Incomplete Information Processing: A Solution to the Forward Discount Puzzle[1]

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Abstract

The uncovered interest rate parity equation is the cornerstone of most models in international macro. However, this equation does not hold empirically since the forward discount, or interest rate differential, is negatively related to the subsequent change in the exchange rate. This forward discount puzzle implies that excess returns on foreign currency investments are predictable. In this paper we investigate to what extent incomplete information processing can explain this puzzle. We consider two types of incompleteness: infrequent and partial information processing. We calibrate a two-country general equilibrium model to the data and show that incomplete information processing can fully match the empirical evidence. It can also account for several related empirical phenomena, including that of "delayed overshooting". We show that incomplete information processing is consistent both with evidence that little capital is devoted to actively managing short-term currency positions and with a small welfare gain from active portfolio management. The gain is small because exchange rate changes are very hard to predict. The welfare gain is easily outweighed by a small cost of active portfolio management.

1 Introduction

One of the best established and most resilient puzzles in international finance is the forward discount puzzle.[1] Fama (1984) illuminated the problem with a regression of the monthly change in the exchange rate on the preceding one-month forward premium. The uncovered interest rate parity equation, which is the cornerstone of many models in international macro, implies a coefficient of one. But surprisingly Fama found a negative coefficient for each of nine different currencies. A currency whose interest rate is high tends to appreciate. This implies that high interest rate currencies have predictably positive excess returns. The relationship between excess returns and interest rate differentials is illustrated in Table 1 for five currencies against the U.S. dollar. A regression of the quarterly excess return on a foreign currency on the difference between the U.S. and foreign interest rate yields coefficients ranging from -1.5 to -4.[2] Moreover, as we document below, interest rate differentials continue to negatively predict the excess returns five to ten quarters ahead.

Most models assume that investors incorporate instantaneously all new information in their portfolio decisions. To explain the forward premium puzzle, we depart from this assumption. There are many types of costs associated with active portfolio management, whereby portfolios are adjusted at high frequency based on all available information. If investors delegate these decisions to financial institutions, some of the direct costs incurred are replaced by fees, as well as agency and monitoring costs. The fees charged by active portfolio managers tend to be substantial.[3] For the purpose of our analysis we therefore take as given that there

[1]For surveys see Lewis (1995), Engel (1996), or Sarno (2005). Some of the more recent contributions include Backus, Foresi and Telmer (2001), Beakert, Hodrick and Marshall (1997), Chaboud and Wright (2005), Chinn and Meredith (2005), Chinn and Frankel (2002), Fisher (2005), Flood and Rose (2002), Gourinchas and Tornell (2004), Mark and Wu (1998), Sarno, Valente and Leon (2006) and Verdelhan (2005).

[2]While there are potential statistical problems in these predictability regressions (mainly small sample bias and bias caused by the persistence of the forward discount), these problems usually can only explain a part of the total bias. See, for example, Stambaugh (1999), Campbell and Yogo (2006), or Liu and Maynard (2005).

[3]There is no established statistic on management fees. But everything indicates that *active* portfolio managers, such as hedge funds, charge fees that are often well above 2% of invested funds. An interesting question is why these fees are high. They are likely to reflect three

are some non-negligible costs associated with active portfolio management.

Incomplete information processing can take two different forms: (i) *infrequent* information processing, where investors make portfolio decisions infrequently, and (ii) *partial* information processing, where investors use only a subset of all available information. We will argue that there is extensive evidence on both. We examine the impact of incomplete information processing in a simple two-country general equilibrium model that is calibrated to data for the five currencies in Table 1. Agents are fully rational given the constraints they face (including the costs). We show that even for a quite small cost of making portfolio decisions, most investors do not find it in their interest to actively exploit all available information. We find that such a framework can account for both the sign and size of forward discount bias illustrated in Table 1.

There are two distinct features that are surprising in the forward discount anomaly. The first aspect is the consistent sign of the bias. Why would the excess return be high for currencies whose interest rate is relatively high? This can be explained by infrequent information processing by investors. Froot and Thaler (1990) and Lyons (2001) have informally argued that models where some agents are slow in responding to new information may explain the forward discount puzzle. The argument is quite simple. An increase in the interest rate of a particular currency will lead to an increase in demand for that currency and therefore an appreciation of the currency. But when investors make infrequent portfolio decisions, they will continue to buy the currency as time goes on. This can cause a continuing appreciation of the currency. This is consistent with the evidence documented by Fama (1984) that an increase in the interest rate leads to a subsequent appreciation. It also implies that a higher interest rate raises the expected excess return of the currency.

Infrequent information processing can also explain the dynamic response of currency depreciation, or excess returns, to changes in interest rates. Interestingly,

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elements: (i) the costs associated with collecting and processing information, computing the optimal portfolio, and attracting and distributing funds, (ii) profit margins due to their financial expertise and product differentiation and (iii) a profit sharing component intended to deflect agency and monitoring costs. There exists a substantial literature investigating the compensation of porftolio managers. See for example Berk and Green (2005) or Dybvig, Farnsworth and Carpenter (2004) and references therein.

predictability is not restricted to horizons of a month or a quarter: the forward discount at time t can also predict excess returns at future dates. This feature is typically overlooked in the literature. Consider a regression of a future three-month excess return q_{t+k}, from $t+k-1$ to $t+k$, on the current interest rate differential $i_t - i_t^*$. Figure 1 shows the evidence for the five countries in Table 1, where k increases from 1 to 30. There is significant predictability with a negative sign for five to ten quarters. Over longer horizons, however, the slope coefficient becomes insignificant or even positive. This is consistent with findings that uncovered interest parity holds better at longer horizons.[4] The persistence in the predictability of excess returns is related to the phenomenon of delayed overshooting. Eichenbaum and Evans (1995) first documented that after an interest rate increase, a currency continues to appreciate for another 8 to 12 quarters before it starts to depreciate.[5] As pointed out above, this is exactly what one expects to happen when investors make infrequent portfolio decisions.

The second surprising aspect of the forward premium puzzle is that investors do not exploit the predictability of excess returns. The standard explanation is that an excess return reflects a risk premium. But many surveys written on the forward discount puzzle have concluded that explanations for the forward discount puzzle related to time-varying risk premia have all fallen short.[6] Our analysis shows that, given the high risk involved, a small asset management cost discourages investors from exploiting the predictability.[7] This risk is illustrated in Figure 2, which shows for one currency, the DM/\$, a scatter plot of the excess return on DM against the U.S. minus German interest rate differential. The negative slope of the regression line represents predictability. It is clear though that predictability is largely overshadowed by risk.[8] Since there is so much uncertainty, potential welfare gains from actively exploiting the predictability are very small and can easily be

[4]See for example Chinn and Meredith (2005), Boudoukh et al. (2005), or Chinn (2006).

[5]Gourinchas and Tornell (2004) explain both predictability and delayed overshooting with distorted beliefs on the interest rate process.

[6]See Lewis (1995) or Engel (1996). Recently Verdelhan (2005) has more success based on a model with time-varying risk aversion due to habit formation.

[7]Verdelhan (2004) shows that foreign exchange excess returns lead to small and unstable Sharpe ratios in the short-term.

[8]More formally, this is reflected in the low R^2 for excess return regressions in Table 1, which is on average 0.09.

outweighed by portfolio management costs. This means that for many investors it is simply not worthwhile to actively trade on excess return predictability. Even for those who do actively trade on the excess return predictability, the high risk limits the positions they will take. We will show in the context of the model that a small fraction of financial wealth actively devoted to forward bias trade will not unravel the impact of infrequent decision making.

It is the combination of infrequent and partial information processing that is key to our results. Infrequent information processing by itself leads to predictability of the right sign, but does not fully match the data quantitatively. On the other hand, partial information processing by itself does not lead to predictability. We find that it is the combination of the two perspectives that matches the data very closely. The distinction between partial use of information and infrequent information processing is also found in the recent literature on rational inattention (or inattentiveness) in macro models. One strand of the literature, based on Sims (1998, 2003), considers continuous but partial information processing due to (Shannon) capacity constraints. In another strand of the literature, e.g., Mankiw and Reis (2002), there are time-dependent decision rules, where information is processed infrequently.[9] Although the two types of approaches are related, they have a different impact in an asset pricing context.

Our theoretical analysis is also related to recent developments in the stock market literature.[10] On the one hand, several studies show how asset allocation is affected by predictability.[11] On the other hand, some recent papers examine the

[9]There is a growing literature in macroeconomics based on rational inattention, in particular in the context of price setting by firms and consumption decisions by households. Examples of papers where agents process partial information due to information capacity constraints are Sims (1998, 2003) and Mackowiak and Wiederholt (2005). Examples of papers where agents process information infrequently due to explicit information processing costs are Begg and Imperato (2001), Bonomo and de Carvalho (2003), Moscarini (2004), and Reis (2004a,b). Carroll (2003), Dupor and Tsuruga (2005) and Mankiw and Reis (2002) assume exogenously that new information arrives, and is processed, at a certain rate (either with a fixed probability or at fixed intervals).

[10]Evidence of excess return predictability has been extensively documented for stock and bond markets (e.g. see Cochrane, 1999).

[11]See for example Kandel and Stambaugh (1996), Campbell and Viceira (1999), or Barberis (2000).

impact of infrequent portfolio decisions due to limited attention in asset markets.[12] However, the literature has not linked predictability with infrequent trading: those papers that examine the impact of predictability assume it exogenous, while papers that examine infrequent portfolio decisions do not examine its impact on asset prices. Our paper departs from the existing literature by incorporating both predictability and infrequent portfolio decisions and by showing that the latter can cause the former. Our methodological contribution to the literature is to solve endogenously for an asset price in a model with time-varying expected returns.

To what extent do the assumptions of infrequent and partial information processing match the behavior of investors? It is well known that individual investors make infrequent portfolio decisions. The Investment Company Institute (2002) reports that of U.S. investors who have mutual fund investments or hold stock directly, 60% made no transactions in 2001.[13] As many as 85% of investors report that they follow a buy-and-hold strategy. Systematic evidence is typically not recorded for the foreign exchange market, but trade in the foreign exchange market is closely related to international trade in stocks, bonds and other assets. Infrequent portfolio reallocation across markets is consistent with the evidence of Froot, O'Connell, and Seasholes (2001). They show that cross-country equity flows react with lags to a change in returns, while the contemporaneous reaction is muted.

Moreover, there are indications that the size of the market that actively trades on expected excess return opportunities in currency markets is only a tiny fraction of cross border financial holdings. This market consists of hedge funds exploiting forward discount bias and financial institutions that provide such services to individual clients.[14] Interviews that we have conducted with financial institutions that conduct these trades suggest that worldwide the capital devoted to it is currently about $200 billion. This is only 0.3% of global cross border financial holdings and

[12]Duffie and Sun (1990), Lynch (1996), and Gabaix and Laibson (2002) have all developed models where investors make infrequent portfolio decisions because of a fixed cost of information collection and decision making.

[13]For a discussion of evidence on infrequent trading see Bilias et al. (2005) and Vissing-Jorgenson (2004).

[14]The latter include currency overlay managers, commodity trading advisors and leveraged funds offered by established asset management firms. See Sager and Taylor (2006) for a recent description of the foreign exchange market.

even far less as a fraction of overall financial wealth (in 2004 external assets were \$56.6 trillion). Mutual funds do not actively exploit excess returns on foreign investment since these funds only trade within a certain asset class and cannot freely switch between domestic and foreign assets.[15] Lyons (2001) points out that as a result of the excessive risks involved most large financial institutions do not even devote their own proprietary capital to currency strategies based on the forward discount bias.[16]

Regarding the partial use of information, anecdotal evidence suggests that even the most active traders use only a very small fraction of the available information to predict future exchange rates. Interviews with institutions that actively conduct speculative trades exploiting the forward discount bias suggests that exchange rate expectations are formed based on very simple rules. Many institutions do not bother forecasting at all and expect the future spot rate to be the same as the current spot rate. Other institutions use a simple factor model, with four or five factors, to predict future exchange rates. These factors may include the forward discount or interest rate differential, equity returns, some measure of risk-appetite and past currency changes. Others mainly use some form of technical analysis. There is no uniform practice in developing these forecasts and at most a very small subset of the available information space is used. This is not surprising because of the well-known difficulty in consistently outperforming the random walk when predicting exchange rate changes over short horizons.[17] It is likely that investors making infrequent decisions about currency holdings process even less information.

The remainder of the paper is organized as follows. Section 2 describes a two-country general equilibrium model where all investors make infrequent portfolio decisions. The model is calibrated to data for the five currencies in Table 1. In section 3 we discuss the implications of the model for the forward discount and delayed overshooting puzzles. We also consider an extension of the model to partial information processing and to investors that actively manage their portfolio each

[15] This may be partly endogenous since we show that the gains from actively reallocating the portfolio between different currencies is small.

[16] Banks, which are the most active institutions on the foreign exchange market, do not conduct these trades either. Two thirds of trade in the foreign exchange market is done among banks that are foreign exchange dealers (BIS, 2004). While these dealers follow the markets very closely during the course of a trading day, they hold little foreign exchange overnight.

[17] See Meese and Rogoff (1983) and more recently Cheung et al.(2005).

period. Section 4 relates our analysis to the existing literature on the forward discount puzzle. Section 5 concludes.

2 A Model of Infrequent Decision Making

In this section we present a model of the foreign exchange market where investors make infrequent portfolio decisions. We first describe the basic structure of the model and the solution method. We then discuss under what cost of active portfolio management it is optimal for all investors to make infrequent portfolio decisions. Some technical details are covered in the Appendix, with a Technical Appendix available on request providing full technical detail.

2.1 Model's Description

2.1.1 Basic Setup

We develop a one good, two-country, dynamic general equilibrium model. Our overall approach is to keep the model as simple as possible while retaining the key ingredients needed to highlight the role of infrequent decision making. There are overlapping generations (OLG) of investors who each live $T+1$ periods and derive utility from end-of-life wealth. Each period a total of n new investors are born, endowed with one unit of the good that can be invested in assets described below. We model the infrequent decision making by assuming that investors make only one portfolio decision when born for the next T periods. Below we will derive the threshold cost where it becomes optimal to make infrequent portfolio decisions.

This OLG setup is easier to work with than the alternative where agents have infinite horizons and make portfolio decisions every T periods. In that case we also would have to solve for optimal savings-consumption decisions, which depend on assumptions made about the frequency of those decisions. We have abstracted from saving decisions by assuming that agents derive utility from end-of-life wealth. This allows us to focus on portfolio decisions.[18] We want to emphasize though that

[18] An infinite horizon setup would complicate matters in other ways as well. The optimal portfolio would be hard to compute since it depends on a hedge against changes in expected returns. There would also be the standard complications associated with the stationarity of wealth and the distribution of wealth.

while such an infinite horizon setup would be more complicated, the mechanisms at work would be similar to that in our simpler OLG framework. The crucial element is that information is incorporated gradually into portfolio decisions because only a limited fraction of agents make new portfolio decisions each period. In both the infinite horizon and OLG setups a fraction $1/T$ of agents makes a new portfolio decision each period. The fact that in the OLG setup these decisions are made by a new "generation" of investors is of little relevance for what follows.

The model contains one good and three assets. In the goods market we assume purchasing power parity: $p_t = s_t + p_t^*$, where p_t is the log-price level of the good in the Home country and s_t the log of the nominal exchange rate. Foreign country variables are indicated with a star. The three assets are one-period nominal bonds in both currencies issued by the respective governments and a risk-free technology with real return $\bar{r}$.[19] Bonds are in fixed supply in the respective currencies.[20]

We first describe the monetary policy rules adopted by central banks, then optimal portfolio choice, and finally asset market clearing.

2.1.2 Monetary Policy

We assume that the Home country central bank commits to a constant price level. This implies zero Home inflation, so that the Home nominal interest rate is $i_t = \bar{r}$. The foreign interest rate is random, $i_t^* = -u_t$ where

$$u_t = \rho u_{t-1} + \varepsilon_t^u \quad \varepsilon_t^u \sim N(0, \sigma_u^2) \tag{1}$$

The error term captures foreign monetary policy innovations. The forward discount is:

$$fd_t \equiv i_t - i_t^* = u_t + \bar{r} \tag{2}$$

These assumptions imply that there are in essence only two assets, one with a risk-free real return $\bar{r}$ and one with a stochastic real return. The latter is Foreign bonds, which has a real return of $s_{t+1} - s_t + i_t^*$. This setup leads to much simpler

[19] This is necessary to tie down the real interest rate since the model does not contain saving and investment decisions.

[20] One can think of the governments that issue the bonds as owning claims on the riskfree technology whose proceeds are sufficient to pay the interest on the debt. The remainder is thrown in the water or spent on public goods that have no effect on the marginal utility from private consumption.

portfolios than we would get under symmetric monetary policy rules, in which case the real return on Home and Foreign bonds would both be stochastic.[21]

2.1.3 Portfolio Choice

We now turn to the optimal portfolios. Since PPP holds, Foreign and Home investors face the same real returns and therefore choose the same portfolio. We assume constant relative risk-aversion preferences over end-of-life consumption, with a rate of relative risk-aversion of γ. Investors born at time t maximize $E_t W_{t+T}^{1-\gamma}/(1-\gamma)$, where W_{t+T} is end-of-life financial wealth that will be consumed. Investors make only one portfolio decision when born, investing a fraction b_t^I in Foreign bonds.[22] End of life wealth is then

$$W_{t+T} = \prod_{j=1}^{T} R_{t+j}^p \tag{3}$$

where R_{t+j}^p is the gross investment return from $t+j-1$ to $t+j$,

$$R_{t+j}^p = (1-b_t^I)e^{i_{t+j-1}} + b_t^I e^{s_{t+j}-s_{t+j-1}+i_{t+j-1}^*} \tag{4}$$

In order to solve for optimal portfolios we adopt a second order approximation of log portfolio returns.[23] We define $q_{t+j+1} = s_{t+j+1} - s_{t+j} + i_{t+j}^* - i_{t+j}$ as the excess return on Foreign bonds from $t+j$ to $t+j+1$ and $q_{t,t+T} = q_{t+1} + .. + q_{t+T}$ as the cumulative excess return from t to $t+T$. In Appendix A.1 we show that the optimal portfolio rule is

$$b_t^I = b^I + \frac{E_t q_{t,t+T}}{\gamma \sigma_I^2} \tag{5}$$

where b^I is a constant and σ_I^2 is defined as

$$\sigma_I^2 = \left(1 - \frac{1}{\gamma}\right) var_t(q_{t,t+T}) + \frac{1}{\gamma} \sum_{j=1}^{T} var_t(q_{t+j}) \tag{6}$$

[21] Without having to introduce nominal rigidities, from the point of view of the Home country it also captures the fact that exchange rate risk is far more substantial than inflation risk.

[22] We assume that the portfolio share is held constant for T periods, which fits reality better than investors deciding on an entire path of portfolio shares for the next T periods.

[23] We maximize the objective function after replacing the log portfolio returns by their second order approximation. An alternative solution method is to start from the first order condition for portfolio choice and then substitute a first order approximation of the log portfolio return. This gives the exact same solution. The latter is the approach adopted by Engel and Matsumoto (2005) to solve for optimal portfolios in a general equilibrium model with home bias.

The optimal portfolio therefore depends on the expected excess return over the next T periods, with less aggressive portfolio choices made when either agents are more risk averse or there is more uncertainty about future returns.

2.1.4 Liquidity Traders

There is another group of investors that we refer to as liquidity traders. These are modeled exogenously. In the noisy rational expectations literature in finance it is very common to introduce exogenous noise or liquidity traders since this noise prevents the asset price from revealing the aggregate of private information. Here we do not have private information, but the exogenous liquidity traders are introduced to disconnect the exchange rate from observed macroeconomic shocks. It is well known since Meese and Rogoff (1983) that observed macro fundamentals explain very little of exchange rate volatility for horizons up to 1 or 2 years. This is what Lyons (2001) has called the exchange rate determination puzzle. In the absence of shocks to liquidity trade the exchange rate would only be driven by interest rate shocks in the model, in clear violation of the empirical evidence.[24] Related to this, introducing liquidity shocks can account for the low R^2 of regressions of the excess return on the forward discount. While changes in exchange rates are predictable by the forward discount, the extent of this predictive power is very limited, as discussed in the Introduction.

The real value of Foreign bond investments by liquidity traders at time t is $(\bar{x} + x_t)\bar{W}$, where $\bar{W}$ is aggregate steady state financial wealth and x_t follows the process:

$$x_t = C(L)\varepsilon_t^x = (c_1 + c_2 L + c_3 L^2 + ...)\varepsilon_t^x \quad \varepsilon_t^x \sim N(0, \sigma_x^2) \tag{7}$$

We choose the process of liquidity trade to match two key features of the data. First, we choose the magnitude of the shocks to match observed exchange rate volatility in the data, which affects optimal portfolios. Second, we choose the polynomial $C(L)$ such that in equilibrium the impact of these "noise" shocks on

[24] Bacchetta and van Wincoop (2004, 2006) show that in the presence of heterogenous information small liquidity shocks can have a large effect on exchange rates movements, so that exchange rates are disconnected from macroeconomic fundamentals. The exogenous "noise" that is generated by liquidity supply shocks can also be modeled endogenously, without any implications for the results. See Bacchetta and van Wincoop (2006) in the context of the foreign exchange market.

the exchange rate is very persistent and the exchange rate is close to a random walk, as widely documented. We will return to this below when discussing the solution method. Apart from being realistic, this is also of importance in the decision of whether to actively manage the portfolio each period or not. If there were large predictable components to exchange rate changes, the gain from active portfolio management would obviously be larger.

It is important to note that liquidity trade shocks do not contribute to excess return predictability associated with the forward discount. The reason is that we do not allow these shocks to affects interest rates, either directly or indirectly.[25]

2.1.5 Market Clearing

We finally discuss the Foreign bond market clearing condition. There is a fixed supply B of Foreign bonds in the Foreign currency. The real supply of Foreign bonds is $Be^{-p_t^*} = Be^{s_t}$, where we normalized the Home price level at 1 (so that $p_t = 0$). Investors are born with an endowment of one, but their wealth accumulates over time. Let $W_{t-j,t}^I$ be the wealth at time t for an investor born at $t-j$. This is equal to the product of total returns over the past j periods, $W_{t-j,t}^I = \prod_{i=1}^j R_{t-j+i}^p$. The market clearing condition for Foreign bonds is then

$$n\sum_{j=1}^{T} b_{t-j+1}^I W_{t-j+1,t}^I + (\overline{x} + x_t)\bar{W} = Be^{s_t} \tag{8}$$

We will set $\bar{x}$ such that the steady state supply of Foreign bonds relative to total financial wealth, $Be^{\bar{s}}/\bar{W}$, is equal to b, which is set exogenously. Without loss of generality we will assume that the nominal supply B is such that this holds for a zero steady state log exchange rate: $\bar{s} = 0$.

Several non-linear terms show up in the market clearing condition. Portfolio demand depends on the product of portfolio shares and wealth, with the latter being a function of past portfolio shares and returns. The supply is also a non-linear function of the log exchange rate. We linearize this budget constraint around the point where the log exchange rate and asset returns are zero and portfolio shares

[25] In a previous version of the paper, we assumed an interest rate rule reacting to the exchange rate. In that context, liquidity trade contributes to the forward bias puzzle since liquidity shocks are correlated with the interest rate. For this impact to be large, however, the interest rate must be very sensitive to the exchange rate. This is the mechanism emphasized by McCallum (1994).

are equal to their steady-state values. Details can be found in Appendix A.2 and the Technical Appendix. We will think of liquidity demand shocks as equivalent to exogenous supply shocks, so that the linearized net supply after dividing by steady state wealth is $bs_t - x_t$.

2.1.6 Solving the Model

From the market equilibrium condition, we can derive the equilibrium exchange rate. The details of the solution method are discussed in Appendix A.2 and in the Technical Appendix. Here we describe the main elements. We conjecture the following equilibrium exchange rate equation:

$$s_t = A(L)\varepsilon_t^u + B(L)\varepsilon_t^x \tag{9}$$

where $A(L) = a_1 + a_2 L + ...$ and $B(L) = b_1 + b_2 L + ...$ are infinite lag polynomials. Conditional on this conjectured exchange rate equation we compute excess returns as well as their first and second moments that enter into the optimal portfolios. One can then solve for the parameters of the polynomials by imposing the linearized bond market equilibrium condition.

But rather than solving for $A(L)$ and $B(L)$ given the model and the process for interest rate and supply shocks, we instead choose $A(L)$, b_1 and $C(L)$ (process of supply shocks) such the that (i) the Foreign bond market equilibrium condition is satisfied and (ii) $\hat{x}_t = B(L)\epsilon_t^x$ follows an AR process:

$$\hat{x}_t = \rho_x \hat{x}_{t-1} + b_1 \epsilon_t^x \tag{10}$$

The latter implies $b_i = \rho_x^{i-1} b_1$ for $i > 1$. Rather than taking the process of supply shocks as given, we therefore choose it such that the impact of these shocks on the exchange rate will follow a highly persistent AR process.

As discussed in the Appendix, the parameter b_1 and the parameters of the polynomial $A(L)$ can be solved jointly. Once b_1 and $A(L)$ are solved, the parameters of the polynomial $C(L)$ follow immediately from the market clearing condition. But $C(L)$ is not consequential for the rest of the analysis. Since the polynomial $A(L)$ has an infinite number of parameters, and solving b_1 and $A(L)$ therefore requires solving an infinite number of non-linear equations, we truncate the polynomial $A(L)$ after $\bar{T}$ lags. We set $a_s = 0$ for $s > \bar{T}$ and solve $b_1, a_1, .., a_{\bar{T}}$ from $\bar{T} + 1$

non-linear equations. Since interest rate shocks are temporary, their impact on the exchange rate dies out anyway, making this approximation very precise for large $\bar{T}$. In practice we set $\bar{T}$ so large that increasing it any further has negligible effect on the results.

2.2 On the Optimality of Infrequent Decision Making

So far we have exogenously assumed that traders make infrequent portfolio decisions. We would like to know under what circumstances such a passive portfolio management strategy is optimal. There is a trade-off between the higher expected returns under active portfolio management and the cost involved. Assume that the cost of active portfolio management is a fraction τ of wealth per period. The question then is how large τ needs to be for it to be optimal for all traders to make decisions infrequently. We will refer to the level of τ where expected utility is the same under active and passive portfolio management strategies as the *threshold cost*. As long as τ is above this threshold, it is optimal for traders to make infrequent portfolio decisions. For now we will assume that all traders face the same cost τ. In the next section we will also consider a case where the cost τ that agents face differs across agents, so that it is possible that some choose to actively manage their portfolio while others make infrequent portfolio decisions.

In order to determine the threshold cost, we must consider the alternative where traders make portfolio decisions each period.[26] An investor with an actively managed portfolio must solve a more complicated multi-period portfolio decision problem. Since equilibrium expected returns are time varying, the optimal dynamic portfolio contains a hedge against changes in future expected returns. A technical contribution of the paper is to derive an explicit analytical solution to the multi-period portfolio decision problem with time-varying expected returns. Here we briefly describe the method, leaving the details to Appendix A.1 and the Technical Appendix.

We start by conjecturing that the value function at time $t+s$ $(s = 0, .., T)$ of an agent born at time t is

$$V_{t+s} = e^{Y'_{t+s} H_s Y_{t+s}} (1-\tau)^{(1-\gamma)(T-s)} W_{t+s}^{1-\gamma} / (1-\gamma) \tag{11}$$

[26] We will abstract from scenarios where agents make portfolio decisions at intervals between one and T.

Here W_{t+s} is wealth at $t+s$, H_s is a matrix and Y_{t+s} is the state space. The latter consists of $Y_{t+s} = (\varepsilon^u_{t+s}, .., \varepsilon^u_{t+s+1-\bar{T}}, \hat{x}_t, 1)'$. Since in principle the state space is infinitely long, for tractability reasons we have truncated it after $\bar{T}$ periods, with $\bar{T}$ very large. This in effect means that agents ignore interest rate shocks that happened a very long time ago. The key conjecture is that the term in the exponential of the value function is quadratic in the state space.

At time $t+s$ the optimal portfolio is chosen by maximizing $E_{t+s}V_{t+s+1}$. We first substitute $W_{t+s+1} = (1-\tau)W_{t+s}e^{r^p_{t+s+1}}$ into the expression for V_{t+s+1}, where r^p_{t+s+1} is a second order approximation of the log portfolio return from $t+s$ to $t+s+1$. We then maximize with respect to the portfolio at $t+s$. We show that $V_{t+s} = E_t V_{t+s+1}$ indeed takes the conjectured form in (11). Starting with the known value function at $t+T$, $V_{t+T} = W^{1-\gamma}_{t+T}/(1-\gamma)$, which corresponds to $H_T = 0$, we then solve the value function for earlier periods with backward induction, until we have computed the value function at time t.

The solution to this portfolio problem yields the following optimal portfolio share invested in Foreign bonds at time $t+s$ for an investor born at time t:

$$b^F_{t,t+s} = \bar{b}^F(s) + \frac{E_{t+s}(q_{t+s+1})}{(\gamma-1)\hat{\sigma}^2_F(s) + \sigma^2_F} + D^s Y_{t+s} \tag{12}$$

The first term, $\bar{b}^F(s)$, is a constant. The second term depends on the expected excess return over the next period. In the denominator $\sigma^2_F = var_t(q_{t+1})$. The term $\hat{\sigma}^2_F(s)$ is defined in the Appendix but in practice is very close to $var_t(q_{t+1})$, so that the denominator is close to $\gamma var_t(q_{t+1})$. The third term is a hedging term that captures a hedge against changes in future expected returns. D^s is a vector of constant terms, so the hedge term is linear in the state space.

Assume that each new generation consists of n_F agents that make frequent portfolio decisions, actively managing their portfolio each period, and n_I agents that make infrequent portfolio decisions, with $n = n_I + n_F$. The market equilibrium condition then becomes

$$n_F \sum_{j=1}^{T} b^F_{t-j+1,t} W^F_{t-j+1,t} + n_I \sum_{j=1}^{T} b^I_{t-j+1} W^I_{t-j+1,t} + (\bar{x} + x_t)\bar{W} = Be^{s_t} \tag{13}$$

where $W^F_{t-j+1,t}$ is the wealth at time t of agents born at time $t-j+1$ that actively manage their portfolio.

In section 3.3 we will consider the case where the fraction of agents that actively manages their portfolio is positive. For now we focus on the case where it is optimal for all agents to make infrequent portfolio decisions. In that case $n_F = 0$ in equilibrium and $n_I = n$. This will be the case as long as the cost of active portfolio management is higher than the threshold cost.

The threshold cost τ is determined such that the expected utility of an investor who makes frequent portfolio decisions is the same as that of an investor who makes infrequent portfolio decisions.[27] Above we have already discussed how the value function is computed under active portfolio management. Since each investor starts with wealth equal to 1, the value function at birth is $e^{Y_t' H_0 Y_t}(1-\tau)^{(1-\gamma)T}/(1-\gamma)$. For an investor making only one portfolio decision for T periods, the time t value function is $V_t = E_t W_{t+T}^{1-\gamma}/(1-\gamma)$. After substituting $W_{t+T} = e^{r_{t+1}^p + .. + r_{t+T}^p}$, maximization with respect to b_t^I yields the optimal portfolio (12) and a time t value function that takes the form $e^{Y_t' H Y_t}/(1-\gamma)$. When born, investors need to decide whether to actively manage their portfolio before observing the state Y_t.[28] We therefore compare the unconditional expectation of the time t value functions for the two strategies, where the expectation is with respect to the unconditional distribution of Y_t. The threshold cost τ is such that expected utility is the same under both strategies.

2.3 Parameterization

We calibrate the model to data for the five currencies on which Table 1 and Figure 1 are based. Consistent with the quarterly excess returns in Table 1 and Figure 1, a period is set equal to one quarter. The AR process for the forward discount, and therefore u_t, is estimated for the countries and sample period corresponding to the excess return regression reported in Table 1.[29] We set ρ_u and σ_u equal to the average across the countries of the estimated processes. This yields $\rho_u = 0.8$

[27] The full details can be found in the Technical Appendix.

[28] In a more realistic framework where agents have infinite lives and make portfolio decisions every T periods, this corresponds to agents deciding on the frequency of portfolio decisions before observing future states when portfolio decisions are actually made. In other words, it corresponds to a time-dependent decision rule.

[29] We use three-month Euro-market interest rates from Datastream between December 1978 and December 2005.

and $\sigma_u = 0.0038$.

The process for the supply $x_t = C(L)\epsilon_t^x$ cannot be observed directly. We set the standard deviation σ_x of the innovations to this process such that the implied exchange rate volatility in the model matches that in the data. To be precise, σ_x is set such that the standard deviation of $s_{t+1} - s_t$ in the model is equal to the average standard deviation of the one quarter change in the log exchange rate for the five currencies and time period of the excess return regression reported in Table 1. The average standard deviation is 0.057. We choose the polynomial $C(L)$ such that the exchange rate is close to a random walk, consistent with extensive empirical evidence. To be precise, we choose $C(L)$ such that $\hat{x}_t$ follows an AR process as in (10) with AR coefficient $\rho_x = 0.99$. This means that the exchange rate is close to a random walk since supply shocks dominate exchange rate volatility.

In the benchmark parameterization we set $T = 8$. This implies that agents make one portfolio decision in two years, so that half of the agents change their portfolio during a particular year. While it is hard to calibrate this precisely to the data, it corresponds well to evidence for the stock market reported in the introduction indicating that only about 40% of investors change their stock or mutual fund portfolios during any particular year. It also corresponds well to evidence reported by Parker and Julliard (2005) and Jagannathan and Wang (2005) that Euler equations for asset pricing better fit the data when returns are measured over longer horizons of one to three years. In section 4 we will further discuss that evidence and its connection to our model.

The final two parameters are b and γ. We set $b = 0.5$, corresponding to a two-country setup with half of the assets supplied by the US and the other half by the rest of the world. The rate of relative risk aversion is set at 10. This is in the upper range of what Mehra and Prescott (1985) found to be consistent with estimates from micro studies, but consistent with more recent estimates by Bansal and Yaron (2004) and Vissing-Jorgenson and Attanasio (2003).[30] A risk-aversion of 10 also reduces the well known extreme sensitivity of portfolios to expected excess returns in this type of model.[31]

[30]The estimates in Bansal and Yaron (2004) are based on a general equilibrium model that can explain several well known asset pricing puzzles. The estimates in Vissing-Jorgenson and Attanasio (2003) are based on estimating Euler equations based on consumption data for stock market participants.

[31]Other ways to improve this feature include loss aversion preferences, habit formation pref-

3 Explaining the Forward Premium Puzzle

We now examine the model's quantitative implications for excess return predictability. We will show that the model indeed generates such predictability. We first present the results in our benchmark case and provide the intuition on the mechanism leading to predictability. We show that this is closely related to the phenomenon of delayed overshooting. We then report the threshold cost of active portfolio management such that investors are equally well off adopting a passive or active portfolio management strategy. We show that the threshold cost is very small and certainly below any reasonable value of the true cost of active portfolio management. This justifies the infrequent decision making by all investors.

While the model is able to explain excess return predictability, the regression coefficient in the excess return equation is smaller than in the data. We show that we cannot match this moment even if we drastically change the values of γ and T. Drawing on a large number of small sample simulations, we also show that the difference with the data cannot be explained by small sample bias. However, we report two potential explanations that quantitatively line up the model to the data. First, when we simulate the model over 25-year samples, we obtain a wide range of regression estimates. While the mean of the estimated predictability coefficients is less than in the data, a relatively large proportion of the regression coefficients are at least as large as in the data. Second, we show that we can match the estimated coefficient in the data when we additionally assume partial information processing. Under partial information processing we consider the case where investors either assume that the exchange rate is a random walk or only use the current interest rate differential to optimally predict future exchange rates.

3.1 Benchmark Results

Panel A of Figure 3 reports results when regressing excess returns q_{t+s} on the forward discount fd_t, similar to Figure 1. While standard models predict coefficients around the zero line, the model is able to generate negative coefficients for small values of s, followed by positive coefficients for larger s. The usual one-period

erences, parameter uncertainty, transaction costs, and portfolio benchmarking. These would substantially complicate the model though.

ahead coefficient is equal to -0.95. Panel B shows a scatter plot of interest rate differentials against subsequent one-period excess returns for one simulation of the model over 100 periods, which corresponds to 25 years. The scatter plot is similar to what is found in the data as shown in Figure 2. The interest differential predicts excess returns, but both in the model and the data the predictability is largely out-shadowed by risk. To summarize, the benchmark parameterization delivers significant excess return predictability in the right direction, but the extent of the predictability is less than in the data. In the data the regression coefficient is close to -2.5. We will now give some intuition both for why this predictability occurs and what limits the extent of the predictability.

Delayed Overshooting

Figure 4 provides the key intuition behind our findings. Panel A shows the impulse response of the exchange rate to a one standard deviation decrease in the Foreign interest rate. It compares the benchmark case with the case where all investors make portfolio decisions each period. In the latter case there is standard overshooting, i.e., the lower Foreign interest rate causes an immediate appreciation of the Home currency, followed by a gradual depreciation. In that case the excess return predictability coefficient is close to zero (-0.014).[32] With infrequent portfolio decisions, however, we find delayed overshooting, consistent with the empirical findings of Eichenbaum and Evans (1995). The initial appreciation is now smaller, but the Home currency continues to appreciate in the following several quarters, after which it starts to gradually depreciate.

The continued appreciation is a result of the delayed portfolio response of investors. Investors making portfolio decisions at the time the shock occurs sell Foreign bonds in response to the news of a lower Foreign interest rate. The next period a different set of investors adjust their portfolio. They too will sell Foreign bonds in response to the lower interest rate, leading to a continued appreciation of the Home currency. The currency continues to appreciate for three quarters.

Panel B shows the evolution of the forward discount and the excess return (computed using the path of the exchange rate in Panel A). The Figure shows that initially the drop in the excess return is larger than the rise in the forward

[32]The fact that it is not exactly zero is because the change in the exchange rate changes the real supply of the foreign asset, Be^{-s_t}, which has a small risk-premium effect.

discount. The reason is that the excess return $s_{t+1} - s_t - fd_t$ decreases both because of the rise in the forward discount (lower Foreign interest rate) and the subsequent appreciation of the Home currency (negative change in the exchange rate). However, the Figure also shows that this is not long-lasting. Within three quarters the absolute decline in the excess return is less than the rise in the forward discount and at $T = 8$ quarters they both go in the same direction. This limits the magnitude of the negative excess return predictability coefficient. Related to that, the delayed overshooting in panel A only lasts 3 quarters, while Eichenbaum and Evans (1995) report empirical evidence indicating delayed overshooting lasting for two to three years.

The reason why the delayed overshooting does not last longer than 3 quarters is that at that point investors start buying Foreign bonds again. Investors know that the Foreign interest rate will continue to be lower than the Home interest rate, but they also realize that eventually the Home currency will depreciate. The reason is that the investors who sold Foreign bonds at the time the shock happened will increase their holdings of Foreign bonds 8 quarters later when they adjust their portfolio again.[33] After all, the interest rate differential in favor of Home bonds is expected to be much smaller 8 quarters later. Three periods after the shock the expected depreciation of the Home currency over the next 8 quarters is sufficient to more than offset the expected interest differentials in favor of the Home bonds. Investors will then start buying Foreign bonds again, causing the Home currency to gradually depreciate. This of course assumes very careful forward looking behavior on the part of investors, processing all available information to predict the exchange rate two years into the future. This information processing capacity may be unrealistic, an issue to which we will turn below.

Threshold Cost

Following the method described in section 2.2, we find an annualized threshold cost of 0.27% of wealth. This means that it is indeed optimal for all investors to make infrequent portfolio decisions when the cost of active portfolio management is at least 0.27% of wealth. This number is far below fees charged by active portfolio managers, which do not even include additional agency and monitoring costs when

[33]More precisely, and leading to the same outcome, they are replaced by a new generation that chooses a new portfolio.

delegating these decisions to fund managers and the transactions costs associated with frequent portfolio adjustments.

The reason that the threshold cost is so small is that there is so much uncertainty about future returns. Since the component of the exchange rate that depends on supply shocks is close to a random walk, virtually the entire predictability comes from interest rates. Panel B of Figure 3 illustrates that the predictability of excess returns by interest differentials is simply overwhelmed by uncertainty, as is the case in the data. Therefore only a small cost of active portfolio management is sufficient for investors to not actively exploit the predictability.

Small Sample Results

In order to allow for better comparison to results based on the data reported in Table 1 and Figure 1, we have also simulated a 25-year period for the model. Based on 1000 simulations of a 25-year period, we find that the average excess return predictability is very close to the population moment of -0.95. This means that there cannot be a systematic small sample bias. However, the excess return predictability varies quite considerably across simulations. This is consistent with empirical evidence that shows that the excess return coefficient tends to be unstable over time. Panel A of Figure 5 reports the frequency distribution. In 12% of cases we find an excess return predictability coefficient of less than -2. This means that the findings in the data are well within reach of the model.[34]

Panel B reports the average of the regression coefficients of q_{t+s} on fd_t ($s = 1,..,30$) for the 10% of simulations (100 simulations) generating the lowest coefficient for $s = 1$. The picture is very similar to Figure 1 based on the data. The average predictability coefficient is -2.6 for $s = 1$. It continues to be negative for about six quarters, dropping in absolute size as s increases.

Alternative Parameterizations

Table 2 presents results on the one-period ahead predictability coefficient and the threshold cost for some alternative values of the rate of risk aversion γ and the frequency T of decision making. The excess return predictability coefficient is larger for higher values of γ and T, but not enough to match the data. Based

[34]In contrast, the probability of this being the case is only 1.1% when all investors make portfolio decisions each period.

on population moments generated by the model, it is not possible to match the empirical estimate of about -2.5 even when we substantially increase γ and T. It remains the case though that for a large range of parameters there is a substantial probability that the excess return predictability coefficient is less than -2 in simulations of a 25-year period.

We also see that the threshold cost remains quite low for a wide range of parameters. It is highest for a low rate of risk-aversion of $\gamma = 1$ since agents are then less averse to the risk associated with exploiting excess return predictability.

3.2 Partial Information Processing

Although investors in the model make infrequent portfolio decisions, we have assumed that they use all available information when they make those decisions. In other words, investors have rational expectations and are able to determine the future behavior of other investors and the full path of future returns based on all information available today. As explained above, it is this forward looking behavior that leads investors to start buying Foreign bonds after three periods, which limits the extent of delayed overshooting.

However, as shown in the rational inattention literature, in the presence of costly information processing it may be optimal for investors to only process partial information.[35] Such partial information processing also corresponds better to the description of the actual behavior of investors. For example, investors may simply rely on the Meese and Rogoff (1983) evidence that no simple model can beat the random walk to predict nominal exchange rates. Many large financial institutions do not bother to try to outperform the random walk when forming expectations of the exchange rate one month or more into the future. If they do, they tend to use very simple forecasting rules.[36]

[35]Consistent with that Fama (1991) suggests that "a weaker and economically more sensible version of the efficient market hypothesis says that prices reflect information to the point where the marginal benefits of acting on information do not exceed the marginal cost".

[36]Kasa (2006) shows that such behavior can be optimal even in the absence of information processing costs. He presents an example where robust filtering theory and Shannon capacity constraints are observationally equivalent. In other words, introducing a limited information processing capacity in a framework where agents know the model is equivalent to introducing model uncertainty. There is an equivalence between using partial information with limited information

We will consider two relevant cases of partial information processing. In the first case investors form optimal expectations of future spot exchange rates and interest rates on the basis of only current interest rates. They do not use all past interest rates and liquidity supply shocks to form expectations. Investors therefore optimally exploit the findings from excess return predictability regressions reported in Figure 1, which only have current interest rate differentials on the right hand side. In the second case investors continue to predict future interest rates on the bases of current interest rates, based on the AR process, but they expect future spot rates to be equal to the current spot rate.[37] We will focus on the first case and briefly mention the results for the random walk assumption towards the end.

More Predictability

Figure 6 shows the main results. We keep all the parameters as in the benchmark parameterization. The usual one-period ahead regression coefficient of the excess return on the forward discount is now -2.1. This is close to the average regression coefficient found in the data and reported in Table 1. Panel A of Figure 6 shows that the coefficient continues to be negative for 5 quarters, declining in absolute size, then turns positive and eventually back to zero for very long lags. This closely matches the data reported in Figure 1.[38] Panel C shows the frequency distribution of the one-period ahead predictability coefficient, again based on 100 simulations of a 25-year period. In 41% of simulations the coefficient is now less than -2.5. Panel B shows that a scatter plot of excess return observations versus the forward discount, based on a 25-year simulation of the model, is again very similar to what we found in the data reported in Figure 1.

More Delayed Overshooting

The more negative regression coefficient with partial information processing can

processing capacity and only using robust information with model uncertainty.

[37]While this second case is very attractive in the context of the widely reported difficulties in outperforming the random walk, and the actual practice by many investors, it has the theoretical flaw that investors use current interest rates to predict future interest rates but not future exchange rates.

[38]In the data this coefficient continues to be negative for about 10 quarters, but its coefficient is insignificantly different from zero after about 5 quarters. Also, the decline of this coefficient back to zero in the data happens after 30 quarters, not reported in Figure 1.

be explained by more delayed overshooting. Panel A of Figure 7 shows that after a drop in the Foreign interest rate, the Home currency appreciates for eight quarters. In contrast to the full information case, investors continue to sell Foreign bonds for eight quarters. The expected excess return over 8 quarters is now proportional to the interest rate differential, with a coefficient of $\beta_1 + .. + \beta_8$, where β_s is the regression coefficient in $q_{t+s} = \alpha_s + \beta_s fd_t$. The sum of the first eight coefficients is -2.6. This means that the expected excess return over the next eight quarters is -2.6 times the current forward discount. Investors therefore will continue to sell Foreign bonds during the first eight quarters when the lower Foreign interest rate raises the forward discount. After eight quarters investors start buying Foreign bonds again because the first group of investors selling Foreign bonds when the shock happened is replaced by another generation. Foreign bonds are by then more attractive than they were eight quarters earlier since the interest rate on Foreign bonds has gradually increased over time.

Under full information processing the expected eight-period depreciation of the exchange rate gradually rises after the shock because investors know that the delayed overshooting (appreciation phase) is temporary. This leads them to switch from selling Foreign bonds to buying Foreign bonds quite soon, so that the delayed overshooting does not last so long. But under partial information processing investors do not condition their expectations on the entire history. The expected depreciation over the next eight quarters is only conditioned on the current interest differential. Because the interest differential is declining over time, the expected depreciation of the home currency over the next eight quarters is also declining with it, quite the opposite of what happens under full information processing.[39]

Threshold Cost

The choice to process partial information is fully rational when the cost of full information processing outweighs the benefits. Therefore we like to know under what cost of information processing it is optimal to form expectations only based on the current interest rate rather than the full information set. We find that when this cost is at least 0.58% of wealth on an annualized basis, it remains optimal for investors to only base expectations on the current interest rate. Since this is quite small, an equilibrium based on partial information processing appears reasonable.

[39] The expected eight-period depreciation is 1.58 times the forward discount.

We can now also again ask what the threshold cost of active portfolio management is. In doing so we will assume that under active portfolio management expectations of future excess returns are also based only on the current interest rate. We then find that the annualized threshold cost is 0.68%.

Two Final Comments

Two final comments are in order. First, when investors assume that the exchange rate follows a random walk we find a one-period ahead excess return coefficient that is even somewhat more negative, -2.54. In that case investors continue to sell Foreign bonds to an even greater extent over the first eight periods because they do not expect the domestic currency to depreciate at any time in the future. There continues to be delayed overshooting for eight periods in this case. Second, partial information processing by itself does not generate much excess return predictability. If all investors make portfolio decisions each period, using only current interest rates to forecast future excess returns, the one-period ahead excess return predictability coefficient would be -0.07.

3.3 Investors with Actively Managed Portfolios

We now introduce investors with actively managed portfolios into the model. As discussed in the introduction, the current size of the industry that actively manages short-term currency positions is tiny. The assumption that we have made so far, that no investors actively manage their currency positions, is therefore currently (and certainly over the past 25 years) a good approximation. Nonetheless this market does exist and has been growing substantially in recent years. A natural question is therefore how large this market needs to become in order for it to start eroding the excess return predictability.

In order for some investors to choose to have their currency positions actively managed, while others choose not to do so, there must be some difference across investors. It is possible that investors differ in their expectations of excess returns, perceptions of risk, degree of risk-aversion or the covariance of currency speculation returns with returns on other financial positions they hold. To simplify, here we assume that the cost of active portfolio management differs across investors. One group faces a cost of active portfolio management below the threshold, while the other group faces a cost above the threshold. To the extent that fees depend on the amount of capital invested, the first group would consist of traders with larger investments.

As already reported above, as the proportion of investors that make frequent portfolio decisions goes to 1, predictability disappears. Panel A of Figure 8 shows how the predictability coefficient changes when the proportion of investors with actively managed portfolios goes from 0 to 10%. Both cases of full and partial information are shown. In the latter case all investors form expectations based on current interest rates. Panel B reports the threshold cost such that expected utility is the same for actively and passively managed portfolios.

Figure 8 shows that the excess return predictability coefficient drops significantly in absolute size as the fraction f of investors with actively managed portfolios increases. The model still generates substantial excess return predictability when 1% of wealth is actively managed. This corresponds to 2% of steady state external financial holdings in the model, or about seven times the current size of the industry that actively manages currency positions (0.3% of external holdings). It is therefore not likely that the forward discount predictability will disappear any time soon.

When 10% of financial wealth is actively managed the excess return predictability is significantly reduced. This is not surprising as investors with actively managed portfolios devote significant resources towards exploiting excess return predictability. Even with a rate of risk aversion of 10, these investors are very aggressive. With an excess return predictability coefficient of -2.5 (as in the data), a two standard deviation increase in the Foreign interest rate will lead active investors to increase their holdings of Foreign bonds from a steady state of 50% of wealth to 132% of wealth.

There is a natural limit to the size of the industry that actively manages currency positions. This is illustrated in Panel B of Figure 8. It shows that the threshold cost declines rapidly as the fraction of actively managed wealth increases. This is not surprising because of the reduction in excess return predictability. The profit opportunities left unexploited go down with the increase in actively managed portfolios. It would therefore not be optimal for too many investors to actively manage their currency positions.

4 Discussion

In this section, we relate the previous analysis to three distinct aspects of the existing literature on the forward premium puzzle. First, how does the model connect to risk-premium based explanations of the forward discount puzzle? Second, how does the model relate to survey evidence of predictable expectational errors? Third, how can the model shed light on a variety of other stylized facts associated with excess return predictability in the foreign exchange market?

Connection to Risk Premium Explanations

The standard assumption in finance is that expected excess returns reflect a risk premium.[40] This assumes that agents continuously rethink the optimality of their portfolios. In this paper we have deviated from this by considering the implications of infrequent decisions about portfolios due to a cost of making such decisions. However, this does not mean that the model is completely disconnected from risk-premium explanations. First, in section 3.3 we have introduced investors who do make decisions each period. From the perspective of these investors the expected excess return is identical to a risk premium. The risk premium is negatively correlated with the forward discount, which is what is needed to get a negative coefficient when regressing the excess return on the forward discount. A higher Foreign interest rate, which lowers the forward discount, raises the fraction invested in Foreign bonds. This increases the dependence of next period's wealth on the excess return of Foreign bonds and therefore raises the risk premium.

It should be emphasized though that it is the infrequent decision making by the great majority of investors that generates this time varying risk premium. As a result of passive portfolio management, a higher Foreign interest rate leads to an expected appreciation of the Foreign currency, leading active investors to increase their holdings of Foreign bonds and therefore an increase in the risk premium they demand. In the absence of passive investors, the higher Foreign interest rate would be followed by an expected depreciation of the Foreign currency, so that investors making frequent portfolio decisions would change their holdings of Foreign bonds

[40]In the context of the foreign exchange market Engel (1996) reviews explanations for the forward discount puzzle based on time varying risk premia. For more recent contributions, see Backus et al. (2001), Beakert et al. (1997) and Verdelhan (2005).

very little and the change in the risk premium would be very small.

Second, there is also a risk premium for investors making infrequent portfolio decisions. For those investors a T-period Euler equation applies:

$$E_t(c_{t+T})^{-\gamma}q_{t,t+T} = 0 \tag{14}$$

where c_{t+T} is consumption at $t + T$. The standard asset pricing equation equates the expected product of the pricing kernel and excess return to zero. In that case the pricing kernel is the marginal utility of consumption next period and the excess return is also measured over one period. For investors making infrequent portfolio decisions the only difference is that the pricing kernel is the marginal utility of consumption T periods from now and the excess return is measured over T periods. The risk premium for passive investors therefore applies over T periods and is equal to the rate of risk aversion times the covariance of the excess return over T periods and consumption in T periods. For these investors the one-period excess return cannot be associated with a risk premium.

There is evidence that long-horizon Euler equations indeed fit the data better than short-horizon Euler equations. Recently Jagannathan and Wang (2005) and Parker and Julliard (2005) have provided such evidence for stock returns. Jagganathan and Wang (2005) show that the Euler equation fits the data substantially better at a one-year horizon than a monthly horizon. They argue that infrequent portfolio and consumption decisions can account for this. Parker and Juliard (2005) use stock return data to estimate an Euler equation where excess returns are measured over one quarter but consumption growth over multiple quarters. They find that the Euler equation fits the data best with consumption growth measured over three years. They argue that one reason for this may be the "presence of constraints on information flow" and refer to a literature where agents make infrequent portfolio decisions.

Survey Evidence of Predictable Expectational Errors

Many papers on the forward discount puzzle argue that the bias must be the result of either time varying risk-premia or predictable expectational errors (e.g. Froot and Frankel, 1989). The logic of this argument is based on the assumption that all agents make active portfolio decisions each period. In that case the expected excess return is equal to a risk premium and the actual excess return is equal to a risk premium plus expectational error. The bias therefore results from either the risk premium or the expectational error being negatively correlated with the forward discount. This decomposition is no longer valid in our model since the Euler equation does not apply on a periodic basis for investors making infrequent portfolio decisions.

Evidence of predictable expectational errors is nonetheless consistent with the findings of the model. For example, when agents assume that the exchange rate follows a random walk, the expectational error of the change in the exchange rate is predicted negatively by the forward discount. This is consistent with extensive evidence based on survey data.[41] More generally, evidence of predictable expectational errors is consistent with partial information processing. Since there is evidence of predictable expectational errors for large financial institutions, one would certainly expect that individual investors process only a limited amount of information when making portfolio decisions.

Extensions

Several other stylized facts related to the forward discount puzzle have been documented in the literature. The model proposed in this paper certainly cannot account for all of them. However, the analysis can be extended to deal with several of the additional features. We briefly mention three of them.

First, we could introduce long-term bonds. The model would then replicate the empirical evidence showing that the forward discount puzzle tends to go away over long horizons. Meridith and Chinn (2005) provide such evidence using regressions of the change in the exchange rate over a long horizon of 5 or 10 years on the interest rate differential for long-term bonds with corresponding maturity. They find coefficients of respectively 0.67 and 0.68. Without introducing long-term bonds we can conduct a closely related exercise of regressing the average excess return on foreign currency investments over T periods on the forward discount at time t. The resulting coefficient is the average of the coefficients β_s of the excess return regressions $q_{t+s} = \alpha_s + \beta_s fd_{t+k} + \varepsilon_{t+s}$, for s from 1 to T. Both in the model and in the data these average predictability coefficients gradually decline in absolute size

[41] See Froot and Frankel (1989). Bacchetta, Mertens, and van Wincoop (2006) review subsequent papers and present the latest evidence.

as T increases and are close to zero when $T = 20$ (5 years).

A second extension is to modify the monetary policy rules in order to introduce persistent inflation shocks. This will allow the model to account for evidence by Bansal and Dahlquist (2000) that there is less excess return predictability for developing countries. Consider for example a change in Home country's monetary policy from a zero inflation target to a 10% inflation target. The only change that this generates in the model is in the steady state. There will now be a constant 10% steady state depreciation and the Home interest rate will be 10% higher. In deviation from this steady state the solution is the same as before. Such a change in policy therefore raises both $s_{t+1} - s_t$ and fd_t by the same large amounts. One can therefore expect that persistent inflation shocks in the model will lead to a much higher coefficient in a regression of $s_{t+1} - s_t$ on fd_t.

A third extension is to introduce transactions costs. As extensively discussed in Sarno, Valente and Leon (2006), their finding of non-linearities in the relationship between excess return predictability and the size of the interest rate differential can naturally be explained by introducing these costs. This leads to a band of inaction.[42] When interest rate differentials are small, the gains from trading on the expected excess return may not outweigh the transaction cost, so that the excess return remains predictable. But when the interest rate differential gets large enough active traders will take aggressive positions to exploit excess return predictability. Since introducing transaction costs will further reduce the welfare gain from active portfolio management, it provides a reinforcing motive for making infrequent portfolio decisions.

5 Conclusion

The model of incomplete information processing developed in the paper can shed light on many key empirical stylized facts related to the forward premium puzzle. First, it can explain why the amount of capital devoted to actively managing

[42] See Baldwin (1990) and the discussion in Lyons (2001, 206-220). A transaction cost of exchanging home bonds for foreign bonds is quite different from limited participation models where there is a transaction cost of exchanging bonds for money, the latter used for consumption. Alvarez, Atkeson, and Kehoe (2005) use such a model to shed light on the forward discount puzzle. In their model all agents can exchange all bonds at no cost.

short-term currency positions is tiny. The welfare gain from active management of currency positions is very small since exchange rates are notoriously hard to predict. These welfare gains are easily outweighed by a small cost of active portfolio management. Second, infrequent decisions by investors about currency exposures lead to a delayed impact of interest rate shocks on exchange rates. This can explain the phenomenon of "delayed overshooting," whereby the exchange rate continues to appreciate over time after a rise in the interest rate. Third, the delayed overshooting gives rise to excess return predictability of a magnitude consistent with that seen in the data. Fourth, even future excess returns continue to be predictable by the current forward discount, with the magnitude of the predictability declining as time goes on.

Qualitatively similar models can also be developed to account for excess return predictability in other financial markets. For the stock market there is extensive evidence that most investors make infrequent portfolio decisions, in particular when reallocating between stocks and other assets. And in parallel to the delayed overshooting evidence for the foreign exchange market, it is widely documented that stock prices respond with delay to new publicly available information. Stock prices continue to move in the same direction six to twelve months after public events such as earnings announcements, stock issues and repurchases and dividend initiations and omissions.[43]

The model developed here is obviously very stylized. Reality is far more complex, with a much larger information space, time-varying model parameters, uncertainty about the nature of the model itself and information asymmetries between investors and agents. A richer model would therefore provide a more solid foundation for existing costs of actively managing portfolios. However, it is not clear that the main findings would change. First, the mechanism through which delayed overshooting happens in the model would similarly apply in far more complex environments. Second, the gains from frequent portfolio decisions would remain small in any model that captures the well known difficulty of predicting changes in exchange rates.

[43] See Hong and Stein (1999) for references. The literature is most extensive regarding continued stock price appreciation subsequent to a positive earnings announcement, which has become known as "post earnings announcement drift."

A Appendix

In this Appendix, we sketch the main steps to derive the portfolios of both investors making frequent and infrequent portfolio decisions and to solve the model. More details can be found in a Technical Appendix available upon request.

A.1 Optimal Portfolios

We first describe how we derive the optimal of portfolio (5) of investors making infrequent portfolio decisions. For investors born at time t the value function is:

$$V_t = E_t e^{(1-\gamma)(r^p_{t+1}+..+r^p_{t+T})}/(1-\gamma) \tag{15}$$

We adopt a second order approximation for the log return:

$$r^p_{t+s} = \bar{r} + b^I_t q_{t+s} + 0.5 b^I_t (1-b^I_t) var_t(q_{t+s}) \tag{16}$$

Substituting this into the value function, maximization with respect to b^I_t yields

$$b^I_t = b^I + \frac{E_t q_{t,t+T}}{\gamma \sigma^2_I} \tag{17}$$

where

$$b^I = \frac{0.5 \sum_{j=1}^{T} var_t(q_{t+j})}{\gamma \sigma^2_I} \tag{18}$$

and σ^2_I is defined in (6).

For investors making frequent portfolio decisions the optimal portfolio is more complex since it involves a hedge against changes in future investment opportunities. Consider an agent born at time t. We will compute the optimal portfolio and value function at $t+s$ for $s = 0,..,T-1$. We make the following guess for the value function:

$$V_{t+s} = e^{Y'_{t+s} H_s Y_{t+s}} (1-\tau)^{(1-\gamma)(T-s)} W^{1-\gamma}_{t+s}/(1-\gamma) \tag{19}$$

where H_s is a square matrix of size $\bar{T}+2$.

We know that

$$W_{t+s+1} = (1-\tau) W_{t+s} e^{r^p_{t+s+1}} \tag{20}$$

We again adopt a second order approximation for the log return:

$$r^p_{t+s+1} = \bar{r} + b^F_{t+s} q_{t+s+1} + 0.5 b^F_{t+s}(1-b^F_{t+s})\sigma^2_F \tag{21}$$

where σ_F^2 is the conditional variance of next period's excess return. After substituting (20) and (21) into the Bellman equation we have

$$V_{t+s} = E_{t+s} e^{v_{t+s+1}} (1-\tau)^{(1-\gamma)(T-s)} W_{t+s}^{1-\gamma} / (1-\gamma) \tag{22}$$

where

$$v_{t+s+1} = (1-\gamma)\bar{r} + (1-\gamma) b_{t+s}^F q_{t+s+1} + (1-\gamma) 0.5 b_{t+s}^F (1-b_{t+s}^F) \sigma_F^2 + Y_{t+s+1}' H_{s+1} Y_{t+s+1} \tag{23}$$

It is useful to write

$$q_{t+s+1} = M_1^s Y_{t+s} + M_2^s \epsilon_{t+s+1} \tag{24}$$

and

$$Y_{t+s+1} = N_1^s Y_{t+s} + N_2^s \epsilon_{t+s+1} \tag{25}$$

where

$$\epsilon_{t+s+1} = \begin{pmatrix} \epsilon_{t+s+1}^u \\ \epsilon_{t+s+1}^x \end{pmatrix} \tag{26}$$

After substituting (24)-(25) into (23) we can compute $E_{t+s} e^{v_{t+s+1}}$. Maximizing the resulting time $t+s$ value function with respect to b_{t+s}^F yields the optimal portfolio in (12) where:

$$\bar{b}^F(s) = \frac{0.5\sigma_F^2}{(\gamma-1)\hat{\sigma}_F^2(s) + \sigma_F^2} \tag{27}$$

and

$$\begin{aligned}
\hat{\sigma}_F^2(s) &= M_2^s \Omega^s (M_2^s)' && (28)\\
\Omega^s &= (\Sigma^{-1} - 2C_2^s)^{-1} && (29)\\
C_2^s &= (N_2^s)' H_{s+1} N_2^s && (30)\\
D^s &= 2M_2^s \Omega^s (N_2^s)' H_{s+1} N_1^s / [(\gamma-1)\hat{\sigma}_F^2(s) + \sigma_F^2] && (31)
\end{aligned}$$

A.2 Solving the Equilibrium Exchange Rate

Consider the market equilibrium condition (13). The case where all investors make infrequent portfolio decisions (eq. (8)) is easily found by setting $n_F = 0$ and

$n_I = n$. A first order Taylor approximation of (13) gives:

$$n_F \sum_{j=1}^{T} b^F_{t-j+1,t} + n_I \sum_{j=1}^{T} b^I_{t-j+1} + n_F \bar{k}^F + n_I \bar{k}^I +$$
$$\sum_{j=1}^{T-1} (n_F k^F(j) + n^I k^I(j)) q_{t-j+1} + (\bar{x} + x_t)\bar{W} = B + Bs_t \tag{32}$$

where

$$\bar{k}^F = \sum_{s=1}^{T-1} \bar{b}^F(s) s(\bar{r} - \tau)$$
$$k^F(j) = \sum_{s=1}^{T-j} \bar{b}^F(s-1)\bar{b}^F(s+j-1)$$

and

$$\bar{k}^I = \sum_{s=1}^{T-1} \bar{b}^I s\bar{r}$$
$$k^I(j) = (T-j)(\bar{b}^I)^2$$

Steady state financial wealth is defined as total financial wealth when the returns on Home and Foreign bonds are equal to their steady state levels ($\bar{r}$ for Home bonds and 0 for Foreign bonds), $\tau = 0$ and the fraction invested in Foreign bonds is b. Based on that definition we have

$$\bar{W} = wnT \tag{33}$$

where

$$w = \sum_{j=1}^{T} \left(\bar{R}^p\right)^{j-1} / T \tag{34}$$
$$\bar{R}^p = (1-b)e^{\bar{r}} + b \tag{35}$$

The constant term in the portfolio of liquidity traders, $\bar{x}$, is set such that the market clearing condition holds in steady state for a given real interest rate $\bar{r}$. Finally, we subtract the steady state from both sides of (32), we divide it by nT, and use the expressions for optimal portfolio to get an expression in deviation from steady state:

$$f\frac{E_t \tilde{q}_{t+1}}{\gamma\sigma^2} + fDY_t + (1-f)\frac{1}{T}\sum_{j=1}^{T} \frac{E_{t-j+1}\tilde{q}_{t-j+1,t-j+1+T}}{\gamma\sigma_I^2} +$$
$$\sum_{j=1}^{T-1} \frac{1}{T}(fk^A(j) + (1-f)k^I(j))\tilde{q}_{t-j+1} + wx_t = wbs_t \tag{36}$$

where $f = n_F/n$ is the fraction of agents making frequent portfolio decisions, the tilde denotes excess returns in deviation from their steady state and

$$D = \frac{1}{T}\sum_{j=1}^{T} D^{j-1}$$

$$\frac{1}{\sigma^2} = \frac{1}{T}\sum_{j=1}^{T} \frac{\gamma}{(\gamma-1)\hat{\sigma}_F^2(j-1)+\sigma_F^2}$$

We conjecture (9) with

$$A(L) = a_1 + a_2 L + a_3 L^2 + ... \tag{37}$$

$$B(L) = b_1 + b_2 L + b_3 L^2 + ... \tag{38}$$

Substituting (9) into the market equilibrium condition (36), we obtain an equilibrium exchange rate equation. We then need to equate the conjectured to the equilibrium exchange rate equation. We choose the process

$$x_t = C(L)\epsilon_t^x = (c_1 + c_2 L + c_3 L^2 + ...)\epsilon_t^x \tag{39}$$

such that $\hat{x}_t = B(L)\epsilon_t^x$ follows the AR process (10). We normalize such that $c_1 = 1$.

We therefore choose $A(L)$, b_1 and $C(L)$ such the that (i) the Foreign bond market equilibrium condition (36) is satisfied and (ii) $\hat{x}_t = B(L)\epsilon_t^x$ follows the AR process in (10). The latter implies imposing $b_{s+1} = \rho_x b_s$ for $s \geq 1$. Imposing the market equilibrium condition involves computing first and second moments of excess returns based on the conjectured exchange rate process. After that is done both sides of the market equilibrium equation can be written as a linear function of the underlying innovations at time t and earlier. We then need to equate the coefficients multiplying these innovations on the right and left side of the equation, which involves solving a fixed point problem. The overall approach is rather straightforward, but the algebra is a bit lengthy and can be found in the Technical Appendix.

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Table 1: Predictable Excess Returns

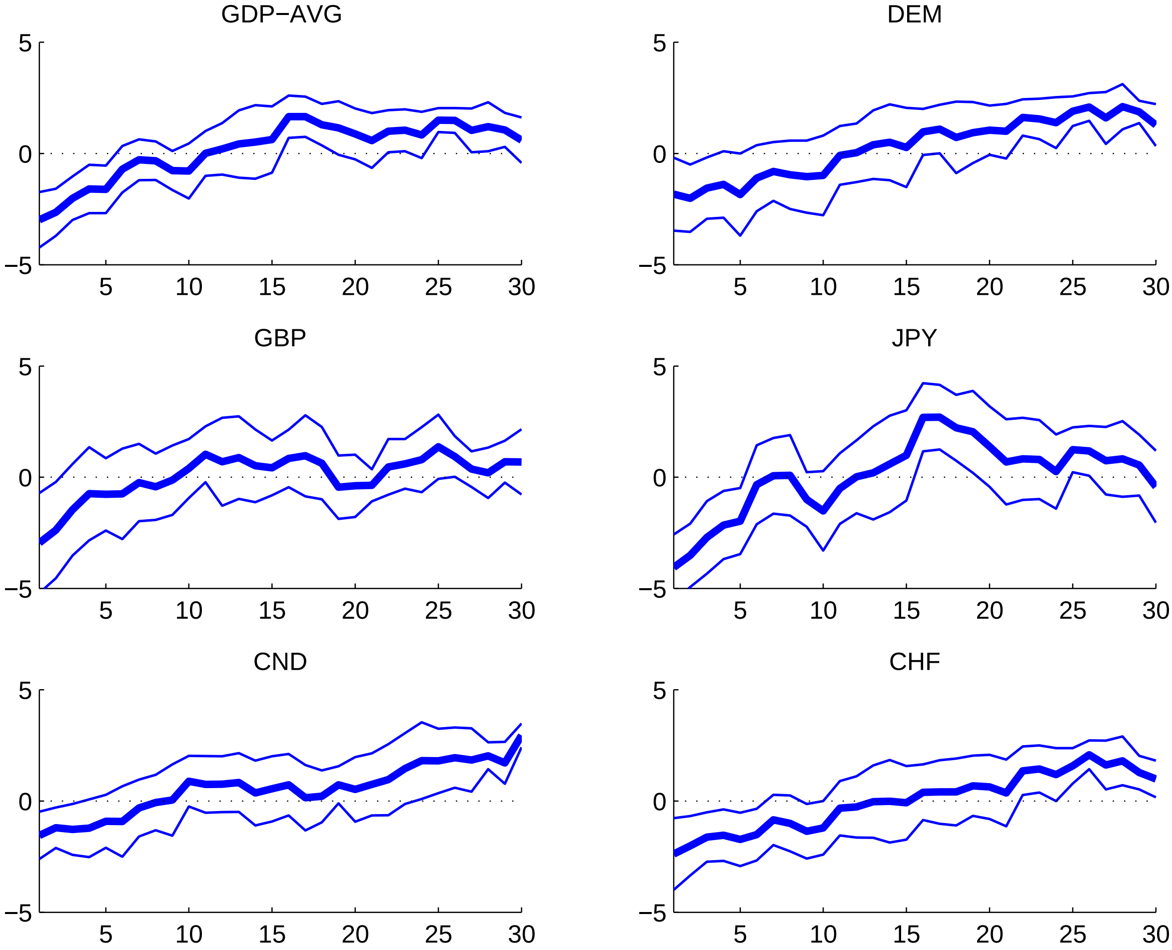
$q_{t+1} = \alpha + \beta(i_t - i_t^*) + \epsilon_{t+1}$			
Currencies	β	$\sigma(\beta)$	R^2
DEM	-1.8344**	0.8189	0.05
GBP	-2.9537***	1.1214	0.10
JPY	-4.0626***	0.7438	0.16
CND	-1.5467***	0.5305	0.05
CHF	-2.3815***	0.8068	0.09
EW Average	-2.5558***	0.6192	0.09
GDP Average	-2.9821***	0.6223	0.11

Note: $q_{t+1} = \Delta s_{t+1} - (i_t - i_t^*)$. Δs_{t+1} refers to the 3-month change in the log exchange rate. The exchange rate is measured as net-of-period rate from IFS. Interest rates are 3-month rates as quoted in the London Euromarket and were obtained from Datastream (Thomson Financial). *** and ** denote significance at respectively the 1% and 5% level. SUR system estimated from 109 quarterly observations over sample from December 1978 to December 2005. Newey-West standard errors with 1 lag. "EW Average" refers to the equally weighted average of the regression coefficients. The last row reports the GDP weighted average.

Table 2: Sensitivity Analysis

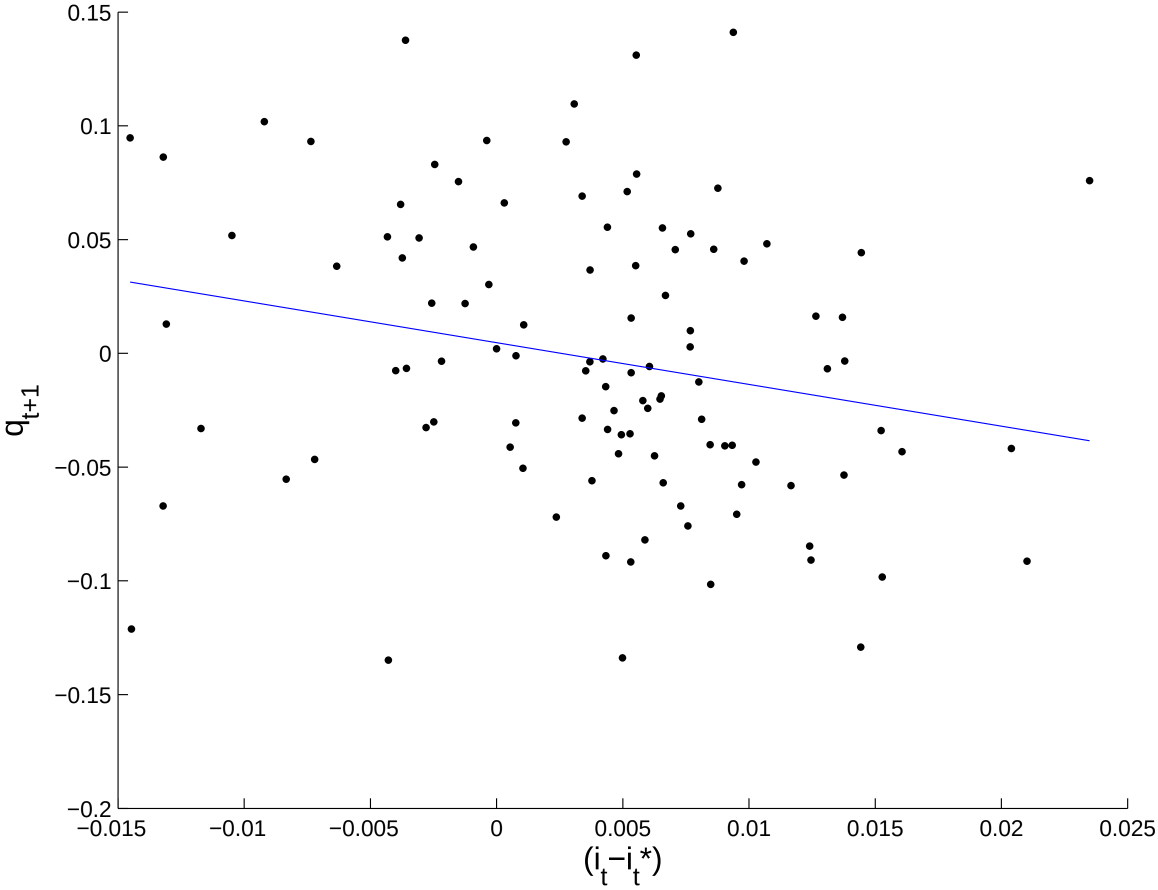
parameters	predictability coefficient β in $q_{t+1} = \alpha + \beta f d_t$	frequency (%) $\beta < -2$ in simulations 25-year period	information processing costs (%)
benchmark ($\gamma = 10$, $T = 8$)	-0.95	12	0.27
$\gamma = 1$	-0.49	4	1.12
$\gamma = 50$	-1.16	18	0.06
$T = 4$	-0.55	5	0.13
$T = 12$	-1.12	15	0.33

Figure 1: Forecasting Slopes



Note: Slopes β_k of regressing $q_{t+k} = \alpha + \beta_k(i_t - i_t^*) + u_{t+k}$ for each currency. Thin lines are standard error bands (+/- 2 s.e.). Same quarterly data as in Table 1. "GDP-AVG" is the GDP weighted average of the five currencies β_k.

Figure 2: Predictability of Excess Return on DEM



Note: Same quarterly data as in Table 1. OLS Slope = -1.8344 (s.e. = 0.8189, computed with 1 Newey-West lag). R^2= 0.05

Figure 3 Excess Return Predictability - Benchmark Parameterization

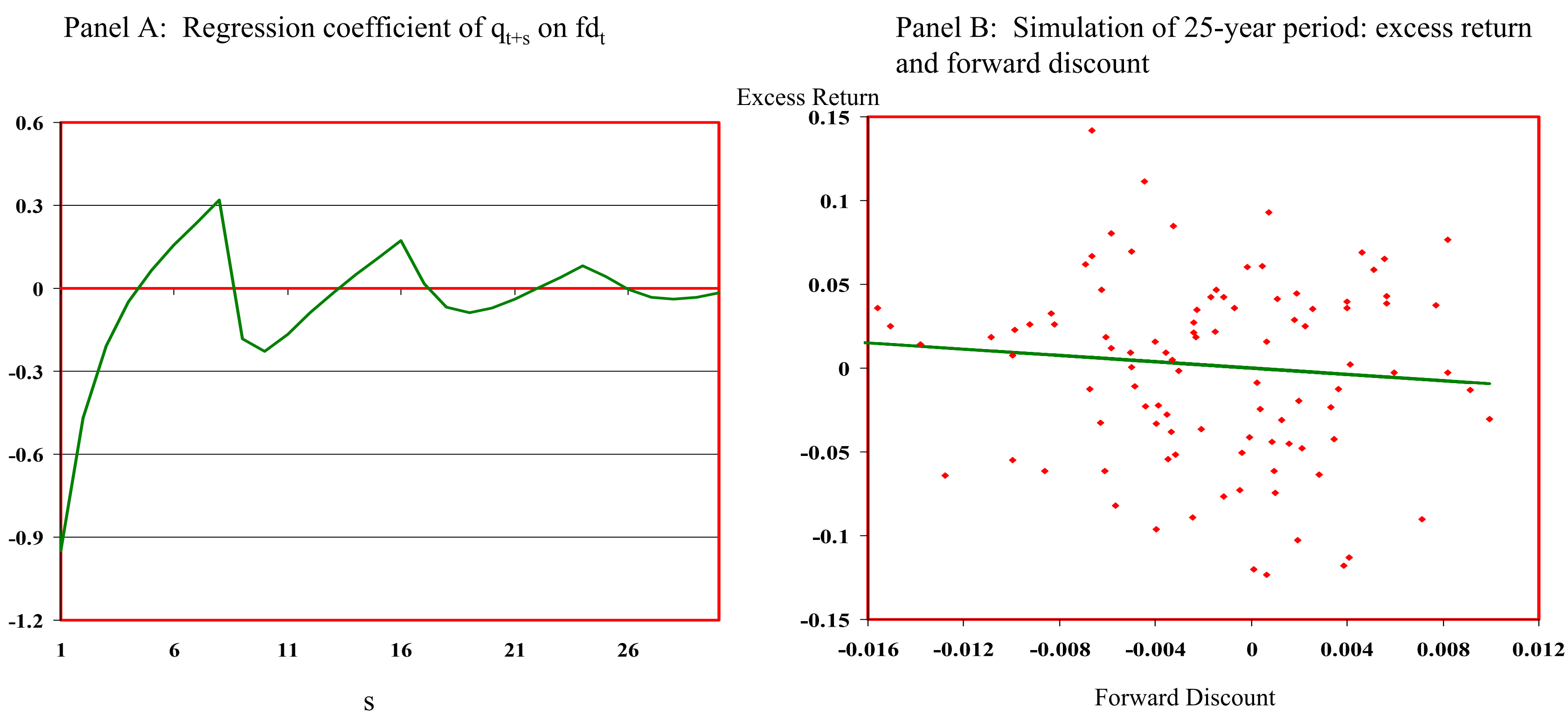
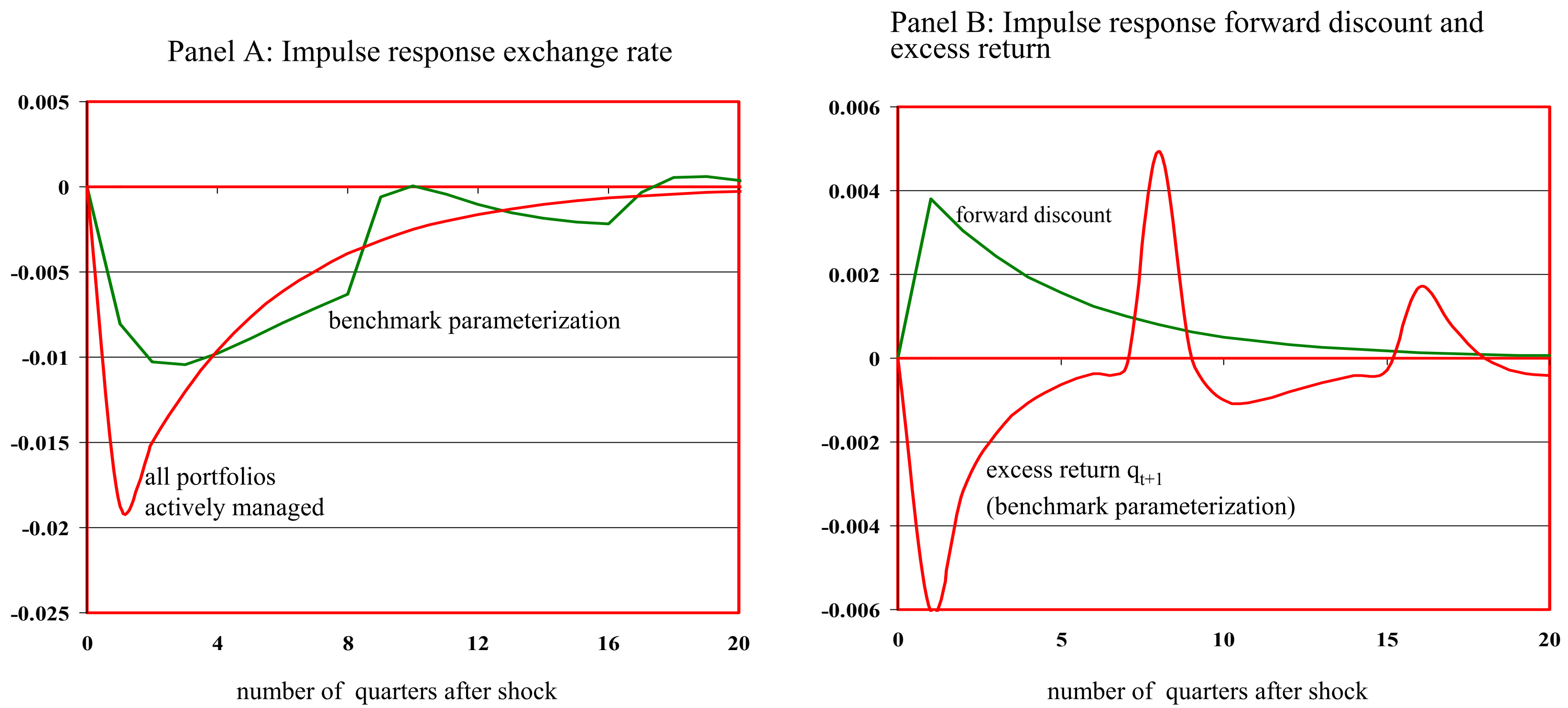


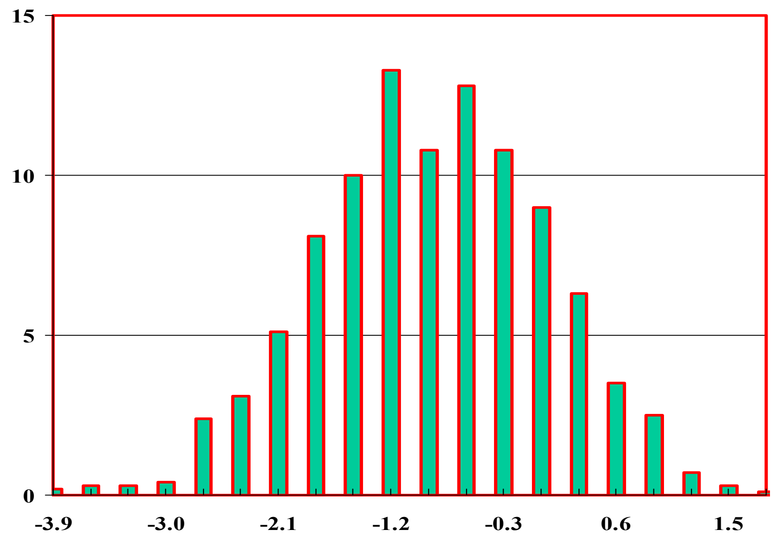
Figure 4 Impulse Responses to Interest Rate Shock under Benchmark Parameterization*



*Panel A shows the impulse response of the log exchange rate to a one standard deviation interest rate shock (decrease in the foreign interest rate) for both the benchmark parameterization and the case where all investors make frequent portfolio decisions. Panel B shows the forward discount and excess return under the benchmark parameterization in response to the same shock.

Figure 5 Small Sample Results - Benchmark Parameterization

Panel A: Frequency distribution of regression coefficient of q_{t+1} on fd_t based on 1000 simulations of 25-year period



Panel B: Average regression coefficient of q_{t+s} on fd_t

10% of simulations of 25-year period with largest predictability

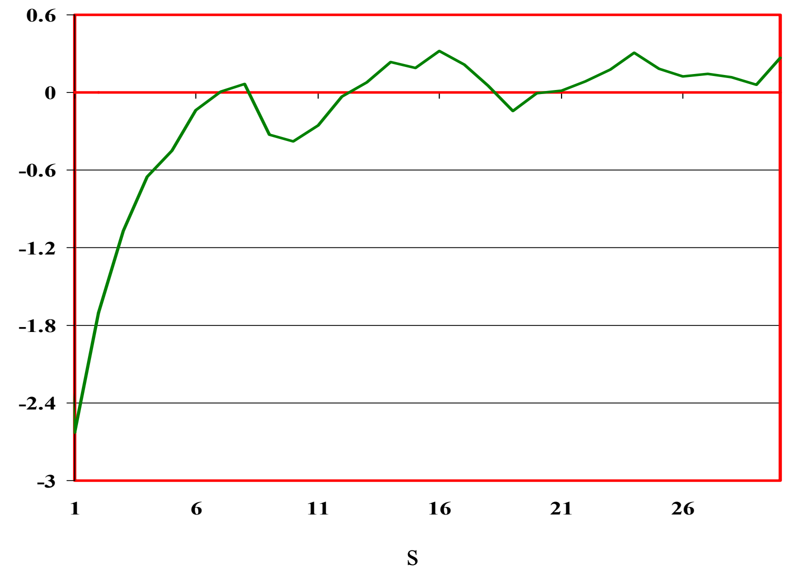


Figure 6 Excess Return Predictability under Partial Information Processing

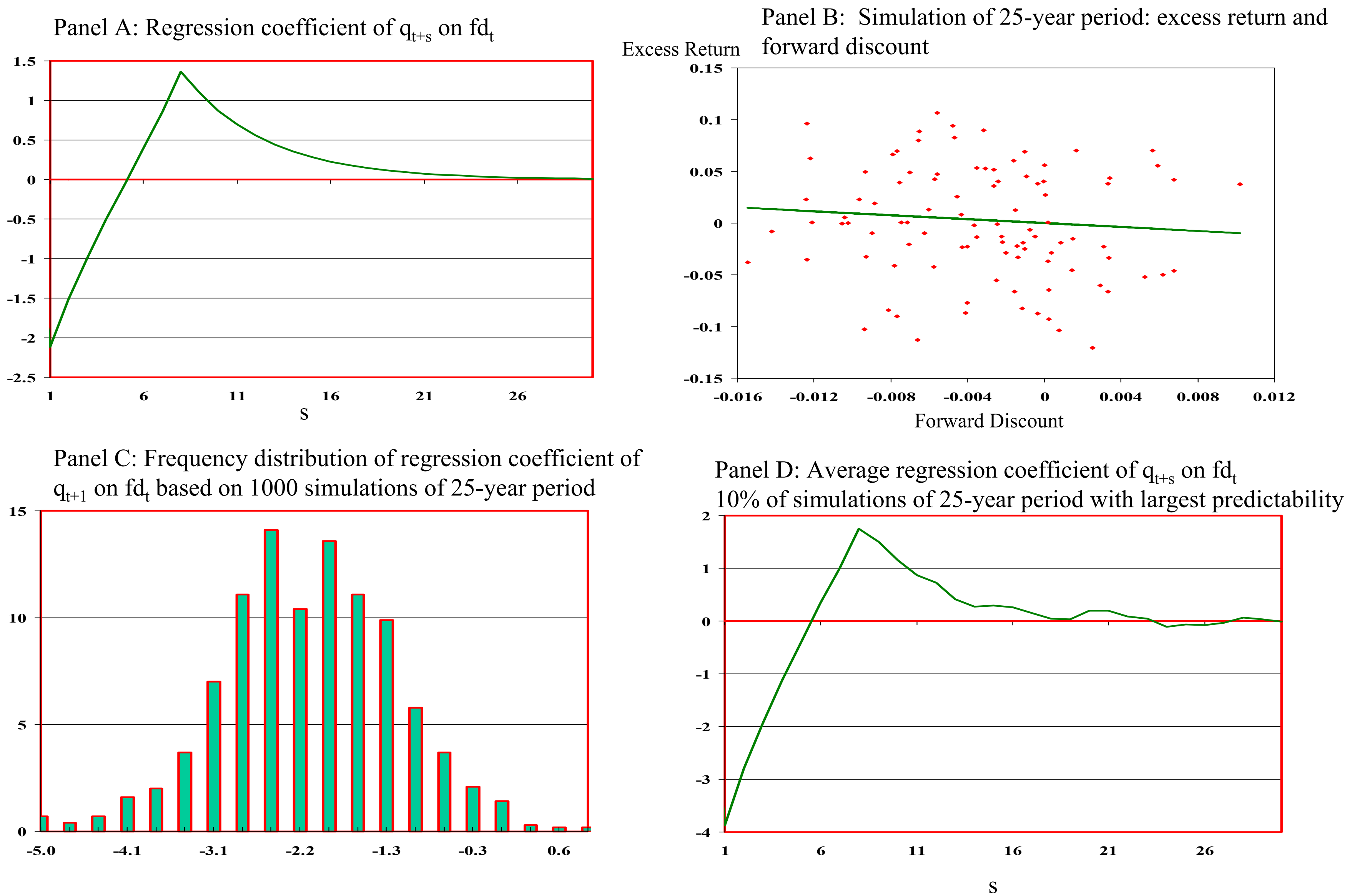


Figure 7 Impulse Responses to Interest Rate Shock under Partial Information Processing

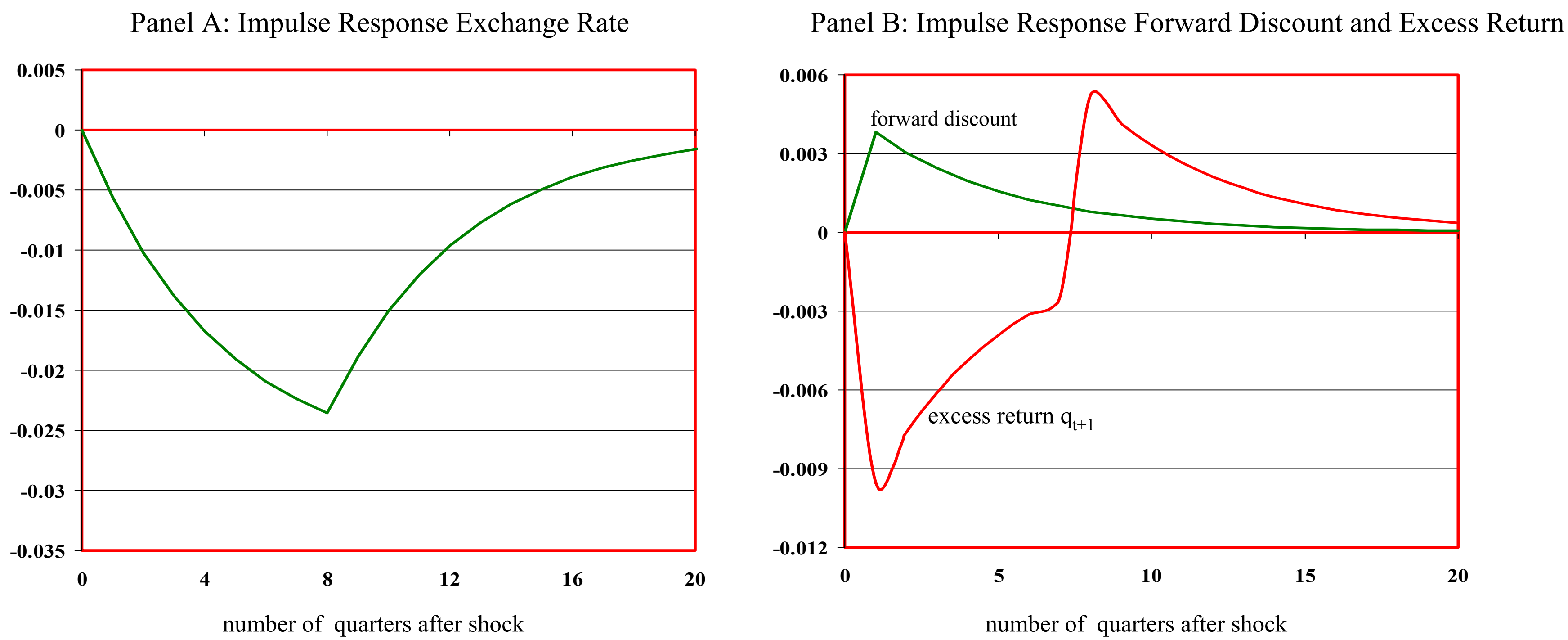
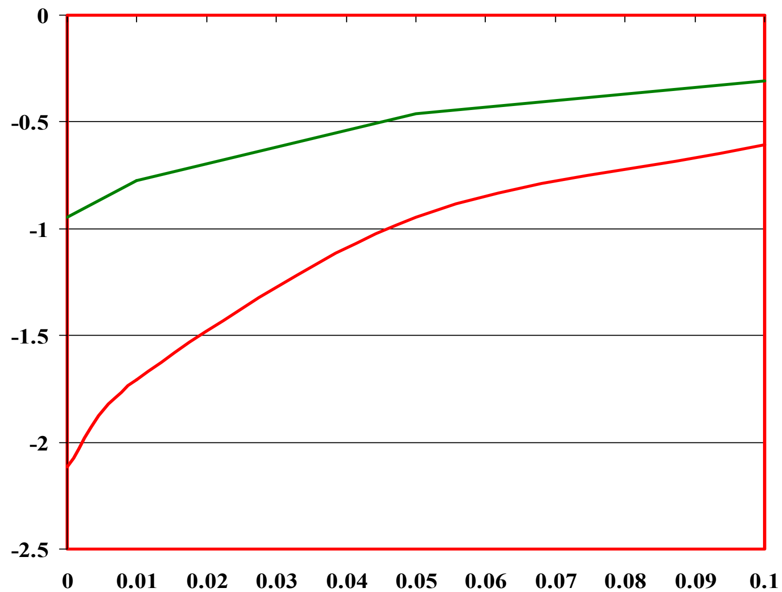


Figure 8 Actively Managed Portfolios: Impact on Predictability and Threshold Cost

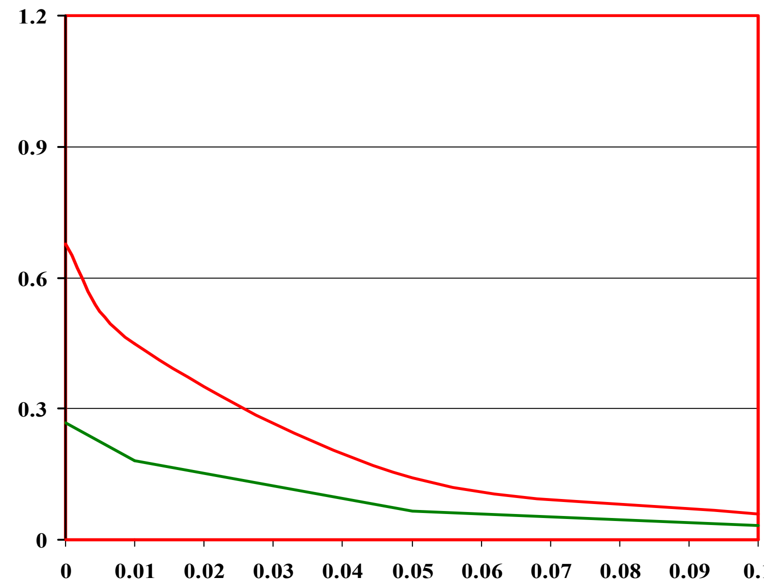
Panel A: Predictability coefficient β of regression

$$q_{t+1}=\alpha+\beta fd_t$$



fraction “f” of investors with actively managed portfolios

Panel B: Threshold Cost



fraction “f” of investors with actively managed portfolios