Spring

1984

Economic Review

Number

2

Federal Reserve Bank of San Francisco

The Federal Reserve Bank of San Francisco's Economic Review is published quarterly by the Bank's Research and Public Information Department under the supervision of Michael W. Keran, Senior Vice President. The publication is edited by Gregory J. Tong, with the assistance of Karen Rusk (editorial) and William Rosenthal (graphics).

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Pricing Mortgages: An Options Approach

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Options theory has provided a framework for valuing financial instruments with contingent claims features. This paper uses a simple numerical options pricing technique to price adjustable and fixed rate mortgages containing prepayment options. The simulations performed illustrate the sensitivity of mortgage prices to mortgage features. They also underscore the risk-return tradeoff made by a lender who chooses to emphasize the origination of adjustable rate mortgages.

Lenders in the residential mortgage market were among those caught unprepared by the high and volatile interest-rate environment of the 1970s. The fixed rate, long-term mortgage that was then dominant limited the ability of financial institutions to adapt to high and rising interest rates. Most institutions that specialized in mortgage lending confronted deteriorating net worth and cash flow positions as the value of their mortgage portfolios declined while the costs of their deposit liabilities rose.

The industry's response was to re-examine their marketing of the conventional fixed rate mortgage and to introduce new instruments, such as the adjustable rate mortgage (ARM), that passed some interest-rate risk to the borrower. Unfortunately, there were few guides to help mortgage lenders "price," that is, set initial contract rates, on these new instruments.

The purpose of this paper is to illustrate how the options pricing model developed in the theory of finance can be applied to the problem of pricing mortgage instruments. Although the technique is quite general, we illustrate it with two relatively simple examples—a conventional fixed rate mortgage with a prepayment option and a special type of adjustable rate mortgage. These two applications demonstrate the usefulness of the options pricing model, and illustrate how mortgage contract rates are determined both by the specific provisions of the contract and the underlying assumptions about further interest rate movements.

The model reproduces fairly accurately those mortgage rates observed in the secondary mortgage market, and demonstrates how those rates would be affected by different contract provisions for prepayment penalties and for "caps" on how much interest rates can be varied on adjustable rate mortgages. It also is capable of explaining observed spreads between rates on GNMA pass-through securities and other riskless rates. Finally, the model provides estimates of the value to mortgage lenders of the interest rate risk protection offered by adjustable rate mortgages. These estimates suggest that current techniques for pricing adjustable rate mortgages may result in overpricing these instruments.

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I. Basic Mortgage Instruments

The mortgage instruments examined here are a conventional fixed rate mortgage (FRM) and an ajustable rate mortgage (ARM). On the FRM, the contract rate is fixed for the life of the loan (we will be using 30 years), and the payment is simply that which will amortize the face value of the loan over its life. The typical fixed-rate mortgage contains a number of additional features. One of the most common, and the one we will concentrate on, is the option the borrower has to pre-pay the remaining principal of the loan before the end of its life. This option often carries a penalty if exercised early in the mortgage's life.

In its purest form, the ARM is a loan on which the contract interest rate is continuously varied. The periodic payment at any time is a payment that will fully amortize the remaining balance of the loan at the contract rate over the remaining life of the mortgage. In essence, it is a sequence of very short-term loans of varying contract rates. Because the interest rate on the pure ARM is continuously adjusted, it should always sell at par— that is, at a price equal to the remaining principal. This is the attraction of the ARM to institutions desiring to avoid the effects of interest rate movements on the net worth of their portfolio.

In reality, most ARMs are "impure." That is, the contract rate typically is adjusted only at intervals, and the size of the individual or total adjustments may be "capped." Later on, we shall discuss how these features can be incorporated into the pricing exercise.

II. The Options Pricing Model

Our pricing simulations rely on the observation that a mortgage may be viewed simply as a coupon-type bond with certain options attached to it. This equivalence allows us to use the bond option pricing model discussed in a previous *Economic Review*[1] to analyze the valuation of mortgages. It is useful to summarize briefly the essential steps in the model. The accompanying Box provides a numerical illustration of these steps, and a detailed description of the process is provided in the Appendix.

In essence, options pricing models rely on the observation that if a portfolio of options and the underlying security on which they are based can be constructed to yield the riskless rate of return, it is possible to infer the price of the option from the value of the underlying security and the riskless interest rate.[2] The actual mechanism for doing so involves three steps.

First, the possible future outcomes for interest rates must be specified. Options have a value only in a world in which there is uncertainty about future interest rates, that is, a world in which there is more than one possible outcome for them. The pricing simulation approach taken here begins with the assumption that the short-term, riskless interest rate is drawn from a log normal probability distribution. This diffusion process for interest rates can be approximated by a binomial representation and produces a "tree" of interest rate possibilities over time like the one depicted in the Box.

The nature of interest rate movements is controlled by the parameters of the underlying binomial distribution, the mean and the standard deviation. In the context of an interest rate process, the last two can be interpreted, respectively, as the annual rate of geometric drift (M) and the annual standard deviation or "uncertainty" of interest rates (S).

The second step is to recognize that, given the interest rate tree, it is possible to price a debt security like a bond by using these interest rates to calculate its present, or discounted, value. The process is relatively complicated when there is more than one future price for the bond, and this future price goes into the calculation of the bond's present value. Investors in this situation are assumed to calculate the present value of the bond by averaging the different future outcomes for its price. In our simulations, the calculations are made much easier by assuming a binominal distribution for the evolution of interest rates, which means that for each period there are only two possible future values for the bond.

If investors are risk-neutral, they are indifferent about the *dispersion* of possible future bond prices and use only their expected value (the average calculated using their probabilities as the weights) in calculating today's price. If investors are risk-averse, they will prefer an investment with a smaller dispersion of future outcomes over one with a larger dispersion, given that both have the same expected value. As the Appendix shows, risk aversion can be taken into account by introducing a risk aversion parameter, L, into the formula for calculating the present value of the bond. In this particular formulation, L can be shown to represent the "price of risk" as articulated by Dothan.[3]

The final step is to calculate the price of the option on the debt instrument. An option on the underlying instrument is simply a right to purchase or sell the underlying instrument at any time during the life of the option at a specified price, called the exercise price. If the option is a right to purchase, it is termed a "call" option; if it is a right to sell, it is temed a "put." The price of an option depends upon one of two things. First, it may be equal to the proceeds of exercising the option, which is the difference between the value of the underlying instrument and the option's exercise price. However, an option may have a greater value if it is held and exercised later. In that case, its price is determined by the value rather than the current exercise price.

The notion that permits us to estimate the value of an option before it is exercised is the notion of a riskless hedge. In particular, Black showed that by constructing a portfolio (consisting of options and their underlying instruments) that yields the riskless interest rate, the implicit value or price of the option can be inferred.[4] The actual computation is elaborated in the Appendix.

The sequence of steps outlined in this Section yields a mechanism for pricing a debt instrument and an option on that instrument in an uncertain interest rate world. All that is necessary to implement this model for the purpose of pricing mortgages is to recognize that a mortgage is a debt instrument and that many of its features can be viewed as options on a simple mortgage instrument.[5]

Estimating the Model Parameters

The model just described has three main parameters: the expected interest drift rate (M), interest rate uncertainty (S) and the risk aversion parameter (L), all of which must be estimated. The method adopted here to determine the relevant values of M, S, and L is to search over alternative values of these parameters and to compare the model's simulations of the yield on a simple, riskless debt instrument without options with that actually observed in the real world. The set of parameters that best replicates the actual series of yields is used in our subsequent analysis. More specifically, we use the set of parameters that minimize the sum of squared differences between the actual and simulated yields.

This procedure was employed using observations on ten-year U.S. Treasury Notes from 1982 and 1983. The model was used iteratively to find the coupon that generated a par valued instrument for each interest rate tree; the implicit yield to maturity was computed and compared to actual yields. Theoretical and empirical considerations allowed us to simplify the estimation process by setting M, the drift parameter, equal to zero.[6] The resulting estimates for the risk aversion parameter (L), and the uncertainty parameter (S), were .05 and .20 respectively.

The small but positive risk aversion parameter implies that the marketplace is characterized by risk aversion rather than a risk-neutral or risk-taking relationship between utility and wealth. This parameter obviously interacts with the uncertainty parameter in the computations, but it is informative to break them out separately since the former is a basic parameter of behavior, and as such is more likely to be stable over time than the uncertainty parameter, which is likely to be heavily influenced by ambient interest rate variability.

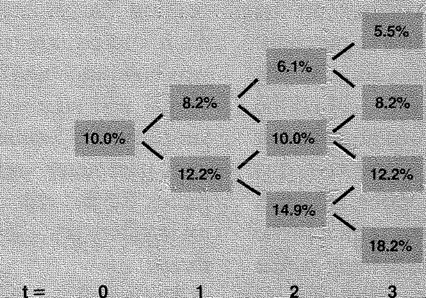
In any case, it is inappropriate to assign excessive meaning to the specific magnitudes of the parameter estimates. Both the simplifications inherent in the model and the estimation procedure suggest that they are at best useful as guidelines of market-wide parameters that may have been relevant to financial market behavior in 1982 and 1983. However, efforts were made to test the sensitivity of our simulated results to alternative values of these parameters. Also, we compare the model's simulations of yields on an actively-traded mortgage instrument with the instrument's actual yields as another way of checking the model's assumptions.

Pricing An Option

1. Future Interest Rate Movements

As the text describes, interest rates are a random variable. The investor, however, knows the probability distribution that generates interest rates; he can predict possible outcomes of future interest rates. For example, he knows that in the next period (t=1), interest rates either fall to 8.2 percent or rise to 12.2 percent, each with a probability of half. Similarly, in the following period (t=2) there is an equal probability that rates will either fall or rise. Thus, between t=0 and t=2 interest rates could follow one of four possible paths: rise in both periods, fall in both, rise then fall, or fall then rise. These outcomes are shown in the interest rate "tree" of Figure A, which also shows the possible outcomes in period 3, the period in which the bond matures.

Figure A
Interest Rate Tree

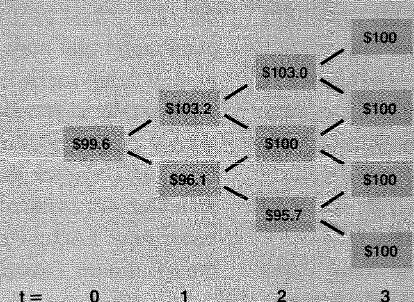


2. Pricing the Bond

The investor next needs to calculate the different prices the bond could possibly sell for in each period. The bond has a face value of $100, which is paid when the bond matures in period 3. In addition, a coupon of $10.00 is paid in periods, 1, 2 and 3. The only time when there is no uncertainty about the price of a bond is the last period, t=3. Here the bond is worth $110—its last coupon of $10 plus redemption value of $100—no matter what interest rates turn out to be.

In the next-to-last period (t=2), however, there are three possible outcomes, depending on what interest rates turn out to be. The price of the bond is calculated as the present (discounted) value of its future payments. Thus, in period 2, the future payments are the $110 to be paid in period 3. Using each of the three possible interest rate outcomes in period 2—6.7, 10.0 and 14.9 percent—to discount the $110, yields the three bond prices shown in Figure B.

Figure B
Bond Price Tree

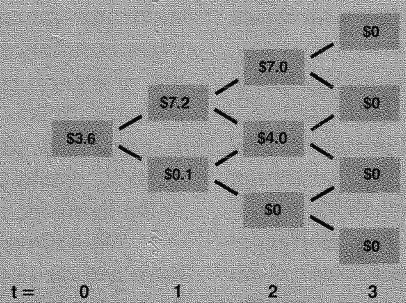


(In periods t=0 and t=1, things are a little more complicated because for each interest rate, there are *two* possible future values for the bond. In this example, investors are assumed to be risk-neutral, so that they *average* the discounted future values of the bond using as weights the probability attached to each to calculate its current price.)

3. Pricing the Option

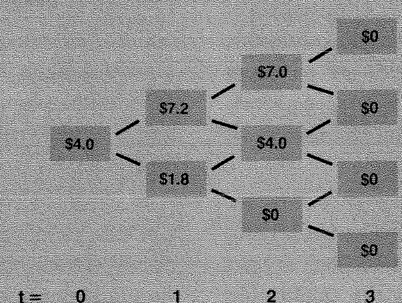
Recall that the option has an exercise price of $96, that is, the holder of the option has the right to buy the bond at a price of $96. Figure C shows the profit from exercising the option for each of the possible outcomes of bond prices. For example, if the bond sold for $100 in period 2, exercising the option then would yield a profit of $4.00 because the option holder could buy the bond for $96 and then sell it in the market for $100.

Figure C
Exercise Value Tree



Clearly, the option would never sell for less than this exercise value, as it is called. It could sell for more, however, because it may pay to hold the option and exercise it later. For example, in period 1, the exercise value is only ten cents if interest rates turn out to be 12.2 percent. If, instead, the option is held and exercised in the next period (t=2), there are two possible outcomes (see Figure D)—an exercise profit of either $4.00 or zero. Averaging these two possible outcomes and discounting them back to t=1 yields a value of $1.80. The option therefore will sell for $1.80 rather than ten cents in this case.

Figure D
Exercise Value Tree, If Option Held



III. Valuing the Fixed Rate Mortgage

We will first illustrate the pricing of a conventional fixed rate mortgage with a pre-payment option. Such an instrument can be viewed as a constant coupon bond with a call provision, and valued accordingly using the numerical bond and option pricing model. We assume that the pre-payment option can be exercised for a price equal to the remaining balance of the loan plus any pre-payment penalties.

We want first to see if the model can replicate observed pricing behavior in the market for fixed rate mortgages. There is no good source of data on origination rates on conventional mortgages. Therefore, we must employ data from the secondary mortgage market to test the model. One useful secondary mortgage market instrument is the Government National Mortgage Association Mortgage Pass-through Security (GNMA-PS).

The GNMA-PS, is a bond-like instrument that is guaranteed by the GNMA and which is based on a group of mortgages originated by private lenders largely under Federal Housing Administration (FHA) and Veterans Administration (VA) regulations. These are thirty-year, fixed rate mortgages with a prepayment option for which there is no penalty. The pass-through security essentially passes through to the owner of that "bond" the periodic interest and principal payments made by the mortgagees. The pass-through securities offer a number of advantages from our standpoint as a source of actual observations on the behavior of the mortgage market. First, the underlying mortgages are all of a similar type. Second, the GNMA-PS can be bought and sold like a conventional bond. In addition, the principal and interest payments are guaranteed by GNMA, making the instrument essentially free from default risk. This combination of features gives us a series of actual market valuations of a consistent set of mortgages with pre-payment options. The market's valuation of FHA-type mortgages should be reflected in the behavior of GNMA-PS yields.

To simulate the GNMA pass-through yields, we must first use the model to value the underlying mortgages. Each GNMA security states the contract mortgage rate that is in force on the underlying mortgages. Using this contract rate and the assumption of a thirty-year mortgage life, the periodic mortgage payment (that is, the interest and principal repayment), can be calculated using a simple mortgage amortization formula. From the viewpoint of the bond and option pricing model, this payment is the bond "coupon." The prepayment option, which is inherent in these mortgages, may be exercised without penalty. Thus, the exercise price of this call option at any time during the life of the mortgage is simply the remaining mortgage balance.

This information makes it possible to simulate both the current price of the underlying mortgage and the price of the prepayment option for any given set of interest rate diffusion assumptions, given the current short term interest rate. The net value of a mortgage with a prepayment option in the marketplace is simply the difference between the price of the mortgage and the price of the option that it contains. This is because, as far as the marketplace is concerned, the mortgage borrower receives a valuable option at the time that he obligates himself to the mortgage payments.

We will call the difference between the bond price and the option price the net price. This is the price at whith the GNMA-PS should sell if the model and the interest rate assumptions are appropriate. In fact, of course, mortgage "prices" are usually quoted for convenience as implicit yields rather than as prices. Quoted GNMA-PS yields are derived on the assumption that the net price applies to an instrument with a 12-year life, that is, the mortgages are assumed to be prepaid in 12 years.

Given the net price of the mortgage and its contractual periodic principal and interest payments, we can calculate the implicit yield of a 12-year GNMA-PS. The yield is simply the discount rate which, when applied to the principal payment made in the terminal period and the stream of coupon payments, yields a discounted present value equal to the simulated net price of the mortgage.

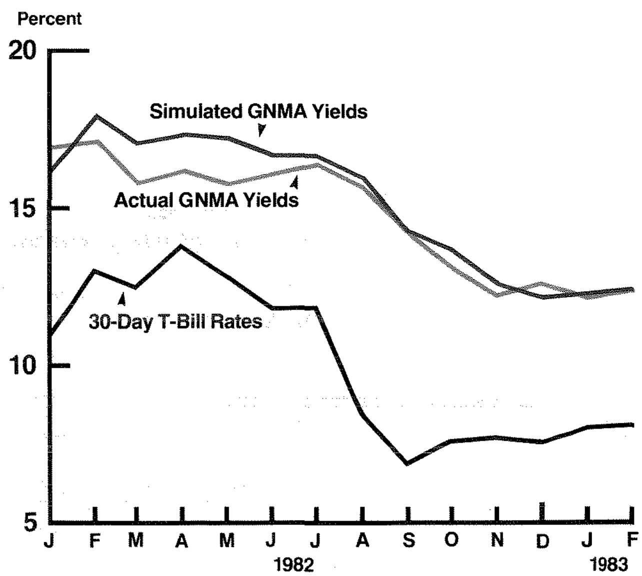
In Chart 1, we present simulated and actual GNMA yields produced by the bond and option pricing model over a period of 14 months. The simulations use the actual 30-day T-bill rate as the starting point for each simulated interest rate tree and the interest rate diffusion parameters and the risk aversion parameter estimated earlier.[7] The actual and forecast yields are quite similar, despite the fact that there was considerable variation in short-term interest rates over the period of simulation—1982 to early 1983. Clearly, many purely statistical models could perform this replication as well as or better than our model. The advantage of our model is that it permits simulation of hypothetical instruments, which a purely statistical model might not. The performance of the model, in replicating yields on an actual instrument is encouraging and provides some empirical basis for believing that the simulations that follow may synthesize what would occur in the real world.[8]

Further Explorations

Unlike the FHA mortgages examined in the last section, most FRMs contain a penalty for prepayment of the mortgage principal. A typical prepayment penalty applies only for the first five years of a mortgage and is usually stipulated to be six months' interest at the mortgage contract rate or less, but under current regulations, the lender is free to set the penalty conditions at will.[9] In this section, we examine the sensitivity of FRM yields to variations in the prepayment penalty conditions. We also examine how sensitive FRM yields are to the underlying interest rate and risk aversion parameters of the model.

Since we are simulating a hypothetical mortgage, the steps in this simulation are somewhat different from those in the GNMA-PS case. For example, unlike the GNMA case, the mortgage contract rate is not an administered rate but, rather, will itself be determined as part of the simulation. For the given interest rate diffusion assumptions, the contract mortgage rate that yields the par value of the mortgage without an option is computed first. The price of a pre-payment option on this "mortgage" is then computed and subtracted from the pure (par value) mortgage value to get the net price. Once again, however, mortgage "prices" are usually stated as contract rates, not prices. Thus, we need to build the value of the option into the mortgage contract rate. To do this, the mortgage contract rate is increased by an arbitrary, small amount in the option computation until the net price calculation equals original mortgage par value. We are thus able to determine the contract rate spread between a mortgage without a prepayment option and one with the stipulated

Chart 1
30-Day T-Bill Rates and Simulated and Actual GNMA Yields



option and penalty.[10]

Simulations of this type were carried out over a wide range of model parameters and for three mortgage penalty configurations: a penalty of zero, a penalty equal to six months interest for the first five years, (typical of conventional penalties) and a penalty equal to six months interest applicable to the full 30-year life of the mortgage. The results of these simulations along with that of the simple mortgage without a prepayment option, are presented in Chart 2.

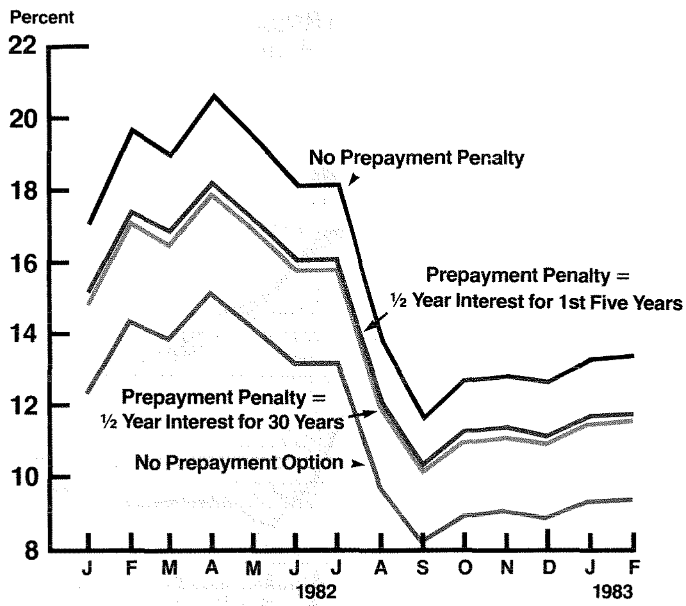
The Chart illustrates clearly the importance of appropriate pricing of the prepayment option. For example, with the "market" estimates for risk aversion and interest rate uncertainty, the market yield differential between a mortgage without any prepayment option and one with an option but no prepayment penalty, is nearly 400 basis points (Chart 2). Charging the conventional penalty reduces this spread to less than 250 basis points, and charging a penalty equal to six months' interest for the lifetime of the instrument decreases it a further 25 basis points.

Chart 3A illustrates the sensitivity of the appropriate mortgage yield to the anticipated level of interest rate uncertainty. With no interest rate uncertainty, the option—which provides the borrower with protection against uncertainty—has no value and penalty variations are, of course, meaningless. As interest rate uncertainty increases, the value of the prepayment option increases and should be manifested in higher market yields.

The results depicted in Chart 3B are perhaps of more interest to the modeler than the maker of pricing policy. They test the sensitivity of our simulated results to the parameter that describes the assumed level of risk aversion that prevails in the economy. The sensitivity of the model to this parameter indicates the hazards of incorrect parameterization of the model. Our own investigations, discussed earlier, suggest that L should be approximately .05, if the model is to approximate closely the Treasury Note yields actually observed during the 1982 estimation period. The importance of this parameter to the simulations, however, suggests that more refined procedures for estimating L may be desirable.[11]

Chart 2

Simulated Fixed Rate Mortgage (FRM) Contract Rates

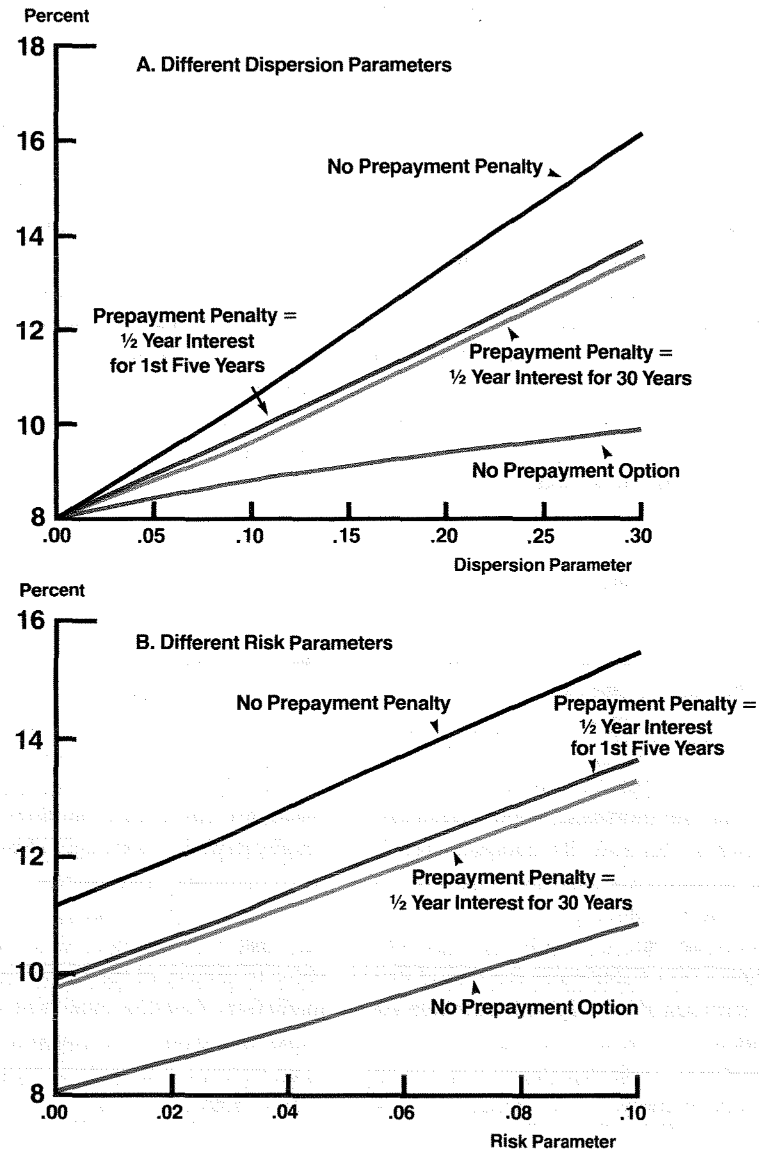


Some Qualifications

There are, of course, many qualifications to these findings that should not be overlooked by the reader. First, because we ignored the inherent option available to the borrower to default on the mortgage, the mortgage yields reported in Charts 2, 3A and 3B are very likely understated. However, the process of modelling the conditions under which a default option will be exercised by the borrower are complex and beyond the scope of this paper. In addition, lender losses from default are extremely small in practice suggesting that the existing explicit and implicit cost to the borrower or default make that option seldom worth exercising, and therefore very likely of low value.

Second, the mortgage instrument modelled above only allows for "economic" prepayment of a mortgage. That is, we implicitly assume that the

Chart 3
Simulated Fixed Rate Mortgage (FRM) Contract Rates



prepayment option is exercised only when it makes sense because of the relative market value of the mortgage and the exercise price of the option. We do not allow for "exogenous" motives for prepayment such as death, changes in taste about the underlying real estate, job transfers, and so on. These factors may be important in the real world and affect equilibrium market mortgage yields. In essence, we assume there is no value associated with the sale of the underlying real estate prior to the maturity of the loan. Thus, if the mortgage contains a "due on sale" clause, which is essentially an option owned by the lender, our model has essentially assumed that the value of this option is zero. If in fact, exogenous forces do precipitate sale of the underlying real estate, then the "due on sale" option would have a value greater than zero and our simulations would overstate the market yield. (An option with positive value owned by the lender would be incorporated into a lower mortgage contract rate.) This is a shortcoming of the simulations, but at the present time there is insufficient data to model the "premature sale" phenomenon. Moreover, the ability of the model to simulate GNMA-PS yields offers some justification for ignoring this shortcoming of the model.

Third, the model ignores transactions costs. This criticism affects both the bond and option pricing model itself, and the mortgage simulations presented earlier. Most such costs are likely to be relatively minor and therefore unlikely to affect the results of the simulation substantially. Other "transactions costs" such as the points typically paid by the borrower at the time the mortgage is originated, are not really transactions costs but rather a different way of pricing a mortgage. We have assumed that the lender and the borrower are indifferent between the pricing of a mortgage feature via yield premia and by "up front" money in the form of points. Thus, all of our simulations assume no payment of points. In fact, of course, tax and cash flow considerations may make it more attractive for a lender to receive payments in the "up front" form. These considerations are too cumbersome to be usefully modelled here and, again, are unlikely to affect the simulated results in a substantial way.

Finally, our simulations abstract from any general equilibrium consequences of mortgage market behavior on interest rates in general. The model takes as given the initial and anticipated future short-term riskless interest rate and assumes that there is no important feedback from the mortgage market to this rate structure. Such an assumption seems reasonable in the limited context of our efforts here.

IV. Fixed Rate Versus Adjustable Rate Mortgage Pricing

Because many mortgage lending institutions are using the adjustable rate mortgage to insulate their portfolio from the interest rate risk inherent in fixed rate instruments, it would be interesting to compare the simulated fixed rate mortgage results with those that apply to an adjustable rate instrument. For a "pure" adjustable rate mortgage, such a comparison is quite simple: because its contract rate is assumed to be adjusted continuously and with a ceiling or floor, the instrument always sells at par and its initial contract rate is simply the then-prevailing short rate. Chart 4 illustrates the spread that would prevail between the initial contract rate on such an instrument and the contract rate on a conventional 30-year fixed-rate mortgage with typical prepayment terms (namely, a prepayment penalty equal to 6 months' interest if prepayment is made in the first 5 years).

The large spread between the two contract rate graphs shown in Chart 4 demonstrates that the advantages of insulation from interest-rate risk offered by the adjustable-rate mortgage are only obtained through significant reduction in the rate of return obtained on the mortgage. (Since the expected drift of short-term interest rates over the thirty-year period is 0, the difference is due entirely to interest rate risk.) In essence, this finding illustrates the value to society of the traditional interest rate intermediation function that had been performed by banks and other financial institutions. Conversion of an institution's portfolio to adjustable rate instruments (both on the assets and liabilities side of the balance sheet) is tantamount to abandoning the interest rate intermediation function. What the simulation suggests is that the expected earnings of such risk-insulated institutions will be much lower than those that continue to offer interest rate intermediation service.

A second observation to be made from Chart 4 concerns a practical aspect of ARM pricing. In our simulations, there are no transactions costs, operating costs, or other costs of administering a mortgage lending business. Thus, it should be kept in mind that the simulations presented, even if fortuitously correct in other aspects, underestimate the actual market yield that would be observed. Rather than use an arbitrary figure to account for these omissions, we simply underscore this inherent assumption of our model.

Impure ARMs

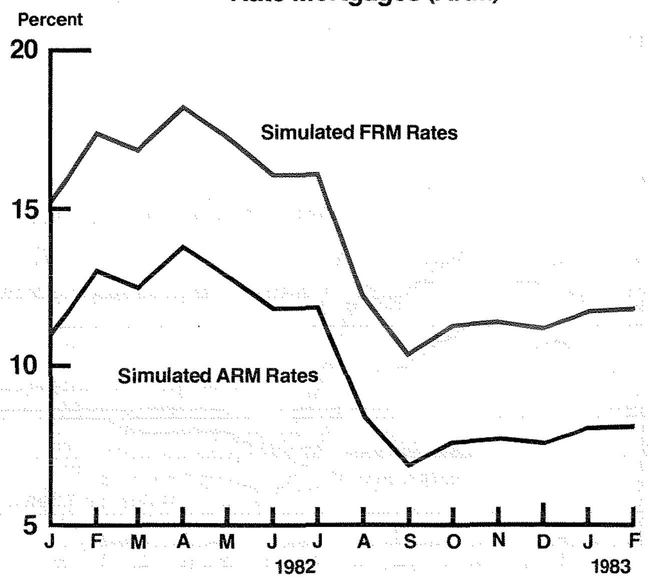
An obvious liability of the adjustable rate mortgage simulations presented above is that they do not incorporate features typical of such mortgages in the real world. In particular, most real world adjustable rate mortgage contracts do not permit continuous and unbounded adjustment of the contract rate. Rather, the rate is usually adjusted only at intervals (say, every six months) and the upward range of adjustments is often "capped" so that the rate may rise only some maximum amount over the life of the instrument. This cap is often expressed as a certain number of percentage points above the initial contract rate.[12]

Qualitatively, such features would appear to make the ARM more nearly a fixed rate mortgage. Thus, such "impure" ARMs would tend to have a contract rate somewhere between the pure ARM rate and the rate on a fixed rate mortgage.

Simulating precisely the impact of such features on ARM contract rates is not a trivial exercise, but it can be addressed in concept by the bond and option pricing model employed here. To illustrate how such simulations might be carried out, we focused on a simplified "capped" adjustable rate mortgage. We ignore the complication of infrequent rate adjustment and continue to assume that rates can be adjusted in every period of the simulation. We assume, as in the fixed rate mortgage simulations, that there is a prepayment option but that the

Chart 4

Simulated Contract Rates for Standard Fixed Rate Mortgages (FRM) and Pure Adjustable Rate Mortgages (ARM)



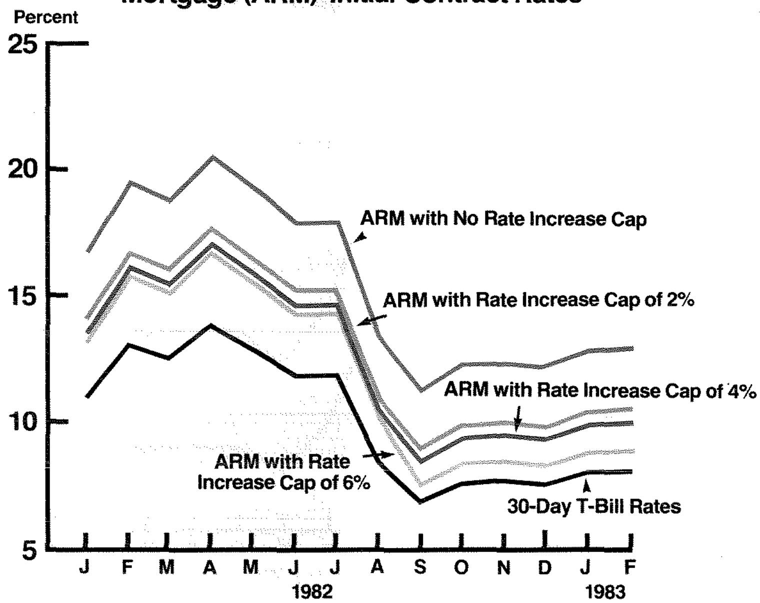
penalty, as is common practice, is zero. In addition, we assume that if there is a "due on sale" clause, the value of this option is zero. (That is, the exercise of the prepayment option is always an "economic" consideration rather than one based on exogenous real estate trading motives.)

The modelling of variable rate mortgages is made easier computationally if a special variant of this instrument is employed in lieu of the "pure" instrument described earlier. In particular, we described an adjustable rate mortgage earlier as an instrument which continuously recomputed the periodic payment using a short-term rate as the contract rate, the remaining life of the mortgage, and the remaining principal at each period. A similar but computationally less cumbersome variant is a loan whose periodic payment is based on a simple interest rather than amortization rate computation; specifically, we model a loan which would probably best be called a "floating rate" loan rather than a conventional adjustable rate loan. That is, we assume that the principal amount of the loan is paid off in equal periodic increments but that interest is paid each period at the current (short) rate on the remaining principal. This loan is quite similar to that employed in commercial lending, and should serve to demonstrate the basic elements of the pricing of fluctuating rate instruments.[13]

Except for the payment adjustment convention described above, such an instrument resembles once again a coupon-type bond, and the basic bond and option pricing approach described earlier can be employed. It should be noted, however, that the contract rate cited in the results reported below is the initial contract rate necessary to give the instrument par value; this rate is adjusted up or down over the life of the mortgage in direct proportion to the changes in short market rates, with a maximum value equal to the "cap" rate when applicable.[14]

The results of simulations of these instruments with several cap alternatives are presented in Chart 5. They lead to a number of interesting observations. First, variation in the cap provision of the variable rate mortgage has a significant effect on the simulated initial contract rate of the variable rate instrument. As expected, the less binding the mortgage rate cap, the lower the effective contract rate of the mortgage. Conceptually, as was pointed out above, a pure, uncapped ARM would have an initial contract rate equal to the prevailing short-term in-

Chart 5
30-Day T-Bill Rates and Simulated Adjustable Rate Mortgage (ARM) Initial Contract Rates



terest rate. It is interesting to note that the spreads between the contract rates are smallest when the prevailing short-term interest rates are high, and greatest when short-term rates are low. This is a result of the use of additive interest rate caps which allow for greater relative movements in the value of low-rate mortgages than in the value of high-rate mortgages. Thus, in some sense, a six percent cap on a six percent mortgage is "less binding" than a six percent cap on a twelve percent mortgage. This suggests that "mark-up" rules of thumb in pricing variable rate mortgages with various caps probably should not be employed by mortgage lenders. (Note, in addition, that these simulations assume that all the other parameters of the simulation except the short-term interest rate are unchanged.)

Second, the spread between the simulated contract rate and the prevailing short rate—even for caps as low as 2 percent—is within several hundred basis points of the short-term interest rate. These results are at variance with what is—albeit anecdotally—observed in the real world. Lenders appear to assume that even essentially "uncapped" (that is with caps of 6 percent and greater) variable rate mortgage loans should be priced at several hundred basis points above short rates. These results indicate that the simplistic ARM pricing mechanisms that have been observed in the market have resulted in "over-pricing" of ARMs.[15] Because of the simplifications employed in the model, it is easy to make too much of this observation. However, it may help to explain why ARMs were not widely accepted in the marketplace when initially offered in their "pure" form.

This analysis also illustrates an important point about the use of ARMs by lenders who hope to limit the consequences of interest rate risk. Because they offer the borrower no protection against interest rate changes, the lender is in essence performing no interest rate intermediation function and the market "price" of variable rate instruments contains no implicit compensation for this role. In the real world, any compensation above the short-term interest rate offered by variable rate mortgages will be compensation for other functions performed by the lender, such as denomination intermediation and assumption of default risk. Neither of these functions is implicitly or explicitly captured in our model, but they are certainly minor relative to the role of interest rate intermediation. The results thus illustrate an important lesson about a lender seeking protection from interest rate risk through origination of variable rate instruments: there is very little income potential to such activity.

Some Further Qualifications

A few additional qualifications are in order because of the simplifications inherent in the simulated instruments. We assume, for example, that the variable rate mortgages' contract rate can be adjusted continuously. In the real world, the lender can also elect to limit both the frequency and the amount of individual contract rate adjustments. In general, such features will tend to raise the appropriate contract yield above that presented in Chart 5. On the other hand, many real world variable rate mortgages contain a limitation on the rate of downward adjustment of interest rate as well. The effect of such a provision will be to lower the appropriate initial contract yield of a variable rate mortgage. Although it is perfectly feasible to incorporate such features in the simulations, we have chosen not to do so for simplicity of presentation and our preference to focus on the major features of these instruments.

A second qualification concerns the particular type of adjustable rate instrument employed in our simulations. It should be recalled that the instrument modelled here does not really re-amortize the remaining mortgage principal as the contract rate is adjusted; the principal repayment schedule remains the same, with only the interest component of the payment changing as the "contract" rate changes over time. From some experimental simulations, it was determined that the computational advantages of this assumption far outweigh any imprecision that was introduced. Nonetheless, it should be kept in mind that the ARM instrument modelled in this paper approximates the instruments employed in the real world. However, we believe that the approximations are good, at least for the parameter range presented in Chart 5.

Just as in the case of the FRMs, the simulated results are quite sensitive to the risk and dispersion parameters. In general, the larger the assumed level of risk aversion or the level of future interest rate uncertainty held by the marketplace, the greater is the yield on the individual instruments and the greater the spread between their contract rates. Increased interest rate uncertainty has a smaller effect proportionately on the fixed rate instrument and the capped adjustable rate instrument. This result is to be expected because of the relative immunity from changes in value that are enjoyed by variable rate instruments when interest rates change.

V. Summary and Conclusions

This paper has applied a simple, numerical bond and option pricing technique to the problem of pricing mortgage instruments. The model was applied to the problem of pricing fixed rate mortgages with prepayment options and to both ''capped'' and ''uncapped'' variable rate mortgages. As a crude test of the basic robustness of the model, it was used to simulate the yields on GNMA pass-through certificates and performed quite well.

The results of our investigation have a number of analytical and policy implications. First, the results suggest that the model used here can be a helpful guide to determining appropriate mortgage pricing policy for many typical instruments. For example, a lender could use these techniques to explore the effects that changes in mortgage features will have on average mortgage yields. In such a case, the modeller would obtain and employ market estimates of the parameters of the model. The model also gives its user the flexibility of comparing simulations using the market's perception of interest-rate variability with simulations incorporating the user's own assessment. In this way, the user can evaluate the wisdom and consequences of pricing the instruments at the ''market'' rate.

Second, the model underscores the importance of considering contingent claims features of debt instruments when examining their behavior in the marketplace. The fact that the yields on GNMAs, for example, are typically higher than other risk-free instruments has sometimes been ascribed to differences in the liquidity of GNMAs versus Treasury instruments. The model simulation suggests, however, that the spread between GNMA and Treasury instruments is explained by the value of the prepayment option implicity in the mortgages that underlie the certificates. (In fact, if our simulations are accurate, this is the major explanation for the difference in the yields of these two classes of instruments.)

A third, more tentative finding of the simulations is that the early problems encountered in marketing adjustable rate mortgages may have been due to their ''overpricing'' relative to existing short-term market rates of interest. There is some evidence that lenders price even quite ''pure'' variable rate mortgages by simply adding a few hundred basis points to the short-term rate. Our simulations suggest that such compensation cannot be justified on the basis of interest rate risk considerations. (Alternatively, of course, fixed-rate mortgages may have been ''underpriced,'' but this implication is inconsistent with the model's close replication of the yields of these instruments in the secondary market.) These observations must obviously be regarded as tentative since our simulations employ a number of simplifying assumptions. It is useful to note, however, that the ''pure'' adjustable rate mortgage has thus far failed to obtain a major presence in the marketplace; what recent growth has taken place in the popularity of ARMs has coincided with more binding caps on these mortgages, making them more nearly fixed rate instruments. It is conceivable that these developments represent the marketplace's (inadvertent) evolution toward a proper pricing strategy for these instruments.

A final and related point concerns the use of adjustable rate instruments in lenders' portfolios as a means of avoiding interest rate risk. Our simulations indicate the magnitude of the trade-off between higher portfolio yields and the interest rate risk inherent in these portfolios. Although adjustable rate mortgages offer the lenders protection against interest rate risk, they do so at considerable sacrifice of expected yield. Financial institutions must decide for themselves whether their function is simply one of denomination intermediation and default risk assumption, or whether they wish to provide interest rate intermediation services in the residential mortgage market.

Appendix: Details of the Bond and Option Pricing Model

As stated in the text, short-term real interest rates are assumed to be drawn from a log normal distribution approximated by a binomial period. Starting from the current short-term riskless rate, the alternative paths of future short-term riskless rates are determined by combinations of up-jump and down-jump ratios. That is, the interest rate in period T can take one of the following values:

$R^u_t = UR_{t-1}$

$R^d_t = DR_{t-1}$

where

U = Jump-up (rise in interest rates)

D = Jump-down (fall in interest rates)

We assume that the ratios of the two possible interest rate movements are constant. This makes the relation between interest rates over time multiplicative and enables us to use an interest rate tree for which every period t has t elements instead of one with 2^t elements.

Given these alternative interest rate paths, bond prices at any instant are defined as

$$B(t) = [.5(1+L) \times B(t+1)^U + .5(1-L) \times B(t+1)^D + C]/(1+R_t)^{1/N}$$

where

L = risk aversion parameter

$B(t+1)^U$ = Price of bond in period t + 1 if interest rates rise

$B(t+1)^D$ = Price of bond in period t + 1 if interest rates fall

C = per period coupon payment

R = prevailing interet rate

N = Number of periods per year

Thus, the greater the market's risk aversion, (that is, the greater is L) the more weight is given to the up-jump state, and for a given coupon the lower the bond price.

The proceeds from exercising a call option on such a "bond" in period t are equal to B(t) − E(t), where E(t) = the exercise price of the option in period T.

The proceeds for a put option can be expressed as E(t) − B(t).

In both cases the option is assumed to be of the American type, that is it is able to be exercised at any time during its life. The price (OP) of an option at any point in time is therefore the maximum of the proceeds from exercise or its value if held for future exercise (FV). More precisely,

$$OP(t) = Max[E(t) - B(t), FV(t)]$$

From the notion of a riskless hedge, it can be shown that the value of holding the option for future exercise is equal to:

$$FV(t) = (.5 \times OP(t+1)^U + .5 \times OP(t+1)^D)/(1+R_t)^{1/N} \qquad (1)$$

where $OP(t+1)^U$ = the option price in period t + 1 if interest rates rise

$OP(t+1)^D$ = the option price in period t + 1 if interest rates fall

since the price of the option is known with certainty only at the end of its life (that is, its price is zero at that time) solving for the current price of the option involves working "backwards" in time using the above relationships. The authors have written FORTRAN programs that perform this general numerical computation procedure.

The procedure described above is entirely general and may be applied to any financial instrument that can be described as a finite series of "coupon" payments, however irregular. In addition, the exercise price and exercise conditions may be varied at will permitting quite complex instruments to be valued in a simple manner.

Most of our applications in this paper were directed at valuing mortgage type (that is, self-amortizing) instruments. We employ standard formulae for computing the periodic payments for a self-amortizing instrument and for computing its remaining principal balance. The periodic payment is assumed to be

$$C = (PR \times CR)/1-(1+CR)^{-NPT}$$

and the remaining balance (RB) in period t can be computed from the formula

$$RB(t) = PR \times (1 + CR)^{t-1}(1 - D)$$

where

PR = Principal

CR = Contract Rate

$$D = \frac{1 - (1 + CR)^{-(t-1)}}{[1 - (1+CR)^{-NPT}]}$$

NPT = Total number of periods in the life of the investment

The relationships and procedures presented in this Appendix represent the basic computations employed in the various simulations presented in this paper. As this paper suggests, however, the computational details are influenced by the type of instrument simulated and the objectives of the simulation exercise. In some cases, only the bond pricing computations are necessary. In others, both the bond pricing and option pricing procedures are employed.

FOOTNOTES

1. Randall J. Pozdena and Ben Iben, "Pricing Debt Instruments: The Options Approach," Federal Reserve Bank of San Francisco **Economic Review** Summer 1983. pp. 19-30.

2. See Richard Rendleman and Brit Bartter, "The Pricing of Options on Debt Securities," **Journal of Financial and Quantitative Analysis**, March 1980, pp. 11-24.

3. See Brennan and Schwartz, "Bond Pricing and Market Efficiency," **Financial Analysts Journal**, September-October 1982, p. 49-56.

4. The original Black and Scholes paper is, Fisher Black and Myron Scholes, "The Pricing of Options and Corporate Liabilities," **Journal of Political Economy**, May 1972, pp. 637-654.

5. For a review of options terminology, see Pozdena and Iben, **ibid**. p. 20.

6. In our earlier work, we estimated interest rate drift and uncertainty parameters using a simple time series estimation technique on actual short-term interest rates. That investigation yielded an estimate of interest rate drift for the period studies here of approximately zero. In addition, if interest rate movements are viewed as being generated by a mean reverting process, there may be theoretical justification for assuming that the annual rate of interest rate drift is zero over a long horizon. Finally, as a practical matter, our experience with the model suggests that the qualitative findings of our simulations would not be significantly affected by the use of a non-zero drift parameter and the presentation of the results would be made significantly more cumbersome if a third parameter dimension were incorporated.

7. The data on actual GNMA yields was obtained from various issues of the **Weekly Bond Report**, Solomon Brothers, New York.

8. It should be noted, however, that there is one sense in which this simulation overstates the performance of the options model. One of the pieces of information used in creating the simulated GNMA yields is the contract rate on the mortgages that underlie the pass-through certificate. Although this rate is an administered rate, it is adjusted periodically as conditions in the mortgage market in general change. Thus, it is not a purely arbitrary figure, but rather, contains some market information. Since this coupon stream is incorporated into our valuation, our estimated yields are probably somewhat better than they otherwise would be.

9. At the time of this writing, the Federal Home Loan Bank Board has removed regulations affecting prepayment penalty clauses in mortgage contracts. Mortgages made by state chartered institutions may not be similarly deregulated at this time.

10. This procedure of "capitalizing" the value of the option onto the contract rate of the mortgage in an iterative procedure and, although convergent, is carried out in our computation a limited number of times. Therefore, our estimates are themselves approximations and contain small approximation errors.

11. Our estimation procedure was a semi-manual one. A more sophisticated approach would incorporate the simulation model directly in a three variable optimization program.

12. In practice, "caps" often apply to movements in rates in both directions. The contract rate on a mortgage with an initial rate of 10% and a cap of 4% is thus restricted to the range of rates between 6% and 14%. We do not incorporate the downside rate limitation feature in our simulations here. The implications of this simplification are discussed below.

13. Many commercial loans are so-called "floating rate" loans. In general, these are "bullet" type loans which obligate the borrower to payments of interest during the life of the loan with repayment of principal at the end of the loan's life. Often, however, there are either explicit provisions or incentives for earlier repayment of a portion of the principal value of these loans. In this sense, the type of loan specified here is a variant of such a floating rate loan. We are simply more explicit about the principal repayment schedule, linking it to the repayment schedule that would apply on a fixed rate self-amortizing instrument.

14. The actual simulation procedure is quite cumbersome and can only be outlined briefly here. Essentially, the aim of the simulation is to discover an initial contract rate for the ARM which par values the instrument, recognizing that the mortgages contains a prepayment option which must be "capitalized" into the contract's yield. In the initial period of its life, the adjustable rate mortgage has payments that are precisely those that would obtain on a thirty year, fixed rate mortgage with similar prepayment option features.

15. A forthcoming survey of mortgage loan features conducted by the Federal Home Loan Bank of San Francisco supports these observations.

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