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Pricing Debt Instruments: The Options Approach

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As interest rates have become more volatile, participants in financial markets have become more aware of the need to accommodate interest rate uncertainty in the design of their portfolios. This increased awareness has led to a rise in the demand for mechanisms capable of transferring interest rate risk between the parties to a transaction.

One of these mechanisms is the trading of options on debt securities such as Treasury bills and Treasury notes. These instruments, traded on organized exchanges, give the holder the option to buy or sell a debt security at a predetermined price within a specific time frame. As such, they help a market participant avoid the effect of interest rate risk on the value of his portfolio. However, the investor in debt options must be able to determine whether the option is "over-" or "underpriced" from his standpoint compared to the price determined by the market.

A similar observation may be made concerning the pricing of liability products by depository institutions. Fixed rate bank or savings and loan time deposits, for example, traditionally offer a fixed return over the term with significant penalties for premature liquidation. In a period of volatile interest rates, the choice of the combination of the deposit rate and the early withdrawal penalty can critically affect the marketability of the fixed rate deposit instrument in comparison to a more nearly variable rate instrument such as money market mutual fund shares.

Thus, financial institutions, like investors in debt securities, also face the difficulty of determining the appropriate price of an option—in their case, the early withdrawal option inherent in their fixed rate deposits. There is certainly some "price" at which a financial institution with a given set of interest rate expectations would be willing to market a deposit with given early withdrawal features. But many financial institutions may not have enough confidence in their ability to translate their interest rate forecasts into appropriate prices.

The purpose of this paper is to present the results of some new experiments with a debt instrument pricing methodology. The methodology is based on options theory and recently developed pricing techniques. Its application is illustrated first by pricing the recently approved "put" options on government securities and comparing simulated results with market outcomes. The methodology is then used to illustrate the applicability of options pricing in an indirect context, namely, that of evaluating policy regarding early withdrawal penalties on deposit liabilities. In each case, options are involved and the methodology relates the "prices" of these instruments to interest rate forecasts. The empirical estimates presented are not intended to apply directly to a particular options pricing problem. They are intended instead to illustrate the sensitivity of rational debt instrument prices to interest rate forecasts (and the features of the instruments) and the usefulness of the options perspective to both investment analysis and policy problems.

Two specific results come from our analysis. First, in the simulation of options on Treasury notes, the options prices obtained by the pricing model are good approximations of the prices at which options on Treasury notes have recently traded. Second, our estimates of early withdrawal option prices suggest that the combination of regulated rates and the penalty structure that existed in the 1970s (particularly on fixed rate deposit instruments of less than one year) may have put depository institutions at a severe competitive disadvantage in marketing their deposit services against other instruments in the marketplace.

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The remainder of the paper is divided into four sections. In the first, the basic theory of options pricing is presented. The second section expands this discussion and focuses on the pricing of options on debt securities. In addition, we present the methodology incorporated in our computations in this section. The valuation methodology is tested by pricing Treasury note options. In the third section, the methodology is applied to the valuation of early withdrawal penalties. The fourth section concludes the paper with a discussion of the policy implications and limitations of current pricing methodologies.

I. Options Theory

We often think about options in the context of marketed options, such as those on corporate shares traded on organized exchanges since 1973, or options on certain Treasury securities that have been traded on selected U.S. exchanges since 1982. In its most general form, however, an option is simply a contract—or stipulation within a contract—that gives the owner of the option the right to trade in some asset at a defined price any time on or before a given date (the "exercise" date). From this perspective, many conventional financial agreements contain options, and these implicit options can be analyzed in the same fashion as explicitly traded options.

For example, a corporate bond that is issued with a call provision giving the corporation the right to buy the bond back at a stipulated price, in essence, contains an option. Specifically, it contains a call option because it gives the *owner* (the corporation) the right to acquire ("call away") an underlying security (the bond) from the lender (who would also be called the option *writer*). The price stipulated in the bond indenture is called the *exercise* price.

Similarly, the ability to withdraw funds from a deposit account gives the depositor the option to force the borrower of the funds (the depository institution) to buy back the deposit instrument—in options terminology, to "put" the deposit on the borrower. The early withdrawal feature is thus a *put option* owned by the depositor.

Whether or not a particular option is traded in the market as an independent security, it should have value. If it is also part of a more complicated securities contract, it should influence the price of the underlying securities contract. Indeed, options in some sense are the fundamental building blocks of more complex financial instruments. Thus, even a complete instrument could be valued if it were decomposed into its constituent options, each of which would be a simpler instrument and easier to value.

Pricing Options

The price of an option, whether explicitly traded as a separate security or not, depends upon expectations of future economic conditions as these affect the value of the underlying security. In the abstract, the prospective and contingent nature of options would appear to make evaluation of an option extremely difficult because future conditions are never known with certainty. Nonetheless, financial economists have devised methods of evaluating such contingent claims.

The analytical breakthrough in this area came in 1973 with the work of Black and Scholes.[1] They reasoned that an option could be valued by inference from the value of portfolios that contained the option. Specifically, Black and Scholes used the idea of a riskless hedge—a portfolio consisting of the option and its underlying security constructed to yield the riskless return. The price that the investor will be willing to pay for the options necessary to construct a riskless version of such a hedge will depend upon the riskless return available elsewhere as well as the anticipated scenario of the stock price movements.

Although the underlying security may take on a wide range of values, Black and Scholes were able to derive an analytical formula for the price of an option on corporate stock by relying on very general assumptions about the stochastic nature of stock price movements, the current price of the stock, and an assumed riskless real rate of return.[2] Although the user of this formula must provide the estimates of the variability of the stock's future price movements, the implication of the Black/Scholes work is that the price of the option is otherwise unambiguous.

The Black/Scholes formula applies only to options on corporate stock, but the notion of inferring options values from riskless hedges and alternative future values of the underlying security has enabled other researchers to apply the idea to the valuation of options on debt securities as well. The application of options valuation to these instruments, however, is somewhat more complex because the value of the underlying security is likely to move in a complex fashion as interest rates change. In other words, although it may be reasonable to assume that interest rates move in a random fashion about some trend, for example, it is not reasonable to assume that the value of debt securities moves similarly. If the underlying instrument is a bond, for example, the response of the value of a new bond to movements in interest rates may be very complex depending upon the bond's features (e.g., the number and timing of coupon payments); nevertheless, the value of the bond is virtually known with certainty toward the end of the bond's life.

As a result, it is difficult to derive a purely analytical debt option valuation methodology, except for specific underlying securities.[3] The generalizable approaches thus tend to consist of numerical approximation techniques.

Numerical Approximation

Numerical approximation techniques rely upon the observation that a continuous process (such as movements in the value of a security) can be divided into discrete steps without losing the essential features of the process. Since there are theoretical relationships between certain discrete statistical processes and continuous statistical processes, the valuation can be made to depend upon a few simple parameters, much in the spirit of the original Black/Scholes analytical approach.

An example may help to clarify this point. Let us assume that risk-free nominal interest rates move in equally probable discrete steps or jumps (either up or down) and that the magnitude of the movements up or down does not change over time. In Chart 1, for example, we show alternative paths for the interest rate over three future time periods, assuming that a jump "up" always multiplies the interest rate by a factor of 1.1 and a jump "down" by .9. This tree of movements in interest rates can be translated into a tree of prices for a debt instrument because its value in any given period depends upon the path which interest rates may take between now and some future time period. By working backwards from the date the instrument matures (when the value of the bond is known with certainty), all values of the debt corresponding to each interest rate value on the interest rate tree can be computed.

With the tree of values of the underlying debt, the *value* of the option on the debt instrument, were it to be exercised, can be computed for each branch of the tree as well. With this information and the basic notion of a riskless hedge, it will be shown below that it is possible to derive the option's appropriate *price* at any branch of the tree.[4] Thus, if the discrete interest rate movement process could be shown to approximate a continuous process in a reasonable fashion, options on instruments with quite complex features could be valued.

This approach to pricing options has been used by a number of analysts,[5] but most notably by Rendleman and Bartter.[6] The arithmetic of the computations is quite simple. The contribution of these authors was in relating this simple binary interest rate movement process to the continuous movement that can be expected in the real world. In particular, they showed that the "up" and "down" jump ratios can be manipulated to incorporate various underlying assumptions about the trends in interest rates and the variance of their movement about that trend.

The numerical approximation technique, therefore, yields a valuation methodology that uses inputs that are almost as simple as those required by the original Black/Scholes formula for corporate stocks—only forecasts of the trend and variance of interest rates need be provided. Once the precise features of the underlying security and its option are imbedded in the procedure, the value of the option under different interest rate movement scenarios can be computed easily.[7]

II. Valuing Options on a Bond

In this section, we will make our presentation more specific by pricing a put option on a coupon bond. A coupon bond is a general instrument and procedures applicable to it are broadly applicable to other instruments. For ease of exposition, we focus on a specific option and bond: a put option with the same life as the underlying bond. In other words, lives of the option and the bond are both assumed to be T periods. Later on, we will use the methodology to value bond options that are actually traded, in which case the option life is shorter than the bond maturity. We emphasize, however, that the procedure is a very general one despite the simplifying assumption of the example. Our numerical approximation technique is a modification of the methodology of Rendleman and Bartter[8] and involves a sequence of modelling steps beginning with the specification of the process of interest rate movements.

Interest Rate Pattern

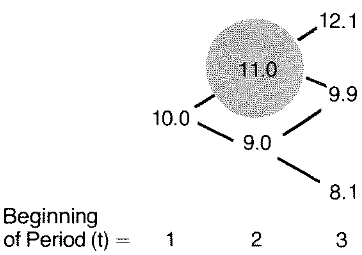
We will use the following example to illustrate the process. As in other examples discussed earlier, we shall assume that the short term risk-free interest rate one period in the future can take one of two values: Z^- denoting a fall in the interest rate and Z^+ denoting a rise in the interest rate. (In this paper a plus superscript indicates an increase in a variable and a minus superscript a decrease). The risk-free interest rate in period t takes one of the following values:

$$I^-_t = Z^- I_{t-1}$$
$$I^+_t = Z^+ I_{t-1}$$

We assume further that the values of Z^- and Z^+ are constant over time, and that each year there are only a finite number of times that interest rates can move (N). In addition, the life of the bond is assumed to expire at the beginning of the T + 1 year. In the general case, a binomial tree would generate 2^{TN} values in the last period. However, since we assume that the Z^+ and Z^- ratios remain constant over time and that the relationship between interest rates over time is multiplicative, we obtain the following interest rate tree which has only TN values in the final period:

Chart 1

Interest Rate Tree



To find values for Z^+ and Z^-, we need to make assumptions regarding the statistical properties of interest rate movements. If we assume that the probabilities of the two states are constant over time, then the logarithm of Z (denoted z) can be said to be drawn from a binomial distribution with annual mean of

$$M = N\,[Z^+W + Z^-(1-W)]$$

and annual variance of

$$S^2 = N[(Z^+-Z^-)^2\,W\,(1-W)]$$

where

N = Number of periods per year and
W = Probability of Z^+.

The result is important for two reasons. First of all, for large N and for W = .5 (an assumption employed in our work), the binomial distribution approaches that of the lognormal. This means that under these assumptions, the interest rate jump process approximates a state of the world in which the instantaneous riskless interest rate follows a lognormal distribution. Second, if one knows M and S one can solve for Z^+ and Z^-.

On a more intuitive level, M can be thought of as a measure of the annual geometric drift[9] of the mean interest rate, while S is a measure of the dispersion of interest rates around the mean. The drift and dispersion parameters, a part of the option pricing process, must be forecast by the pricer of the option. In the empirical work that follows, we obtain simple forecasts of the drift and dispersion of the interest rate by econometrically estimating the historical drift (and dispersion about that drift) of short-term interest rates.[10] We vary these parameters considerably in our analysis.

The purpose of employing an empirically derived forecast is simply to provide a benchmark for the various values of the mean and drift employed. These values are by no means offered as sophisticated or definitive forecasts.

Bond Pricing Under Uncertainty

In pricing the bond, we employ the pure expectations hypothesis under which, at any time t, the bond should be priced so that its expected return (over time t−1 to t) is the rate earned on a default-free discount bond. That is, its value in period t−1 should be equal to its expected value (plus any coupons) in period t, discounted at the risk free rate, or

$$D_{t-1} = \frac{E(D_t) + C/N}{(1 + I_{t-1})^{1/N}}$$

where

D_t is the bond price in period t
$E(D)_t$ is the expected bond price in period t
C is the annual coupon payment
N is the number of periods per year.

Since the expected bond price in the terminal period (that is, T+1) is its face value ($100), the bond pricing formula for period T is:

$$D_T = \frac{100 + C/N}{(1 + I_T)^{1/N}}$$

As discussed earlier, the interest rate in period T can take on T possible values. Thus, there are T possible bond prices in this period. In the preceding periods, the expected bond price is the discounted average of the two possible bond prices in the next period. The bond prices can then be determined recursively using a tree of the same form as the interest rate tree.

Pricing the Option

Now we have all of the elements necessary for pricing the option. As stated earlier, the basic pricing method is based upon the notion that one can form a riskless hedge by purchasing the right combination of a bond and its option. If the price of the bond increases or decreases, that of the option will move in the opposite direction, offsetting the effect of the bond price movements in the yield of the portfolio, at least for small bond price changes. In the case of put options, for every $1 invested in a bond at time $t - 1$, the number of put options that should be purchased is

$$\frac{(H_t^- - H_t^+)}{(V_t^+ - V_t^-)}$$

where

$$H_t^+ = (D_t^+ + C/N)/D_{t-1}$$
$$H_t^- = (D_t^- + C/N)/D_{t-1}$$

and

V_t^+ = value of option if the price of the bond increases,

V_t^- = value of option if the price of the bond decreases,

Simply stated, this formula tells us that as the possible variability in bond prices increases (i.e., as D_t^+ and D_t^- diverge) so does the need for a hedge. If there were no possibility of variation in bond prices, H_t^+ would equal H_t^- and no options would be needed to form a riskless hedge.[11]

Since the joint investment is riskless, the option should be priced to earn the riskless rate of interest. Rendleman and Bartter have shown that this price is

$$P_{t-1} = \frac{V_t^+ [(1+I_{t-1})^{1/N} - H_t^-] + V_t^- [H_t^+ - (1+I_{t-1})^{1/N}]}{(H_t^+ - H_t^-)(1 + I_{t-1})^{1/N}}$$

To use this formula, we must first calculate the value of the option (V) in each period. In doing so, we must consider the alternatives facing the option holder at the beginning of each period. If he exercises the option, its value is just the difference (call it VEXER) between the exercise price and the market price of the bond. Otherwise, the option's value is the price at which he could sell it. Since the investor is assumed to be rational, the value of the option to the investor will be the larger of these two numbers.

We know that the price of the option is zero in the last period of its life, since it cannot be exercised after it has expired. And we know VEXER because the price of the bond and the exercise price are known for each period. Thus, by beginning the option pricing process in the last period of the option's life, it is possible to use the pricing formula above to determine the option price one period earlier. We repeat this process until we reach period 1 and obtain the initial price of the option.

Options on Treasury Notes

As a simple test of the methodology presented above, we will use the valuation technique to price a real world instrument. There are a number of options on currently traded government debt securities that could be priced using this technique, but we have chosen options on 10-year Treasury notes being traded on the American Stock Exchange (AMEX) because the features of these options are most like those of the hypothetical instrument described.

Before proceeding with the demonstration, some differences between the Treasury note options and our hypothetical options should be noted. First, the options on T-notes which Amex is trading are available with March, June, September, and December exercise dates, with the option expiring the Saturday following the third Friday of the expiration month. This differs from the hypothetical instrument discussed above in that the life of the option is not the same as that of the bond. Second, the bond pays interest semiannually, while the equivalent coupon in our hypothetical model is paid continuously. Third, in the real world, costs such as brokerage and settlement fees must also be considered.

To keep the exposition as straightforward as possible, we will not attempt to incorporate all these differences into our model. Instead, we will employ the model as outlined in the previous section, with the exceptions that we shall assume the life of the option to be 3 months and, of course, give the note a 10-year maturity instead of the arbitrary maturity used before. Finally, there are several exercise prices for the option offered in the AMEX instrument. These are stated as if the value of the underlying instrument were \$100. For simplicity, we employ only the exercise prices 96, 100, and 104 dollars because they are the major exercise prices on recently traded instruments.

Interest Rates and Option Prices

As we have already stated, the primary function of an option is to protect the owner against future interest rate fluctuations. In the case of a put option on a T-note, for example, the owner is protected against upward movements in interest rates. Given that expected interest rate changes play such a central role, two features of these changes should each be a major factor in determining the price of an option: the magnitude of expected changes and their associated uncertainty. In other words, the higher one expects interest rates to rise during the life of the option, and the more uncertain one is about future interest rates, the more one would value protection against such fluctuation and the higher the price one would be willing to pay for the option. Thus, one of the factors which we shall test is the sensitivity of the price of the option on the T-note to different interest rate scenarios. These results will then be compared to the prices of options actually traded.

Table 1
Estimated Interest Rate Drift and Dispersion: Varous Periods, 1978–1983

Estimation Period (Year, Month)	**Drift (M)**	**Dispersion (S)**
81.04 - 83.03	−.10	.42
81.02 - 83.01	−.06	.41
80.11 - 82.10	.02	.40
80.08 - 82.07	.08	.34
80.05 - 82.04	.10	.33
80.02 - 82.01	.16	.34
79.11 - 81.10	.24	.29
79.08 - 81.07	.23	.28
79.05 - 81.04	.18	.22
79.02 - 81.01	.23	.20
78.11 - 80.10	.25	.21

Source: see text

Analysis and Results

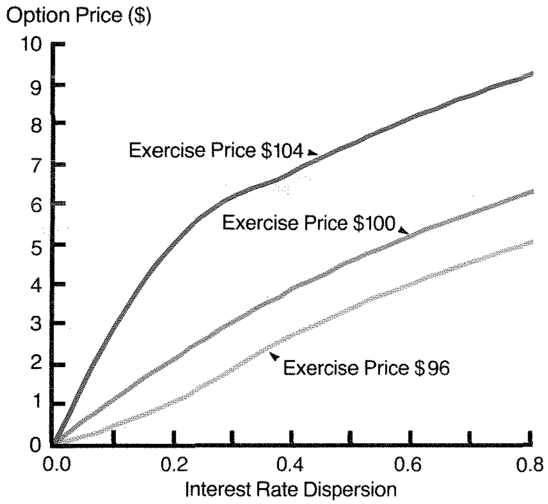
Although we are not attempting to price particular options, the sensitivity analysis is more useful when based on a realistic interest rate range. To devise a forecast for interest rate movements that simulates what the market may be using, we use a simple time-series econometric model and extract from it the two main parameters of the option pricing model, M and S.[12] The first parameter represents the trend or drift followed by the interest rate, while the second represents the variability of uncertainty about the forecasted trend. The results of this estimation process for various periods are presented in Table 1. Because T-note options were first traded in late 1982 and we wish to value the instruments as of December 1982, we choose the values .02 for M and .40 for S from the table. The final parameter needed is the initial interest rate. We use 8.53 percent because it was the interest rate on one-month commercial paper in December 1982.

The sensitivity analysis performed uses these parameters. We start the analysis by allowing the forecast dispersion, S, to change while holding the trend or drift term, M, constant. The results from the exercise are presented in Chart 2, and are consistent with simple intuition about options pricing. First, as expected, the option price increases as interest-rate dispersion increases. Second, and also as expected, the option has no value even when there is a positive interest rate trend. The latter indicates that even an expected rise in interest rates would not give an option value if the rise were known with certainty.

Next, we vary the trend term while holding the

Chart 2

Option Price/Interest Rate Dispersion with Historic Interest Rate Trends*



*This graph assumes that the underlying instrument has a face value of $100.

dispersion term constant. The results, presented in Chart 3, are again what one would expect. The price of the option increases when the trend is toward higher interest rates, since the likelihood of the option being exercised profitably is higher when interest rates are expected to rise. One should note that even with a negative trend term, the option will have a positive value if the dispersion term is large enough. This means that market participants may wish to purchase an option to protect themselves against the possibility that interest rates may rise when they expect rates to fall, if they were not certain interest rates will in fact fall.

Finally, we compare the option prices estimated by the model with those actually traded. The price of the note and its option calculated from the model using the forecasted interest rate parameters (i.e., M = .02 and S = .40) with strike prices of 96 and 100 are $2.73 and $3.88 per $100 of face value of underlying note, respectively. The prices of options on a similar note actually traded on AMEX on December 21, 1982, were $.87 and $2.69, respectively. Clearly, the model produces simulations that are within the same order of magnitude as the actual prices of the options traded.

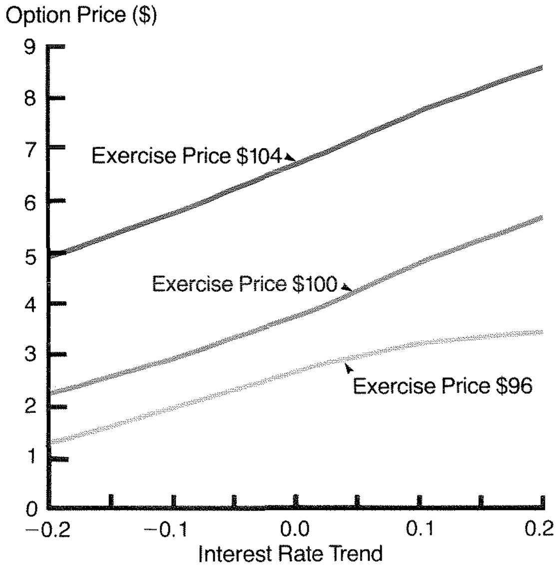
Some differences between actual and simulated prices should not be surprising. For one, the model does not incorporate transactions costs. The exclusion of such costs tends to bias th[illegible] tions upward, since the net proceeds of exercising the option would otherwise be lower by the amount of the costs. Transactions costs are particularly important in light of the current thinness of the existing AMEX options market.

A second possible source of divergence is our forecast of interest rate trend and dispersion. As the previous section on sensitivity analysis indicates, the option prices calculated by the model are greatly affected by the interest rate parameters used. Thus, if our forecast of interest rate trend and dispersion is different from that of the market, then the option price calculated by the model will differ from the actual option price. In this regard, however, we have another piece of data to corroborate the results: the price of the underlying T-note. The simulated note prices are within one percent of the prices of the notes underlying the actual options traded. This does not guarantee that our forecast parameters are correct, but it does suggest that the model is at least pricing the option and the note consistently.

A more subtle potential problem with the model is that by employing the pure expectations hypothesis, it leaves no room for specifying the utility

Chart 3

Option Price/Interest Rate Trend with Historic Interest Rate Dispersion*



structure of wealth. That is, the degree of risk-aversion is not a parameter of the model. Although explicit inclusion of a risk aversion parameter in the bond pricing formulae may be desirable (and, indeed, has been tried by other investigators), the addition of a third parameter makes our presentation more cumbersome and is unlikely to yield radically different results when applied to short-term instruments.[13]

In light of the imperfect means available to incorporate the factors relevant to options pricing, the Rendleman-Bartter model, on the whole, approximates the real world reasonably well.

III. An Application to Early Withdrawal Penalties

Besides simulating the prices of traded options on debt securities, the options pricing methodology developed in Section II can be applied to more general policy issues regarding debt instruments. An example is the evaluation of early withdrawal penalty features of fixed rate deposit instruments.

Fixed rate deposits, unlike other instruments of the same maturity offered in the marketplace, include an early withdrawal provision that allows the depositor to liquidate the account and obtain the par value of the account less a pre-stipulated penalty. In the presence of uncertainty regarding future interest rates, the early withdrawal provision should increase the value that investors place on the underlying instrument, while the pre-payment penalty, which exercising the provision entails, should have a negative effect. Thus, the early withdrawal structure as well as the rate structure are important factors in pricing a deposit account in relation to other investment opportunities.

Historically, early withdrawal penalties and deposit rates were largely determined by regulation. Deposit rates were either regulated directly or linked by regulation to a market rate. The latter mechanism allowed deposit rates to reflect, at least partly, changing market conditions. This was not true of pre-payment penalties which were determined without any attempt to measure the effect of the penalty structure on the marketability of fixed rate deposit instruments. Thus, the regulation of pre-payment penalties prevented banks from tailoring the interest rate and pre-payment features offered on their deposit accounts to investors' interest rate risk/return preferences.

Valuation Method

The option to withdraw funds from a deposit account is a put option on a debt security. The security is a deposit account which pays coupons whose magnitudes are determined by the deposit rate. The holder of such an account—one that permits early withdrawal—essentially owns the option to "put" this "bond" on the depository institution for an exercise price equal to the principal value of the account less any predetermined penalty. (We assume there is zero risk of default on the part of the institution.)

The value of such an option derives from the possibility of a larger than expected rise in interest rates. That is, if the rise in interest rates makes the yield on the fixed rate deposit less than the yields of competing instruments, the market value of the deposit account will fall below its par value. Since the early withdrawal option enables the depositor to get back the par value of his deposit, he may, depending on the penalty associated with early withdrawal, profit from exercising the option.

In such a context, the exercise price (X) can be considered the deposit's principal amount (F) minus the early withdrawal penalty (E). The value or proceeds of the exercise (VEXER) are then $X - D - F - E - D$ where D is the market value of the deposit. The price of the early withdrawal option can then be determined using the general option value formula described earlier.

This valuation method is applied here to hypothetical deposit accounts whose deposit rates and early withdrawal penalties are tied to the yield on risk-free market instruments. A comparable real-world example, of course, is the 6-month money market certificate (MMC) introduced in 1978. Its deposit rate is fixed for the six-month period and is linked to the yield on newly issued Treasury bills in the week the deposit is created. It presently has an early withdrawal penalty of 3 months interest. We will also include valuation of the early withdrawal options on one-year, 2½-year, and 4-year deposit instruments in our analysis. We assume that the yields for all of these instruments are linked to discount-type Treasury security yields of similar maturity (an assumption that departs from reality but makes our analysis more consistent across instruments.)

There are three major steps involved in valuing the early withdrawal option on such instruments. First, since the deposit rates are linked to market yields, it is necessary to simulate the Treasury security yield using the (assumed) market interest rate tree. We use the bond valuation approach described in Section II to price the relevant Treasury security and to obtain the appropriate implicit yield.

The second step in the process is to link the implicit yield to the deposit instrument, since we assume that the deposit rate is linked to this yield by regulation. We use the implicit yield to derive the stream of "coupons" offered by the MMC account as well as the interest penalty. In keeping with the actual practice in the case of MMCs, we assume that the deposit pays simple interest only, so the "coupon" or periodic interest payment is constant throughout the instrument's life. Thus, the deposit can be seen as a "bond" which pays a constant coupon each period and returns the principal amount at the end of its life. Finally, using this representation of the deposit and the early withdrawal penalty, we price the early withdrawal put option using the formulae described earlier.

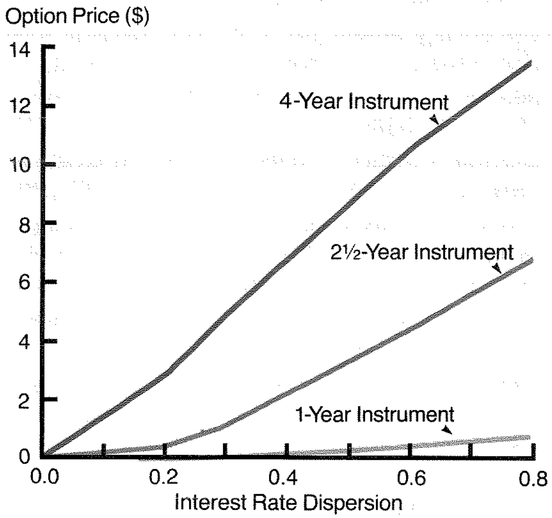
Assumed Penalty Structure

Current early withdrawal penalty policy requires, in most cases, a sacrifice of 3 months interest on the 6-month and one-year instruments and a sacrifice of 6 months simple interest on instruments greater than one year in maturity. Since one of the goals of the analysis is to examine the interaction between regulated rates and early withdrawal penalties, these penalty structures are the ones assumed in our analysis.

Using the valuation techniques and assumptions described above, the prices of the par-value withdrawal options inherent in our hypothetical deposit instruments result from a variety of interest rate and penalty environments. For reference purposes, we employ M = .2 and S = .3. As Table 1 indicates, these values are consistent with the "forecasts" produced by our simple econometric model in the

Chart 4

Option Price/Interest Rate Dispersion with Current Penalty and Historic Interest Rate Trend*



*This graph assumes that the underlying instrument has a face value of $100.

1979 to 1981 period. We also employ a starting interest rate of 12 percent, which is approximately the rate that prevailed in late 1981.

Effects of Interest Rate Uncertainty

Chart 4 demonstrates the effect of interest rate uncertainty (dispersion) on the value of the early withdrawal option (using the assumed penalty structure and a historic forecast of drift of .2). Only the 1-year, 2½-year, and 4-year instruments are sensitive to changes in uncertainty in a broad range around the historic dispersion of .3. The withdrawal feature on the 6-month instrument takes on zero value throughout except at high uncertainty levels. (We will see later that this is due to the very high penalty implicit in the 6-month instrument.)

In general, however, the greater the uncertainty about interest rate movements the greater is the value of the early withdrawal option inherent in a deposit-type instrument. The value of the option is also greater, at any given level of uncertainty, for a longer maturity deposit. Both are results that we would expect to follow from the very nature of an option as a hedge against an uncertain world. The more uncertain that world or the longer the investment must be in place, the more valuable the option to liquidate becomes.

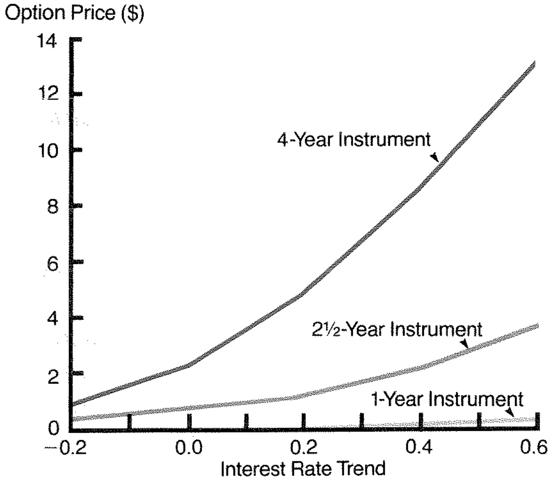
From the standpoint of the policy behavior of financial institutions, it is obvious that, if they could, they should pay lower rates of interest (or find other ways of charging for the option) during periods of high interest rate volatility, everything else being equal. At the historic dispersion, the 2½-year certificate contains an option worth approximately $1.25 per $100 of face value. The 4-year certificate contains an option worth approximately $4.75 per $100 of face value. Whether charged "up front" or in the form of lower deposit rates, such options represent a valuable service provided by depository institutions and should be an important source of income and an important means of differentiating the deposit product from other financial instruments.

Effects of Interest Rate Drift

Chart 5 illustrates the effect of another important interest rate forecast parameter: anticipated drift in interest rates. The more widely held is the view that interest rates are going to rise (i.e., the greater is the forecast of drift), the greater is the value of an option to liquidate (and reinvest). Holding the level of uncertainty (dispersion) at its historic level and using the current withdrawal penalty structure, we find that by varying the drift, the early withdrawal option indeed increases dramatically in value as anticipated drift increases. This is particularly true for the instruments of longer maturity.

Chart 5

Option Price/Interest Rate Trend with Current Penalty and Historic Dispersion*



We can conclude that during periods of widely anticipated (but uncertain) increases in interest rates (such as the late 1970s), long-term deposit-type instruments with withdrawal options should be very attractive in comparison to their bond-market competition paying roughly comparable rates. (Recall that the deposit rate on these instruments is linked to the yield offered by a government debt security.)

In addition, we can surmise that if anticipated drift is significantly negative (i.e., interest rates are anticipated to fall sharply over the life of the instrument), the value of the early withdrawal option rapidly approaches zero. This, too, is a common sense consequence of the nature of the early withdrawal option. Within a declining rate scenario, the exercise of the option prior to maturity would expose the investor to investment alternatives with lower returns than the existing instrument. The option is thus less likely to be of value to the investor.

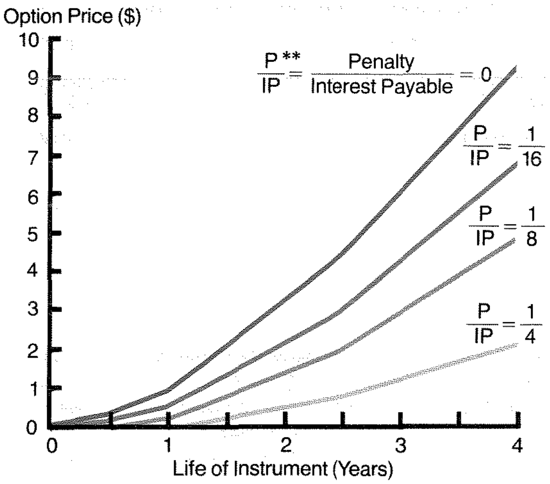
Even under conditions of zero anticipated drift, the early withdrawal options on the 1-, 2½- and 4-year instruments have positive value in an uncertain world. For the 4-year instrument, the option's value is approximately $2.50 for every $100 in deposit value, representing a significant feature of par-value deposits that should not be ignored in pricing these instruments competitively.

Effects of Early Withdrawal Penalties

Note that in all of the previous figures, the option price of the shortest term deposit instruments reacted least (if at all) to changes in drift and dispersion. To some extent, this is to be expected. The shorter the intended term of the investment, the less important the option to liquidate and reinvest in response to changing economic conditions. However, a less obvious but major factor in the behavior of the 6-month and 1-year instruments is the prevailing penalty structure. The current penalty structure appears to eliminate the distinction between par-value deposit instruments and bonds for those deposit instruments whose yields are linked to Treasury market yields. This is illustrated clearly in

Chart 6

Option Price/Penalty with
Historic Interest Rate Trend and Dispersion*



*This graph assumes that the underlying instrument has a face value of $100.

**The assumption here is that the penalty for early withdrawal is a constant fraction of the total interest payable. For example, for a two-year instrument with a penalty of six months' interest, P/IP = ¼.

Chart 6 where the valuation of the early withdrawal option on all four instruments (at the historic drift and dispersion) for a variety of early withdrawal penalties expressed in terms of months of simple interest is plotted. Even at penalties considerably smaller than those presently enforced, the early withdrawal option has no value for deposits of less than one year.

Calculations from the options model suggest that at the conventional penalty of 3 months' interest, the early withdrawal option has zero value for the 6-month deposit. In fact, the penalty must be reduced to approximately a ½ month's interest before it takes on a non-zero value. This suggest that, for our simulated interest rate uncertainty, the existing penalty structure on the 6-month instrument prevented banks from using the pre-payment option to differentiate their deposit account from bond-like instruments available on the marketplace and, therefore, to compete for funds. It also restricted the ability of investors to choose between instruments offering a higher return and those offering protection against interest rate risk but with a lower return.

Obviously, our valuations depend critically upon interest forecast assumptions and ignore transactions costs and convenience aspects of deposit-type instruments and their alternatives. Nonetheless, the results do strongly suggest that under conditions of rising interest rates and high levels of uncertainty, the existing structure of early withdrawal penalties on short-term instruments (combined with regulated rates) has been extremely onerous. It may have contributed to the difficulties that depository institutions faced in the late 1970s and early 1980s in competing effectively against money market funds and direct investment in Treasury securities.

V. Conclusion

This paper has applied a simple option-pricing model to two instruments: traded options on Treasury notes and fixed rate deposit instruments. The first application illustrated the importance of interest rate forecast parameters to the valuation of traded options. An investor (or seller) of such options can use such pricing techniques to help relate his view of the future path of interest rates to option prices and, thereby, to help him take positions in the options.

The second application illustrates the usefulness of the options perspective to setting general policies regarding deposit-instrument pricing. Although the option pricing exercises were based on somewhat hypothetical instruments, they highlight the sensitivity of proper instrument prices to the features of the instrument and to alternative future interest rate scenarios.

A specific implication of the simulations is that the regulation of early withdrawal penalties prevented financial institutions from using the early withdrawal feature of their deposits to compete for funds. That is, they were prevented by the prohibitively high penalty structure from using the early withdrawal option to enable them to offer a lower return on their deposit accounts than those of comparable instruments (such as T-bills) without withdrawal provisions. Our analysis of MMC rates provided empirical evidence to support this point at least in the case of the 6-month instrument. Despite the fact that the regulated deposit rate was simply a *ceiling* rate (that is, the institutions could offer a lower rate if they desired), virtually all MMCs in recent years have been marketed at the ceiling rate. This suggests that under the regulated penalty structure (and the interest rate environment), the early withdrawal option was not an attractive investment feature.

Whatever the specific nature of financial instruments, the increased complexity of financial markets will require more sophisticated and careful accommodation of interest rate risk. The options perspective appears to be a useful and practical mechanism for analyzing the effects of alternative future interest rate paths on financial instruments and their markets. Although much work needs to be done to improve options pricing models, our exercises with one model suggest that they are likely to have broad applications for financial institutions managing their portfolios in a highly competitive and uncertain world.

FOOTNOTES

1. Fisher Black and Myron Scholes, "The Pricing of Options and Corporate Liabilities," **Journal of Political Economy**, May 1973, pp. 637–654.

2. More precisely, it is a function of the current price of the underlying stock, the life of the option, the variance of the continuously compounded annual rate of return on the stock, the exercise price and the continuously compounded riskless rate of return. The stochastic assumption is that the continuously compounded rate of return follows a normal distribution with constant variance. (See Black and Scholes, **op. cit.**).

3. Some of the complexity of the analytical pricing approach is revealed by the work of Michael Brennan and Eduardo Schwartz, "Savings Bonds, Retractable Bonds and Callable Bonds," **Journal of Financial Economics,** August 1977, pp. 67-88.

4. This process is elaborated upon below.

5. William Sharpe appears to have been the first to suggest the general type of numerical approximation technique presented here (albeit in the context of options on stock) in his text **Investments**, (Prentice-Hall), 1978, pp. 366-371. A useful review of numerical techniques, however, is presented in Robert Gerske and Kuldeep Shastri, "Valuation by Approximation: A Comparison of Alternative Option Valuation Techniques," **Working Paper 13-82**, University of California, Los Angeles, August 1982.

6. Richard Rendleman, Jr. and Brit Bartter, "The Pricing of Options on Debt Securities," **Journal of Financial and Quantitative Analysis**, March 1980, pp. 11-24.

7. See Rendleman and Bartter or Geske and Shastri, **ibid.**

8. The major theoretical changes are a continuous-time discounting procedure and a slightly different time-dating convention necessitated by the computer simulation program written by the authors. However, to assist readers in relating this paper to the work of Rendleman and Bartter, similar terminology and nomenclature are employed where possible. We wish to thank Dr. Rendleman for his helpful comments at several points in this adaptation of their work.

9. It is important to emphasize the difference between arithmetic and geometric drift. Since arithmetic drift, A, is equal to $(M+S^2)/2$, readers must be cautious about the empirical inferences drawn about the sensitivity analysis presented in this paper. Constant arithmetic drift—the conventional notion of "flat" interest rate evolution—is not the same as constant geometric drift when there is some dispersion about the drift. In this paper, the sensitivity analysis is presented in terms of the original parameters of the model, namely M and S, rather than in terms of arithmetic drift to illustrate better the sensitivity of the model to its specified parameters.

10. The interest rate parameters are obtained from a regression of the general form

$$\ln (i_t/i_{t-12}) = a + u$$

where i_t is the monthly commercial paper rate, a is a parameter, and u is an error term. The coefficient a is interpreted as the estimated geometric drift M and the standard error of the equation is interpreted as the dispersion parameter S.

11. This computation is conventionally referred to as the "hedge ratio." Instinctively, the variable H is the gain anticipated between periods t-1 and t in the holding of the underlying security and V is the value of the associated option. Both V and H will depend upon the interest rate state that actually evolves. If the difference in the gain on the underlying security between states differs from the offsetting movements that would occur in the option value, the proportion of options to underlying instruments in the portfolio must be changed accordingly.

12. See footnote 10. The values presented were estimates using a time series dated 11/80 to 10/82.

13. See Richard Rendleman, "Some Practical Problems in Pricing Debt Options," Duke University, August 1982, Mimeo.