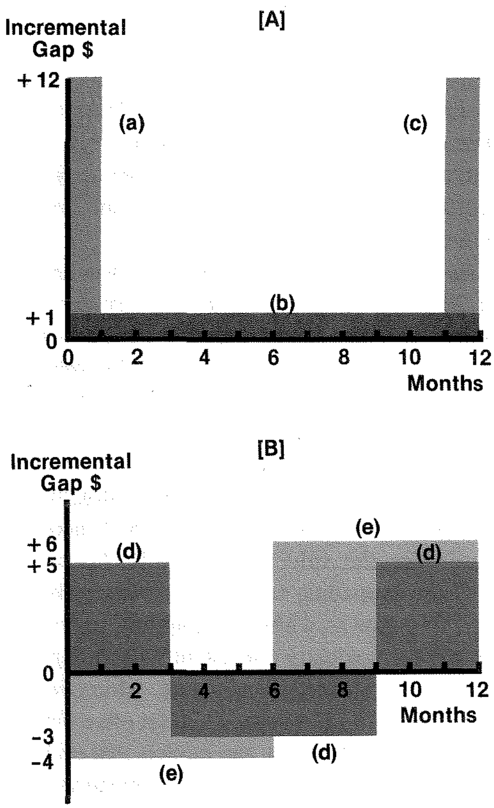
where Δ means "change in," $E(\Delta \text{NII})$ is the expected change in net interest income and $E(\Delta r)$ is the expected change in interest rates. Thus, to get an expected NII greater than the hedged NII, i.e., a positive expected change in NII, one contructs a positive Gap$ when $E(\Delta r)$ is positive and a negative Gap$ when $E(\Delta r)$ is negative.

One issue remains to be addressed: how are assets and liabilities that are automatically repriced a number of times in the gapping period—such as monthly variable rate loans— treated in measuring Gap$? A liability or asset is said to be repriced when the contractual interest rate changes, as when a maturing account is rolled over into a new account within a bank or when rates change contractually, as in a variable rate account. Each such account is included in the values of either RSA$ or RSL$ once, corresponding to its first repricing date provided this date is within the gapping period. This treatment is logically consistent with that given to maturing rate-sensitive accounts. Moreover, it is consistent with an important but not often discussed assumption made in ALL gap models that each interest rate change is treated separately and in sequence.

As an example, suppose the goal is to hedge NII. The Gap$ initially constructed may hedge NII only against the first interest rate change. As time passes in the gapping period and the first repricing date on the asset and/or liability is reached, the funds must be redeployed to make the Gap$ for the *remainder* of the gapping period zero once again. This procedure positions the bank to protect NII expected at the beginning of the year against the next rate change, and after that rate change, against the next, and so on.

The last is an important observation because an understanding of the influence of Gap$ on NII for one interest rate shock implies an understanding of its operation on multiple rate changes. We need, then, explicitly consider only one rate change per gapping period to illustrate *any* gap model. The exposition is simplified further by having the rate change come before the first repricing date in the gapping period.

Figure 1
Alternative Incremental Gap Patterns



"State of the Art" Gap Model

A major problem with the basic gap model is that it computes Gap$ as the difference between RSA$ and RSL$ regardless of when the assets and liabilities are repriced within the gapping period. All that counts in measuring Gap$ is that repricing occurs during the gapping period; it does not matter when during the period the repricing occurs or when it occurs first, as in the case of a variable rate instrument. As an extreme example, suppose all the rate-sensitive assets are repriced on day 1 while all the rate-sensitive liabilities are repriced but only on the last day of the year. Should RSA$ = RSL$ in this instance the basic gap model would falsely indicate, by Gap$ = 0, that NII is protected from rate changes.[9]

The newer literature attempts to solve this intraperiod problem by using a "maturity bucket" approach. The approach calls for the dollar gap to be measured for each of several subintervals (maturity buckets) of the gapping period. Most authors recommend that Gap$ be measured over each 30- to 90-day time increment. These separate dollar gap values are called incremental Gap$s and they sum to the total that is measured by the basic gap model. From now on, this total will be referred to as the cumulative Gap$. Take, for example, a bank with a cumulative Gap$ of $12 for the year. This Gap$ could arise from any number of different incremental (intrayear) gaps. Several of these incremental gap patterns, each of which has a +$12 cumulative Gap$, are depicted in Figures 1A and 1B. The vertical axis in these graphs measures the net asset repricing for the associated month, a month being the assumed maturity bucket.

Suppose that one interest rate shock occurs once before any repricing occurs, pattern (a) in Figure 1A would have a net interest income realized at year-end that differs from the originally expected NII by more than that for pattern (b). Similarly, pattern (b) is more NII risky than pattern (c). The NII risk exposure of gap patterns like those in Figure 1B are more difficult to assess—an issue we will address later. Nevertheless, it should be clear that any cumulative Gap$ can arise from a large number of different incremental gap patterns and, therefore, many different levels of net interest income risk can be associated with one measured cumulative Gap$.[10]

The current gap literature recommends that NII be hedged by setting each incremental gap equal to zero. If rates are expected to rise, positive gaps should be put into place; the opposite holds for expected rate declines. The use of incremental Gap$ rather than just the cumulative Gap$ increases the probability that NII will turn out to be as expected.

There are, however, two common simplifications made in the measurement of incremental Gap$s that can distort the purported accuracy of the incremental gap approach. First, incremental Gap$s are normally measured such that cash flows of interest and periodic principal payments are ignored or attributed to the wrong maturity bucket. For banks that use book values to compute incremental Gap$s, it is common for pre-maturity interest and principal cash flows (i.e., payments or receipts) to be attributed to the maturity bucket that includes the maturity *date* of the instrument.[11] This can falsely attribute pre-maturity cash flows to maturity buckets that occur after these flows are received. For banks that use maturity values rather than book values to compute incremental Gap$s, intervening cash flows are often completely ignored.

The second commonly encountered measurement error in computing incremental Gap$s results from the use of large maturity buckets. Just as with the cumulative Gap$s, each incremental Gap$ will fail to reveal perverse intraperiod repricing. For example, suppose each 90-day incremental Gap$ is zero. If rates change once, as much as one quarter of the annual NII can be exposed to a single rate change.[12] If 30-day gaps are used, as much as one twelfth of the annual NII can be exposed. Continuing along these lines, one can construct examples wherein each bucket has a positive incremental Gap$ but in which if rates increase, contrary to our expectations, NII decreases.

Binder (1982) has described a gap model based on daily incremental gap measurements.[13] His model relies heavily on the use of computer simulations which are essential when the number of incremental gaps to be monitored is large. Even the effects of fairly simple gap patterns, as in Figure 1B, on NII are difficult to predict without computer simulations. We argue later in this paper that such simulations are not needed because there exists a *summary measure* of a bank's incremental gap pattern.

Another major criticism of the basic gap model is that it assumes rate changes for assets and liabilities of all maturities are of the same magnitude when there is overwhelming evidence that rate changes occur in varying magnitudes. The gap literature has handled this issue of different rate change magnitudes by assuming that the volatility of the rates in question is in constant proportion to the volatility of some standard interest rate. If this assumption is reasonably accurate, it should be incorporated into the gap model to improve its performance.

One can use historical interest rate change data on various RSA and RSL to estimate rate change proportionality.[14] These proportional factors measure the rate volatility of RSA and RSL relative to one particular account. This standard account can be anything, but interest rate futures contracts make convenient benchmarks. Suppose a 90-day futures contract is selected as the standard. Furthermore, assume that the proportionality factors for 90-day commercial paper (CP) and 90-day certificates of deposit (CD) are .95 and 1.05, respectively. These numbers indicate that, on average, the CP rate is 95 percent as volatile as the rate on the deliverable contract underlying the futures contract while the CD rate is 105 percent as volatile. If the bank has a $100 obligation in 90-day CD and $500 lent in 90-day CP, then the apparent 90-day Gap$ is +$400. But taking into account the relative volatilities, the "standardized" Gap$ is $370.[15] The bank can hedge its current asset sensitivity (NII increases with rate increases) by buying $370 in futures.[16]

The remaining substantive criticism of the basic gap model, that it pays too little attention to the evolution of NII risk exposure as time passes, has also been corrected in the current gap literature. As asset and liabilities mature, they can be reissued in denominations, maturities and repricing intervals to alter incremental Gap$s that remain in the gapping period. That is, the incremental gap pattern can be dynamically reshaped during the gapping period either towards a more hedged position or a more active one. Suppose the bank has to start the gapping period with a +$1000 Gap$ on day 270. If all other daily Gap$ equal zero and the bank wishes to hedge, it should attempt to reissue maturing RSL to re-mature on day 270 and maturing assets to re-mature after year-end. If the $Gap\$_{270}$ is completely eliminated before rates change, then the NII computed at the beginning of the year will have to be hedged. If rates change before the $Gap\$_{270}$ is completely eliminated, then NII will not have been fully hedged but the risk will have been reduced. Similar treatment can be accorded to expected deposit inflows and the like.

In summary, the "state of the art" gap model computes incremental Gap$s daily or weekly. The outcomes of various interest rate forecasts, given specific incremental gap patterns, are simulated using computers. These incremental Gap$s may have been adjusted or standardized to reflect the relative interest rate volatilities of various RSA and RSL. The dynamic evolution of the gap pattern is considered by allowing banks to interrupt computer simulations during the gapping period to restructure rate risk with maturing RSA and RSL and new accounts. As such, the model is more complete and theoretically pleasing than the basic gap model.

Remaining Criticisms

The "state of the art" gap model, hereafter called the gap model, itself suffers from five deficiencies that are directly or indirectly addressed later in this paper. First, others have implied that the model can hedge NII only by equating each incremental gap to zero. We will show that this hedging condition is unnecessarily strong by revealing the existence of nonzero incremental Gap$ that hedge NII. This is an important point because in Section II we derive a systematic means to discover all hedging incremental gap patterns. This increased number of gap patterns yields more flexibility in simultaneously hedging NII and accommodating customer demands for bank products. Furthermore, what we learn about flexibility in hedging also applies to active NII management.

An example will help illustrate that non-zero incremental gap patterns can hedge NII. Suppose, for simplicity, that all assets and liabilities are currently earning 10 percent. If the RSA repriced on each day of the year (360 days) equals the RSL repriced on the same day, then net interest income (NII) would be zero whether or not rates change. Consider now daily Gap$s that equal zero on every day but three: $Gap\$_{30} = \$1000$, $Gap\$_{90} = -\$2000$, and $Gap\$_{152} = \$1000$. The cumulative Gap$ for the year is zero. The basic gap model would have us believe we are hedged but the more detailed incremental gap model would not. However, in this instance the incremental gap model is misleading. Suppose that on day one, just after this incremental gap pattern has been acquired, rates on all RSA and RSL increase to 12 percent.[17] NII can change only because of the non-zero Gap$s on days 30, 90, and 152. The first such gap causes NII for the gapping period to rise by $18.17.[18] The second and third non-zero Gap$s causes NII to fall by $29.23 and to rise by $11.05, respectively. These three influences cancel. Moreover, this netting out of the three effects is independent of the rate change assumed. An infinite number of other gap patterns also hedge NII. Like the example above, some have non-zero incremental Gap$ values but a cumulative Gap$ of zero. Others, perhaps like those in Figure 1B, have non-zero values for both incremental and cumulative Gap$.[19]

A second problem with the gap model is that when many maturity buckets are used, the model does not generate a single number index of the interest rate risk of the bank. The *basic* gap model provides one in the cumulative Gap$, but we have shown that this measure tells us very little. The gap model would be more appealing if such index numbers existed. These numbers could, in turn, be used to derive risk-return trade-offs or frontiers, a helpful concept elaborated on in Section III.

A third problem arises out of the second problem. Because the gap model does not generate a single number for risk exposure, it cannot easily be used to determine the number of futures contracts that would hedge the overall rate risk of a bank, a calculation of current interest to many bankers. In its current form, the gap model incorporates financial futures in one of two ways, neither of which is particularly appealing. First, it can use financial futures to hedge specific instruments. These individual hedges are then removed from incremental gap computations. Second, one can simulate the effect of a futures contract on NII in the same way *or* at the same time the incremental gap pattern's influence is simulated. By trial and error, the appropriate aggregate hedge can be discovered.

A fourth problem is that stockholders could quarrel with exclusive concern over NII by bank management. Stockholders are interested in share values, an important determinant of which is the market value of bank assets and liabilities. They wish to position their capital based upon their attitudes towards risk and expected return, where expected return is the expected earnings for a given period *plus* the expected change in market value of equity over this period. The gap literature pays attention to expected earnings but not to the influence of interest rates on the market value of assets and liabilities.

A fifth problem lies not with the model *per se*; it arises because proponents of the model almost completely ignore the theory of the term structure of interest rates. This theory suggests that currently available information on how rates are expected to change have already been incorporated in the "yield curve" or term structure. If these rate changes, which represent the market's forecasts, come to pass, any active NII strategy will not improve NII relative to that available by NII hedging.[20] To be successful in actively managing NII, one must have a better interest rate forecast than the market's.[21]

Consider the following example: The market expects interest rates, expressed on an annualized basis, to be 10 percent for the first quarter, 11 percent for the second, 12 percent for the third, and 13 percent for the fourth. This interest rate pattern gives a one-year rate of 11.49 percent[22] and indicates that the market expects rates to rise. A bank might incorrectly infer that it will profit from a gap constructed, say, by booking a one-year loan of $1000 at 11.49 percent and a $1000 90-day CD that will roll over with interest every quarter. Should rates rise as the market expects, NII earned for the year will be zero. Given our assumption of the same rate structure for assets and liabilities, this is the NII obtained by hedging. (The initial negative carry switches mid-year to a positive carry and it does so in such a way that there is no time-value benefit to the initial negative carry.[23]) Only if rates are forecasted by the bank to fall by more than the market forecast would this incremental gap pattern yield a NII in excess of that originally promised. If rates are forecasted by the bank to rise or to fall by less than the above market forecast, then it would be appropriate to construct a negative Gap$ on net for the year.

The final problem with the gap model is its inattention to unexpected deposit withdrawals or loan prepayments. Predictable withdrawals and prepayments, e.g., deposit reductions as bank customers meet seasonal requirements, can easily be addressed in the gap model. These "maturities" are best matched with similarly maturing assets or liabilities. A more difficult problem arises when the amount and timing of withdrawals and prepayments depends upon the spread between their contractual rates and current market rates. Unexpected changes of this sort can substantially affect realized net interest income, yet the gap literature is silent on the appropriate treatment.

II. A Generalized "Duration" Gap Model

We have developed a generalized "duration" gap model to show that the "state of the art" gap model described in Section I is a special and constraining model for measuring and monitoring the interest-rate risk exposure of NII. The technical aspects of the duration gap model are contained in Appendix I but we will briefly review its main features and derivation here.

We start the derivation of the generalized model with a definition of NII when interest rates do not change unexpectedly within a year; this net earnings figure is referred to as NII_0. We then restate net interest income in general terms for cases when rates change unexpectedly. The last step involves finding the overall combination of assets and liabilities that would balance changes in interest income with those in interest expense. The result is that NII_0 will be realized, even if rates change unexpectedly, provided that the weighted sum of the market value of all rate-sensitive assets equals the weighted sum of the market value of the rate-sensitive liabilities where the weights are equal to the fraction of the year from repricing to the end of the year:

$$\sum_{j=1}^{N} MVA_j(1-t_j) = \sum_{k=1}^{M} MVL_k(1-t_k) \quad (3)$$

where MVA_j (MVL_k) is the *market* value at the beginning of the year of a single asset (liability) payment that will be repriced during the year; t_j (t_k) is the *fraction* of the year until this asset (liability) payment is repriced or is first repriced if repricing occurs more than once during the year; and N (M) is the number of separate rate-sensitive asset (liability) payments.[24]

It is important to note that NII_0 and the NII associated with rate changes does not presume that assets and liabilities are valued by the bank at market value. Rather, as shown in Appendix I, all accounts are carried at book value and interest income and expense are computed using these values. The generalized hedging condition stated in equation (3) is in market value terms because of mathematical conditions needed for a NII hedge rather than an assumed accounting convention.

The generalized model in its simplest form relies on the following assumptions: (1) that the gapping period is one year, (2) that the term structure of interest rates is flat (constant) for each type of asset and liability, (3) that the unbiased expectations hypothesis of the term structure governs the expected evolution of interest rates (given the flat term structure assumption, this means that all rate changes are unexpected), (4) that all asset and liability interest rates are affected equally when any unexpected change in rates occurs, and (5) that no deposit withdrawals or loan prepayments take place. Each of these assumptions can be relaxed, and in Appendix II, we do so for several of them.

Many incremental gap patterns are consistent with equation (3). If each repriced asset is matched in timing and amount with a liability, equation (3) will be upheld. But, item by item matching is not necessary. For example, let the market values of all RSA and all RSL be represented as MVRSA and MVRSL, respectively. Equation (3) can be re-expressed as

$$MVRSA\,(1-D_{RSA}) = MVRSL\,(1-D_{RSL}) \qquad (4)$$

$$\text{where } D_{RSA} = \sum_{j=1}^{N} (MVA_j/MVRSA)\, t_j \text{ and}$$
$$D_{RSL} = \sum_{k=1}^{M} (MVL_k/MVRSL)\, t_k \qquad (5)$$

D_{RSA} and D_{RSL} are, respectively, the Macaulay's durations of the RSA and RSL. Duration is a cash flow timing statistic of financial instruments that has been used in bond portfolio management for many years.[25] Duration in our context is the weighted average time to repricing, where individual weights in this average are $MVA_j/MVRSA$ for RSA and $MVL_k/MVRSL$ for RSL.

Equation (4) reveals an interesting alternative to the NII hedging condition as typically expressed in the gap literature. Sufficient, but not necessary, hedging conditions are tnat MVRSA=MVRSL and $D_{RSA}=D_{RSL}$. The first of these is somewhat like equating RSA$ and RSL$, i.e., somewhat like setting the cumulative Gap$ of the basic gap model equal to zero. The second equates the "average" repricing date of the RSA with the "average" repricing date of the RSL.

Reconsider the example given in Section I ("Remaining Criticisms") of a gap pattern that hedges NII even though all incremental gaps are not equal. The details are given in Example 1 of Table 1, where equation (4) is shown to be met by setting MVRSA= MVRSL and $D_{RSA}=D_{RSL}$. Example 2 in Table 1 provides a case where neither market values nor durations are equal yet equation (4) is satisfied. Example 3 will be discussed momentarily.

The full value of equation (4) becomes clear when the accounts currently on the books of the bank violate this hedging condition. To illustrate, define the duration gap (DG) as the size of the departure from the equality given in equation (4), or

$$DG = MVRSA\,(1-D_{RSA}) - MVRSL\,(1-D_{RSL}) \qquad (6)$$

The sign of DG indicates the type of rate risk to which the bank is currently exposed. The larger is DG in absolute value, the greater is the risk. Suppose $DG>0$. This inequality indicates that a fall (rise) in rates will cause realized NII to be less than (greater than) NII_0. This is analogous to a "net positive gap" in the conventional literature.[26] The converse is true when $DG<0$, in which case, we have in some sense a "net negative gap."

The duration gap thus defined yields a single-valued risk index that is not only a convenient statistic but also an indicator of risk as accurate as the risk level derived from computer simulations of the incremental gap pattern embedded in the current value for DG. This is an important point. The use of duration in the gap model results in a gap-type measure of risk that can be as intuitively understandable as the outmoded cumulative Gap$ measure and as accurate as using the measured incremental gap pattern in a computer simulation to

Table 1

Example 1

Assumptions: (a) Each daily Gap$ equals zero except, $\text{Gap\$}_{30} = +\$1000$, $\text{Gap\$}_{90} = -\$2000$ and $\text{Gap\$}_{152} = +\$1000$.

(b) The negative gap is treated as the only RSL and the two positive gaps as the two rate-sensitive assets. This assumption is made for convenience and the results are not specific to it.

(c) The interest rate for any account is 10 percent.

Outcomes: (a) The total current market value of $1000 received on day 30 and received on day 152 is $1953.

(b) The current market value of $2000 received on day 90 is $1953.

(c) Thus, MVRSA = MVRSL

(d) The duration of the RSL is .25 years. (The duration of a single payment is always the time to the payment date.)

(e) The duration of the RSA is .25 years using equation (5).

(f) The hedging condition given in equation (4) is met since the market values of the assets and liabilities are equal as are their weighted average repricing dates (durations).

Example 2

Assumptions: (a) Each daily Gap$ equals zero except, $\text{Gap\$}_{90} = +\$1000$ and $\text{Gap\$}_{180} = -\$1536$.

(b) The positive gap represents an RSA and the negative gap represents an RSL.

(c) The interest rate on any account is 10 percent.

Outcomes: (a) The market value of the RSA is $976.

(b) The market value of the RSL is $1465.

(c) The duration ofthe RSA is .25 years.

(d) The duration of the RSL is .5 years.

(e) The hedging condition given in equation (4) is met even though neither market values nor durations of RSA and RSL are equal.

Example 3

This example is the same as Example 1 except that the $\text{Gap\$}_{90}$ has been changed from −$2000 to −$1500. This makes the bank net asset sensitive in a "duration" sense.

Outcomes: (a) With all other assumptions of Example 1 in force, the reduction of $\text{Gap\$}_{90}$ from −$2000 to −$1500 upsets the NII hedge found in Example 1.

(b) The duration gap (DG) is positive and equals $366.

(c) Equation (7) indicates that $488 in market value of rate-sensitive liabilities of duration .25 will complete the hedge. This additional RSL yields, within a one dollar rounding error, a total market value of RSL of $1953—the same as in Example 1.

(d) Equation (7) also indicates that, among others, NII can be a hedge if (i) $724 in market value of a RSL with duration of .49 year was added; (ii) $367 in market value of a RSL with duration of .003 year (1 day) was added; (iii) $399 in market value of .083 years (30 days) was subtracted from the total RSA; or (iv) $633 in market value of RSA of duration .42 years (152 days) was subtracted from the total RSA.

determine the effect of an interest rate change on NII.

Furthermore, the duration gap can be used to select the appropriate adjustment in RSA and/or RSL to remove NII risk:

If DG<0, then to achieve a NII hedge add $X in market value of *net* rate-sensitive assets with a duration Y where

$$X = \text{absolute value of } DG/(1-Y) \tag{7}$$

If DG>0, then to achieve a NII hedge, add $X in market value of *net* rate-sensitive liabilities with a duration of Y where X and Y are defined above.

Example 3 in Table 1 can be used to illustrate the point. In it, we have taken some of the RSL of Example 1 away from the bank. This makes the bank net asset sensitive, i.e., MVRSA>MVRSL, and the bank will do well if rates rise unexpectedly but it will do poorly if rates fall. Equation (7) can be used to rediscover the amount of RSL we had in Example 1. Alternatively, this equation can be used to restructure RSA or RSL in other ways. Several options are given at the end of Example 3.[27] The asset/liability manager can use the type of flexibility illustrated in Example 3 to achieve the NII hedge with minimal disruption of existing bank accounts and/or maximum accommodation of customer demands.

Fed funds and interest rate futures contracts are particularly useful hedging instruments to use in equation (7). Both can be used to alter, in either direction, NII sensitivity to rate changes of accounts already on the books. Because daily and term Fed funds contracts are paid in lump sum at maturity, the duration of any Fed funds contract is its maturity date. The "duration" of an interest rate futures contract is more complicated. No scheduled cash payments arise from this contract so evaluation of a duration formula like that in equation (5) is impossible. Nevertheless, one can obtain a duration for a futures contract; it has been shown to be the duration of the underlying deliverable contract.[28] Note that the use of futures in the context of equation (7) provides a NII hedge for the entire bank, not just an interest rate risk hedge for a single cash instrument.

As time passes during the gapping period, there will be changes in market values and durations of RSA and RSL. There will also arise a need to restructure the balance sheet as maturing accounts are re-booked. Thus, one must periodically rebalance the assets and liabilities to re-establish the equality in equation (4). At the end of the first month, for example, the hedging condition for the remainder of the year is

$$MVRSA'\,(11/12 - D_{RSA}) = MVRSL'\,(11/12 - D_{RSL}) \tag{8}$$

where the primes indicate market values and durations, measured after the month has passed, for the accounts that are rate-sensitive during the months remaining in the year. If the NII hedge is in place throughout the year, any number of unexpected interest rate changes can occur and yet NII will equal NII_0 at year-end. These re-balancing costs are apt to be low.[29] Re-balancing will be taking place anyway as unexpected inflows and outflows occur during the year. Also, the disposition of maturing contracts alone may provide sufficient flexibility in altering asset and liability durations to re-establish the NII hedge.

Several assumptions made for simplicity's sake at the beginning of this section are actually too strong. The more important generalizations are:

1. Term structures of interest rates are not always or even normally flat, and unexpected changes in rates may be term specific.

2. Rate changes for certain instruments are more volatile than others with like maturities.

3. Deposit withdrawals and loan prepayments are functions of the spread between the rates paid on these accounts and current market rates.

Appendix II addresses each of these generalizations in a preliminary form to show that the model can be reworked to incorporate more realistic assumptions.

III. Use of Gap Models for Purposes Other than NII Hedging

Active Strategies for Net Interest Income

Earlier sections of this paper assumed that a bank wished to hedge its NII completely. This may not be true. Some banks may believe that they can do better than the market in forecasting interest rates, and decide to adopt interest rate risk. For them, the duration gap model described in Section II, can be quite useful.

Almost all that the basic gap model offers for active strategies is contained in equation (2). The equation implies that if the bank expects interest rates to increase (that is, to increase more than the market forecast), it should set the cumulative Gap$ greater than zero in order to increase expected NII; it should set the cumulative Gap$ less than zero for the opposite expectation. As was shown in Example 2 in Table 1, non-zero cumulative Gap$s can be constructed such that NII is hedged. This line of reasoning can be extended to note that a non-zero cumulative Gap$ can have no rate risk. Thus, the basic gap model is as deficient in active NII management as it was in passive NII management.

The extended traditional gap models rely on computer simulations to determine NII under various scenarios of rate changes. Conditional upon different plausible rate forecasts, users of traditional models seek incremental gap patterns that generate computer stimulations with the highest extra NII return. Simulations might also be conducted to determine the downside risk should rates move against the bank.

The duration gap model developed in Section II can easily be used to systematize active NII strategies through the construction of a risk-return trade-off or "frontier." Such a frontier would represent a menu of choices of expected gains of NII above NII_0 accompanied by associated risks. With this frontier, bankers can select a duration gap strategy that is consistent with their risk-return preferences.

Implementation of the chosen duration gap for an active NII strategy would be accompanied by the same asset/liability choice freedom discussed earlier when NII was hedged by setting the duration gap (DG) equal to zero. That is, what ever is revealed to be the optimal DG, there are thousands of incremental gap patterns consistent with this number. The one chosen from the available set can, then, depend solely on the current set of assets and liabilities and on maximizing the accommodation of new customer demands.

A NII risk-return frontier can be derived as follows: An accurate approximation to a one-year holding period return on an asset or liability repriced during the year is[30]

$$i_r = i_0 D + i^*(1-D) \qquad (9)$$

where i_r is the realized one-year return on a RSA or RSL that has a Macaulay duration of D. (D has to be less than one year by the definition of rate sensitivity.) The one-year rate of interest at the beginning of the year is i_0 and the forecasted one-year rate at the repricing date is i^*. An intuitive interpretation of this formula is that i_0 is attained for D portion of the year and i^* is experienced for the remaining (1-D) portion of the year. This equation can be rewritten as $i_r = i_0 + (1-D)\Delta i$, where Δi is the bank forecast of how the rate will change in comparison to the market forecast. (To simplify the presentation, we have assumed that the market forecasts no change.)

Through very simple substitutions in the equations given in Appendix I, one can obtain the result that

$$NII_r - NII_0 = DG\,\Delta i \qquad (10)$$

where NII_r is the NII realized over the one-year gapping period and NII_0 is the NII realized if rates change only as expected by the market. Equation (10) can be used to compute the gains in NII for various DG values. If $\Delta i > 0$, then the bank should set $DG > 0$ and NII_r will increase with the magnitude of DG. (If $\Delta i < 0$, then $DG < 0$.)

It is unlikely that the value or even the sign of Δi will be known with certainty, but we assume the bank can identify all possible outcomes and their probabilities of occurrences.[31] The expected (probability weighted) NII in excess of NII_0 becomes

$$E(NII_r - NII_0) = E(\Delta NII) = DG\,\overline{\Delta i} \qquad (11)$$

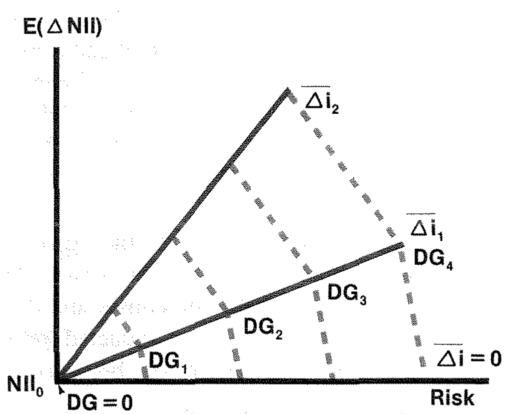
where $\overline{\Delta i}$ is the probability weighted interest rate change.[32] If either $\overline{\Delta i} = 0$ or $DG = 0$, then the expected NII is NII_0, otherwise, $E(\Delta NII) > 0$ whenever DG and $\overline{\Delta i}$ have the same sign.

Equation (11) tells us that if rates are expected to rise (fall) on average, then it is better to set RSA shorter (longer) than RSL. This can be done by having more dollars of RSA than RSL or by setting the duration of the RSA less than that of RSL or both. Notice the close similarity to the intuitively appealing, but incorrect, equation (2).

One can solve for the standard deviation of ΔNII and substitute it into equation (11). The standard deviation is related to the variance of ΔNII outcomes and as such can be viewed as a measure of the riskiness of the NII strategy in force. The result of this substitution is an expression of E(ΔNII)—the risk frontier. A set of such frontiers is given in Figure 2 for $\overline{\Delta i} > 0$. Note that there is a frontier for each probability distribution of Δi's. The more certain a rate increase, the steeper is the frontier. That is, increases in forecast certainty generate larger E(ΔNII) for every risk level. The position on the relevant frontier depends upon DG. The more DG departs from zero, the greater the risk and expected return. Note that for any DG, E(ΔNII) increases and risk decreases as $\overline{\Delta i}$ certainty increases. The bank manager's and/or shareholders' preferences for NII risk and return determine the desired position on the frontier, i.e., the duration gap. This is a subjective choice; the frontiers are, themselves, objectively determined from the interest rate forecasts of the bank.

Figure 2

Risk-Return Frontier



The figure sketches several risk-return frontiers; each one of which is represented as a solid line.

Higher subscripted DG entries have larger positive values than lower subscripted duration gaps. DG > 0 since we have assumed that $\overline{\Delta i}_2 > \overline{\Delta i}_1 > 0$.

Dashed lines indicate the change in expected return and risk, holding DG constant, as interest rate forecasts change —moving us from one frontier to another.

Protecting Market Value

Bank shareholders are ultimately interested in the market value of their shares. The market value of bank capital (MVBC), which is obtained by subtracting the market value of liabilities from the market value of assets, is a prime determinant of share value.[33] In general, shareholders have recently become concerned over their investments because past interest rate movements have caused MVBC to depart substantially from book values in many institutions. Recent empirical work by Flannery and James (1982) reveals that bank stock prices are also sensitive to the degree of current interest rate risk exposure. Under pressure from shareholders on one side, bank management has also recently seen several regulatory proposals concerning the interest rate risk of MVBC. These proposals include mark to market accounting, risk related FDIC/FSLIC premiums, and call report disclosure of gap positions.[34]

The duration analysis presented here can be extended to hedge MVBC. The condition needed for MVBC to be hedged (in the literature, "immunized") is that[35]

$$\text{MVA}\, D_A = \text{MVL}\, D_L \qquad (12)$$

where MVA (MVL) is the market value of *all* bank financial assets (liabilities) and D_A (D_L) is the duration of these assets (liabilities) and MVBC = MVA − MVL. If MVBC is greater than zero, as is normally the case, then equation (12) can hold only if D_L exceeds D_A.

A bank may wish to puts its MVBC at risk. If the bank expects rates to rise more than the market concensus, then it will increase MVBC if MVA D_A > MVL D_L. One can easily use the developments in the first part of Section III to derive MVBC-risk frontiers analogous to those for NII-risk illustrated in Figure 2.

One can tie the immunization or active strategies for MVBC together with the earlier work in this paper on NII. Equation (12) can be rewritten as[36]

$$\text{MVA}_{NS}(D_{NSA}-1) + \text{MVA} - \text{MVRSA}(1-D_{RSA}) = \text{MVL}_{NS}(D_{NSL}-1) + \text{MVL} - \text{MVRSL}(1-D_{RSL}) \qquad (13)$$

or

$$\text{MVA}_{NS}(D_{NSA}-1) + \text{MVBC} = \text{MVL}_{NS}(D_{NSL}-1) + \text{DG} \qquad (14)$$

Here, NS stands for non-rate sensitive. If one wishes to hedge NII and immunize MVBC, one sets DG=0 and $MVA_{NS}(D_{NSA}-1)$ + MVA equal to $MVL_{NS}(D_{NSL}-1)$ + MVL. Alternatively, if one wishes to immunize MVBC but to take a fling on NII, one selects a nonzero DG but then fulfills the equality in equation (14). Finally, one can hedge NII but put MVBC at risk by setting DG=0 but not fulfilling the equality in equation (14).

IV. Summary

This paper began by reviewing how banks and thrifts manage the interest rate risk exposure of current bank earnings. Many do so using the "gap" asset/liability management model. Several shortcomings in this model were revealed in Section I. Chief among these were the inability of the model to generate a simple and reliable index of the interest-rate risk exposure of bank and thrift net interest income and the unnecessarily contraining set of assets and liabilities allowed by the model. Section II used more general conditions for hedging bank and thrift net interest income to determine a "duration gap" model that generates a single number to quantify the risk position of the financial institutions. The model also reveals a larger set of asset and liability choices to financial institutions to enable them to establish net interest income hedges not currently in place.

Section III expanded the duration gap model to consider strategies that actively place earnings at risk. Risk-return frontiers were developed to quantify the choices for "better earnings" based on rate forecasts. Finally, it was shown that the duration gap model can be generalized to hedge the market value of bank capital, should the bank wish to do so, against unexpected changes in interest rates. We showed that net interest income and the market value of bank capital can be either simultaneously or independently hedged.

Appendix I

Assume the bank has a one year gapping period. Furthermore, assume the unbiased expectations hypothesis of the term structure holds and that yield curves are flat. Any change in interest rates is, therefore, unexpected. Let NII_o represent net interest income for the coming year should rates not change unexpectedly. This level of net interest income is the goal of the hedging strategy. Mathematically, NII_o can be expressed as:

$$\text{(a) } NII_o = \sum_{j=1}^{N} A_j[(1+r_j)^{t_j}(1+i_j)^{1-t_j} - 1] - \sum_{k=1}^{M} L_k[(1+r_k)^{t_k}(1+i_k)^{1-t_k}-1]$$

A_j is the asset *book* value at the beginning of the year of a cash inflow that will occur at time t_j, where t_j is expressed as a *fraction* of a year. This asset has an associated contractual interest rate of r_j, expressed as an annualized rate. Upon repricing, this cash inflow is expected to earn a new rate of i_j. There are N repriced asset flows. (A long-term mortgage with monthly payments of $500, a contractual interest rate of 5 percent and a new interest rate of 10 percent—the rate on new mortgages—would have twelve flows represented in the above summation. The X^{th} month flow has an A_j of $500/1.05^{X/12}$, $t_j=X/12$, $r_j=.05$ and $i_j=.10$.) If an asset does not generate any flows during the one year gapping period, then it influences net interest income only in an accrual sense and $t_j=1$. Similar definitions apply to L_k, one of M liability repricing flows.

Consider the impact on NII of an unexpected change in all current rates by an additive amount λ, where λ can be positive or negative. For mathematical convenience, assume the interest rates change before any cash inflows or outflows occur. NII now becomes functionally dependent on λ.

$$\text{(b) } NII(\lambda) = \sum_{j=1}^{N} A_j[(1+r_j)^{t_j}(1+i_j+\lambda)^{1-t_j} - 1] - \sum_{k=1}^{M} L_k[(1+r_k)^{t_k}(1+i_k+\lambda)^{1-t_k} - 1]$$

If $NII(\lambda)$ is to be hedged, then a set of asset and liability flows must be found that leaves $NII(\lambda)$

equal to NII_o. For this to occur, it must be true that there be no change in NII(λ) as λ departs from zero by some small amount. Mathematically, this means that the derivative of NII(λ) with respect to λ equals zero in the neighborhood of λ=0. Now,

$$\text{(c) } \partial NII(\lambda)/\partial\lambda = \sum_{j=1}^{N} A_j (1+r_j)^{t_j}(1+i_j+\lambda)^{-t_j}(1-t_j)$$

$$- \sum_{k=1}^{M} L_k(1+r_k)^{t_k}(1+i_k+\lambda)^{-t_k}(1+t_k)$$

If we evaluate this derivative at λ=0 and set the result equal to zero, we obtain

$$\text{(d) } \sum_{j=1}^{N} A_j(1+r_j)^{t_j}(1+i_j)^{-t_j}(1-t_j) =$$

$$\sum_{k=1}^{M} L_k(1+r_k)^{t_k}(1+i_k)^{-t_k}(1-t_k)$$

But $A_j(1+r_j)^{t_j}/(1+i_j)^{t_j}$ is the current market value (MV) of a contractual flow of $A_j(1+r_j)^{t_j}$ dollars t_j periods from now. Thus, the first order condition for a NII hedge is that

$$\text{(e) } \sum_{j=1}^{N} MVA_j(1-t_j) = \sum_{k=1}^{N} MVL_k(1-t_k)$$

This is equation (3) in the text from which equation (4) was derived. The second order conditions in this development prove to be more complicated than informative and are not treated here.

A remaining question is whether or not the hedging condition expressed in equation (e) holds for non-infinitesimal changes in λ. Fulfillment of the hedging conditions does not exactly equate NII(λ) with NII_o for very large changes in λ, say of ± 300 basis points. These errors are, however, small.

Appendix II

Three strong assumptions made in Section II are that (1) the term structure is flat and unexpected changes in rates keep it so, (2) the rate change on an instrument is as volatile as on any other instrument of similar maturity, and (3) deposit withdrawals and loan prepayments are not interest dependent. One at a time each of these assumptions is relaxed in this Appendix to determine how the hedging condition previously expressed will change.

A. Nonflat term structures that shift in a nonparallel fashion:

If term structures are not flat, each instrument has an annualized interest rate of $h(0,t_n)$, where $h(0,t_n)$ is an element in a term structure for the n^{th} type of instrument with repricing at date t_n. The NII_o for this model is the same as equation (a) in Appendix I, except that $h(0,t_j)$ replaces i_j and $h(0,t_k)$ replaces i_k. The stochastic process affecting interest rates must be specified. Suppose, as did Fisher and Weil (1971), that $1+h^*(0,t) = (1+h(0,t))(1+\lambda)$, where $h^*(0,t)$ is the new term structure after an unexpected interest rate shock. NII(λ) is now

$$\text{(f) } NII(\lambda)=\sum_{j=1}^{N} A_j[(1+r_j)^{t_j}(1+h(0,t_j))^{1-t_j}(1+\lambda)^{1-t_j}-1]$$

$$-\sum_{k=1}^{M} L_k[(1+r_k)^{t_k}(1+h(0,t_k))^{1-t_k}(1+\lambda)^{1-t_k}-1]$$

Differentiate NII(λ) with respect to λ, evaluate at λ=0, and set the result equal to zero. This gives

$$\text{(f) } \Sigma A_j(1+r_j)(1+h(0,t_j))^{1-t_j}(1-t_j) =$$

$$\Sigma L_k(1+r_k)(1+h(0,t_k))^{1-t_k}(1-t_k)$$

or

$$\Sigma MVA_j(1+h(0,t_j))(1-t_j) =$$

$$\Sigma MVL_k(1+h(0,t_k))(1-t_k)$$

A reexpression of this hedging condition using a duration measure evolves in a less direct manner than before. Nevertheless, one can rewrite this last equation as

$$\text{(g) } MVRSA\ T_{RSA} = MVRSL\ T_{RSL}$$

where $T_{RSA}=\Sigma MVA_j(1+h(0,t_j))(1-t_j)/MVRSA$, etc. Equality of T_{RSA} with T_{RSL} is equivalent to setting a weighted average repricing date of RSA with that of RSL. The weights differ from those used in equations (4) and (5). This reflects the more general assumptions on the term structure and the stochastic process affecting it.

B. Relative Interest Rate Changes:

Return to the assumption of flat term structures that shift in a parallel fashion, but relax the assumption that the sizes of the unexpected rate changes are equal for instruments with the same maturity.

Assume instead that all unexpected rate changes are perfectly correlated but have differing magnitudes across securities. Thus, $\lambda_j = p_j\lambda$ and $\lambda_k = p_k\lambda$, where p_j and p_k are constants. These values may be found by examining historical series. NII(λ) becomes

$$\text{(h) } NII(\lambda) = \Sigma A_j[(1+r_j)^{t_j}(1+i_j+p_j\lambda)^{1-t_j} - 1] - \Sigma L_k[(1+r_k)^{t_k}(1+i_k+p_k\lambda)^{1-t_k} - 1]$$

Differentiate this equation with respect to λ, evaluate the result for $\lambda=0$ and set it equal to zero. This gives

$$\text{(i) } \Sigma MVA_j(1-t_j)p_j = \Sigma MVL_k(1-t_k)p_k$$

Again, implicit in this equation is the possibility of hedging by equating MVRSA with MVRSL and a weighted average repricing date of RSA with that of the RSL.

C. Rate Sensitive Withdrawals and Prepayments

The current interest rates on mortgages, consumer CD's, etc. help determine the rate of loan prepayments and early deposit withdrawals. Let A_j and L_k become functionally dependent on the unexpected change in interest rates. The hedging condition becomes

$$\text{(j) } MVRSA(1-D_{RSA}) + \Sigma \partial A_j/\partial\lambda|_{\lambda=0}(1+i'_j) = MVRSL(1-D_{RSL}) + \Sigma \partial L_k/\partial\lambda|_{\lambda=0}(1+i'_k)$$

where $(1+i') = (1+r)^t(1+i)^{1-t}$.

FOOTNOTES

1. Rose (1982b) p. 1.

2. Rose (1980) notes that banks' requirement of variable rates on loans offered institutional customers was a prime motivation for their shifting to the commercial paper market.

3. Among the first authors in this area are Baker (1978), Binder (1979), and Clifford (1975).

4. Most of the gap literature focuses on managing net interest margin (NIM) rather than net interest income. Net interest margin is NII ÷ Earning Assets. Since there are very few instances when growth in earning assets is explicitly treated and because it eases mathematical developments throughout the paper, NII not NIM will be used. If one understands the model in terms of NII, one also understands it with respect to NIM.

5. One can compute the dollar amount of the rate-sensitive assets (RSA$) and the dollar amount of the rate-sensitive liabilities (RSL$) using either book values at the beginning of the year or the dollar values as of the repricing dates. Both have been used in the literature. On both expositional and theoretical grounds, the second method is preferable and will be used here. The qualitative conclusions, however, do not depend on which of these methods of valuation is used.

6. These two accounts may well be rate-sensitive because, should rates rise, they may be (1) withdrawn from the bank, (2) deposited in higher earning accounts, or (3) paid a higher implicit return by the provision of additional "free" banking services. The gap model normally does not take these effects into account.

7. Rose (1980) p. 90.

8. When Gap$ = 0, the NII earned equals the NII for no change in rates because Gap$ = 0 hedges NII.

9. The influences of dissimilar repricing schedules probably did not give proponents of the gap model much in the way of "surprises" in realized net interest income over the 1974–79 period during which time the gap model was introduced. More recently, however, the increase in interest rate variability has made the timing of asset and liability repricing within the gapping period a significant influence on net interest income.

10. As time passes within the gapping period, incremental gaps change in ways that depend upon the characteristics of the rate-sensitive accounts. This becomes important when there are multiple rate changes. For example, pattern (a) in Figure 1a could arise either because there is a $12 asset maturing in one month or because there is a one year loan with an interest rate set monthly equal to the market rate. If it is the maturing asset, then the bank in month two is as susceptible to rate changes in month two as in the variable rate loan case only if the maturing asset is rolled over into a 30 day loan. Should the maturing asset be rolled over into a one year fixed rate loan, the bank's exposure to additional rate changes this year will be zero unlike the continued exposure under the variable rate loan scenario. As mentioned in Section I each rate change is conceptually addressed separately and, as such, multiple rate changes do not greatly complicate the analysis.

11. See footnote 5.

12. Suppose the bank has only a $100 loan maturing on day one and an equal amount in a 90-day deposit. The 90-day incremental Gap$ is zero yet NII is at risk for 1/4 of a year. Note that the risk would be even higher if this situation existed every quarter and there were multiple changes in rates.

13. Accurate information on the repricing structures of assets and liabilities is costly to come by. Once the repricing dates are recorded, however, it would seem to matter little in terms of costs on how this information is grouped into maturity buckets.

14. See Baker (1981) or Dew (1981) for the evidence in gap literature on relative rate volatilities and the methods of incorporating this type of information into the gap model.

15. The $500 in CP is equivalent to the volatility of .95 × $500 = $475 in 90 day futures. The $100 in CD is equivalent

to the volatility of $105 in 90 day futures. This method of standardization can be extended to measure the incremental gaps at dates other than the maturity length associated with the standard contract. It is beyond the scope of this paper, however, to provide the details here. Dew (1981) hints at how such a procedure might work.

16. If rates rise NII will also because of the +$370 standardized Gap$. The rise will be less than the naive +$400 Gap$ would have us believe. It is possible for a positive naive gap to be consistent with a negative standardized gap. If $370 in futures were purchased, the rate increase will cause a completely offsetting fall in the futures contract value.

17. We assume equal rates and rate changes for assets and liabilities to simplify the exposition.

18. Given that $1000 more assets than liabilities are repriced on day 30, this bank earns on net for the remainder of the year $\$1000 \times (1.12)^{330/360}$ rather than $\$1000 \times (1.10)^{330/360}$.

19. The example in the text has a cumulative Gap$ of zero. This does not mean that the basic gap model is superior to the incremental gap model. The following provides an example where neither the basic nor the "state of the art" gap model would indicate a NII hedge but where in fact such a hedge exists. Let $Gap\$_{90} = +\$1000$ and $Gap\$_{180} = -\$1536$. The cumulative Gap$ of −$536 indicates, according to the basic gap model, that NII rises when rates fall. Let rates fall from 10 percent to 8 percent. The influence of the rate change and $Gap\$_{90}$ on NII is $\$1000[1.08^{.75} - 1.10^{.75}] = \$14.70$. The magnitude of the second Gap$'s influence on NII is $-\$1536[1.08^{.5} - 1.10^{.5}] = -\$14.70$.

20. The term structure may contain a liquidity premium. This can conceptually be treated as an issue separate from using the current term structure to predict future rate changes. If such a liquidity premium exists and is positive, one would wish to be somewhat shorter in the times to liability repricing than otherwise would be the case. Nevertheless, conditional on the current value of the liquidity premium, the bank's asset and liability position is still one that depends on a difference between the bank's interest rate forecast and the market's.

21. For information on the ability of forecasters to outperform the market in interest rate predictions, see Prell (1973) and Throop (1981).

22. The one year interest rate of 11.49% is found by evaluating $1.10^{.25}\, 1.11^{.25}\, 1.12^{.25}\, 1.13^{.25} - 1$.

23. The CP repriced on day 90 has a value of $\$1000 \times 1.10^{.25}$, at day 180 it will be $\$1000 \times 1.10^{25} \times 1.11^{.25}$. Continuing in this fashion gives a year end CP value of $1114.90. This equals the one year $1000 loan with accumulated interest. Assets return 1.49% more than liabilities, expressed on an annualized basis, during the first quarter. This falls to .49 then −.51 percent and then −1.51 percent in the next three quarters, respectively. The initial negative carry on the asset (the positive spread between earnings and costs) eventually turns into a positive one. See Kaufman (1972) and Rose (1982b) for more details.

24. Note that repricing amounts that occur after the end of the gapping period (one year) do not enter equation (3). This is as it should be. These flows are not rate-sensitive and, just as in the conventional gap models, do not influence banker decisions on structuring NII rate risk for the assumed one year gapping period.

25. A convenient introduction to duration is provided by Weil (1974) and Hopewell and Kaufman (1973).

26. We can assume that MVRSA=MVRSL. Under this assumption, a positive DG is equivalent to $D_{RSA} < D_{RSL}$; that is, the average repricing date on RSA is shorter than the average repricing date on RSL. Another way of interpreting a positive DG is to assume $D_{RSA} = D_{RSL}$. Now DG < 0 because MVRSA < MVRSL; that is, at the average repricing date more RSA are repriced than RSL. Under either interpretation, the bank is "net asset sensitive" when the duration gap is positive.

27. Example 3 (c) has us book an additional $488 in RSL with D = .25. This money has to alter the balance sheet by more than this entry. The $488 could "refinance" another liability with duration in excess of one year, or it could finance a new asset with cash flows beyond one year, etc. The same consideration applies to Example 3 (d). Notice that as one tries to hedge NII, one may alter the NII to be hedged or may push some asset and liability choices into periods outside the current gapping period, potentially exacerbating problems associated with hedging NII in future years.

28. Kolb and Chiang (1981) develop the reasoning on this duration issue and show how futures contracts which have no defined net present value would enter equations (4) and (5). See also, Bierwag, Kaufman and Toevs (1983b).

29. Simple simulations in the bond portfolio literature suggest that rebalancing need not be undertaken compulsively; once a month is probably more than sufficient. Bierwag, Kaufman and Toevs (1983a) provide, in the context of a bond portfolio, a graphical interpretation on how this sequential hedging protects against multiple rate changes.

30. Babcock (1976) was the first to point out this relationship in terms of duration.

31. For realism, at least one Δi must be of the opposite sign of the others.

32. Let each of V possible interest rate outcomes have an associated probability v_j, where $\Sigma_{j=1}^{V} v_j = 1$ and $\Sigma_{j=1}^{V} v_j \Delta i_j = \overline{\Delta i}$.

33. Market participant expectations on future earnings, expected monopoly rents, expected changes in regulations, etc., can cause bank stock values to depart from the proportionate share of the market value of bank capital.

34. Gap reports can be viewed as more fully disclosing the interest rate risk exposure of the bank. As such, bank stock values may be affected as could be the value of future new stock issues.

35. This immunization condition was developed by Redington (1952) and expanded upon by Grove (1974). This formula is derived under the assumption that term structures are flat and shift due to unexpected causes to new positions parallel to the original ones.

36. $D_A = (MVA_{NS}\, D_{NSA} + MVRSA\, D_{RSA})/MVA$ and $D_L = (MVL_{NS}\, D_{NSL} + MVRSL\, D_{RSL})/MVL$. Thus,

$$MVA_{NS}\, D_{NSA} + MVRSA\, D_{RSA} = MVL_{NS}\, D_{NSL} + MVRSL\, D_{RSL}$$

when equation (12) holds. Add and subtract MVA on the righthand side of this last equation ($MVA = MVA_{NS} + MVRSA$) and do the same for MVL on the lefthand side ($MVL = MVL_{NS} + MVRSL$). This gives equation (13).

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