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The Analytics of German Monetary Unification

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This paper studies a situation in which two previously isolated countries decide to unite their currencies and their fiscal policies. We assume that initially there is a "soft currency" country and a "hard currency" country. Given fiscal policy, we study the range of exchange rates of "soft" for "hard" currency that are feasible set. The inflation rate under the new consolidated government depends on the fiscal policy it follows, but does not depend on the exchange rate selected.

On July 2, 1990, East and West Germany became united through a common currency. The West German *Deutsche Mark* (DM) became the only legal tender on both sides of the border, and debts and payments denominated in the East German *Ostmark* (OM) were converted to DM at rates stipulated in an agreement signed by both governments on May 2.

The monetary union of East and West Germany raises a variety of issues, including the consequences of choosing one conversion rate over other possible rates, the price level implications of the conversion, and the welfare implications of the conversion for citizens of the two countries. To shed light on some the issues involved, this paper provides a theoretical analysis of German monetary unification.

Our analysis relies on a standard model of money, specifically, the overlapping generations model of Samuelson (1958). Although other models, such as the cash-in-advance model, are available, our key conclusions depend on aspects of the model that would appear in virtually any model of money, namely, the budget constraints of the two governments and the demand for fiat currency in each of the two countries being a function of the rate of return on currency. Thus, very similar results would emerge from these other models.

We analyze two countries which initially manage to isolate themselves, so that neither country trades with or borrows from the other, nor do the residents of one country hold the currency of the other. One country balances its budget and thereby supports a zero-inflation monetary system. There is also a country that runs a persistent government deficit and finances the deficit by a combination of inflation tax and repressed inflation. We model repressed inflation as a legal restriction or rationing scheme that forces citizens to hold more currency than they voluntarily would. This produces a "currency overhang" and repressed inflation. These legal restrictions are to be interpreted in the manner of Bryant and Wallace (1984) as devices to increase the base of the inflation tax.

We refer to the first country as the "hard currency country" because the value of its currency is stable over time (there is zero or low inflation), and people hold and exchange its currency voluntarily. We refer to the other country as the "soft currency country" because its currency lacks one or both of those attributes: the value of its currency is deteriorating over time, and/or particular classes of people (typically, citizens of the soft currency country) are required to hold some of its currency involuntarily, either through explicit savings requirements or as a consequence of a commodity rationing scheme.

We compare the initial situation with a second one which we call monetary union: in the former soft currency country, the controls that forced residents to hold the soft currency are dismantled. The currency and credit markets are united with those of the hard currency country. In the process, the new, consolidated government chooses a rate at which the old, soft currency will be exchanged for the new, single currency. We study how the inflation rate in the unified monetary system depends on the fiscal policy of the new government. We show that there is a range of rates that can be sustained as equilibrium exchange rates, and we study the welfare consequences of a choice in this range.

I. Overview

In this section, we provide a brief overview of our arguments and results. Our reasoning exploits properties of two basic relationships: a demand function for government-issued currency, and the government's budget constraint.

In the model we use, money is held voluntarily by agents to an extent determined by the return on currency. Since currency does not pay explicit interest, the real rate of return on currency is the change in its purchasing power. Since we prefer to work with *gross* rates of return (one plus the net change), we denote the rate of return on currency from t to $t+1$ as $R_t = p(t)/p(t+1)$, where $p(t)$ is the price level at t. We assume that the real demand for currency in a country is an increasing function of R_t, which we denote by $f(R_t)$; the nominal supply, or stock of currency at t is $H(t)$, and $f(R_t) = H(t)/p(t)$.

A government can raise real revenues by generating inflation, thereby imposing an inflation tax on people who hold currency from t to $t+1$. The *base* for the tax is $f(R_t)$, the real amount of currency held, while the *rate* of the tax is $1-R_t$. The government's budget constraint at t can be written as

$$\frac{H(t) - H(t-1)}{p(t)} = D, \tag{1}$$

where D is the real value, assumed constant over time, of that portion of the deficit financed by currency creation. This budget constraint can be written as

$$\frac{H(t)}{p(t)} - \frac{H(t-1)}{p(t-1)}\frac{p(t-1)}{p(t)} = D$$

or

$$f(R_t) - f(R_{t-1})R_{t-1} = D.$$

In a steady state situation, $R_{t-1} = R_t = R$, so the above equation becomes

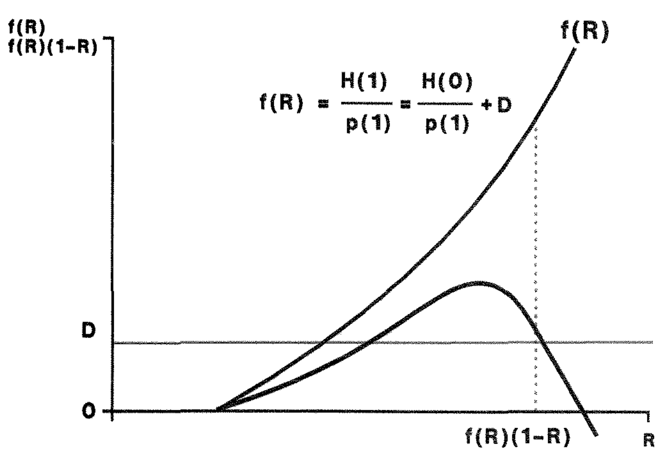
$$f(R) \times (1-R) = D$$

base of inflation tax — rate of inflation tax

which decomposes the amount of inflation tax collected into the product of the base for the tax and the tax rate.

When the demand for currency is an increasing function of R, the inflation tax revenue function $f(R)(1-R)$ is as depicted in Figure 1. As R rises from some low value, $f(R)(1-R)$ initially rises because the base of the tax $f(R)$ rises faster than the rate $1-R$ falls. Eventually, however, as R rises toward 1, that is, as inflation falls to 0, $f(R)(1-R)$ begins to fall toward 0. Notice that, as a result of the curve's shape, if there exists one tax rate that finances a

Figure 1



steady state deficit D, then there are in general two such rates. For reasons indicated below, we will assume that we are always in the "good" equilibrium (with a higher R or, equivalently, a lower inflation rate).

For a single closed economy, Figure 1 can be used to determine the steady state equilibrium value of R, and an initial price level $p(1)$ at some time $t=1$. First, the equilibrium R is determined by the intersection of $f(R)(1-R)$ with the deficit D. Then, given that value of R, equation (1) written at $t=1$ can be manipulated to yield an equation that determines $p(1)$ as a function of some initial inherited currency stock $H(0)$:

$$\frac{H(1)-H(0)}{p(1)} = D$$

or

$$f(R) = \frac{H(1)}{p(1)} = \frac{H(0)}{p(1)} + D.$$

This equation can be solved for $p(1)$ as a function of D and $H(0)$. We can use Figure 1 to pick off the value of $f(R)$ associated with the equilibrium R.

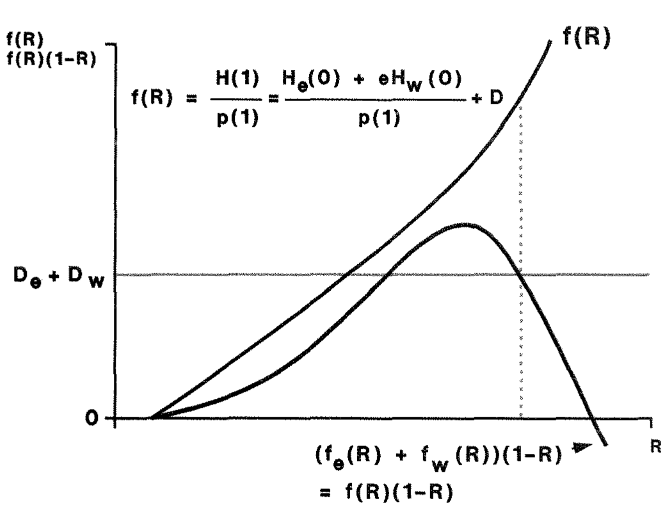
Our model of East and West Germany before unification describes the two separate economies using two versions of Figure 1, one with a very low D, the other with a high D. The country that runs a low deficit D attains a high return on money R and a low inflation rate. The country with a higher D attains a lower R, assuming it is willing to allow the price level to be determined freely by the supply of and demand for its currency. Later in this paper, we describe some measures that a government can take to enhance artificially the demand for its currency. Using a version of Figure 1, we shall show how such measures can be used to raise the base of the inflation tax and reduce the tax rate needed to finance a given deficit. We represent East Germany as having resorted to such measures.

Our approach to studying currency unification can be summarized by constructing a figure as the vertical summation of the two versions of Figure 1. At some time $t=1$, we suppose that the two countries open their borders and consolidate both their currencies and their government budgets. The stock of the new currency is the sum of the old western currency and the old eastern currency multiplied by an exchange rate e: the old eastern currency is, in effect, exchanged for the new currency at a rate of e DM per OM. This means that the currency stock inherited at time $t=1$ from the old regime is $H_W(0) + eH_E(0)$, where the subscripts W and E refer to West and East, respectively. We want to study the consequences of alternative values of e. The unified monetary-fiscal authority assumes the old deficits, so that the deficit of the unified government is simply $D = D_E + D_W$. The demand for the new currency is $f(R) = f_E(R) + f_W(R)$, so that the inflation tax revenue is $(1-R)\,[f_E(R) + f_W(R)]$.

Figure 2 depicts the equilibrium values for R and $p(1)$ in the new regime. Inspection of that figure shows that whether an equilibrium exists in the new regime does not depend on the value of the exchange rate e. Indeed, if an equilibrium exists, there are many values of e compatible with that equilibrium.[1] A stationary equilibrium depends only on the size of $D_E + D_W$ relative to the maximum height attained by the inflation tax revenue function $(1-R)(f_E(R) + f_W(R))$. When a stationary equilibrium exists, the value of e influences the value of the price level $p(1)$: the higher is e, the higher $p(1)$ will be. Thus, our apparatus distinguishes sharply between the "level" and "rate of change" effects. The setting of e is irrelevant for the steady state inflation rate under the new regime, but e does influence the "one-time" inflation at the start of the new regime.

In the remainder of this paper we use this model to elaborate on the consequences of the move to monetary unification. We study what difference the choice of e makes, and to whom. We find that the choice of e matters to easterners and westerners who enter unification with either assets or debts denominated in either former currency, but that it doesn't affect the welfare of others. Although the exact detail of who wins and loses in the process of unification may depend on our particular model (which is the overlapping generations model of Samuelson, as noted above), the general macroeconomic features

Figure 2



of our results, are much more robust, because they depend only on features of the demand for money and the government budget constraint that are embodied in Figures 1 and 2.

The remainder of this paper is organized as follows. Section II presents the basic economic model we use to describe a closed monetary economy, and some of the policy options open to the monetary-fiscal authorities. In Section III we indicate which options are assumed to have been chosen by the authorities of the two countries. Section IV describes the consequences of a monetary unification hitherto unforeseen and suddenly implemented. Section V examines the effects of an anticipated monetary unification. Section VI examines anticipated unification when there is uncertainty about the exact terms of unification. Finally, Section VII discusses some qualifications.

II. The Model

We will be using an overlapping generations model of a simple kind. Models of the type used in this paper were used by Wallace (1980), Sargent and Wallace (1981,1982), Bryant and Wallace (1984) and Sargent (1987). The presentation in this paper most closely follows Sargent (1987).

Time is discrete and starts at $t=1$. Each period, a generation is born, which is destined to live two periods, and is indexed by the subscript t; also, in period 1, there are agents called the initial old, who live only one period. There is a single consumption good in each period. The agents' identical preferences are defined over consumption in each period of their lives; these preferences are represented by $u(c_t(t), c_t(t+1))$; the initial old have preferences $u_0(c_0(1))$. The vector of endowments in both periods is represented by the pair $(\tilde{\omega}_t^h(t), \tilde{\omega}_t^h(t+1))$, where h indexes the agent. We allow the possibility that some agents have different endowments from others. There is no production in this model, nor is there any uncertainty.

There are two countries, called East and West. Variables that are specific to either country carry an E or W subscript. Each country has a constant population of size $2N_i$ for $i \in \{E, W\}$. Before the monetary reform, each country has a government which can collect lump-sum taxes on agent h of generation t. After-tax endowments will be called $(\omega_t^h(t), \omega_t^h(t+1))$. Our intention is to focus on the changes in fiscal policy that will be feasible after unification; for this reason, we consider the tax schedule prevailing before unification as given, and subsume it in the after-tax endowments. Later, we will analyze departures from this initial state.

A government can also issue intrinsically useless pieces of paper called East or West Marks (and denoted EM or WM). The total amount of currency outstanding at the end of period t is written $H_i(t)$. The initial old in both countries are endowed with an aggregate amount $H_i(0)$ of their currency. Governments purchase the consumption good in the amounts $G_i(t)$, and dispose of it in ways that procure utility for no one.

Each period, there is a market for the consumption good in each country, and the price of the good in *Marks* is written $p_i(t)$. There is also a market for loans among young agents. We will assume that these loans are denominated in *Marks*, and carry a nominal interest rate denoted r_t.[2] The real interest rate on these loans, by definition, is $R_t = r_t p(t)/p(t+1)$.

We assume that an impermeable separation stands between the two countries (a Wall), so that no interaction takes place between East and West. This Wall was erected before period 1, and is initially expected to stand indefinitely.

We begin the analysis with a study of some of the policies that the two governments can conduct. For simplicity, we represent a government's task as the financing of a constant deficit of taxes with respect to expenditures, denoted $D \geq 0$. A government can require the young in each generation to hold a minimum amount $\lambda \geq 0$ of the currency in real terms. The parameter λ is a policy instrument that is designed to influence the base of the inflation tax.[3]

We will study two possible regimes; in the first one, λ is set equal to 0, so that constraint (2), below, is only the traditional one which forbids agents to issue currency. In the second regime, λ is positive, and the corresponding constraint is binding. These options are available in either country, and this section sets forth the analytics in the context of a single, closed economy with general endowment patterns. We will later specify which regime will prevail in each country.

All young agents solve the following problem:

$$\max_{c_t(t),\, c_t(t+1),\, l(t)} \quad u(c_t(t),\, c_t(t+1)) \qquad (P)$$

subject to the constraints

$$c_t(t)+\frac{m(t)+l(t)}{p(t)} \leq \omega_t(t)$$

$$c_t(t+1) \leq \omega_t(t+1) + \frac{m(t)+r_t l(t)}{p(t+1)}$$

$$\lambda \leq \frac{m(t)}{p(t)} \tag{2}$$

where $l(t)$ denotes the amount lent (or borrowed, if negative) by the young agent, and $m(t)$ the agent's choice of money holdings.

The equilibrium is the solution to the agents' maximization problem, the goverment's budget constraint, as well as an equilibrium condition in the credit market.

Regime 1: Either $\lambda = 0$ or the currency constraint is never binding

A classic arbitrage argument shows that equilibrium requires

$$r_t = 1 \text{ or } R_t \geq \frac{p(t)}{p(t+1)} .$$

Agents' decisions can be represented by a saving function, which is the solution to the maximization problem above. Letting $f_t^h(R_t)$ be the saving of agent h of generation t, we have

$$f_t^h(R_t) = \omega_t^h(t) - c_t^h(t),$$

where $R_t = p(t)/p(t+1)$ is the rate of return on money holdings. The function f_t^h will be strictly increasing in R_t, under the assumption of gross substitutability of consumption in the two periods. It should be kept in mind that this function depends on the after-tax endowment of each agent.

The government's budget constraint is

$$D = \frac{H(t)-H(t-1)}{p(t)}, t \geq 1$$

$$= \frac{H(t)}{p(t)} - R_{t-1}\frac{H(t-1)}{p(t-1)}, t \geq 2 \tag{3}$$

and the equilibrium condition in the credit market is

$$\sum_h f_t^h(R_t) \equiv Nf(R_t) = \frac{H(t)}{p(t)} . \tag{4}$$

This equation states that the net saving of generation t equals the net dissaving of generation $t-1$ and of the government.

Equation (3) defines a one-to-one mapping between R_t and $h(t)=H(t)/Np(t)$. We use it to replace $H(t)/p(t)$ in (3). Writing $d=D/N$, we express condition (3) as

$$f(R_t) = R_{t-1}f(R_{t-1}) + d \tag{5}$$

$$R_t = f^{-1}(R_{t-1}f(R_{t-1}) + d)$$

$$= \phi(R_{t-1}).$$

An equilibrium sequence $\{R_t\}_{t=1}^{\infty}$ will solve this first-order non-linear difference equation.

The function ϕ can take many forms, depending on the utility function u. In the case where u takes the form

$$u(c_t,c_{t+1}) = ln(c_t) + ln(c_{t+1})$$

f is found to be

$$f(R) = \frac{\Omega_1}{2} - \frac{\Omega_2}{2R} \tag{6}$$

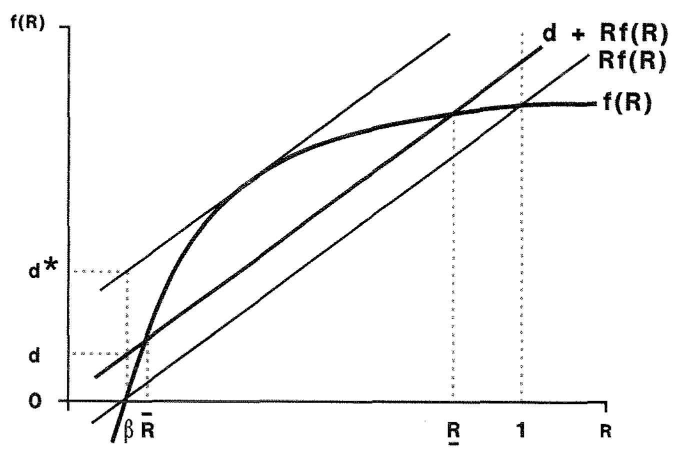
where $\Omega_i = \sum_h \omega_i^h$ for $i \in \{1,2\}$, and (5) becomes

$$\frac{\Omega_2}{R_t} + 2d - \Omega_1 - \Omega_2 + \Omega_1 R_{t-1} = 0$$

which is shown in Figure 3. If $\Omega_1 > \Omega_2$ holds, then for values $0 \leq d \leq d^* = (\sqrt{\Omega_1} - \sqrt{\Omega_2})^2$ there are two stationary solutions for R (and for h), found by intersecting the graph of $d+Rf(R)$ with that of $f(R)$. Figure 4 shows the function $(1-R)f(R)$, and the two stationary solutions can be found for any deficit $d \leq d^*$. In the case $d=0$, the two solutions are β and 1, where we define $\beta = \frac{\Omega_2}{\Omega_1} < 1$.

Under rational expectations dynamics, the lower gross rate of return on currency, $\underline{R}$, is stable, while the higher $\overline{R}$, is unstable. Any path starting at $h(1) \in [0,\overline{h}]$ (respectively $R_1 \in [\frac{\Omega_2}{\Omega_1}, \overline{R}]$) will converge to $\underline{h}$ (respectively $\underline{R}$). Paths starting at $h(1) > \overline{h}$ (respectively $R_1 > \overline{R}$) are not feasible because they eventually drive $h(t)$ to $+\infty$, which would

Figure 3



eventually mean negative consumption. Hence R_t is necessarily in $[\frac{\Omega_2}{\Omega_1}, \overline{R}]$.

Notice that the comparative dynamics associated with the "stable" stationary values $\underline{R}$ are in some sense perverse: an increase in the deficit *raises* $\underline{R}$, and *lowers* inflation. Thus, we can not rely on the rational expectations dynamics of this model to focus attention on government deficits as a cause of inflation. However, it has been shown in several contexts, both theoretical and experimental, that learning reverses the stability of the stationary points $(\underline{R}, \overline{R})$ relative to the rational expectations dynamics.[4] Such learning schemes suggest that we select the higher stationary point $\overline{R}$ as our equilibrium. Point $\overline{R}$ is associated with "classical" comparative dynamics: a higher deficit lowers $\overline{R}$, and thus raises the inflation rate. We appeal to these learning dynamics as our justification for focusing on the $\overline{R}$ stationary equilibrium.

A young agent's budget set is depicted in Figure 5: point C is attained when an interest rate of 1 prevails (in other words when the price level is constant) whereas point B is attained for $R < 1$. The seigniorage function $f(R)(1-R)$ can be read as the distance $A\omega$, when the line AB has a slope of -1.

Regime 2: $\lambda > 0$, and the currency constraint is always binding

We now consider a regime in which λ is positive and binding.

Evidently, if the currency constraint is binding, $h(t) = \lambda$ for all $t \geq 1$, and

$$d = \lambda(1-R_t) \quad \text{or} \quad R_t = R = 1 - \frac{d}{\lambda}.$$

Thus, the inflation rate is unique, constant, and positive. Note that increasing λ raises R, thereby lowering the

Figure 4

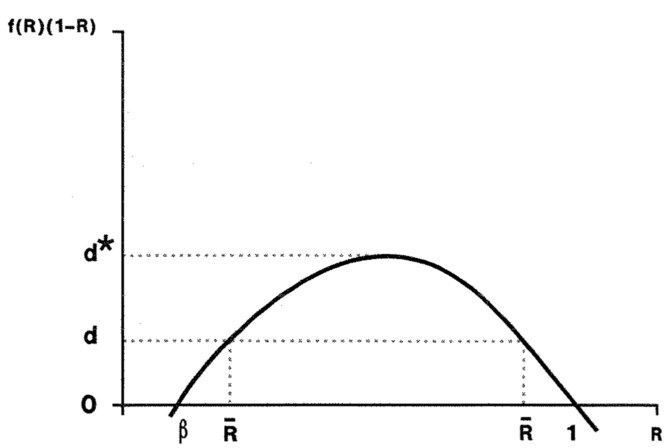
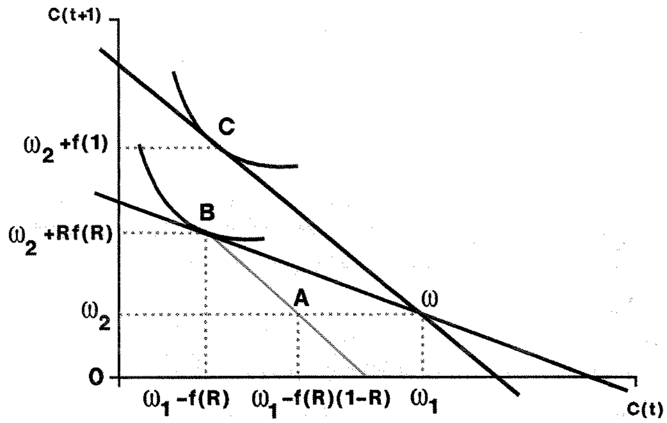


Figure 5



inflation rate. Note also that the nominal amount of forced savings per capita grows with time, since it is $\lambda\, p(t)$. Chart 1 shows the actual data for East Germany.[5] For the constraint to be binding, we must verify that

$\lambda \geq f_t^h(R_t)$ for all h and $t \geq 1$,

which translates into the condition

$$d \geq (1-R)f(R). \tag{7}$$

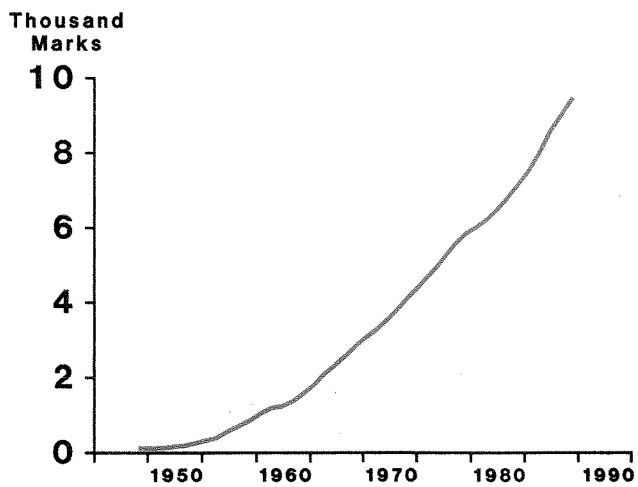
Another condition must also hold, namely, that consumption remain positive. This imposes on λ the condition that

$\lambda < \min_h (\omega_1^h) = \underline{\omega}_1,$

which translates into the following condition on R:

$$R < 1 - \frac{d}{\underline{\omega}_1} = \overline{R}^*$$

Chart 1
Nominal Savings Per Capita in East Germany (1949 - 1989)



Thus R is bounded above, away from 1; furthermore, R must lie in the regions of $(0, \bar{R}^*)$ where condition (7) is satisfied.

In the case of the logarithm utility function, (7) is satisfied if: a) $d > d^*$, and then it is true for all $R \in (0, \bar{R}^*)$; or b) $0 \leq d \leq d^*$, and then it is true for $R \in (0, \underline{R}) \cup (\bar{R}, \bar{R}^*)$. Note that a) corresponds to values of the deficit that cannot be financed in regime 1. Moreover, in b) the return on money R can be chosen to be higher than in regime 1.

Figure 6 illustrates this: the seigniorage function $(1-R)f(R)$ is represented and the region below that curve is shaded. When the deficit is d_2, it cannot be financed by voluntary holdings of money. A solution with forced savings can be found as the intersection of the d_2 line with the graph of $\lambda(1-R)$, with the resulting rate R_2. If the deficit is d_1, it can be financed with or without the currency constraint; with the constraint, a rate such as R_1 can be achieved, which is higher than $\bar{R}$. With a lower value of λ, lower rates of return are achieved, such as R_3.[6]

It is possible, depending on the utility function and endowments, that every agent would prefer regime 2 to regime 1. This situation is illustrated in Figure 7: point A is that attained in regime 1, point B in regime 2: the utility level is higher under the forced savings regime. Thus regime 2 could be justified on two grounds, depending on the level of deficit the government has chosen to finance via inflation: that this deficit is too high to be financed with voluntary holding of money by agents, or that the government can improve agents' welfare by moving from regime 1 to regime 2.

Repressed Inflation

There are two senses in which we can speak of repressed inflation in regime 2: one is that the rate of return on money can be made higher (and the inflation rate lower) in regime 2, as we saw. The other is that the initial price level $p(1)$ is higher in regime 1.

To see this, we solve for $p(1)$. The government budget constraint at $t=1$ is

$$d = \frac{H(1)-H(0)}{Np(1)} .$$

In regime 1, the equilibrium condition yields

$$p(1)\frac{\Omega_1}{2} - p(2)\frac{\Omega_2}{2} = \frac{H(1)}{N}$$

$$p(1)\left(\frac{\Omega_1}{2} - \frac{\Omega_2}{2R_1}\right) = p(1)d + \frac{H(0)}{N}$$

$$p(1) = \frac{H(0)}{N(\Omega_1/2 - \Omega_2/2R_1 - d)} = \frac{H(0)}{N(f(R_1)-d)} .$$

In regime 2, it yields

$$Np(1)\lambda = H(1)$$

$$p(1) = \frac{H(0)}{N(\lambda - d)} .$$

Thus, as long as the legal constraint on money holdings is binding, the initial price level is higher in regime 1.

This result can be reformulated in the following terms: suppose that regime 2 has been in force from $t=1$ on, and that, at time $t=t_0$, the legal restriction on money holdings is removed unexpectedly, all other parameters of the problem remaining unchanged. Then, either the deficit is too high to be financed and money becomes worthless immediately, or else it can be financed, in which circumstance the actual price level $p(t_0)$ is higher than was previously expected, and the inflation rate is higher from t_0 on than at any time before. This is the content we give here to the phrase "repressed inflation."

Figure 7

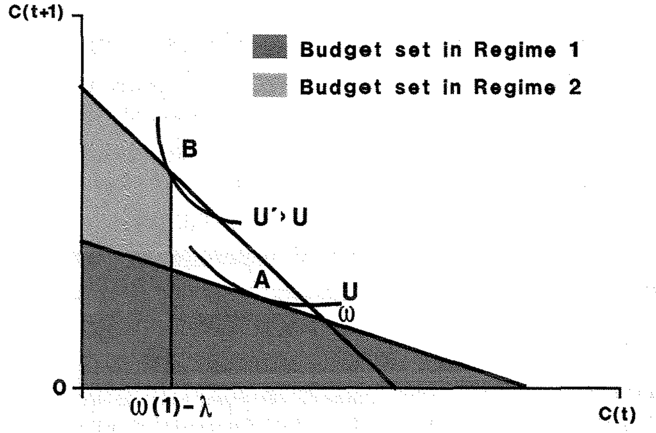
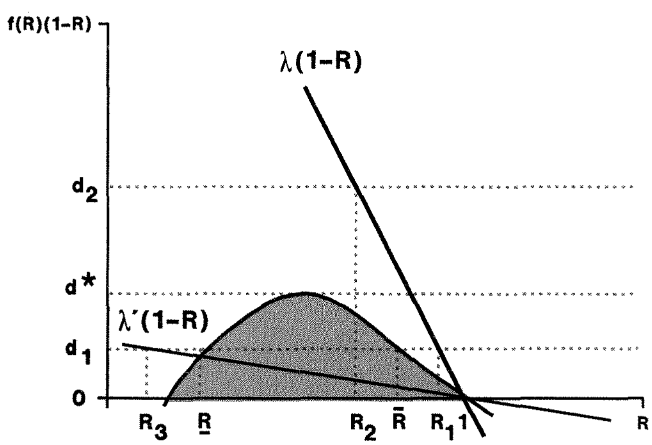


Figure 6



III. East and West before Unification

In country East, appropriate social arrangements ensure that all agents receive identical after-tax endowments (γ_1, γ_2), $\gamma_1 > \gamma_2$, in all generations $t \geq 1$. Agents within a generation are identical in preferences and endowments, which implies that there will be no intra-generational lending: each agent chooses $l^h_E(t) = 0$.

The government of East faces a constant positive deficit of tax revenues with respect to its expenditures, so that for all $t \geq 1$

$$G_E(t) - \sum_h \tau^h_t(t) - \sum_h \tau^h_{t-1}(t) = D_E$$

with $D_E > 0$. It has chosen to resort to a currency constraint, so that regime 2 as described above prevails in East. This means that the equilibrium price level path is of the form:

$$p_E(t) = p_E(1)\left(\frac{1}{R_E}\right)^{t-1}$$

with $R_E = 1 - \frac{d_E}{\lambda} = 1 - \frac{D_E}{N_E\lambda}$,

$$p(1) = \frac{H_E(0)}{N_E(\lambda - d_E)}.$$

In country West, N_1 agents have the endowment (α_1, α_2) while $N_2 = N_W - N_1$ agents have the endowment (β_1, β_2). We assume that

$\alpha_1 > \alpha_2$ and $\beta_2 > \beta_1$,

which makes the first type of agents (indexed by $W\alpha$) "lenders" and the second type (indexed by $W\beta$) "borrowers". A consequence of this assumption will be to introduce some distributional effects of the events which will happen in Sections V and VI. It is assumed that

$$\frac{N_1\alpha_2 + N_2\beta_2}{N_1\alpha_1 + N_2\beta_1} = \frac{\Omega^W_2}{\Omega^W_1} < 1,$$

which insures existence of equilibria with valued fiat currency.

Agents solve the maximization problem (P) referred to above and choose to hold private debt as well as money: since we still assume that private debt is not indexed, the budget constraint of a young agent in the West endowed with (ω^h_1, ω^h_2) is

$$c^h_t(t) + \frac{m^h_W(t) + l^h_W(t)}{p_W(t)} \leq \omega^h_1$$

$$c^h_t(t+1) \leq \omega^h_2 + \frac{m^h_W(t) + l^h_W(t)}{p_W(t+1)}$$

$$m^h_W(t) \geq 0.$$

Lenders are indifferent between holding money or private debt, while borrowers will set $m^h_{W\beta}(t) = 0$ and $l^h_{W\beta}(t) \leq 0$.

The government of country West is assumed to be running a "tight" policy: the deficit is set to $D = 0$ in all periods, and the money stock is constant, $H(t) = H(0)$ for all t. Taxes are set so as to achieve this goal.

This is merely a particular case of regime 1, with $D = 0$; with the logarithmic utility functions, we know that there may be two stationary solutions β and 1. Indeed, the equilibrium condition is

$$\sum_h \frac{l^h_W(t) + m^h_W(t)}{p_W(t)} = \sum_h f^h_W(t) = \frac{H_W(0)}{p_W(t)}, t \geq 1 \quad (8)$$

and with logarithmic utility functions (8) becomes

$$\frac{\Omega^W_1}{2} - \frac{p_W(t+1)}{p_W(t)}\frac{\Omega^W_2}{2} = \frac{H_W(0)}{p_W(t)}. \quad (9)$$

The general solution to this first-order difference equation in $p(t)$ is found to be

$$p_W(t) = \bar{p}_W + (p_W(0) - \bar{p}_W)\left(\frac{1}{R_s}\right)^t \quad (10)$$

where we defined

$$\bar{p}_W = \frac{2H_W(0)}{\Omega^W_1 - \Omega^W_2} > 0 \text{ and } R_s = \frac{\Omega^W_2}{\Omega^W_1} < 1.$$

The constant $\bar{p}_W$ is the unique non-inflationary solution, in which $R_t = 1$. For all other solutions, $R_t = R_s < 1$ is a constant, and $lim_{t\to\infty} p(t) = \infty$. The same argument about stability under learning, as described above, will serve to select the non-inflationary equilibrium, in other words the one with the highest return on money. We will consider this equilibrium to be prevailing in West.

IV. Monetary Unification

We consider the following situation. At some date, which we renormalize to be $t=0$, the Wall separating East and West unexpectedly disappears. The two countries unite, and become provinces of a single country. The two governments merge to form a single government. This new government inherits the stream of expenditures and pre-unification taxes, and has the power to impose new taxes on the citizens of both (former) countries. We will assume that the new government enacts the following rule: residents of each half of the new country may move to the other half, in which case they will receive an endowment of (0,0).[7] This ensures that the distribution of population remains the same after unification: agents will not move between the two provinces, and they can be taxed at different rates, depending on prior citizenship (that is, on their current place of residence). The single government also has the ability to issue a currency called the Mark (denoted M). These arrangements prevail for $t \geq 1$. At the beginning of period 1, all West Marks are exchanged for Marks one for one, and all East Marks are exchanged at the rate of one EM for e M. The government chooses e, and sets $\lambda=0$, which means that in the East the compulsion to hold currency has been eliminated.

Our purpose in this section is to describe the class of exchange rates, interest rates, price levels, and inflation rates that are consistent with these new arrangements. We establish the following:

1. If the consolidated government adopts the fiscal policies of the two preexisting governments, so that the deficit of the consolidated government is simply the sum of the deficits of the two prior governments, it may or may not be feasible to effect monetary unification without fiscal changes, depending on how big the consolidated deficit is.

2. If it is feasible for the new government to effect monetary unification under a fixed policy that simply consolidates the deficits of the two countries, then there is a large number of admissible exchange rates. For young people born at $t \geq 1$, welfare is identical for any feasible choice of an exchange rate. For the old at $t=1$, who bring their old East and West Marks into the new unified system, the choice of the exchange rate matters. Easterners are better off, the higher the value chosen for e.

3. If the fiscal policy of the new government simply consolidates and continues the deficits of the old governments, the move to monetary unification raises the inflation rate in the West and may or may not reduce it in the East, depending on the real value of the constraint previously imposed. All western lenders born at $t \geq 1$ are made better off by this change. Western borrowers born at $t \geq 1$ are made worse off by this change.

4. The increased inflation rate imposed on westerners by the monetary unification can be avoided by reducing the deficit of the consolidated government. The consequences for different citizens' welfare of this deficit reduction depends on precisely which people's taxes are raised.

The new government has the possibility to depart from prior taxing practices; any new taxes it decides upon will be denoted $\tau_t^h(i)$ (tax on agent h of generation t in period $i \in \{1,2\}$ of his life). The resulting after-tax endowment will be denoted $\tilde{\omega}_t^h(i)$. The aggregate tax burden on the young (respectively old) in period t is denoted $T_1(t)$ (respectively $T_2(t)$). Our assumptions imply that the government may forever tax young and old in each (former) country separately; therefore both $T_1(t)$ and $T_2(t)$ may carry E and W superscripts.

The old generation at time $t=1$, who are indexed 0, have the budget constraints

$$\text{eastern borrowers:} \quad c_E^h(1) \leq e\,\frac{m_E^h(0)}{p(1)} + \tilde{\gamma}_2$$

$$\text{western lenders:} \quad c_W^h(1) \leq \frac{m_W^h(0) + l_W^h(0)}{p(1)} + \tilde{\alpha}_2$$

$$\text{western borrowers:} \quad c_W^h(1) \leq \frac{m_W^h(0) + l_W^h(0)}{p(1)} + \tilde{\beta}_2$$

The young in all generations will henceforth face the following problem:

$$max\ u(c_t(t),\, c_t(t+1))$$

subject to the constraints

$$c_t(t) + \frac{m(t)+l(t)}{p(t)} \leq \tilde{\omega}_t(t)$$

$$c_t(t+1) \leq \tilde{\omega}_t(t+1) + \frac{m(t)+l(t)}{p(t+1)},$$

the solution to which is represented by the saving function $f_t^h(R_t) = (m^h(t)+l^h(t))/p(t)$.

The government faces the budget constraint

$$D(t) = \frac{H(t)}{p(t)} - R_{t-1}\frac{H(t-1)}{p(t-1)},\ t>1 \qquad (11a)$$

$$D(1) = \frac{H(1)}{p(1)} - \frac{H_W(0)+eH_E(0)}{p(1)} \qquad (11b)$$

$$D(t) = D_W(t)+D_E(t) = (-T_1^W(t)-T_2^W(t-1)) + (D_E - T_1^E(t)-T_2^E(t-1)).$$

The equilibrium condition is, for all $t \geq 1$:

$$F_t(R_t) \equiv N_1 f_t^{W\alpha}(R_t) + N_2 f_t^{W\beta}(R_t) + N_E f_t^E(R_t) = \frac{H(t)}{p(t)}. \quad (12)$$

The following proposition is a straightforward application of the Kareken and Wallace (1981) result on the indeterminacy of exchange rates.

Proposition 1. *Given an equilibrium* $\{\bar{R}_t, \bar{p}(t), \bar{H}(t), (\bar{\tau}_{t-1}^h(t), \bar{\tau}_t^h(t))_h, \bar{c}_{t-1}^h(t), \bar{c}_t^h(t), \bar{e}\}_{t=1}^{\infty}$, *for any* $\hat{e} \in (0,\infty)$ *there exists another equilibrium satisfying* $\bar{R}_t = \hat{R}_t$, $\bar{\tau}_{t-1}^h(t) = \hat{\tau}_{t-1}^h(t)$, $\bar{\tau}_t^h(t) = \hat{\tau}_t^h(t)$, $\bar{c}_t^h(t) = \hat{c}_t^h(t)$, $\bar{c}_t^h(t+1) = \hat{c}_t^h(t+1)$ *for all h; and* $\bar{p}(t) \neq \hat{p}(t)$, $\bar{H}(t) \neq \hat{H}(t)$, *for all t*, $\bar{c}_0^h(1) \neq \hat{c}_0^h(1)$.

Proof:

We take as given that a monetary equilibrium exists; the macron-bearing equilibrium, $\{\bar{R}_t, \bar{p}(t), \bar{H}(t), \bar{D}(t), \bar{e}\}_{t=1}^{\infty}$, solves (11) and (12). For any choice of $\hat{e} \in (0,\infty)$, we can construct a caret-bearing equilibrium as follows. Given a choice of $\hat{e}$, combine (11*b*) and (12) into

$$\bar{D}(1) = F_1(\bar{R}_1) - \frac{H_W(0)+\hat{e}H_E(0)}{\hat{p}(1)}.$$

Solve this equation for $\hat{p}(1)$ to get

$$\hat{p}(1) = \frac{H_W(0)+\hat{e}H_E(0)}{F_1(\bar{R}_1)-\bar{D}(1)}. \quad (13)$$

Since the macron-bearing equilibrium solves (11) and (12) with positive money stocks, the denominator on the right hand side of (13) is positive, and (13) can be solved for $\hat{p}(1)$. Then $\hat{p}(t+1) = \hat{p}(t)/\bar{R}_t$, and (12) gives $\hat{H}(t) = F_t(R_t)\hat{p}(t)$. Since $\hat{H}(t)/\hat{p}(t) = \bar{H}(t)/\bar{p}(t)$, (11*a*) will be satisfied.[8]

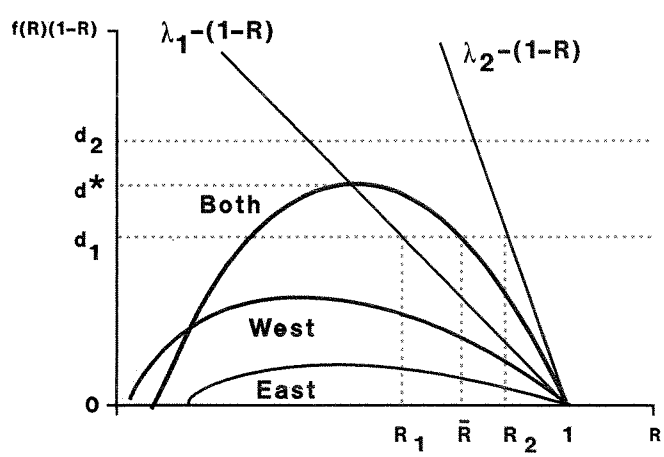
One can interpret this proposition in the following sense: for a given fiscal policy $\{(\bar{\tau}_{t-1}^h(t), \bar{\tau}_t^h(t))_h\}_{t=1}^{\infty}$ such that money has value in equilibrium, there are corresponding sequences of "real" variables $\{\bar{D}_t, \bar{R}_t, (\bar{c}_t^h(t), \bar{c}_t^h(t+1))_h\}_{t=1}^{\infty}$. There is a continuum of price paths $\{p(t)\}_{t=1}^{\infty}$ (and corresponding paths $\{H(t)\}_{t=1}^{\infty}$) consistent with these sequences, indexed by $p(1)$; the choice of $e \in (0,\infty)$ is sufficient to select the price path via equation (13) (which gives $p(1)$ as an affine[9] function of e), without altering any other aspect of the equilibrium. The existence itself of the equilibrium is a disjoint issue from the choice of the exchange rate, and is amenable to the same analysis as was conducted in Section II. Moreover, since the welfare of generations $t \geq 1$ depends only on R_t and not on the specific price level path, the choice of e affects only the consumption of the old at $t=1$. For the latter, each choice of e corresponds to a particular distribution of consumption good.

When does a monetary equilibrium exist? Figure 8 will be helpful in this context. The seigniorage functions of both provinces $f^E(R)(1-R)$ and $f^W(R)(1-R)$ have been represented, as well as the sum $F(R)(1-R)$. Since the unified country will not resort to the λ constraint, a monetary solution is found as the intersection of the $y=d$ line with the graph of $F(R)(1-R)$. If the new government simply consolidates East's deficit without raising taxes, that is, $D(t)=D_E$, then a monetary equilibrium may or may not exist. In Figure 8, the deficit d_2 cannot be financed, although it was financed by East under regime 2. On the other hand, d_1 can be financed. The value d^* is the largest deficit that can be financed.

If an equilibrium exists in the unified country, the inflation rate will rise in West, simply because it was 0 previously ($R_W=1$), and because $R=1$ is incompatible with a positive deficit. As for East, the inflation rate may be higher or lower, depending on the choice of λ that was made initially. For λ_1, the new rate of return $\bar{R}$ will be higher than R_1, and conversely for R_2. It is also apparent that, should the deficit be lowered, the inflation rate may be made lower. How this affects agents' welfare, however, will depend on who is taxed to finance this deficit reduction.

Thus, if we compare the welfare of generations $t < 0$ with that of generations $t \geq 1$ (and assume that taxes are unchanged), we need only consider real rates of return, and

Figure 8



we see that while western lenders will necessarily suffer (and western borrowers benefit) from the unification and the ensuing increase in inflation, easterners can be better or worse off. Which way easterners' welfare goes does not depend on the exchange rate chosen, but rather on the extent to which they were constrained initially. We refer again to Figure 7 on this question.

Welfare implications for the $t=0$ generation

We now consider the welfare implications of monetary unification for the old at time $t=1$. For all save the first generation, welfare is identical under all the equilibria of Proposition 1 above. For the old at time $t=1$, on the other hand, the reallocation effects of varying the exchange rate are important, simply because they are exchanging their old money for the new one, in both provinces. To see this, rewrite the eastern old's consumption in period 1 as

$$c_E^h(1) = \gamma_2 + e\frac{p_E(1)}{p_W(1)}\;\frac{p_W(1)}{p(1)}\;R_E f_E(R_E)$$

where $p_E(1)$ denotes the price level which would have prevailed had the Wall not fallen. For the western old, consumption is

$$c_W^h(1) = \omega_2 + \frac{p_W(1)}{p(1)}\;R_W f_W(R_W) = \omega_2 + \frac{\bar{p}_W\, p(1)}{f_W(1)}$$

Remembering that $f_W(1) > 0$ for lenders, it is clear that the welfare of lenders worsens, the higher the actual price level is in period 1, and conversely for borrowers (inflation benefits debtors). Whether they are better off than if the Wall hadn't fallen depends on whether $\bar{p}_W = p_W(1) > p(1)$. The eastern old's welfare falls when $e/p(1)$ falls; whether they are better off without the Wall depends on whether $ep_E(1)/p(1) > 1$. Note that the eastern old's interests do not necessarily conflict with that of either class of western old.[10]

Thus, to evaluate the welfare consequences of the move to monetary union, we need to specify what fiscal policy the new government adopts. This fiscal policy will determine the new equilibrium return on currency R, as well as the the price level $p(1)$ as a function of e. To compute solutions for various fiscal policies, we return to the assumption that preferences are identical in both countries and of the logarithmic form studied above.

Let us consider the case where the new government decides to tax the young of all generations and of both provinces by an amount $T_1 = \Sigma_h \tau_1^h$ in the aggregate, and the old by an aggregate amount $T_2 = \Sigma_h \tau_2^h$, for $t \geq 1$ so as to set a constant deficit $D = D_E - T_1 - T_2 \geq 0$ for all $t \geq 1$ (recall that the previous deficit paths were D_E for East and 0 for West).

With logarithmic preferences, the saving function for each consumer is

$$f_t^h(R_t) = \frac{\omega_1^h - \tau_1^h}{2} - \frac{\omega_2 - \tau_2^h}{2R_t}.$$

The equilibrium condition becomes

$$\sum_h f_t^h(R_t) = \frac{\Omega_1 - T_1}{2} - \frac{\Omega_2 - T_2}{2R_t} = \frac{H(t)}{p(t)}, \tag{14}$$

and the government's budget constraint

$$D = \frac{H(t) - H(t-1)}{p(t)}. \tag{15}$$

Equations (14) and (15) imply a second-order difference equation in $p(t)$

$$(\Omega_1 - T_1)p(t+1) - (\Omega_1 - T_1 + \Omega_2 - T_2 - 2D)p(t) + (\Omega_2 - T_2)p(t-1) = 0 \tag{16}$$

which, under a boundedness condition on D, has solutions of the form

$$p(t) = a\left(\frac{1}{R_1}\right)^{t-1} + b\left(\frac{1}{R_2}\right)^{t-1} \text{ with } R_1 > R_2,$$

where a and b are subject to the condition that $p(t)$ remains positive, as well as to the initial condition

$$Dp(1) = \frac{\Omega_1 - T_1}{2}p(1) - \frac{\Omega_2 - T_2}{2}p(2) - H_W(0) - eH_E(0). \tag{17}$$

A stationary or constant-inflation equilibrium corresponds to $a = p(1)$, $b = 0$ or to $a = 0$, $b = p(1)$. In both cases, the path $\{p(t)\}_{t=1}^{\infty}$ is of the form

$$p(t) = p(1)\left(\frac{1}{R_i}\right)^{t-1},\quad i \in \{1,2\}$$

and imposing (17) determines $p(1)$ as

$$p(1) = \frac{2(H_W(0) + eH_E(0))}{\Omega_1 - T_1 + (\Omega_2 - T_2)/R_i - 2D}. \tag{18}$$

Thus, $p(1)$ is an affine function of the exchange rate chosen. From Section IV, we know that

$$p_W(1) = \frac{2H_W(0)}{\Omega_1^W - \Omega_2^W}.$$

Hence

$p(1) > p_W(1)$ if and only if

$$\frac{2H_E(0)e}{N_E(\gamma_1 - \gamma_2/R_i) + \Omega_2(1 - 1/R_i) - 2D_E + T_1 + (2 + 1/R_i)T_2} > p_W(1)$$

It appears that there exists a critical value

$$e^* = \frac{H_W(0)}{H_E(0)} \times$$

$$\frac{N_E(\gamma_1 - \gamma_2/R_i) + \Omega_2(1-1/R_i) - 2D_E + T_1 + (2+1/R_i)T_2}{\Omega_1 - \Omega_2}$$

such that $p(1) > p_W(1)$ if and only if $e > e^*$.

Note that e^* may possibly be negative. But if it is positive, and if the government chooses $e < e^*$, a relative deflation in the West[11] will take place in period 1, western lenders will be made better off and western borrowers worse off than with the Wall. Conversely, for $e > e^*$, a relative inflation will occur in period 1. This critical value of the exchange rate does not depend on the price level in country East (which is determined by λ) but rather on the ratio of money stocks, on endowment and population parameters, and on the fiscal policy chosen. In particular, the value $e^\bullet = p_W(1)/p_E(1)$ is irrelevant to the occurrence of inflation in the West in period 1, and to the welfare of the western old. However, $e^\bullet$ matters for the eastern old's welfare, which will be higher than with the Wall if and only if $e/e^\bullet > p(1)/p_W(1)$. The value $e^\bullet$ can be thought of as representing a "black market exchange rate" at the time of unification.

We can consider a few examples: one possibility open to the government is simply to leave after-tax endowments identical to what they were before unification. In other words, the East's deficit is left intact and financed by inflation, and $T_1 = T_2 = 0$. We then rewrite (18) as

$$p(1) = 2\frac{H_W(0) + eH_E(0)}{\Omega_1 - \Omega_2/R_i - 2D_E}$$

The critical value is

$$e_1^* = \frac{H_W(0)}{H_E(0)} \frac{N_E(\gamma_1 - \gamma_2/R_i) + \Omega_2(1-1/R_i) - 2D_E}{\Omega_1 - \Omega_2}$$

Another possibility is for the government to tax only the young of each generation so that $T_2 = 0$, in which case

$$e^*(T_1) = \frac{H_W(0)}{H_E(0)} \times$$

$$\frac{N_E\gamma_1 - N_E\gamma_2/R_i(T_1) + \Omega_2(1-1/R_i(T_1)) - 2D_E + T_1}{\Omega_1 - \Omega_2}.$$

We must keep in mind that R_i will change with T_1. If $T_1 = D$, which corresponds to a balanced budget policy, then $R = 1$ or $R = \Omega_2/\Omega_1$.

These examples illustrate the way in which the government has the ability to choose an initial inflation or deflation (i.e., $p(1) > p_W(1)$ or $p(1) < p_W(1)$), once it has chosen a fiscal policy.

V. The Effects of an Anticipated Unification

We now examine the consequences of a delay between the announcement of monetary unification and the time at which it is implemented. We make the following assumptions.

All arrangements described in the first paragraph of Section IV are announced at time 1 to be prevailing for $t \geq T$. In periods $t = 1, \ldots, T-1$, the same arrangements as before are maintained, that is, both countries remain separate, government spending and taxes are unchanged, East still imposes savings restrictions, and so on.

We assume that at $t = 1$ a fiscal policy is specified for periods $t \geq T$, by which we mean that $\{(\tau^h_{t-1}(t), \tau^h_t(t))_h\}^\infty_{t=T}$ are announced; a rate e, at which East Marks will be received at $t = T$ in exchange for new Marks, is also announced at $t = 1$. Agents can therefore compute the equilibrium allocations and price paths.

At time T, everything will proceed exactly as in Section IV; E and W subscripts will disappear, the old of generation $T-1$ will exchange their monies for mint-fresh Marks, markets will open, a price level $p(T)$ (which can be computed given the fiscal parameters) will prevail.

The young of generation $T-1$ in the West will thus face problem (P):

$$\max u(c_{T-1}(T-1), c_{T-1}(T))$$

subject to the constraints

$$c_{T-1}(T-1) + \frac{m(T-1) + l(T-1)}{p_W(T-1)} \leq \bar{\omega}_1$$

$$c_{T-1}(T) \leq \bar{\omega}_2 + \frac{m(T-1) + l(T-1)}{p(T)},$$

the solution to which is again represented by the saving function $f^h_{T-1}(p_W(T-1)/p(T))$. The equilibrium condition can then be written

$$\sum_h f^h_{T-1}\left(\frac{p_W(T-1)}{p(T)}\right) = \frac{H_W(0)}{p_W(T-1)} \tag{19}$$

which is then solved for $p_W(T-1)$ as a function of $p(T)$. Young agents of previous generations $1 \leq t \leq T-1$ will be solving the same problem, and the path $\{p_W(1), \ldots,$

$p_W(T-1)\}$ can be computed through a backward recursion.

In the case of logarithmic utility functions, (19) takes the form

$$\frac{\Omega_1^W}{2} - \Omega_2^W \frac{p_W(T)}{2p_W(T-1)} = \frac{H_W(0)}{p_W(T-1)} \text{ or}$$

$$p_W(T-1) = R_s^{-1} p(T) + \frac{2H_W(0)}{\Omega_1^W}. \qquad (20)$$

This is solved backward to give

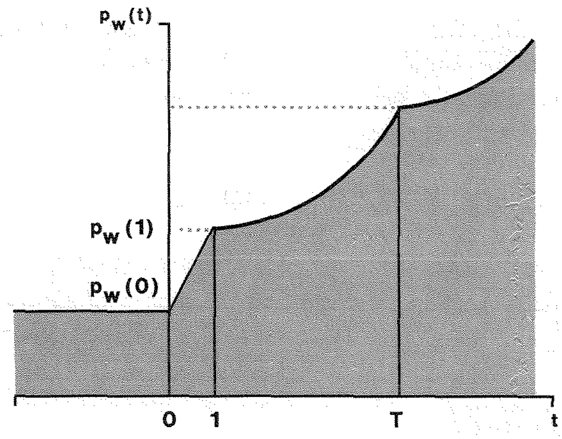
$$p_W(t) = \bar{p}_W + (p(T) - \bar{p}_W)(R_s)^{T-t} \text{ for } 1 \leq t \leq T-1 \qquad (21)$$

which is just another version of (10), with a specific starting condition. Therefore, if $p(T) > \bar{p}_W$ (as in the examples at the end of Section IV), there will be a progressive increase in the price level until it reaches $p(T)$; and $p(t)$ will increase at an accelerating rate as unification approaches. During that period, the inflation rate increases but remains bounded above by $1/R_s$. The time path of $p(t)$ is shown in Figure 9. The initial bout of inflation at the time unification is announced is

$$\frac{p_W(1)}{p_W(0)} = 1 + \left(\frac{p(T)}{\bar{p}_W} - 1\right)(R_s)^{T-1},$$

which is increasing in $p(T)$, and, given $p(T)$, is decreasing in T. It can be shown that $R_s > .5$ is a sufficient condition for inflation to be higher in period 1 than in period 2, as illustrated by Figure 9.

Figure 9



Clearly, if $p(T) = \bar{p}_W$, then the price level remains constant, and if $p(T) < \bar{p}_W$ the price level will fall increasingly rapidly as T approaches.

It should also be noted that the value of $p(T)$ determines which path of price levels will prevail in the period $t=1, \ldots,$ T, and therefore the interest rates which agents of generations 1 to T face. This means that the choice of the exchange rate affects the real allocations of all agents in generations 1 to T, the same way consumption of the old at time of unification depended on the exchange rate in the previous section.

VI. Anticipated Unification with Uncertainty

We now add a new wrinkle to the previous set-up, by introducing some uncertainty over the exchange rate to be chosen at time T.

At time 1, the same announcements are made as in Section V: the two countries will unite at time T, a consolidated government will take charge of both streams of government expenditures, and tax residents of both provinces. A fiscal policy is announced, which supports a monetary equilibrium. All parameters of the policy are made known, except for the exchange rate e. It is announced that the government will randomly choose among n possible exchange rates $(e_1, \ldots, e_n)$, with probabilities $(\pi_1, \ldots, \pi_n)$ where $\Sigma_i \pi_i = 1$. The choice will be made at the beginning of period T. These induce n states of the world in period T. There is no other uncertainty.

As Proposition 1 makes clear, the information available to agents allows them to compute the equilibrium sequences of consumptions and interest rates, for $t \geq T$, which will be identical in all states of the world. The price and money stock sequences, however, will depend on the (random) exchange rate: in particular, n possible price levels may prevail in period T, namely $(p_1(T), \ldots, p_n(T))$, computed from e_1 and e_2 by using (13):

$$p_i(T) = \frac{H_W(T) + e_i H_E(T)}{F_T(R_T) - D(T)} \text{ for } i = 1, \ldots, n.$$

The probabilities attached to the price levels are $(\pi_1, \ldots, \pi_n)$. It is more helpful to think of this distribution in terms of the value money may have in each state, that is, the reciprocals of the price levels $(1/p_1(T), \ldots, 1/p_n(T))$.

We will assume that agents maximize expected utility, and that utility is additively separable, of the form

$$u(c(t), c(t+1)) = u(c(t)) + u(c(t+1)).$$

We assume that financial markets available to agents of generation $T-1$ can be represented by n markets for claims on one unit of consumption in state i. We denote q_i as the prices of these claims, and s_i^h as the quantity of such claims bought (or sold) by the agent. The price of a real loan and the price of a nominal loan can be derived from these n securities prices as

$$\sum_{i=1}^{n} q_i = \frac{1}{R_{T-1}} \tag{22}$$

$$\sum_{i=1}^{n} \frac{q_i}{p_i(T)} = \frac{1}{p(T-1)} \tag{23}$$

Money is therefore one of the assets available to the agent for purposes of transfering wealth across time and states of the world.

We will again proceed by backward recursion, starting from the generation born right before unification, at time $T-1$. The problem solved by an agent of generation $T-1$ will be

$$max\, E\{u(c^h(T-1)) + u(c^h(T))\} = u(c^h(T-1)) + \sum_{i=1}^{n} \pi_i u(c_i^h(T))$$

subject to the constraints

$$c^h(T-1) + \sum_{i=1}^{n} q_i s_i^h \leq \omega_1^h \tag{24}$$

$$c_i^h(T) \leq \omega_2^h + s_i^h \tag{25}$$

Note that the agent now has $n+1$ budget constraints, which can be consolidated into a single budget constraint

$$c^h(T-1) + \sum_{i=1}^{i} q_i c_i^h(T) \leq \omega_1^h + \frac{\omega_2^h}{R_{T-1}}. \tag{26}$$

The first order conditions are (26) and

$$\text{for } i = 1, \ldots, n \quad \frac{\pi_i}{q_i} u'(c_i^h(T)) = u'(c^h(T-1)). \tag{27}$$

Equations (26–27) describe each agent's behavior.

The market-clearing conditions on all financial markets

$$\sum_{h=1}^{n} s_i^h = \frac{H(0)}{p_i(T)}$$

can be written in the form

$$\sum_h (c_i^h(T) - \omega_2^h) = \frac{H(0)}{p_i(T)} \tag{28}$$

$$\sum_h (\omega_1^h - c^h(T-1)) = \frac{H(0)}{p(T-1)}. \tag{29}$$

Equation (29) is redundant but convenient. Equilibrium is characterized by conditions (26–28).

Once $p(T-1)$ and R_{T-1} are solved for using these equations, the next steps are identical to those taken in Section V. An agent of generation $T-2$ faces a pair of prices $(p(T-2), p(T-1))$ and an interest rate R_{T-2} (which must equal $p(T-2)/p(T-1)$ to preclude arbitrage). His saving function can be derived the same way as before, equilibrium will impose

$$\sum_h f_{T-2}^h\left(\frac{p(T-2)}{p(T-1)}\right) = \frac{H(0)}{p(T-2)}$$

which allows us to compute $p(T-2)$ given $p(T-1)$, and so forth to $p(1)$. The only generation to face uncertainty is generation $T-1$.

In the case of the logarithmic utility function $u(c) = ln(c)$, (27) becomes

$$c_i^h(T) = \frac{\pi_i}{q_i} c^h(T-1). \tag{30.i}$$

When these values are substituted into (26) we find

$$c^h(T-1) = \frac{\omega_1^h}{2} + \frac{\omega_2^h}{2R_{T-1}}. \tag{31}$$

Equation (29) becomes

$$\sum_h c^h(T-1) = \frac{1}{2}\left(\Omega_1 + \frac{\Omega_2}{R_{T-1}}\right) = \Omega_1 - \frac{H(0)}{p(T-1)}$$

$$\frac{2H(0)}{p(T-1)} = \Omega_1 - \frac{\Omega_2}{R_{T-1}} \tag{32}$$

This equation relates $p(T-1)$ and R_{T-1}.

We can use (28) and (30) to obtain

$$\sum_h c^h(T-1) = \frac{q_i}{\pi_i} \sum_h c_i^h(T)$$

$$\frac{q_i}{\pi_i}\left(\frac{H(0)}{p_i(T)} + \Omega_2\right) = \frac{q_j}{\pi_j}\left(\frac{H(0)}{p_j(T)} + \Omega_2\right) \; \forall \; i,j$$

$$\text{or } \frac{q_i}{p_i(T)k_i} = \frac{q_j}{p_j(T)k_j}$$

where we denote

$$k_i = \frac{\pi_i}{H(0) + \Omega_2 p_i(T)},$$

and use (22) to solve for q_i as functions of R_{T-1}:

$$q_i = \frac{1}{R_{T-1}} \frac{k_i p_i(T)}{\sum_{j=1}^{n} k_j p_j(T)} \text{ for } i = 1, \ldots, n \tag{33}$$

We then invoke (23) to obtain another relation between $p(T-1)$ and R_{T-1}:

$$p(T-1) = \rho R_{T-1} \tag{34}$$

with

$$\rho = \sum_{i=1}^{n} \left(\frac{k_i}{\Sigma_{i=1}^{n} k_i} \right) p_i(T).$$

Equations (32) and (34) at last allow us to solve for $p(T-1)$:

$$p(T-1) = \frac{2H(0)}{\Omega_1} + \frac{\Omega_2}{\Omega_1}\rho \tag{35}$$

Note the formal analogy between (20) and (35). This will allow an easy comparison with the case under certainty.

Since $p(T-1)$ is solved as a function of the distribution of $(p_1(T), \ldots, p_n(T))$, the price sequence $\{p(1), \ldots, p(T-2)\}$ can be solved for recursively, using equation (20):

for $1 \leq t \leq T-1$,

$$p(t) = \bar{p} + (p(T-1) - \bar{p})\left(\frac{\Omega_1}{\Omega_2}\right)^{t-T+1} \tag{36}$$

with $\bar{p}$ being the zero-inflation price level prevailing before $t=0$.

We establish the following result:

Lemma. *In the logarithmic utility case, for any distribution $(p_1(T), \pi_1; \ldots; p_n(T), \pi_n)$, the following holds:*

$$\rho > \left(E\frac{1}{p(T)}\right)^{-1}.$$

Proof:

We wish to prove that

$$\sum_{i=1}^{n} \frac{\pi_i}{p_i(T)} > \left(\sum_{i=1}^{n} \lambda_i p_i(T)\right)^{-1}$$

$$\left(\sum_{i=1}^{n} \frac{\pi_i}{p_i(T)}\right)\left(\sum_{i=1}^{n} k_i p_i(T)\right) > \sum_{i=1}^{n} k_i$$

$$\left(\sum_{i=1}^{n} \pi_i \frac{1}{p_i(T)}\right)\left(\sum_{i=1}^{n} \pi_i \frac{p_i(T)}{H(0) + \Omega_2 p_i(T)}\right)$$
$$> \sum_{i=1}^{n} \pi_i \frac{1}{p_i(T)} \frac{p_i(T)}{H(0) + \Omega_2 p_i(T)} \quad ;$$

if we denote $\alpha_i = 1/p_i(T)$, $\bar{\alpha} = \Sigma_{i=1}^{n} \alpha_i$ and $f(x) = 1/(H(0) + \Omega_2 x)$, we want to prove

$$\left(\sum_{i=1}^{n} \pi_i \alpha_i\right)\left(\sum_{i=1}^{n} \pi_i f(\alpha_i)\right) > \left(\sum_{i=1}^{n} \pi_i \alpha_i f(\alpha_i)\right);$$

Note that f is strictly decreasing in x: therefore

$\alpha_j \geq \bar{\alpha}$ iff $f(\alpha_j) \leq f(\bar{\alpha})$

$(\alpha_j - \bar{\alpha})(f(\alpha_j) - f(\bar{\alpha})) < 0$ for all j

$$\sum_{j=1}^{n} (\alpha_j - \bar{\alpha})(f(\alpha_j) - f(\bar{\alpha})) < 0$$

$$\sum_{j=1}^{n} (\alpha_j - \bar{\alpha})(f(\alpha_j) - \sum_{i=1}^{n} \pi_i f(\alpha_i)) < 0$$

$$\sum_{j=1}^{n} \alpha_j f(\alpha_j) - \bar{\alpha}\left(\sum_{i=1}^{n} \pi_i f(\alpha_i)\right) < 0$$

We are now in a position to compare two possible policies. First, the government may announce a nondegenerate distribution of possible exchange rates $(e_1, \pi_1; \ldots; e_n, \pi_n)$. This distribution induces a distribution of price levels $(p_1(T), \pi_1; \ldots; p_n(T), \pi_n)$, and a distribution of values of money $(1/p_1(T), \pi_1; \ldots; 1/p_n(T), \pi_n)$. We call the mean value of money $E(1/p(T)) = \Sigma_{i=1}^{n} \pi_i/p_i(T)$. This results in the equilibrium sequence $\{p(1), R_1, \ldots, p(T-1), R_{T-1}\}$ which we just computed, and which we call the *equilibrium under uncertainty*.

Alternatively, the government, exactly as in Section V, may announce that an exchange rate $\tilde{e}$ will be chosen with certainty at time T: we denote $\{\tilde{p}(1), \tilde{R}_1, \ldots, \tilde{p}(T-1), \tilde{R}_{T-1}, \tilde{p}(T)\}$ the corresponding equilibrium sequence, which we call the *equilibrium under certainty* for short. We consider the case where $\tilde{e}$ is such that $1/\tilde{p}(T) = E(1/p(T))$.

The lemma implies:

Proposition 2. *Assume logarithmic utility functions. In the equilibrium under uncertainty, the price levels for $t=1, \ldots, T-1$ are higher, and the rates of return lower, than in the equilibrium under certainty with $1/\tilde{p}(T) = E(1/p(T))$.*

Proof:

The lemma establishes that $\rho > \tilde{p}(T)$. From (35), it is apparent that $p(T-1) > \tilde{p}(T-1)$, and from (34) that $R_{T-1} < \tilde{R}_{T-1}$. Since equation (36) describes both paths of price levels in both equilibria, it must be that $p(t) > \tilde{p}(t)$ for $1 \leq t \leq T-2$ as well. As for the rates of return,

$$R_t = \frac{p_{t-1}}{p_t} = \frac{\bar{p} + (p(T-1) - \bar{p})(\Omega_1/\Omega_2)^{t-T}}{\bar{p} + (p(T-1) - \bar{p})(\Omega_1/\Omega_2)^{t-T+1}}$$

and $\Omega_1/\Omega_2 > 1$ ensures the result.

The proposition confirms what intuition might suggest: we compare a world where money will have a certain value at time T, to one where the future value of money is uncertain, but on average the same. In other words, in the second situation we have introduced some randomness in the value of money, around a given mean. The same way a risk-averse agent will prefer to receive with certainty the mean value of a lottery, rather than the lottery itself, we find that in our model the demand for money (which is

$H(0)/p(T-1)$, with $H(0)$ identical in the two experiments) will fall when uncertainty is introduced. The price level, and the inflation rates, will be higher in all periods between the announcement and the implementation of monetary union, because of the added uncertainty on the future value of money.

The proposition is set forth in terms of distributions of price levels at time T, and is not linked to the particular way in which randomness is introduced in the price level at time T. Other forms of randomness may be considered. Suppose, for example, that the exchange rate is determined with certainty at time 1 ($e=1$, say), but fiscal policy remains indeterminate. Assuming that the aggregate deficit can be financed by inflation, and that the government will choose to finance some constant fraction $\delta \in [0,1]$ of that deficit, the price level at time T is given by equation (13), where the denominator $F(R_T) - \delta D = R_T F(R_T)$ is positive by assumption, and decreasing in δ, as Figure 3 makes clear. Thus the uncertainty over δ, if the government does not commit to a specific value before time T, will induce a distribution of possible values of money, the lowest one associated with a $\delta = 0$ and the highest one with a balanced budget.

The same result then applies: the added uncertainty has the effect of increasing the price levels and the inflation rates in all periods prior to unification, when compared to a choice of fiscal policy which would set the value of money $1/p(T)$ at the mean of the possible values of money.

VII. Final Comments

The model we used in this paper has, as any model must have, a number of limitations. Some are the inevitable drawbacks which characterize any overlapping generations model; they are well known, and this is not the place to discuss them. We might mention that they often plague other workable models of money. We rather wish to point out drawbacks that are specific to the model we used, which should be borne in mind when trying to find similarities between this model and actual persons or events.

In our model, the country once unified remains closed, in the same sense the two countries were originally taken to be closed: there is no rest of the world, and consequently no foreign trade. As a result, we lose the ability to discuss consequences of monetary union on trade, and we miss an important consideration in the determination of the initial inflationary shock at unification. As some have pointed out, the DM is convertible, whereas the OM is not. East Germans endowed with hard *Marks* would presumably buy goods from abroad as well as from West Germany, and this may have a mitigating effect on inflation.

In our model, there are only two periods in agents' lives; therefore, at the time of unification only old people come in from the East to exchange their soft Marks for hard Marks, and these old people, by construction, only wish to spend their balances. Although the demographic structure of East Germany isn't extremely different from that of West Germany,[12] in actuality some East Germans may not want to spend all their freshly minted DM on bananas. Again, this reduces the strength of inflationary forces.

Our model simply assumes that the new government converts all OM instantaneously into freely expendable Marks, and at a single exchange rate. The plan which will be implemented in Germany will not have this feature, although any legal restriction on the expendability of East German savings will have to be easily enforceable.[13] A possible feature would have East Germans buy the State's capital stock with their savings; another would freeze part of their holdings for a period of time left to the Bundesbank's discretion. It is also possible that a fraction of East Germans' money holdings will be convertible at a rate, and the remainder at another, less favorable rate.

We have assumed that the good with which Easterners are endowed is of the same nature as the good available for purchase in the West. One might object to such a ruthless subsumption of BMWs and Trabants as identical commodities, and want to allow for less than perfect substitution. To illustrate the argument, the results of Section IV can be re-examined with $\gamma_1 = \gamma_2 = 0$, in other words with the assumption that goods produced in country East are considered worthless for consumption purposes, once agents are given a choice. Taking this consideration into account would reinforce the inflationary factors. We have also assumed that the Easterners' endowments would not change after unification. Incorporating such a feature would change conclusions about inflationary forces, but would also leave Proposition 1 unchanged.

On a theoretical level, one might object that we have assumed perfect foresight on the part of our agents, before as well as after, unification. But we have shown our agents expecting the Wall to remain in place indefinitely in Section III, and we have then betrayed their expectations in

Section IV (the element of surprise was of course crucial for the trick played on the old people at time 1). We would answer that we in fact assumed a particular probability distribution, namely that the status quo would remain with probability $1-\epsilon$, and that the Wall would come down with probability ϵ (the latter is understood to be as small as usual). We would further argue that this representation is but a stylized version of most observers' probability distributions until the early days of October 1989.

NOTES

1. As we remark later, this result is simply an application and interpretation of the reasoning on which the exchange rate indeterminacy result of Kareken and Wallace (1981) is based.

2. Models of this type usually specify that loans are denominated in the consumption good (e.g. Sargent (1987)). A departure from this usage does not matter in a model with perfect foresight, such as ours, until such time as an unanticipated change in policy occurs.

3. It is possible to interpret the restriction on real balances as the outcome of a commodity rationing scheme which forces the young to hold more money than they would want by limiting the goods available for purchase. Notice that the scheme we use leaves old agents free to spend their accumulated cash balances.

4. See Marcet and Sargent (1989) and Arifovic and Sargent (1990) for some theoretical work on learning schemes in the context of this model. See Marimon and Sundar (1989) for some experimental evidence.

5. "The growth of the total balance of savings is the expression of the people's trust in the socialist development of the German Democratic Republic, and in the stability of its money" (*DDR Handbuch* (1979)).

6. The two regimes described here obviously do not cover all possibilities. For a given value of the deficit $d_1 \leq d^*$, and when λ is set as low as λ_2 in Figure 5, then there are three stationary equilibria, one in which the constraint is binding with $R=R_3$, and two in which it is not binding, with $R=R_1$ or $R=R_2$. Thus, when the deficit can be financed by inflation alone, imposing the constraint does not necessarily imply that it will be binding, because multiple equilibria are possible.

7. This assumption is not excessive, in view of the severe restrictions recently placed on eligibility of East German citizens for social benefits in West Germany.

8. The allocations of the old at time 0 will be affected by $p(1)$: at an extreme, for low values of $p(1)$ the deflation could be so severe that Western borrowers would be unable to honor their commitments. In a sense, this is irrelevant because the only economic forces determining the equilibrium values of variables are the decisions of the young of generations $t \geq 1$. However, a government wishing to spare the original old Western borrowers this predicament would choose e within a range $(\underline{e}, +\infty)$, where $\underline{e}$ verifies

$$\frac{H_W(0)+\underline{e}\,H_E(0)H_W(0)+\underline{e}\,H_E(0)}{F_1(\bar{R}_1)-\bar{D}_1} = \frac{|f^h_{W\beta}(R_W)|}{\tilde{\beta}_2}$$

so that old Western borrowers' consumption after repayment of loans remains positive.

9. A variable y is said to be an *affine function* of variables $x_1, x_2, \ldots, x_n$ if there exist constants $b_0, b_1, \ldots, b_n$ such that $y=b_0+b_1x_1+\cdots+b_nx_n$.

10. Had we followed the usual practice of denominating private debt in real terms rather than nominal terms, western borrowers would have been unaffected by the unification, and western lenders would have been affected through their holdings of money only.

11. By relative deflation in the West we mean that $p(1) < p_W(1)$, that is, the price level actually prevailing at time 1 is lower than it would have been, had the Wall remained in place.

12. One East German out of four is over the age of 50, compared to one West German out of three.

13. This paper was written before the details of the currency unification were worked out.

Data Appendix

The following summarizes some of the available data on the German economies. All amounts (except population figures) are in billions of *local* currency. Sources are *Statistisches Jahrbuch für die BRD* 1989, *Deutsche Bundesbank monthly report* Apr. 1990, *Encyclopædia Britannica Yearbook* 1989.[1]

Federal Republic of Germany	
population (88)	60.8m
GNP (89)	2260.4
govt spending (89)	699.5
monetary base (end 89)	216.6
M1 aggregate (end 89)	450.6
M2 aggregate (end 89)	776.4
M3 aggregate (end 89)	1255.5

German Democratic Republic	
population (88)	16.6m
TSP (87)	789.5
govt spending (88)	291.0
currency stock (end 87)	15.0
savings accounts (end 87)	141.9
—(end 89)	151 to 157
black market exchange rate (OM/DM)	4:1 to 6:1
—(as of late March 1990)	4.40:1

1. TSP is Total Social Product (the socialist version of GNP, which excludes services, etc.). The 1990 figures for savings in East Germany and the black market exchange rate are the ones commonly cited (e.g. *New York Times* March 14, 1990; *International Herald Tribune* Feb.10–11, 1990; *die Welt*, March 6, 1990; *Frankfurter Rundschau*, April 2, 1990).

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