Inflation and the Stock Market: Understanding the "Fed Model"*

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Abstract:

The so-called Fed model postulates that the dividend or earnings yield on stocks should equal the yield on nominal Treasury bonds, or at least that the two should be highly correlated. In US data there is indeed a strikingly high time series correlation between the yield on nominal bonds and the dividend yield on equities. This positive correlation is often attributed to the fact that both bond and equity yields comove strongly and positively with expected inflation. While inflation comoves with nominal bond yields for well-known reasons, the positive correlation between expected inflation and equity yields has long puzzled economists. We show that the effect is consistent with modern asset pricing theory incorporating uncertainty about real growth prospects and also habit-based risk aversion. In the US, high expected inflation has tended to coincide with periods of heightened uncertainty about real economic growth and unusually high risk aversion, both of which rationally raise equity yields. Our findings suggest that countries with a high incidence of stagflation should have relatively high correlations between bond yields and equity yields and we confirm that this is true in a panel of international data.

*This work does not necessarily reflect the views of the Federal Reserve System or its staff. In particular, our use of the term "Fed Model" reflects only conventional parlance among finance practicioners for this common valuation model.

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1 Introduction

The so-called Fed model[4] postulates that the dividend or earnings yield on stocks should equal the yield on nominal Treasury bonds, or at least that the two should be highly correlated. Both investment professionals (see for instance Asness (2003)) and academics (see for instance Thomas (2005)) have long been struck by the strength of the empirical regularity. Figure 1 shows a graph of the yield on a long-term nominal bond and the equity yield (using dividends) for the US aggregate stock market. While some investment professionals are using the Fed model as a model of equity valuation (see references in Estrada (2005)), both practitioners and academics have concluded that the model is inconsistent with a rational valuation of the stock market (see for instance, Asness (2003), Feinman (2005), Campbell and Vuolteenaho (2004), Cohen, Polk and Vuolteenaho (2005), Ritter and Warr (2002) and Sharpe (2002)).

The difficulty in squaring the model with rational valuation can be illustrated using a simple decomposition of the dividend yield and the nominal bond yield. Using the Gordon model, we can write the equity cash yield, EY, on the aggregate stock market as consisting of three components:

$$EY = -EDIV + RRF + ERP \tag{1}$$

where $EDIV$ is the expected growth rate of real equity dividends, RRF is the real risk free rate of interest and ERP is the equity risk premium. Similarly, the yield on a nominal bond is:

$$BY = EINF + RRF + IRP \tag{2}$$

where $EINF$ is expected inflation, RRF is again the real interest rate, and IRP is the inflation risk premium. The high correlation between dividend yields and nominal bond yields is difficult to reconcile with rational models because expected inflation is a dominant source of variation in nominal yields and the extant literature seems to have concluded that it is impossible for expected inflation to have a large (rational) effect on any of the *real* components that drive the equity cash yield. In fact, the aforementioned authors all resort to the simple behavioral model proposed by Modigliani and Cohn in 1979 to explain the empirical regularity: inflation (or money) illusion. Inflation illusion suggests that when expected inflation increases, bond yields duly increase, but because equity investors incorrectly discount real cash flows using nominal rates, the increase in

[4] The Fed Model may have gained its moniker from Prudential Securities strategist Ed Yardeni in 1997 who noted that in the Federal Reserve Humphrey Hawkins Report for July 1997, a chart plotted the time series for the earnings-price ratio of the S&P 500 against the 10-year constant-maturity nominal treasury yield.

nominal yields leads to equity underpricing (the equity yield rises with bond yields and is now too high) and vice versa. Alternatively, one can view equity investors as correctly discounting nominal cash flows and using nominal discount rates, but failing to increase expected nominal cash payouts in response to increases in expected inflation.

The importance of this conclusion extends beyond the narrow confines of testing the Fed model. If behavioral biases induced by inflation cause misvaluation in the equity market, then the potential exists for informed practitioners to devise trading strategies to take advantage of the mispricing. For policy makers, if money illusion causes undue variation in equity prices during periods of inflation uncertainty, this suggests another motive for inflation stabilization policies, as Campbell and Vuolteenaho (2004) point out.

In this article, we carefully re-examine the evidence constructing dynamic versions of Equations (1) and (2) in a vector autoregressive (VAR) framework, building on Campbell and Shiller's (1988) seminal work. In these computations, we construct the risk premium components of yields as residuals since they are not directly measurable. We find that bond yields are indeed highly positively correlated with the dividend yield and that expected inflation is the primary bond yield component responsible for the high stock-bond yield correlation . In the context of a rational model, expected inflation must be positively correlated with the dividend yield through some combination of positive correlation with the real rate and the equity risk premium, or a negative correlation with expected cash flow growth. We find that only a relatively small portion of the overall comovement between expected inflation and the dividend yield can be ascribed to the correlation between expected inflation and real rates. A somewhat larger but still not dominant piece is due to a negative covariance between expected inflation and expected cash flows.[5] The bulk of the positive covariance between the dividend yield and expected inflation comes from positive comovement between expected inflation and the equity risk premium. Importantly, because we measure the equity premium as a residual, these initial results do not identify whether money illusion-induced misvalaution or rational equity risk premiums are responsible for the high correlation expected inflation.

Our subsequent analysis strongly supports the latter explanation. We demonstrate that the high correlation between expected inflation and the dividend yield is almost entirely due to the positive correlation between expected inflation and two plausible proxies for rational time-varying risk premiums: a measure of economic uncertainty (the uncertainty among professional forecasters regarding real GDP growth) and a consumption-based measure of risk aversion. These measures of rationally time-varying risk premiums feature prominently in recent asset pricing articles showing that they help to explain a number of salient asset return features.

[5] This confirms Modigliani and Cohn's careful work that the effect is not due to expected real cash flow growth rates being adversely affected by expected inflation.

Bansal and Yaron (2004, BY henceforth) have stressed the importance of economic uncertainty and Campbell and Cochrane (1999, CC henceforth) have built a model of external habit, leading to a measure of time-varying risk aversion that can be constructed from current and past consumption data and is counter-cyclical. Bekaert, Engstrom and Xing (2008) combine both measures in one model. Consequently, a rational channel seems at work in explaining why the Fed model "works:" high expected inflation coincides with periods of high risk aversion and/or economic uncertainty. We also provide an out-of-sample test of our interpretation of the US data. Specifically, our results suggest that the correlation between equity and bond yields ought to be higher in countries with a higher average incidence of stagflation. We confirm that this is the case. We also make sure that our US results are robust, investigating a wide variety of alternative VAR specifications. The concluding section ties our findings to an older literature on inflation-stock market linkages and discusses some issues for future investigation.

2 Empirical Methodology

2.1 Yield Decompositions

Our goal is to construct dynamic versions of Equations (1) and (2). Beginning with the latter task, we simply assume the nominal yield decomposition relationship holds at each point in time using continuously compounded rates, which we denote using the lower case. In particular, we model by_t, the continuously compounded bond yield, as,

$$by_t = einf_t + rrf_t + irp_t. \tag{3}$$

where rrf_t is a real risk free rate assumed to have maturity equal to that of the nominal bond, $einf_t$ is the average (annualized) expected inflation over the life of the bond, and irp_t is the inflation risk premium associated with the bond. In principle, all three components are unobserved. We achieve identification by finding observable proxies for the real rate and expected inflation, and use equation (3) to infer the inflation risk premium. We describe all empirical variable definitions and data sources briefly in the next section and in more detail in Appendix 7.2.[6]

To decompose the equity yield into its components, we use the Campbell-Shiller (1988, CS henceforth)

[6] In a robustness exercise, we also conduct our main analysis using a different identification scheme for real rates that assumes we can measure the inflation risk premium more directly as a function of inflation uncertatinty. See Section 5 for details.

decomposition. CS arrive at the following formula for the equity yield, ey_t:

$$ey_t = -\frac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty}\rho^j\left(r_{t+j+1} - \Delta d_{t+j+1}\right)\right]. \tag{4}$$

where k and ρ are linearization constants, r_t is the one-period real return to holding equity, and Δd_t is one-period real dividend growth. Without loss of generality, we can split the expected rate of return on equity into risk-free and risk premium components,

$$E_t\left[r_{t+1}\right] = rrf_t + erp_t \tag{5}$$

where erp_t is the continuously compounded one-period equity risk premium. Given the definition of rrf_t in Equation (3), this premium is defined relative to a long-term real risk free rate. Substituting,

$$ey_t = -\frac{k}{1-\rho} - E_t\sum_{j=0}^{\infty}\rho^j\Delta d_{t+j+1} + E_t\sum_{j=0}^{\infty}\rho^j rrf_{t+j} + E_t\sum_{j=0}^{\infty}\rho^j erp_{t+j} \tag{6}$$

which is the dynamic version of Equation (1). Here too, the risk premium component will be treated as the residual, with the two other components constructed empirically using our assumed data generating process, described next.

2.2 Empirical Model: VAR

To model the joint dynamics of stock and nominal bond yields and their components, we stack the following variables into a vector, Y_t,

$$Y_t = \left[einf_t, rrf_t, \Delta d_t, erp_t, irp_t, x_t'\right]', \tag{7}$$

with x_t denoting a vector of time-t observable information variables that will be useful in interpreting the results:

$$x_t = \left[ra_t, vr_t, \Delta ern_t, gern_t^{su}\right]'. \tag{8}$$

Hence, there are a total of nine variables in the VAR[7]. The first two elements of the information vector, x_t, are designed to capture rational components of the equity risk premium, erp_t. First, ra_t, is a measure of rational risk aversion based on the specification of external habit persistence in CC. Second, vr_t is a measure of uncertainty about real economic growth. BY use uncertainty in the context of a data generating process for

[7] Section 5 considers some robustness checks with respect to the variables included in the VAR.

dividend and consumption growth and demonstrate that a modest amount of time-varying uncertainty about real growth can, under some assumptions about investor preferences, generate nontrivial variation in the equity risk premium. The other two variables in x_t represent contemporaneous realized real earnings growth, Δern_t, and a subjective measure of expected earnings growth, $gern_t^{su}$. These variables allow us to compare objective and subjective forecasts of profit growth, which is useful for assessing the possible impact of subjective biases about profit prospects, such as money illusion, in the comovement between stock and bond yields. They may also be useful predictors of future dividends, and thus important for understanding the dynamics of ey_t.

We proceed by assuming a simple data generating process for Y_t, and using the fully observable vector,

$$W_t = [einf_t, rrf_t, \Delta d_t, ey_t, by_t, x_t']' , \tag{9}$$

to identify the dynamics of Y_t. Specifically, we assume a first-order VAR for Y_t,

$$Y_t = \mu + AY_{t-1} + \Sigma\varepsilon_t \tag{10}$$

where μ is a vector of constants with the same dimension as Y_t, A is a square matrix of parameters governing the conditional mean of Y_t, Σ is a lower triangular square matrix of parameters governing the covariance of shocks to elements of Y_t (that is, Σ is the the Cholesky decomposition of the covariance matrix of shocks) and ε_t is a vector of i.i.d. shocks. Once the Y_t dynamics are specified to take this form, a simple linear translation between Y_t and the observable vector, W_t is available. Moreover, we can obtain estimates of μ, A and Σ by first estimating a VAR on W_t. More concretely, we first estimate

$$W_t = \mu^w + A^w W_{t-1} + \Sigma^w \varepsilon_t \tag{11}$$

to obtain estimates of $\left\{\widehat{\mu^w}, \widehat{A^w}, \widehat{\Sigma^w}\right\}$ and then calculate $\left\{\widehat{\theta}, \widehat{\Theta}\right\} = F_1 \left\{\widehat{\mu^w}, \widehat{A^w}, \widehat{\Sigma^w}\right\}$ where F_1 is a matrix function such that

$$\widehat{Y}_t = \widehat{\theta} + \widehat{\Theta} W_t \tag{12}$$

Next, we can identify the VAR parameters of Y_t as

$$\left\{\widehat{\mu}, \widehat{A}, \widehat{\Sigma}\right\} = F_2 \left\{\widehat{\mu^w}, \widehat{A^w}, \widehat{\Sigma^w}\right\} \tag{13}$$

where F_2 is a second matrix function. Appendix 7.1 explains, in detail, how these calculations are made.

2.3 Decomposing Yields under the VAR

As stated above, the nominal bond yield is trivially affine in components of Y_t, as the right hand side terms of Equation (3) are direct elements of Y_t. We can also now more explicitly describe our decomposition of the equity yield into three components,

$$ey_t = const + ey_t^{\Delta d} + ey_t^{rrf} + ey_t^{erp} \tag{14}$$

where $ey_t^{\Delta d} = -E_t \sum_{j=0}^{\infty} \rho^j \Delta d_{t+j+1}$ represents the total effect of cash flow expectations, $ey_t^{rrf} = E_t \sum_{j=0}^{\infty} \rho^j rrf_{t+j}$, represents the total effect of real interest rates, and $ey_t^{erp} = E_t \sum_{j=0}^{\infty} \rho^j erp_{t+j}$ represents the total effect of equity risk premiums. We use objective conditional expectations under the VAR to calculate each of these quantities, and because of the simple VAR structure, the three equity yield components are affine in Y_t. For example, ignoring constant terms, and defining $e'_{\Delta d}$ such that $\Delta d_t = e'_{\Delta d} Y_t$,

$$ey_t^{\Delta d} = -e'_{\Delta d} E_t \sum_{j=0}^{\infty} \rho^j Y_{t+j+1} = -e'_{\Delta d} \rho A (I - \rho A)^{-1} Y_t$$

which is indeed a linear function of Y_t.

To determine the source of the high covariance between stock and bond yields, we decompose it into its nine components:

$$\begin{aligned} COV(ey_t, by_t) \quad = \quad & COV\left(ey_t^{\Delta d}, einf_t\right) + COV\left(ey_t^{\Delta d}, rrf_t\right) + COV\left(ey_t^{\Delta d}, irp_t\right) \\ & + COV\left(ey_t^{rrf}, einf_t\right) + COV\left(ey_t^{rrf}, rrf_t\right) + COV\left(ey_t^{rrf}, irp_t\right) \\ & + COV\left(ey_t^{erp}, einf_t\right) + COV\left(ey_t^{erp}, rrf_t\right) + COV\left(ey_t^{erp}, irp_t\right) \end{aligned} \tag{15}$$

Each of these covariances is readily calculated using VAR arithmetic. For instance,

$$COV\left(ey_t^{\Delta d}, einf_t\right) = -e'_{\Delta d} \rho A (I - \rho A)^{-1} COV(Y_t) e'_{einf} \tag{16}$$

where $vec[COV(Y_t)] = (I - A \otimes A)^{-1} vec[\Sigma\Sigma']$. Note that every element of $COV(ey_t, by_t)$ is ultimately a function of the parameters of the observable VAR, $\left\{\widehat{\mu^w}, \widehat{A^w}, \widehat{\Sigma^w}\right\}$.

2.4 Orthogonalizing the Equity Risk Premium

The equity risk premium component of equity yields in our decompositions above, ey_t^{erp}, is essentially a residual, the difference between the observed equity yield and the summed presented values, calculated under the VAR, of future cash flows and real risk free rates. A disadvantage of this approach is that model misspecification could contaminate the equity risk premium estimates. To try to isolate the component of the equity risk premium that is consistent with rational pricing, we draw on recent theoretical advances in the empirical asset pricing literature. CC and BY suggest that erp_t is approximately linear in risk aversion, ra_t, or real uncertainty, vr_t respectively. In the model of Bekaert, Engstrom and Xing (2008), the equity risk premium is a function of risk aversion and real economic uncertainty. We parse ey_t^{erp} into two components: one spanned-by and one orthogonal-to the vector $[ra_t, vr_t]$. Figure 2 plots the two series. Because this vector is a subset of the information variable vector in the VAR, x_t, we can easily decompose ey_t^{erp} into these two components without any further estimation. Conceptually, the process is analogous to running a regression of ey_t^{erp} on ra_t and vr_t and interpreting the regression residual as the orthogonal component, which we denote ey_t^{erp-re}. For example, we calculate

$$
\begin{aligned}
ey_t^{erp-sp} &= \beta^{erp\prime}[1, ra_t, vr_t] \\
ey_t^{erp-re} &= ey_t - ey_t^{erp-sp}
\end{aligned}
\tag{17}
$$

where the coefficients, β^{erp} are given under OLS as, $E\left([1, ra_t, vr_t][1, ra_t, vr_t]'\right)^{-1} E\left(ey^{erp}[1, ra_t, vr_t]'\right)$ and the two unconditional expectations that comprise the coefficients are readily calculated from the VAR. With this additional decomposition, there are twelve potential components to the covariance between stock and bond yields,

$$
\begin{aligned}
COV\left(ey_t^{erp}, by_t\right) = \; & COV\left(ey_t^{\Delta d}, einf_t\right) + COV\left(ey_t^{\Delta d}, rrf_t\right) + COV\left(ey_t^{\Delta d}, irp_t\right) \\
& + COV\left(ey_t^{rrf}, einf_t\right) + COV\left(ey_t^{rrf}, rrf_t\right) + COV\left(ey_t^{rrf}, irp_t\right) \\
& + COV\left(ey_t^{erp-sp}, einf_t\right) + COV\left(ey_t^{erp-sp}, rrf_t\right) + COV\left(ey_t^{erp-sp}, irp_t\right) \\
& + COV\left(ey_t^{erp-re}, einf_t\right) + COV\left(ey_t^{erp-re}, rrf_t\right) + COV\left(ey_t^{erp-re}, irp_t\right)
\end{aligned}
\tag{18}
$$

If money illusion were present in the data, we would expect to find a positive covariance between the residual equity yield and expected inflation, $COV\left(ey_t^{erp-re}, einf_t\right)$ as all the other covariances with expected inflation are constructed in a manner consistent with rational pricing.

2.5 Calculating the Subjective Bias in Profit Expectations

We compute the equity premium residual assuming that agents use "correct" cash flow forecasts. However, some descriptions of money illusion suggest that the effect comes through incorrect subjective cash flow predictions by market participants which are correlated with inflation expectations. Of course, in our VAR system, subjective errors in cash flow forecasts would end up in the "residual," the equity premium, and if not related to ra_t and vr_t, they will still be attributed to the residual component of the equity premium, ey_t^{erp-re}. To shed light on whether a subjective bias in cash flow expectations is related to the variation in equity yields and expected inflation, we use our VAR to estimate the bias and then check for comovement of the bias with inflation and equity yields. Specifically, we calculate the subjective bias in profit expectations as the difference between the subjective measure of real profit expectations and an objective growth estimate under the VAR, $gern_t^{ob}$, at the same horizon (four quarters). The latter is readily calculated using VAR mathematics because we include realized real earnings growth, Δern_t, as an element of the information vector in the VAR, x_t. Because the subjective earnings expectations measure predicts annual earnings, and we use quarterly data, we compute (ignoring constant terms):

$$gern_t^{ob} = e'_{\Delta ern}\left(A + A^2 + A^3 + A^4\right) Y_t. \tag{19}$$

We define the subjective bias as

$$bias_t = gern_t^{su} - gern_t^{ob} \tag{20}$$

which is clearly affine in Y_t given that $gern_t^{su}$ is also in the information vector, x_t.

3 VAR Results

3.1 Data and Empirical Methods

We estimate the VAR using quarterly data, extending from the 4th quarter, 1968 through the end of 2007. The data are described in detail in Appendix 7.2. Here we give a short overview. The bond yield is the yield to maturity on a nominal 10 year US Treasury bond[8]. As a proxy for the real rate, we use the estimate for the 5 year zero coupon real rate provided in Ang, Bekaert and Wei (2008). As is well known, real term structures are relatively flat at longer maturities so that this maturity is a reasonable proxy for a coupon bond with duration

[8] While the coupon bonds on which these yields are based have a roughly stable maturity, their duration naturally varies over time. We can roughly gauge the degree of this variation under some simplifying assumptions: If (1), the bonds pay semi-annual coupons, and (2) trade at par, then the bonds' duration is function of yield alone. These calculations yield a Macaulay duration series for the bonds that has a mean of around 7.5 years and a standard deviation of about 0.8 years.

significantly lower than 10 years. There is a voluminous literature on inflation forecasting, but recent work by Ang, Bekaert and Wei (2007) strongly suggests that professional surveys provide the best out-of-sample forecasts of inflation. Therefore, we use a proxy for inflation expectations from the Survey of Professional Forecasters (SPF).

The equity data we use are standard and represent information on the S&P500 Index. In our base results we use dividends not accounting for repurchases, but we discuss results with an adjusted measure in Section 5. Consequently, real earnings, dividend growth and the equity yield all refer to the S&P500 Index. Subjective expectations regarding earnings growth are also extracted from the SPF.

Finally, we need empirical proxies for "fundamental risk aversion" and for economic uncertainty. Our proxy for (the log of) risk aversion takes the specification for local risk aversion in CC based on an external habit model. In this model, risk aversion is negatively related to the consumption surplus ratio, which is the ratio of the surplus of consumption over the habit stock divided by consumption. The stochastic process is autoregressive and the shocks are derived from US consumption data. By starting the process in 1947, the effect of initial conditions has died out by the time our sample starts. The resulting measure is clearly counter-cyclical. Our measure for real uncertainty is also based on SPF data. We combine information from a survey about the probability of a recession the next quarter and from the dispersion across respondents about next year's real GDP growth. The Appendix has all the details.

We estimate the VAR on W_t using OLS. Detailed VAR results are available on request. Our data sample is comprised of 157 quarterly observations of a nine-variable vector. In addition to the 9 unconditional means, the first-order VAR transition matrix, A^w, has 81 elements and the innovation covariance matrix, $\Sigma^w \Sigma^{w\prime}$, has 45 distinct elements. The "saturation ratio," or the ratio of the number of the total number of data points to the number of estimated parameters, is thus $(157 \cdot 9)/(9 + 81 + 45) = 10.5$. This is satisfactory but suggests many VAR coefficients may not be statistically significant. To make sure our results are not due to over-fitting the robustness section will consider VARs with insignificant coefficients zeroed out and smaller VARs.

In the results discussion, we immediately focus on the comovements statistics derived from the VAR. Because all of these statistics are functions of the VAR parameters, it is possible to derive standard errors for them using the parameter standard errors and the delta method. However, there are many reasons to suspect asymptotic theory may not work well in this context: some of the variables are persistent, the saturation ratio is not exceedingly large and the residuals are likely fat-tailed. Therefore, we use standard errors derived from a bootstrap procedure, which is described in Appendix 7.3.

3.2 Main Results

Table 1 contains the main results. In Panel A, the top line simply reports the variance of the bond and equity yields, their covariance and their correlation. The heart of the puzzle is that the correlation between ey_t and by_t is 78 percent. Under the VAR point estimates, a (bootstrapped) 90 percent confidence interval for this correlation ranges from 34 to 90 percent. This is puzzling because, as shown under the variance decompositions for the two yields, 55 percent of the variance of the bond yield is driven by expected inflation, whereas 78 percent of the variation of the equity yield is driven by the equity risk premium. For the yields to comove so strongly, expected inflation, a nominal concept, must correlate highly with the equity premium, a real concept. This is confirmed in the covariance decomposition on the right side of Panel A. More than half of the comovement comes from the correlation between expected inflation and the equity premium. The other two relatively large contributors are the covariance between the real rate and the equity premium, which is positive and contributes 16 percent to the $ey_t - by_t$ covariance, and the covariance between expected inflation and the cash flow component of the equity yield, which contributes 12 percent. The latter effect implies that expected inflation is on average positively correlated with periods of low cash flow expectations, as the cash flow component of the equity yield is negatively related to cash flow projections. This in itself already suggests that above-average inflation in the US has occurred often at times of depressed earning (and dividend) expectations. Finally, expected inflation and the real rate are positively correlated, which contributes 7 percent to the comovement between the bond and equity yield. While this number is small, it is relatively precisely estimated. This result is inconsistent with the well-known Mundell-Tobin effect that suggests a negative relation. However, our measures here are long-term (proxying for a 5 to 10 year horizon) and Ang, Bekaert and Wei (2008) also find a positive correlation between expected inflation and long-term real rates.

Looking at the last row of the covariance decomposition matrix, we note that 79 percent of the comovement between equity yields and bond yields comes through the equity premium, a residual in the equity yield decomposition. While it is tempting to conclude that irrational forces are at work, the next panel proves otherwise. In Panel B, we decompose the equity yield into a part spanned by risk aversion and uncertainty and an unspanned part; 80 percent of what the equity premium explains of the total $ey_t - by_t$ covariance comes from the spanned, rational part[9]. If we focus on $COV\left(ey_t^{erp}, einf_t\right)$, the expected inflation component, about 86 percent can be ascribed to the rational component, $COV\left(ey_t^{erp-sp}, einf_t\right)$ with the rest, potentially, coming from money illusion.

In panel C, we explore the comovements among equity yields, expected inflation, and the subjective earnings

[9]Calculated as the sum of the first line in Panel B divided by the sum of the last line in Panel A (0.64/0.81).

bias. On the left side, we see that the subjective earnings bias is barely correlated with either the equity yield or expected inflation. This suggests that subjective bias in cash flow expectations (1) is not an important driver of the equity yield and (2) does not comove strongly with expected inflation. Both of these effects are in sharp contrast with the assumptions of money illusion. Still, equity yields *are* highly correlated with expected inflation. On the right hand side of Panel C, we decompose this comovement because the Fed model puzzle essentially is due to the high correlation between expected inflation and equity premiums. The Panel shows that about 10 percent of their comovement comes from the positive comovements of real rates and expected inflation, 16 percent of the comovement can be ascribed to the negative correlation between expected inflation and cash flow expectations, but 63 percent can be ascribed to the fact that risk aversion and uncertainty are high in times of high expected inflation. The unexplained residual is a paltry 10%, which severely limits the potential role of money illusion.

4 International Results

4.1 Motivation

Given previous results in the literature, our findings are perhaps surprising. For example, Campbell and Vuolteenaho (2005, CV henceforth) perform a closely related VAR-based analysis and interpret their findings as clearly suggestive of money illusion. How can their results be so different from ours? We believe there are four main reasons. First, CV treat cash flows as residuals. All unexplained variation is hence assigned to cash flow variation. In contrast, we attempt to measure cash flows directly and leave the equity premium as the residual component. We prefer the latter method because, although they are highly seasonal cash flows are clearly measurable. Second, CV measure the equity risk premium with a variable due to Cohen, Polk and Vuolteenaho (2005) that may be subject to considerable measurement error and it not, to date, widely used in the literature. Third, CV work directly in terms of excess returns, and therefore ignore one potentially important rational source of common variation in the two yield variables: real rates. Our results in Table 1 indicate that they therefore "miss" about 20 percent of the comovement between equity and bond yields. Finally, subsequent research has found that Campbell and Vuolteenaho's results are not robust to the post-war subsample on which we focus (Wei and Joutz, 2007).

Nevertheless, both their work and ours analyzes one US based data set, with one history of inflation, bond yields and equity yields. Using this data set alone, it is likely hard to definitively exclude the money illusion story in favor of our story. We believe that international data offer an interesting out-of-sample test of our hypothesis.

Essentially, we argue that the US experienced high correlations between equity yields and bond yields because higher inflation happened to occur during recessions, so that in recessions equity and bond premiums are both relatively high. In other words, the Fed model "works" in countries with a high incidence of stagflation.

Estrada (2006) shows that there is indeed substantial cross-sectional variation in the strength of the correlation between bond and equity yields across countries. He focuses on statistical problems in interpreting the correlations in a panel of international data. We now explore the possibility that 'stagflation incidence' accounts for part of the cross-sectional variations in stock-bond yield correlations using data similar to the Estrada sample. Specifically, we collect four variables for 20 countries over the period from December 1987 to June 2005. First, we use the dividend yield, $ey_{i,t}$, provided by Thomson for each country's equity index. The measure is not perfectly available, but 97% of all possible country-months are populated. We also use a long term risk free local currency nominal bond yield, $by_{i,t}$, from Thomson. Third, we measure the inflation rate for each country-month as reported by the local governments, $infl_{i,t}$. Where available, we use the continuously compounded change in the CPI index. If no such series is available for a particular country, we use the GDP deflator. If this variable is available only quarterly, we divide the quarterly inflation rate by three and use repeated values for months in that quarter. Finally, we measure real activity using the recession indicator $recess_{i,t}$ published by the Economic Cycle Research Institute, which provides monthly indicator series for the incidence of recession. Where recession indicators are not available (8 countries and in 2005 for all countries), we define recessions as two consecutive quarters of negative real GDP growth.

4.2 Cross-Country Analysis

We start with a heuristic analysis of the cross-sectional association between "Fed Model effect intensity" and "stagflation intensity." To capture the intensity of the Fed model effect, we compute the time series correlation between the dividend yield and the nominal long bond yield for each country. To measure the intensity of stagflation for a country, we similarly compute the time series correlation of the recession indicator with inflation for each country. Figure 3 plots each country along these two dimensions. Although there are only 20 country observations, a positive relationship seems evident. In fact, the cross-sectional correlation between fed model intensity and stagflation intensity on this plot is 0.50, and significant at the 5 percent level (not accounting for the sampling uncertainty in the time series correlations). Moreover, a cross sectional OLS regression of fed model intensity on stagflation intensity produces a positive slope coefficient of 1.35 which is also significant at the 5 percent level (again, not accounting for the sampling uncertainty in the time series correlations). The significance of the slope coefficient is robust to the (sequential) exclusion of Japan and Austria, potential outliers.

We interpret these results as supportive of a positive relationship. The relationship exists even though the U.S. itself has not exhibited stagflation in the post-1987 sample while retaining a high $by_t - ey_t$ correlation.

We add more statistical formality to this analysis by estimating two sets of cross-sectional regressions with the cross-section of countries' stock-bond yield correlations as the dependent variable. The results for both sets of regressions are reported in Table 2. The first regression set (numbers on the left of the table) focuses on the incidence of stagflation, defined as the percent of observations where a recession occurs simultaneously with high inflation. Our cut off value for high inflation is 10%, but we also conducted the analysis using an inflation level of 5% as the cut-off with largely similar results. Regression (3) shows that stagflation by itself has a huge effect on the equity –bond yield correlation: a country with 1% higher stagflation incidence than the average has a 21 percentage point higher equity-bond yield correlation. Of course, the stagflation effect could be due to its separate components, recession or simply inflation. Regressions (1) and (2) show that the percent of high inflation months by itself does increase the equity yield-bond yield correlation whereas a high frequency of recessions actually reduces it, but the latter effect is not significant. Regression (4) includes all three dependent variables in one regression. This regression provides a nice test of our stagflation story versus just money illusion. If money illusion drives the correlation, the coefficient on inflation should be significant, and there is little reason for stagflation to have a particular effect on the bond-equity yield correlation. However, we find that inflation has an insignificant effect on the correlation. The recession effect is still negative but not significant, and the stagflation effect is large and significantly different from zero. While the associated t-statistic is large, the regression suffers from three econometric problems. First, the sample is small (20 observations). Second, the regressors and regressands involve pre-estimated statistics. Third, the different observations arise from correlated time series. Therefore, we conduct a Monte Carlo analysis, described in detail in the Appendix 7.4, and generate a small sample distribution for the t-statistics in the regressions. Significant t-statistics according to the small sample distribution are indicated with asterisks. The stagflation coefficient remains significant when using the small sample distribution for the t-statistics.

The second set of regressions, replace "high inflation incidence" by average inflation, and "stagflation" by the interaction of inflation and the recession indicator. The univariate regression, Regression (5), reveals that countries with high average inflation do have significantly higher equity yield-bond yield correlations, but when this variable is added to a regression that includes the inflation-recession interaction, Regression (7), the direct effect of inflation disappears. The inflation-recession interaction comes in very significantly and the significance survives at the 5% level under the small sample distribution. The direct effect of the frequency of recessions continues to be negative but insignificant.

5 Robustness Checks

This section describes the set of robustness exercises against which we have tested our main results.

5.1 VAR Specification Tests

Table 3 reports a few specification tests on the VAR residuals. In Panel A, we report the standard Schwarz (BIC) and Akaike (AIC) criteria. The BIC criterion clearly selects a first-order VAR whereas the AIC criterion selects a second-order VAR. In the second panel, we report Cumby-Huizinga (1987) tests on the residuals of a first and second-order VAR for each variable separately. We use 4 autocorrelations. While the selection criteria in Panel A suggest that a VAR(1) adequately describes the dynamics of the data, the Cumby-Huizinga tests in Panel B suggest some serial correlation remains with a first-order VAR and that a second-order VAR is more appropriate. Therefore, we repeat the analysis using a VAR(2) data generating process. All the results in Table 1 are essentially unchanged.

5.2 VAR Robustness Exercises

Table 4 summarizes a number of robustness exercises. We only focus on the critical statistics: the percent contribution of the covariance between expected inflection and the equity premium to the total yield covariation, and the percent contribution of the covariance between expected inflation and the non-spanned, residual part of the equity premium, erp_t^{re}. For ease of comparison, the first line repeats the results from the main VAR reported in Table 1, and the second line reports the results from a VAR(2).

1. The use of a large VAR (9 variables) may imply that many coefficients are insignificant, yet still influence the statistics of interest. Our bootstrapping procedure for calculating standard errors should address this issue to a large extent, but we also conduct two exercises to directly verify the robustness of the point estimates:

   (a) We calculate the results presented in Table 1 after zeroing-out any element of A which has an OLS t-statistic less than one. The results are largely unchanged.

   (b) We also repeat the calculations using a smaller VAR excluding the information variables, that is dropping x_t. While this precludes us from decomposing the equity risk premium and calculating the subjective earnings bias, all the results in Panel A of Table 1 are essentially unchanged.

2. We conduct three exercises to check the robustness of results to alternative bond yield decompositions.

(a) We add an additional information variable to the VAR, a measure of inflation uncertainty based on SPF data (using a procedure similar to that which we used for real uncertainty). The VAR results remain robust.

(b) We substitute a longer-term measure of survey-based inflation expectations (our standard measure looks ahead only four quarters) as our measure of expected inflation. The longer-term measure is not available early in the sample, so we must first filter its early values (see data appendix for a description of this procedure). Our results are robust to this change.

(c) We use a completely different measure of the real rate, by assuming we can measure the inflation risk premium directly – as proportional to inflation uncertainty. Specifically, we subtract long-term inflation expectations and a constant times inflation uncertainty from nominal rates. We use the residual as an alternative real rate measure. We choose the constant of proportionality to match the unconditional mean of the real rate to that of our standard measure from Ang, Bekaert, and Wei (2008). Our main results are not materially affected by this change.

3. We also conduct the analysis presented in Table 1 using two alternative measures of the cash flow from equity.

(a) First, we use earnings instead of dividends, both for constructing cash flow growth and calculating the equity yield. That is, we now investigate the earnings yield. We are motivated to do this, in part, because practitioners overwhelmingly focus on earnings as the unit of fundamental analysis for equity valuation. However, to do formal analysis using earnings in the CS framework, we make the not-entirely satisfactory assumption of a constant payout ratio. The results for earnings-based equity yields are largely consistent with our main results. (1) The stock-bond yield covariance is very high, (2) the majority of the comovement comes through the covariance of the equity yield with expected inflation, and (3) very little of the covariance involves the ey_t^{erp-re} component of the equity yield. One difference from our main results is that the contribution of $COV\left(ey_t^{\Delta d}, einf_t\right)$ to the total $ey_t - by_t$ covariance is substantially larger when using earnings rather than dividends, accounting for 41 percent of the covariance versus just 12 percent under our baseline VAR as reported in Table 1. Hence, rather than the covariance between expected inflation and cash flow growth uncertainty being the driver for the stock-bond yield covariance, it is now comovement between expected inflation and expected cash flow growth. Nevertheless, even if this is the correct interpretation of the data, stagflation remains a critical ingredient: Inflation happens to occur at times of depressed earnings expectations. Note that we use objective, not subjective, earnings forecasts, so that this cannot be caused by money illusion.

(b) Second, we add repurchases to dividends in calculating cash flow, because repurchases have been an important channel by which companies have returned cash to shareholders in the past few decades, and this can have important asset pricing implications (see Boudoukh et al, 2007). The correlation of the resulting equity yield measure with the bond yield remains positive but not statistically significant. This owes to the fact that repurchases have, on a quarterly basis, been extremely volatile, especially over the past few years. The point estimates of our main results are broadly similar to those presented in Table 1, but the estimates of all the $ey_t - by_t$ covariance components are very imprecisely estimated and none are individually statistically different from zero. While this is a disappointing result, it is likely similarly due to the excessive volatility of repurchases.

5.3 International Results

For robustness to our use of dividends as the relevant equity cash flow in the international data, we also conduct the analysis using year-ahead analyst-expected earnings in calculating the equity yield. This change does not affect the results of Table 2 very much. Finally, because the dependent variable in the cross-sectional regressions are correlations and thus limited to the interval $[0, 1]$, we conducted the OLS regressions using a transformation of the correlation, $\ln(1 + corr.) / \ln(2 - corr.)$, which effectively spreads the range of the dependent variable to $(-\infty, +\infty)$. The OLS t-statistics using this transformation are very similar to those reported in Table 2.

6 Conclusion

In this article, we re-examine potential explanations for the surprisingly high correlation between the "real" equity yields and nominal bond yields in US post-war data. We show that the prevailing explanation, money illusion, actually has rather limited explanatory power. We ascribe a large part of this covariation to the rather high incidence of stagflations in the US data. We postulate that in recessions economic uncertainty and risk aversion may increase leading to higher equity risk premiums, increasing yields on stocks. If expected inflation happens to also be high in recessions, bond yields will increase through their expected inflation and, potentially, their inflation risk premium components, and positive correlations emerge among stock and bond yields and inflation. We establish this result using a VAR methodology that uses measures of inflation expectations and two proxies for rational variation in risk premiums, one based on economic uncertainty, one based on the habit model formulated by Campbell and Cochrane (1999). Our confidence in these findings is bolstered by a cross-country analysis that demonstrates that "stagflation incidence" accounts for a significant fraction of the cross-sectional variation in equity - bond yield correlations.

Our findings have potentially important policy implications. If money illusion afflicts pricing in the stock market, inflation stabilization also helps prevent distortions and mis-pricing in the stock market. If money illusion does not affect the stock market, the Federal Reserve's inflation policy has no bearing on the equity market beyond its implications for real economic growth.

Finally, our work is related to but distinct from another "old" hypothesis regarding the relationship between inflation and the stock market: Fama's (1981) proxy hypothesis. Fama argues that the strong negative relationship between stock returns and inflation is due to inflation acting as a proxy for expected real activity. Hence, the hypothesis also relies on stagflation being an important part of US data. Because our VAR allows us to compute cash flow expectations we can directly measure the importance of the "proxy hypothesis." According to Table 1, we find that only 14 percent of the equity-bond yield covariance can be ascribed to the covariance between $einf_t$ and $ey_t^{\Delta d}$ So, while the proxy hypothesis is part of the explanation, our risk-based story clearly dominates.

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7 Appendix

7.1 Recovering the Dynamics of the Latent Factors

By assumption, the partially unobserved state (we denote its dimension as l) vector follows a VAR(1),

$$Y_t = \mu + AY_{t-1} + \Sigma\varepsilon_t. \tag{21}$$

The observable vector, W_t, (also of dimension l) is a linear combination of concurrent values of Y_t as well as expectations of future values of Y_t. The most general relation we consider is

$$W_t = M_0 + M_1 Y_t + M_2 E_t \sum_{j=0}^{\infty} \rho^j Y_{t+j+1} \tag{22}$$

where the matrices M_0 $(l \times 1)$, M_1 $(l \times l)$ and M_2 $(l \times l)$ are comprised of known constants. Under the VAR(1) structure, this has the implication that Y_t and W_t are related by a linear transformation, which we denote as

$$Y_t = \theta + \Theta W_t \tag{23}$$

and we must solve for θ and Θ. Once we have done so, we can use the empirical estimates of the VAR parameters for W_t from the data:

$$W_t = \mu^w + A^w W_{t-1} + \Sigma^w \varepsilon_t$$

to recover estimates of μ, A, and Σ as:

$$\begin{aligned}
\mu &= \theta + \Theta\mu^w - \Theta A^w \Theta^{-1}\theta \\
A &= \Theta A^w \Theta^{-1} \\
\Sigma &= chol\left(\Theta\Sigma^w \Sigma^{w\prime}\Theta'\right).
\end{aligned} \tag{24}$$

where $chol\,(\cdot)$ denotes the Cholesky decomposition. We use the method of undetermined coefficients. Specifically, combining Equations (22) and (23), we obtain,

$$W_t = M_0 + M_1\left(\theta + \Theta W_t\right) + M_2 E_t \sum_{j=0}^{\infty} \rho^j \left(\theta + \Theta W_{t+j+1}\right) \tag{25}$$

Solving for the first term of the above summed expectations, we obtain,

$$E_t \sum_{j=0}^{\infty} \rho^j \theta = (1-\rho)^{-1} \theta \tag{26}$$

For the second piece, we first note that $E_t W_{t+j} = \overline{W} + (A^w)^j \left(W_t - \overline{W}\right)$ defining $\overline{W} = (I - A^w)\,\mu^w$. Using this to expand the second term in the summed expectations,

$$\begin{aligned}
E_t \sum_{j=0}^{\infty} \rho^j \Theta W_{t+j+1} &= \sum_{j=0}^{\infty} \rho^j \Theta \left(\overline{W} + (A^w)^{j+1} \left(W_t - \overline{W}\right)\right) \\
&= \sum_{j=0}^{\infty} \rho^j \Theta \overline{W} + \frac{1}{\rho} \sum_{j=0}^{\infty} \Theta \left(\rho A^w\right)^{j+1} \left(W_t - \overline{W}\right) \\
&= (1-\rho)^{-1} \Theta \overline{W} + \frac{1}{\rho} \left[\Theta \left(\rho A^w\right) \left(I - \rho A^w\right)^{-1} \left(W_t - \overline{W}\right)\right] \\
&= (1-\rho)^{-1} \Theta \overline{W} + \Theta A^w \left(I - \rho A^w\right)^{-1} \left(W_t - \overline{W}\right)
\end{aligned} \tag{27}$$

Putting this all together, we obtain,

$$\begin{aligned}
W_t &= M_0 + M_1 \left(\theta + \Theta W_t\right) \\
&\quad + M_2 \left[\left(\theta + \Theta \overline{W}\right) \phi_1 - \Theta \Phi_1 \overline{W} + \Theta \Phi_1 W_t\right]
\end{aligned} \tag{28}$$

where $\Phi_1 = A^w \left(I - \rho A^w\right)^{-1}$and $\phi_1 = (1-\rho)^{-1}$. Equating W_t coefficients on both sides of the equations yields a solution for Θ:

$$\begin{aligned}
I &= M_1 \Theta I + M_2 \Theta \Phi_1 \\
vec\left(\Theta\right) &= \left(I' \otimes M_1 + \Phi_1' \otimes M_2\right)^{-1} vec\left(I\right)
\end{aligned} \tag{29}$$

Using Equations (24) and (29), we can completely specify the dynamics of Y_t in terms of estimated parameters. The models considered in this article are all special cases of Equation (25).

For the baseline model,

$$\begin{aligned}
M_0^{ey} &= \frac{-k}{1-\rho},\ M_1^{ey} = e_{rrf}' + e_{erp}',\ M_2^{ey} = -e_{\Delta d}' + \rho e_{rrf}' + \rho e_{erp}' \\
M_0^{by} &= 0,\ M_1^{by} = e_{einf}' + e_{rrf}' + e_{irp}',\ M_2^{by} = 0
\end{aligned} \tag{30}$$

where M_0^{ey} denotes the relevant row of M_0 for the equity yield, and similarly for the other superscripts. Also, e_{rrf} denotes a selection vector for Y_t (e.g. $e'_{rrf} Y_t = rrf_t$), etc.

7.2 US Data

The empirical work uses quarterly data over 1968Q4-2007Q4. This section describes our data construction and notation. We begin with a listing of our mnemonics:

- Alternative cash flow growth measures: real dividend/earnings/total payout growth, Δd_t, Δern_t, Δy_t respectively
- Alternative equity yield measures: dividend, dp_t, earnings, ey_t or total payout, yp_t
- Nominal bond yield, by_t
- Real risk free rate, rrf_t
- Expected inflation: four-quarter. $einf_t$, long-term, $einf_t^{lt}$
- Realized inflation, one-period $ainf_t$
- Subjective expected real profits growth, $gern_t^{su}$
- Habit-based risk aversion, ra_t
- Real growth uncertainty, vr_t

7.2.1 Stock and Bond Data

The equity data we use are based on the S&P 500 index. We measure dividends, earnings and repurchases on a quarterly, per-share, seasonally adjusted basis, and price on a quarter-end, per-share basis. The earnings are "as reported" prior to 1985, and "operating" thereafter. Repurchase data are available quarterly from Standard and Poors beginning only in 2001Q2. Prior to that, we estimate repurchases by using estimates (from Boudoukh, et al 2007) of the annual ratio of repurchases to dividends for the Compustat universe, applying this ratio to quarterly dividend series for S&P 500 firms.

We take the quarter-end yield on a constant maturity nominal 10-year Treasury coupon bond from the St. Louis Fed FRED webpage, and estimates of the real risk-free long-term rate provided by Ang, Bekaert and Wei (2007). The rate yield data end in 2004. To extend the series, we filter the missing values using the Kalman filter, assuming a stable VAR describes the comovements of real yields, nominal yields, expected inflation, and inflation uncertainty.

7.2.2 Inflation Data

We measure expected inflation using the Survey of Professional Forecasters (SPF). Specifically, in our main results, we use the median survey response for the four-quarter ahead percent change in the GDP price deflator. As a robustness check in Table 4, we use the 10-year annualized average rate of CPI inflation which is only available since 1980 (to complete the sample, we filter the early sample values using the Kalman filter, assuming four-quarter inflation expectations, long-term inflation expectations, long term nominal rates, and long term real rates evolve according to a stable VAR). We use actual inflation to deflate the equity cash flows. For this we use the GDP deflator (for consistency with the SPF forecast) published by the BEA. We also measure inflation uncertainty using SPF responses in a manner exactly analogous to that used for the construction of the real uncertainty measure (described below).

7.2.3 Subjective Profit Growth Expectations

We measure subjective profit growth expectations using the Survey of Professional Forecasters (SPF). Specifically, we use the median survey response for the four-quarter ahead percent change in the NIPA measure of nominal corporate profits. To calculate a real profit growth measure, we subtract, at the respondent level, the four-quarter rate of expected GDP deflator inflation.

7.2.4 Habit-Based Risk Aversion

We construct a habit-based model of local relative risk aversion following Campbell and Cochrane (1999, CC hereafter). CC use a model of external habit to motivate stochastic risk aversion, the log of which we denote as ra_t. Risk aversion is a function of the log 'surplus consumption' ratio, s_t,

$$ra_t = \ln(\gamma) - s_t \tag{31}$$

where γ is the instantaneous utility curvature parameter. The surplus consumption ratio is defined by CC as:

$$s_t = \ln\left(\left(C_t - H_t\right)/C_t\right) \tag{32}$$

where C_t is real nondurable consumption and H_t is the 'habit stock' which is roughly speaking a moving average of past consumption levels. Rather that modelling H_t directly, CC model s_t as an autoregressive, heteroskedastic process which is perfectly (conditionally) correlated with consumption growth innovations, ε_t^c

$$\begin{aligned} s_t &= (1-\phi)\,\overline{s} + \phi s_{t-1} + \lambda_{t-1}\varepsilon_t^c \\ \lambda_t &= \left\{ \begin{array}{cc} \frac{1}{\overline{S}}\sqrt{1-2\left(s_t-\overline{s}\right)}-1 & s_t \leq s_{\max} \\ 0 & s_t > s_{\max} \end{array} \right\} \end{aligned} \tag{33}$$

where the parameters, $\gamma, \phi, \overline{s}, \overline{S}$ and $s_{\max}$ are calibrated by CC to fit several salient features in the data. We use the parameter values in CC to create our empirical proxy for ra_t. The innovation term, ε_t^c, is the shock to consumption growth, and following CC we use demeaned values for real nondurables and services consumption log growth from the NIPA tables. The sensitivity of s_t to ε_t^c is governed by the λ_t process, which is always non-negative. Consequently, risk aversion tends to behave counter-cyclically. Because the starting point of s_t is not specified, we start the process at its unconditional mean, $\overline{s}$, at the beginning of the consumption growth sample, 1947Q2. Given that our analysis only starts in 1968Q4, the level of s_t is not sensitive to that choice.

7.2.5 Real Growth Uncertainty

We use two imperfect SPF measures of uncertainty about future real growth to generate a real uncertainty index. First, respondents are asked to report their subjective assessment of the probability of negative real GDP growth over the next quarter. Assuming a binomial distribution for real GDP growth (+1.0% growth in expansion, -0.5% growth in contractions), we calculate the standard deviation of real growth for each respondent, and calculate the median response, denoted sd_t. The second measure we use is the dispersion in respondents' expectation for real GDP growth over the next four quarters. The dispersion measure we use is the difference between the 90th percentile response and the 10th percentile of all responses, and is denoted dp_t. To aggregate these two measures, we assume that "true" uncertainty, vr_t^*, follows an AR(1) process, and both empirical measures are noisy indicators of vr_t^*.

$$\begin{aligned} vr_t^* &= b\, vr_{t-1}^* + \varepsilon_t^{vr*} \\ \begin{bmatrix} sd_t \\ dp_t \end{bmatrix} &= \begin{bmatrix} f^{v\pi} \\ f^{d\pi} \end{bmatrix} vr_t^* + \begin{bmatrix} \sigma^{sd}\varepsilon_t^{sd} \\ \sigma^{dp}\varepsilon_t^{dp} \end{bmatrix} \end{aligned}$$

where all variables are demeaned and $\left[\varepsilon^{vr*}, \varepsilon^{ds}, \varepsilon^{dp}\right]$ are distributed i.i.d. $N(0, I)$. Conditional (not smoothed) filtered estimates for vr_t^* are easily estimable by standard Kalman filter methods. We make no attempt to correct for the filtering error.

7.3 Bootstrapping Procedure for Vector Autoregressions

The procedure we employ is as follows. Recall that the VAR we estimate on observed data is

$$W_t = \mu^w + A^w W_{t-1} + \Sigma^w \varepsilon_t \tag{34}$$

1. Calculate, by OLS, point estimates for the VAR parameters, $\widehat{\mu_0^w}, \widehat{A_0^w}$, and $\widehat{\Sigma_0^w}$ using the raw data. Also extract values for the residuals, $\{\widehat{\varepsilon}_t\}_0$
2. Calculate all the reported statistics as $\widehat{\Omega_0}$
3. For 10,000 iterations indexed by i
   (a) randomly shuffle the vector $\{\widehat{\varepsilon}_t\}$ across time to generate $\{\widehat{\varepsilon}_t\}_i$
   (b) Generate a simulated sequence for $\{W_t\}_i$ under the assumed VAR data generating process and the shuffled innovations, $\{\widehat{\varepsilon}_t\}_i$, beginning the $\{W_t\}_i$ sequence at the first data observation, W_1
   (c) Calculate, by OLS, point estimates for the VAR parameters, $\widehat{\mu_i^w}, \widehat{A_i^w}$, and $\widehat{\Sigma_i^w}$ using the drawn data,$\{W_t\}_i$.
   (d) Calculate all the reported statistics as $\widehat{\Omega_i}$
4. Report a confidence interval for $\widehat{\Omega_0}$ as the spread between the 95th and 5th percentile across $\widehat{\Omega_i}$ draws.

7.4 Monte Carlo Procedure for Country Cross-Sectional Regressions

The panel data set is comprised of monthly observations of $ey_{i,t}$, $by_{i,t}$, $\pi_{i,t}$, and $recession_{i,t}$ (as defined in the text) monthly from December 1987 through June 2005 for 20 countries. The regressions we report in Table 2 are of the form,

$$corr_i\left(ey_t, by_t\right) = a + b\,\overline{infl_i} + c\ recess_i^{\%} + d\left(\overline{infl_i} \cdot recess_i^{\%}\right) + u_i \tag{35}$$

where $corr_i\left(ey_t, by_t\right)$ is the time-series correlation between ey and by for country i, $\overline{infl_i}$ denotes the full-sample country-specific mean of inflation and $recess_i^{\%}$ denotes the percentage of observations during which the country was in recession. OLS statistics may be poorly behaved in this regression given (1) the small sample of 20 countries, (2) sampling error in the generated regressors and regressand, and (3) the presence of limited dependent variables (correlations confined to the unit interval). To account for this, we report OLS coefficients and t-ratios in Table 2, but then use the following Monte Carlo procedure to assess the significance of the results.

First, we use the panel data to calculate estimates (and an estimate of their covariance matrix) for the vector,

$$\left\{ corr_i\left(ey, by\right), \overline{infl_i}, recess_i^{\%}, \overline{infl_i} \cdot recess_i^{\%} \right\}_{i=1}^{20}. \tag{36}$$

That is, we jointly estimate 80 statistics: four for each of 20 countries. We use standard GMM techniques allowing for generalized heteroskedasticity and autocorrelation and assume that these estimates are well-behaved[10]. From these estimates and covariance matrix, we generate 10,000 draws from the associated normal distribution. For each draw, we run the OLS regression in Equation (35) and examine the properties of the OLS t-ratios. However, our aim is to simulate the data under the null hypothesis that none of the explanatory variables are related to $corr_i\left(ey_t, by_t\right)$ in the cross-section. Note that the null hypothesis will not necessarily hold in the draws (for instance, if Country X has a high $corr_i\left(ey_t, by_t\right)$ and high $\overline{infl_i}$, in the data sample, this information will be preserved, in expectation, for every draw). To impose the null, we randomize the matching of $corr_i\left(ey_t, by_t\right)$ with $\left[\overline{infl_i}, recess_i^{\%}, \overline{infl_i} \cdot recess_i^{\%}\right]$ cross-sectionally for each draw. For instance, Country X's $corr_i\left(ey_t, by_t\right)$ draw is randomly reassigned to Country Y's draw of the triple, $\left[\overline{infl_i}, recess_i^{\%}, \overline{infl_i} \cdot recess_i^{\%}\right]$. In this way, relationships among the explanatory variables are preserved, but the null hypothesis holds in expectation for every draw.

For each simulated regression, we collect t-ratios for each regression coefficient. We then count the number of times the simulated t-ratios exceed the sample OLS t-ratios. If the portion of simulated t-ratios exceeding the sample t-ratio is greater than 10 percent, we conclude that the estimate is insignificant. If the portion of simulated t-ratios which exceed the sample t-ratio is greater that 5%, but less than 10%, we conclude that the estimate is significant at the 10% level, etc.

[10] This may be justified by noting that the data used for the estimates are comprised of about 240 monthly observations of 4 series (EY, by, π, $recess$) over 20 countries, or about 19,000 data points, whereas the 80 estimates and covariance matrix require 80 + 80*81/2 or about 3000 parameters. The saturation ratio is therefore about 6.

Table 1: U.S. VAR Results

Panel A: Decomposing Yield (Co-)Variation

$VAR\,(by_t)^*$	$VAR\,(ey_t)$	$COV\,(by_t, ey_t)^*$	$CORR\,(by_t, ey_t)$
0.45	0.63	0.22	0.78
$(0.20, 0.60)$	$(0.35, 0.80)$	$(0.03, 0.43)$	$(0.37, 0.90)$

Fractional Contributions

	$VAR\,(by_t)$		$VAR\,(ey_t)$		$COV\,(by_t, ey_t)$ $einf_t$	$COV\,(by_t, ey_t)$ rrf_t	$COV\,(by_t, ey_t)$ irp_t
$einf_t$	0.55	$ey_t^{\Delta d}$	0.14	$ey_t^{\Delta d}$	0.12	0.02	0.00
	$(0.28, 0.71)$		$(-0.10, 0.40)$		$(-0.09, 0.42)$	$(-0.06, 0.08)$	$(-0.13, 0.13)$
rrf_t	0.22	ey_t^{rrf}	0.07	ey_t^{rrf}	0.07	0.02	0.00
	$(0.18, 0.27)$		$(0.02, 0.11)$		$(0.03, 0.11)$	$(0.01, 0.03)$	$(-0.02, 0.03)$
irp_t	0.22	ey^{erp}	0.80	ey^{erp}	0.59	0.17	0.03
	$(0.09, 0.48)$		$(0.52, 1.07)$		$(0.21, 1.16)$	$(0.05, 0.26)$	$(-0.40, 0.26)$

Panel B: Decomposing ey_t^{erp} into ey_t^{erp-sp} and ey_t^{erp-re}

Fractional Contributions

	$VAR\,(ey_t)$		$COV\,(by_t, ey_t)$ $einf_t$	$COV\,(by_t, ey_t)$ rrf_t	$COV\,(by_t, ey_t)$ irp_t
ey^{erp-sp}	0.53	ey_t^{erp-sp}	0.51	0.13	0.00
	$(0.13, 0.76)$		$(0.15, 0.95)$	$(0.06, 0.18)$	$(-0.34, 0.13)$
ey^{erp-re}	0.27	ey^{erp-re}	0.08	0.05	0.03
	$(0.12, 0.73)$		$(-0.01, 0.36)$	$(-0.02, 0.16)$	$(-0.19, 0.22)$

Panel C: Equity Yields, Expected Inflation and Subjective Earnings Expectations Biases

Correlations

$einf_t - bias_t$	-0.04
	$(-0.35, 0.27)$
$einf_t - ey_t$	0.85
	$(0.48, 0.93)$
$bias_t - ey_t$	0.02
	$(-0.29, 0.34)$

Fractional Contributions to $einf_t - ey_t$ Covariance

$ey_t^{\Delta d}$	ey_t^{rrf}	ey^{erp-sp}	ey^{erp-re}
0.16	0.09	0.66	0.10
$(-0.11, 0.53)$	$(0.04, 0.15)$	$(0.25, 0.89)$	$(-0.01, 0.38)$

Results in this table are based on the latent VAR, $Y_t = \mu + AY_{t-1} + \Sigma\varepsilon_t$,where $Y_t = [einf_{tt}, rrf_t, \Delta d_t, erp_t, irp_t, x_t]'$and $x_t = [ra_t, vr_t, \Delta ern_t, gern_t^s]'$, $\varepsilon \sim (0, I)$ and irp_t and erp_t are unobserved. The Y_t system parameters are derived from VAR estimates on the observable vector $W_t = [einf_t, rrf_t, \Delta d_t, ey_t, by_t, x_t]'$ using the data and methodology described in the Appendix. The procedure for decomposing ey_t and by_t into their component pieces (e.g. $ey_t^{\Delta d}$ for ey_t, and rrf_t for by_t) is described in Section 2 as is the procedure for decomposing ey_t^{erp} into parts spanned-by and orthogonal-to proxies of rational equity risk premiums. Bootstrapped 90 percent confidence intervals are reported in parentheses. $*$ denotes that the reported statistic has been multiplied by 100 for readability.

Table 2: Cross-Country Results

Specification	$hinfl_i^{\%}$	$recess_i^{\%}$	$stag_i^{\%}$	$\overline{infl}_i$	$\overline{infl\_rec}$	R^2
(1)	3.95					0.07
	(1.10)*					
(2)		-0.40				0.01
		(0.43)				
(3)			21.37			0.23
			(2.24)**			
(4)	-0.68	-1.59	30.52			0.37
	(0.19)	(1.70)	(2.55)***			
(5)				3.06		0.32
				(2.74)**		
(6)					8.78	0.41
					(3.38)***	
(7)		1.25		-0.50	7.93	0.52
		(0.62)		(0.37)	(1.85)**	

This table presents results for cross-sectional regressions of the general form

$$corr_i\,(ey_t, by_t) = a + b\,hinfl_i^{\%} + c\,recess_i^{\%} + d\,stag_i^{\%} + u_i \tag{37}$$

and

$$corr_i\,(ey_t, by_t) = a + b\,\overline{infl_i} + c\,recess_i^{\%} + d\,\overline{infl\_rec} + u_i$$

where by is the locally nominally risk free long bond yield for country i at time t and ey_t is the dividend yield. The variable $corr_i\,(ey_t, by_t)$ is the time-series correlation between ey_t and by_t for country i. The variable $hinfl_i^{\%}$ denotes the percentage of observations during which the country exhibited high inflation, defined as 10 percent or more (annualized) inflation per month. The variable $recess_i^{\%}$ denotes the percentage of observations during which the country was in recession (the mean of the binary recession indicator variable $recess_{i,t}$). The variable $stag_i^{\%}$ denotes the percentage of observations during which the country exhibited stagflation, defined as the coincidence of high inflation and recession. The variable $\overline{infl_i}$ denotes the full-sample country-specific mean of inflation, $infl_{i,t}$. The variable $\overline{infl\_rec}$ denotes the country-specific time-series mean of the interaction, $infl_{i,t} \cdot recess_{i,t}$. Data are monthly from 1987-2005 for 20 countries. OLS coefficients and t-ratios (in parentheses) are reported. The superscripts *, ** and *** denote significance at the 10, 5, and 1 percent level. Significance is determined using corrections for the small sample and pre-estimation effects of the regressors and regressand utilizing a Monte-Carlo method detailed in the appendix.

Table 3: VAR Specification Tests

Panel A: VAR lag length

	$VAR(1)$	$VAR(2)$	$VAR(3)$	$VAR(4)$
BIC	-73.5	-72.3	-70.7	-70.0
AIC	-75.2	-75.7	-75.7	-75.6

Panel B: Cumby-Huizinga tests (p-values)

	$VAR(1)$	$VAR(2)$
$einf_t$,	0.44	0.38
rrf_t	0.01	0.26
Δd_t	0.04	0.04
ra_t	0.00	0.08
vr_t	0.00	0.57
Δern_t	0.37	0.21
$gern_t^s$	0.04	0.01
ey_t	0.79	0.79
by_t	0.71	0.44

Results in this table are based on the observable VAR, $W_t = \mu^w + A^w W_{t-1} + \Sigma^w \varepsilon_t$,where $W_t = [einf_{tt}, rrf_t, \Delta d_t, ey_t, by_t, x_t]'$ and $x_t = [ra_t, vr_t, \Delta ern_t, gern_t^s]$.' Panel A presents Bayesian information criteria for optimal VAR lag length. The row labeled BIC contains standard Schwartz test results and the row labeled AIC reports results for the Akaike test. In Panel B, p-values for Cumby-Huizinga tests for residual autocorrelation are presented. Each dependent variable is tested separated using the lagged instruments implied by the VAR. We test for autocorrelation at up to four lags.

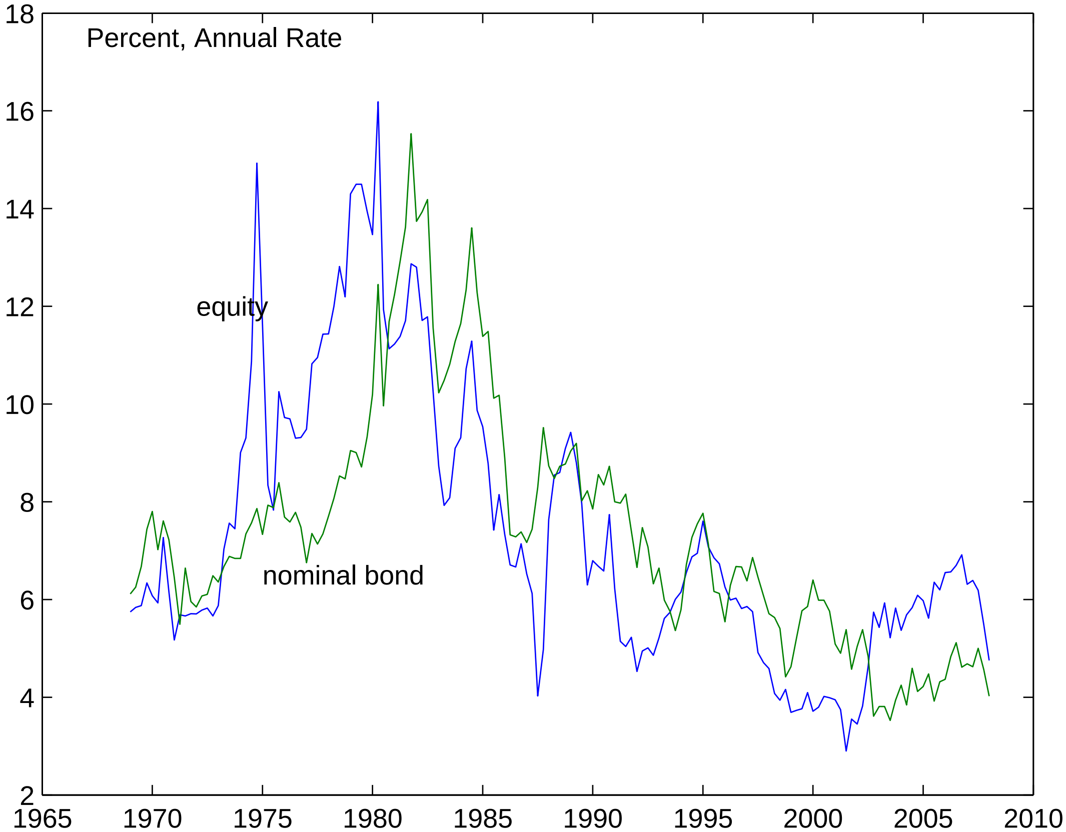
Table 4: U.S. VAR Robustness Exercises

Percent Contribution to $ey_t - by_t$ Covariance under Alternative Specifications

Specification	$COV\left(einf_t, ey_t^{erp}\right)$	$COV\left(einf_t, ey_t^{erp-re}\right)$
Main VAR	0.59	0.08
	$(0.21, 1.16)$	$(-0.01, 0.36)$
VAR(2)	0.58	0.06
	$(0.24, 1.08)$	$(-0.04, 0.25)$
Small VAR	0.47	-NA-
	$(0.19, 1.05)$	-NA-
Zeroed-out	0.56	0.08
	$(0.22, 1.14)$	$(-0.01, 0.15)$
w/inflation uncertainty	0.57	0.07
	$(0.18, 1.11)$	$(-0.01, 0.30)$
long-term inflation exp.	0.47	0.08
	$(0.19, 0.88)$	$(0.00, 0.34)$
alternative real rate	0.58	0.08
	$(0.15, 1.08)$	$(-0.03, 0.28)$
cash flow = earnings	0.42	0.10
	$(-0.20, 1.21)$	$(-0.20, 1.21)$
cash flow = div+repo	0.36	0.35
	$(-3.79, 4.78)$	$(-1.29, 2.37)$

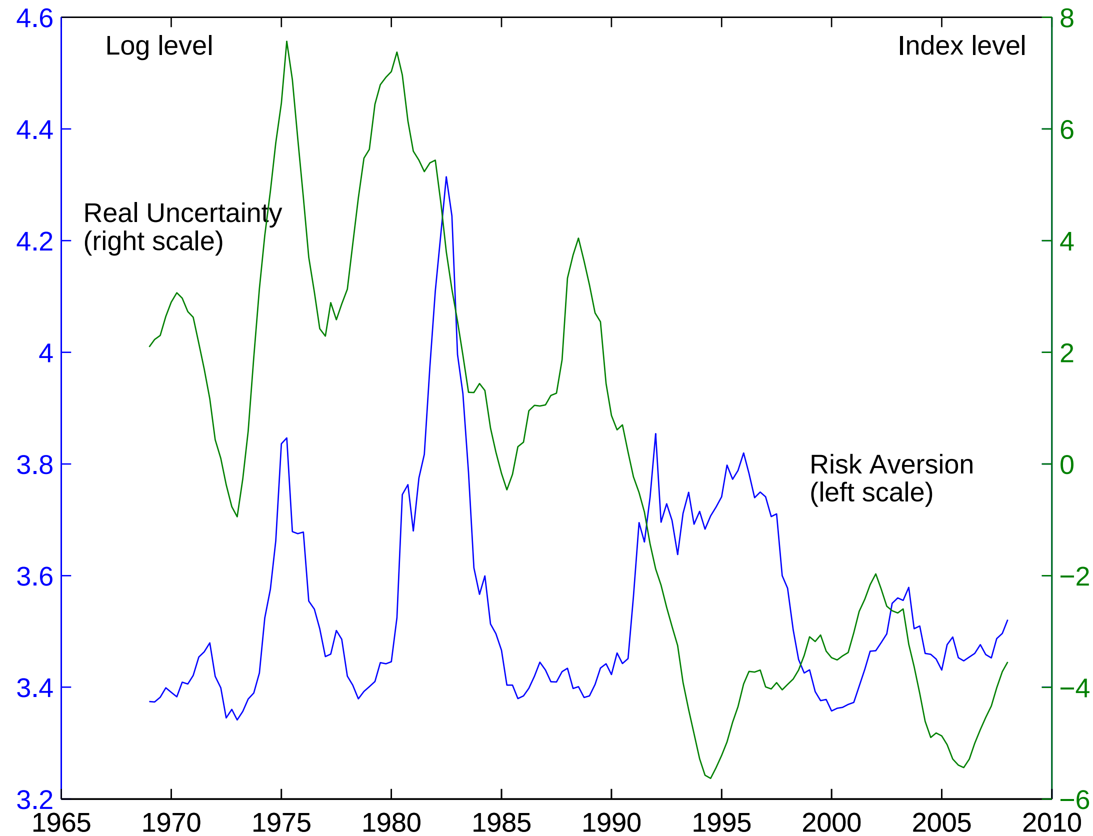
This table reports two key statistics (and their confidence intervals) reported for our main specification in Table 1 under a variety of alternative VAR specifications. The "Main VAR" row simply reproduces the statistics of interest from Table 1: the percent contribution to total $ey_t - by_t$ covariance of $COV\left(einf_t, ey_t^{erp}\right)$ and $COV\left(einf_t, ey_t^{erp-re}\right)$. The "VAR(2)" specification expands the Main VAR to include two lags of all the dependent variables. The "Small VAR" specification drops the x_t vector from the VAR list (without x_t, the $COV\left(einf_t, ey_t^{erp-re}\right)$ contribution cannot be calculated). The "Zeroed-out" specification employs a two-step estimation procedure for our main VAR: first estimate the VAR by OLS, noting all elements of A^W with OLS t-statistics less than 1. In the second step, re-estimate the VAR imposing that the low t-statistic coefficients are zero. The "w/inflation uncertainty" specification adds our measure of inflation uncertainty, $v\pi_t$, to the information variable vector, x_t. The "long-term inflation expectations" specification replace out usual four-quarter expected inflation measure with a longer-term survey-based inflation expectations measure (see data appendix). The "cash flow = earnings" specification replaces the dividend yield and dividend growth in the Main VAR with earnings growth and the earnings-price ratio. The "cash flow = div + repo" specification adds repurchases to dividends before calculating dividend growth and the dividend yield.

Figure 1: Equity and Bond Yield Time Series for the U.S.



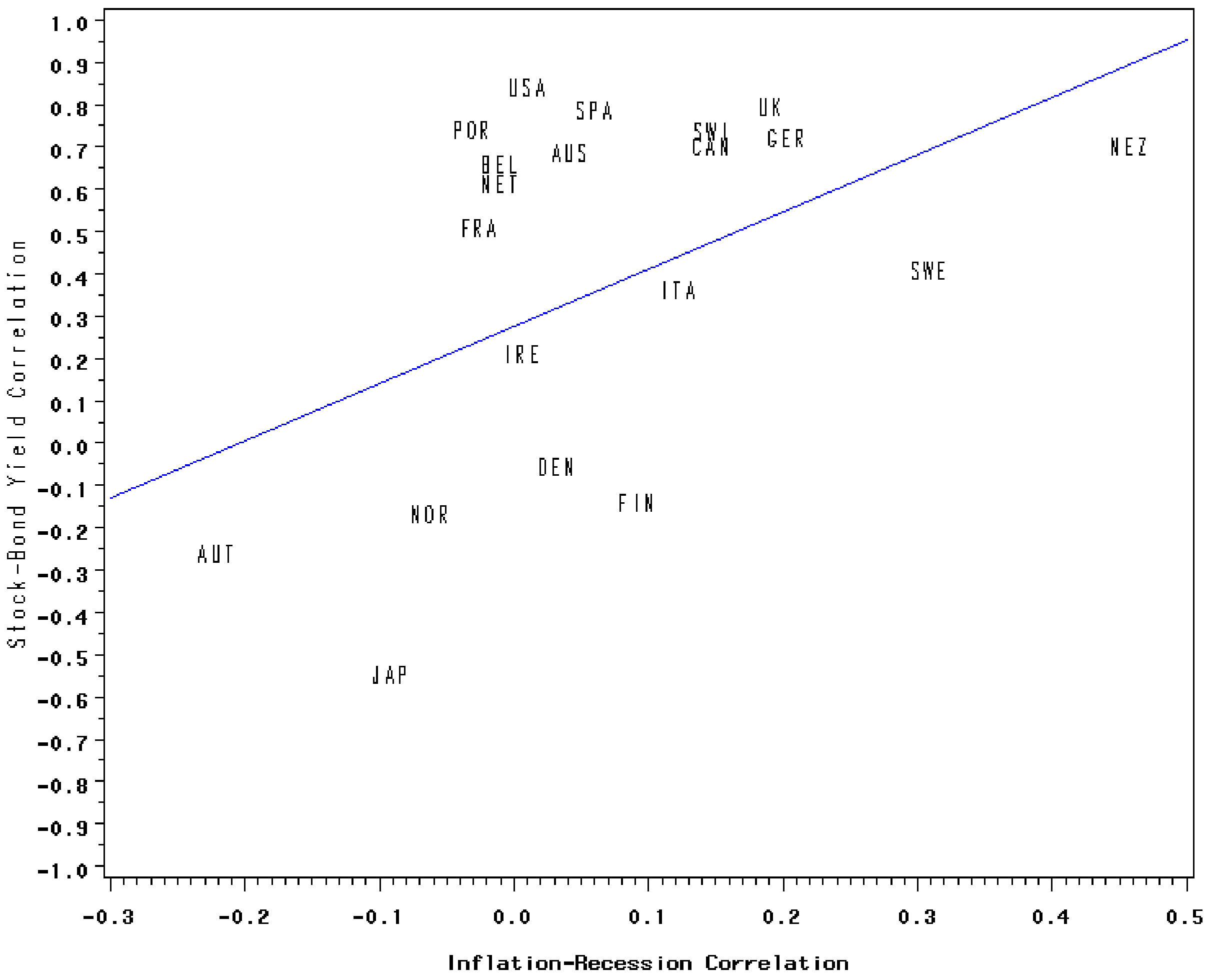
This figure plots time series for the equity yield, ey_t (blue, left scale), and the bond yield, by_t (green, right scale). We measure the equity yield, ey_t as the dividend yield for the S&P500, and the nominal bond yield, by_y, as that of the 10-year constant-maturiy Treasury. For illustration, both yields have been plotted in levels (that is, the ey_t series has been exponentiated), and in units of percentage points, annual rate.

Figure 2: Risk Aversion and Real Uncertainty



This figure plots time series for risk aversion, ra_t (blue, left scale), and real uncertainty, vr_t (green, right scale). Data construction is described in the appendix.

Figure 3: Multi-Country Relationship between Stagflation and the Fed Model



This figure plots countries in the panel data set along two dimensions: (1) the country specific time-series correlation between the dividend yield and the long term (locally risk free) nominal bond yield, and (2) the time series correlation between inflation and a recession indicator. The sample is monthly from December 1987 through June 2005. The slope of the regression line is 1.35 with an OLS standard error of 0.59. A regression (line not shown) estimated excluding the Japan (Austria) observation has a slope of 1.04 with an OLS standard error of 0.54 (1.10 with a standard error of 0.66).