FEDERAL RESERVE BANK OF SAN FRANCISCO

WORKING PAPER SERIES

A Currency Premium Puzzle

Tarek Hassan,
Boston University

Thomas M. Mertens
Federal Reserve Bank of San Francisco

Jingye Wang
Renmin University of China

October 2024

Working Paper 2024-32

https://doi.org/10.24148/wp2024-32

Suggested citation:

Hassan, Tarek, Thomas M. Mertens, and Jingye Wang. 2024. "A Currency Premium Puzzle." Federal Reserve Bank of San Francisco Working Paper 2024-32. https://doi.org/10.24148/wp2024-32

A Currency Premium Puzzle*

Tarek Hassan,[†] Thomas M. Mertens,[‡] and Jingye Wang[§]

October 18, 2024

Abstract

Standard asset pricing models reconcile high equity premia with smooth risk-free rates by inducing an inverse functional relationship between the mean and the variance of the stochastic discount factor. This highly successful resolution to closed-economy asset pricing puzzles is fundamentally problematic when applied to open economies: It requires that differences in currency returns arise almost exclusively from predictable appreciations, not from interest rate differentials. In the data, by contrast, exchange rates are largely unpredictable, and currency returns arise from persistent interest rate differentials. We show currency risk premia arising in canonical long-run risk and habit preferences cannot match this fact. We argue this tension between canonical asset pricing and international macroeconomic models is a key reason researchers have struggled to reconcile the observed behavior of exchange rates, interest rates, and capital flows across countries. The lack of such a unifying model is a major impediment to understanding the effect of risk premia on international markets.

*The views expressed here are solely those of the authors and do not necessarily represent those of the Federal Reserve Bank of San Francisco or the Federal Reserve System. We thank John Campbell, Mikhail Chernov, Riccardo Colacito, Max Croce, Xiang Fang, Rohan Kekre, Moritz Lenel, Pierre De Leo, Jonathan Heathcote, Oleg Itskhoki, Pierlauro Lopez, Hanno Lustig, Andreas Stathopoulos, Adrien Verdelhan, and seminar participants at the University of Southern California, SUFE, Macro Finance Society, WFA 2024, CFRC 2024, NBER SI 2024, and 3rd International Finance/Macro/Trade Conference for helpful comments.

[†] **Boston University**, 270 Bay State Rd, Boston, MA 02215; thassan@bu.edu.

[‡] **Federal Reserve Bank of San Francisco**, 101 Market Street, Mailstop 1130, San Francisco, CA 94105; Thomas.Mertens@sf.frb.org.

[§] **Renmin University of China**, 59 Zhongguancun Road, Haidian District, Beijing 100872, China; jingyewang@ruc.edu.cn.

Introduction

Interest in the role of risk premia in shaping international capital markets has surged in recent years. Numerous studies have examined the ramifications of shocks to both global and country-specific risk premia, investigating their impact on exchange rates, interest rates, capital flows, and financial stability. This research addresses critical phenomena such as the violation of uncovered interest parity (di Giovanni et al., 2022), contagion (Forbes, 2013), the global financial cycle (Miranda-Agrippino and Rey, 2020; Morelli, Ottonello, and Perez, 2022; Boehm and Kroner, 2023; Bai et al., 2023; Oskolkov, 2024),[1] and events such as flight to safety, capital retrenchments (Forbes and Warnock, 2021; Chari, Dilts Stedman, and Lundblad, 2020), and sudden stops (Mendoza, 2010), which are of paramount concern for policymakers.

Despite the intense interest in these topics, constructing quantitative models that can reconcile the observed behavior of exchange rates with large and persistent differences in interest rates across countries has proven difficult. For example, to our knowledge, no quantitative model has been able to reproduce the fact that New Zealand on average has had a risk-free interest rate five percentage points above that of Japan for multiple decades, or that the cost of borrowing in Switzerland is persistently lower than that in Norway. The lack of such a unifying model is a major impediment to understanding the effect of risk premia on the allocation of capital across countries.

In this paper, we show these challenges are rooted in a fundamental tension between the canonical asset pricing models, which have been highly successful in addressing closed-economy asset pricing puzzles (Bansal and Yaron, 2004; Campbell and Cochrane, 1999), and the empirically observed behavior of exchange rates: strong forces in these models require that any large differences in risk premia across countries manifest themselves not as persistent differences in interest rates, but as highly predictable exchange rates. In the data, we see large and persistent differences in interest rates across countries and exchange rates that are notoriously hard to predict (Meese and Rogoff, 1983), putting this class of model at odds with the data. We term this tension the "currency premium puzzle" and argue it is a fundamental reason the literature has struggled to construct a quantitative model that can match both asset prices and macroeconomic quantities across countries.

The classical quantitative challenge in asset pricing is to reconcile a high equity premium (Mehra and Prescott, 1985) with a low and stable risk-free rate (Weil, 1989). The former requires the stochastic discount factor (SDF) to be highly volatile, whereas the latter requires the sum of the mean and variance of the

[1] Also see, e.g., Rey (2015).

logarithm of the SDF (log SDF) to be low and stable. Canonical long-run risk and habit models resolve these puzzles jointly by introducing a negative, functional relationship between the mean and the variance of the log SDF, so that whatever increases the SDF's variance also decreases its mean. This negative functional relationship keeps the risk-free rate low and stable while allowing for a large equity premium.

This mechanical link between the mean and the variance of the log SDF has been highly successful in closed-economy settings, allowing the construction of representative agent models that match both asset prices and macroeconomic quantities, such as aggregate investment and consumption. *We argue the same link is also the fundamental reason why these models struggle in an open economy setting.*

Specifically, in an open economy with complete markets and log-normal SDFs, the means of log SDFs govern expected changes in exchange rates, while the variances of the log SDFs govern the expected return on the currency (Backus, Foresi, and Telmer, 2001). Data on exchange rates and currency premia thus puts bounds on how these moments differ across countries. In particular, a large literature documents that (1) exchange rates are largely unpredictable, which requires the means of log SDFs to be similar across countries (Meese and Rogoff, 1983), and (2) currency premia are quantitatively large, which requires the variances of log SDFs to differ significantly across countries (Hassan and Mano, 2019). Together, these prior findings mean countries' log SDFs should differ widely in their variances but not in their means. This restriction from the international data, however, is difficult to fit with canonical long-run risk and habit preferences, which hardwire an almost one-for-one negative functional relationship between the two moments.

In this sense, large currency risk premia pose a fundamentally different quantitative challenge to these models than the classical closed-economy asset pricing puzzles, and the literature has not widely recognized this challenge.

To illustrate this point, we consider a conventional long-run risk model with complete markets, two representative agents (one in the home country and one in the foreign country), and identical Epstein-Zin (EZ) preferences with parameters equal to those in Bansal and Shaliastovich (2013). Suppose the two countries differ in their risk characteristics (the variance of their log SDFs) for *any* reason, so that one country's currency is riskier than the other. We show that, in this model, 94% of any difference in currency premia between the two countries must manifest as a predictable depreciation of the exchange rate, with only the remaining 6% coming from interest rate differentials. Because of the hard-wired functional relationship between the mean and variance of the log SDFs, this 94:6 split is fully determined by the parameters of the EZ utility function and independent of the features of the economic environment.

In other words, we demonstrate a theorem showing currency risk premia arising in models with these canonical preferences (risk aversion around 10 and elasticity of intertemporal substitution around one) cannot fit the international data.

The same holds for canonical habit preferences (Campbell and Cochrane, 1999), where—again—the link between the mean and variance of the log SDF forces the vast majority of cross-country differences in currency premia to manifest as predictable depreciations, not persistent interest rate differentials.

This finding is all the more problematic because leading theories of why risk premia and interest rates might differ persistently across countries typically point to differences in the economic environment, such as differences in country size (Martin, 2011; Hassan, 2013), their role in global trade (Richmond, 2019), resource endowments (Ready, Roussanov, and Ward, 2017b), their level of indebtedness (Wiriadinata, 2021), or the volatility of shocks (Menkhoff et al., 2012; Colacito et al., 2018a). All of these features of the economic environment are irrelevant for the results above: with complete markets, canonical long-run risk and habit preferences force all of these drivers of currency risk to manifest as large predictable depreciations, not persistent interest rate differentials.

To corroborate our theoretical results, we simulate widely used versions of long-run risk and habit models and show the currency premium puzzle manifests in each of them.

We show the same issue also arises in models that combine disaster risk with Epstein and Zin (1989) preferences (Gourio, 2012; Gourio, Siemer, and Verdelhan, 2013) and in other canonical applications of the rare disaster paradigm.

Moreover, our analytical results show stochastic volatility as in Bansal and Yaron (2004), and departures from log-normality do not substantially alleviate the problem. At a deep level, any complete-markets model in which representative agents in two countries have identical habit or EZ preferences with a preference for early resolution of uncertainty appears to feature the currency premium puzzle.

In sum, the currency premium puzzle we highlight in this paper applies to the most advanced quantitative models that have attempted a synthesis between international asset prices and quantities. We do not see a straightforward resolution, which, of course, is our motivation for writing this paper.

One possibility is that the patterns we observe in the data could be rationalized with systematic differences in preferences across countries. However, in the absence of direct evidence of such differences—such as systematic demonstrations of higher risk aversion among households in New Zealand versus Japan, or in Norway relative to Switzerland—we are unable to evaluate this possibility. Ultimately, any observed economic behavior can be explained by appealing to sufficiently heterogeneous preferences, which is why economists are traditionally skeptical of purely preference-based explanations, favoring instead interpretations grounded in empirical evidence (Samuelson, 1948; Becker, 1976).

In the paper's final two sections, we examine two additional avenues for further developing the literature. First, it is possible that frictions induced by market incompleteness may take a form that solves the tension between large interest rate differentials and unpredictable exchange rates. Because the vast majority of the existing literature relies on the complete-markets assumption, assessing the extent to which this might be possible in general is difficult. However, we are able to study one specific type of incompleteness that arises under incomplete spanning (Lustig and Verdelhan, 2019): when the representative agent in one country cannot trade all assets available in other countries.

Here again we find significant challenges: although incomplete spanning loosens the relationship between marginal utility and exchange rates, it does so in a limited way. In particular, we show that every percentage point that incomplete spanning deducts from exchange rate predictability is also eliminated from that currency's expected return. That is, although it may be possible to construct an (offsetting) pattern of incomplete spanning wedges that eliminates exchange-rate predictability from the models mentioned above, that same friction will then again compress cross-sectional differences in currency returns toward zero, and thus to levels similar to those one would obtain with conventional (constant relative risk aversion) preferences. As a result, incomplete spanning may help eliminate exchange rate predictability, but those models would then likely again fail to generate the large differences in interest rates and currency returns we see in the data.[2]

Finally, we explore synthesizing risk-based models, which have been the focus of recent literature, with the traditional macroeconomic view of cyclical interest rate differentials. In risk-based models, persistent differences in currency returns arise because investing in some countries is inherently riskier than in others. In contrast, the traditional macroeconomic view attributes short-term interest rate differences to temporary variations in expected growth rates and inflation, which are reversed by predictable appreciations (uncovered interest parity holds).[3] If these temporary differences were instead long-lasting, one could construct a synthe-

[2]This result dovetails with several recent papers (Lustig and Verdelhan, 2019; Jiang, 2023; Jiang et al., 2022; Jiang, Krishnamurthy, and Lustig, 2023; Sandulescu, Trojani, and Vedolin, 2021; Chernov, Haddad, and Itskhoki, 2023) that have consistently found various forms of incomplete markets do not necessarily help resolve many of the well-known puzzles in international finance, including the volatility puzzle of Brandt, Cochrane, and Santa-Clara (2006), the Backus and Smith (1993) puzzle, and the exchange rate disconnect puzzle (Meese and Rogoff, 1983).

[3]See Alvarez, Atkeson, and Kehoe (2007) for a discussion of the tension between traditional, macro-based views of short-term interest rates and the risk-based view of exchange rates.

sis where countries with high currency risk premia (and thus expected appreciations) also had persistently higher growth rates or inflation (expected depreciations). Differences in the means and variances of the countries' SDF could then potentially offset each other, transforming predicted appreciations into persistent interest rate differentials. Though this approach has its challenges, we highlight promising work in this area.

Our paper contributes to a large and growing literature at the intersection of international macroeconomics and asset pricing. One important strand of this literature studies in reduced-form the effects of time variation in global and country-specific risk premia, where shocks to global and local risk-bearing capacity motivate variation in global and local borrowing costs, retrenchments, capital flight, and the global financial cycle (Mendoza, 2010; Forbes, 2013; Rey, 2015; Miranda-Agrippino and Rey, 2020; Chari, Dilts Stedman, and Lundblad, 2020; di Giovanni et al., 2022; Forbes and Warnock, 2021; Morelli, Ottonello, and Perez, 2022; Bai et al., 2023; Colacito et al., 2018b).

Several important papers have attempted to microfound such time variation in global risk premia by combining conventional international macro models with long-run risk, habits, and other advances from the asset pricing literature (e.g., Colacito and Croce, 2011, 2013; Colacito et al., 2018a; Verdelhan, 2010; Heyerdahl-Larsen, 2014; Stathopoulos, 2017; Gourio, Siemer, and Verdelhan, 2013). While most of these models are designed to address a variety of quantitative puzzles, ranging from the forward premium puzzle to exchange rate disconnect, none of them are able to match the empirical fact that currency premia are large, whereas exchange rates are largely unpredictable. We contribute to this literature by highlighting this tension as a major obstacle to its further development.

Our paper is also tightly linked to the classic approaches to resolving the equity premium and risk-free rate puzzles. In particular, we show that models that are highly successful in resolving these well-known puzzles (Campbell and Cochrane, 1999; Bansal and Yaron, 2004) face new challenges when confronted with international data.

Our paper also speaks to the growing international macroeconomics literature that has documented large and persistent differences in currency risk, currency returns, and interest rates across countries (Lustig and Verdelhan, 2007; Lustig, Roussanov, and Verdelhan, 2011; Hassan and Mano, 2019).[4] Various papers have related these persistent differences in currency risk to features of the economic environment, such as differences in country size (Hassan, 2013; Martin, 2011), trade centrality (Richmond, 2019), commodity trade (Ready, Roussanov, and Ward, 2017b), indebtedness (Wiriadinata, 2021), fiscal conditions (Jiang, 2021), and

[4]See Hassan and Zhang (2021) for a survey of this literature.

others. We contribute to this literature by showing all of these forces, when evaluated through the lens of a quantitative model with long-run risk or habit preferences, will result in large predicted depreciations, but not the large interest rate differentials we observe in the data.

Lastly, our paper also relates to a growing body of papers that highlights the importance of incompleteness of international asset markets. Since the seminal work of Backus, Foresi, and Telmer (2001), complete markets have been a popular assumption in the international finance literature for the assumption's simplicity. But recent works by Sandulescu, Trojani, and Vedolin (2021), Lustig and Verdelhan (2019), Jiang (2023), Jiang et al. (2022), Jiang, Krishnamurthy, and Lustig (2023), and Chernov, Haddad, and Itskhoki (2023), to highlight a few prominent contributions, have established in a reduced-form way that market incompleteness, or even segmentations and Euler equation wedges, help match salient features of currency markets. By introducing constrained intermediaries into general equilibrium models, Itskhoki and Mukhin (2021) show that shocks to currency intermediation can help resolving a number of puzzles in international macroeconomics[5], while Kekre and Lenel (2024) argue that persistent shocks to interest rate differentials drives most of the exchange rate variations. We show incomplete spanning is also subject to the currency premium puzzle. Although other forms of market incompleteness might help resolve the puzzle, we are not aware of any work that has successfully done so.[6]

The remainder of this paper is organized as follows. Section 1 lays out the basic concepts and framework for analysis. Section 2 discusses the long-run risk model, and section 3 discusses the habit model. Sections 4, 5, and 6 expand the argument by allowing for departures from log-normal distributions, incomplete markets, and heterogeneous growth rates. Section 7 concludes.

1 Basic Framework and Data

We first introduce a simple graphical representation of the currency premium puzzle. We show how data on exchange rates and currency returns put restrictions on the choice of SDFs across countries. For simplicity, we first focus the discussion on log-normally distributed SDFs and complete markets. We show in sections 4 and 5 that the currency premium puzzle also arises in more complex settings with incomplete markets and non-normal shocks.

[5]Also see Gabaix and Maggiori (2015).

[6]Recently, Fang (2021) proposes a model with financial intermediaries, which takes heterogeneous risk-free rates as exogenous and generates cross-country variations in currency premia. In his framework, risk-free rates are the causes instead of the consequences of currency riskiness. However, the drivers of the large cross-country variations in risk-free rates, and how these drivers would interact with currency premia, remain unclear.

1.1 Closed Economy Asset Pricing Puzzles

Our starting point is a complete-markets setup where we assume a log-normal distribution for the SDF M_{t+1}. Absence of arbitrage requires

$$\mathbb{E}_t(M_{t+1}R^a_{t+1}) = 1,$$

where $\mathbb{E}_t$ denotes the mathematical expectation conditional on the information set at time t and R^a_{t+1} denotes the return on an arbitrary asset. Applied to the risk-free rate with log return r_t, this equation relates the mean and variance of the SDF via

$$-\mathbb{E}_t(m_{t+1}) = \frac{1}{2}\,\mathrm{var}_t(m_{t+1}) + r_t. \tag{1}$$

Throughout the paper, lowercase letters denote the logarithm of a variable such that $x = \log(X)$.

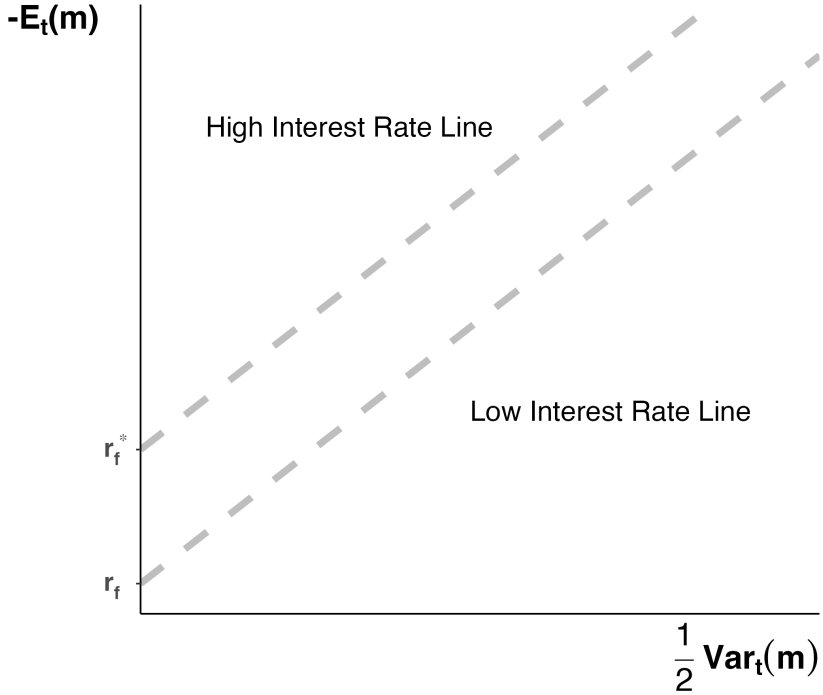
Resolving the equity premium puzzle of Mehra and Prescott (1985) requires the variance of the logarithm of the SDF (log SDF), $\mathrm{var}_t(m_{t+1})$, to be large (Hansen and Jagannathan, 1991). Equation (1) shows this restriction has an immediate implication for the risk-free rate. In the data, risk-free interest rates tend to be low and stable, as is known from the risk-free rate puzzle of Weil (1989).

The joint resolution of the risk-free rate and equity premium puzzles thus requires a mean of the log SDF that moves to accommodate a high, and potentially changing, variance of the SDF to maintain low and stable risk-free rates. As we show in later sections, standard resolutions of the puzzles such as long-run risks and habit utility achieve this feature by imposing a negative functional relationship between the two moments of the log SDF.

Figure 1 represents equation (1) graphically. It plots the scaled (conditional) variance of the log SDF on the horizontal axis and the negative conditional mean on the vertical axis. Each point in this "SDF space" determines a log SDF represented by its first two moments. By equation (1), the first two moments of each SDF pin down the risk-free interest rate. In fact, equation (1) implies any SDF that lies on the same 45-degree line produces the same risk-free rate. We refer to these lines as “iso-risk-free rate (or iso-rf) lines."

Figure 1 shows two examples of iso-rf lines. The intercept of the line with the vertical axis represents the risk-free rate, and higher lines represent higher risk-free rates. We thus have a graphical representation of (1) for different risk-free rates. The equity premium puzzle and the risk-free rate puzzle require a high variance with $(\frac{1}{2}\,\mathrm{var}_t(m_{t+1}), -\mathbb{E}_t(m_{t+1}))$ pairs on or around a particular iso-rf line to maintain a stable risk-free rate.

Figure 1: The SDF Space and Iso-rf Lines



This figure plots two iso-rf lines (grey dashed line) in the SDF space. The higher line represents a higher risk-free rate. The intercept of each line with the y-axis represents the corresponding risk-free rate.

1.2 Exchange Rates and the SDF

Next, we show the logic of the previous section puts additional restrictions on the log SDFs when applied to open economies. Under complete markets, Backus, Foresi, and Telmer (2001) show the expected change in exchange rates, quoted as units of home currency per foreign currency, is given by[7]

$$\mathbb{E}(\Delta s_{t+1}) = [-\mathbb{E}(m_{t+1})] - [-\mathbb{E}(m^{\star}_{t+1})], \tag{2}$$

where Δs_{t+1} is the change in log exchange rates (appreciation rate of the foreign currency), and we use $\star$ to denote the foreign country.

From equation (1), the difference in risk-free rates between the two countries is given by

$$\begin{aligned}\mathbb{E}(r^{\star}_t - r_t) = &\ \mathbb{E}(m_{t+1}) - \mathbb{E}(m^{\star}_{t+1}) \\ &+ \frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1}) - \text{var}_t(m^{\star}_{t+1})).\end{aligned} \tag{3}$$

[7]All equations in this subsection also hold conditionally. We show unconditional equations here because we can only estimate the unconditional moments given data on exchange rates, and we emphasize the cross-country variations of unconditional moments.

Analogous to the equity premium, which is defined as the unconditional difference between returns on equity and the risk-free rate, we define the currency premium as the expected return a home investor would earn if she invests in the foreign risk-free bond versus the home bond.[8] She earns the difference in interest rates net of the depreciation rate of the foreign currency. Using (2) and (3), we obtain the currency premium as

$$\mathbb{E}(rx_{t+1}) = \mathbb{E}(r_t^\star - r_t) + \mathbb{E}(\Delta s_{t+1}) = \frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1}) - \text{var}_t(m_{t+1}^\star)), \tag{4}$$

where rx_{t+1} denotes the currency return earned between t and $t+1$. For ease of exposition, we assume the home country has a lower risk-free rate, so that the currency return coincides with the return on the "carry trade," where the investor borrows in the low-interest-rate currency and lends in the high-interest-rate currency.

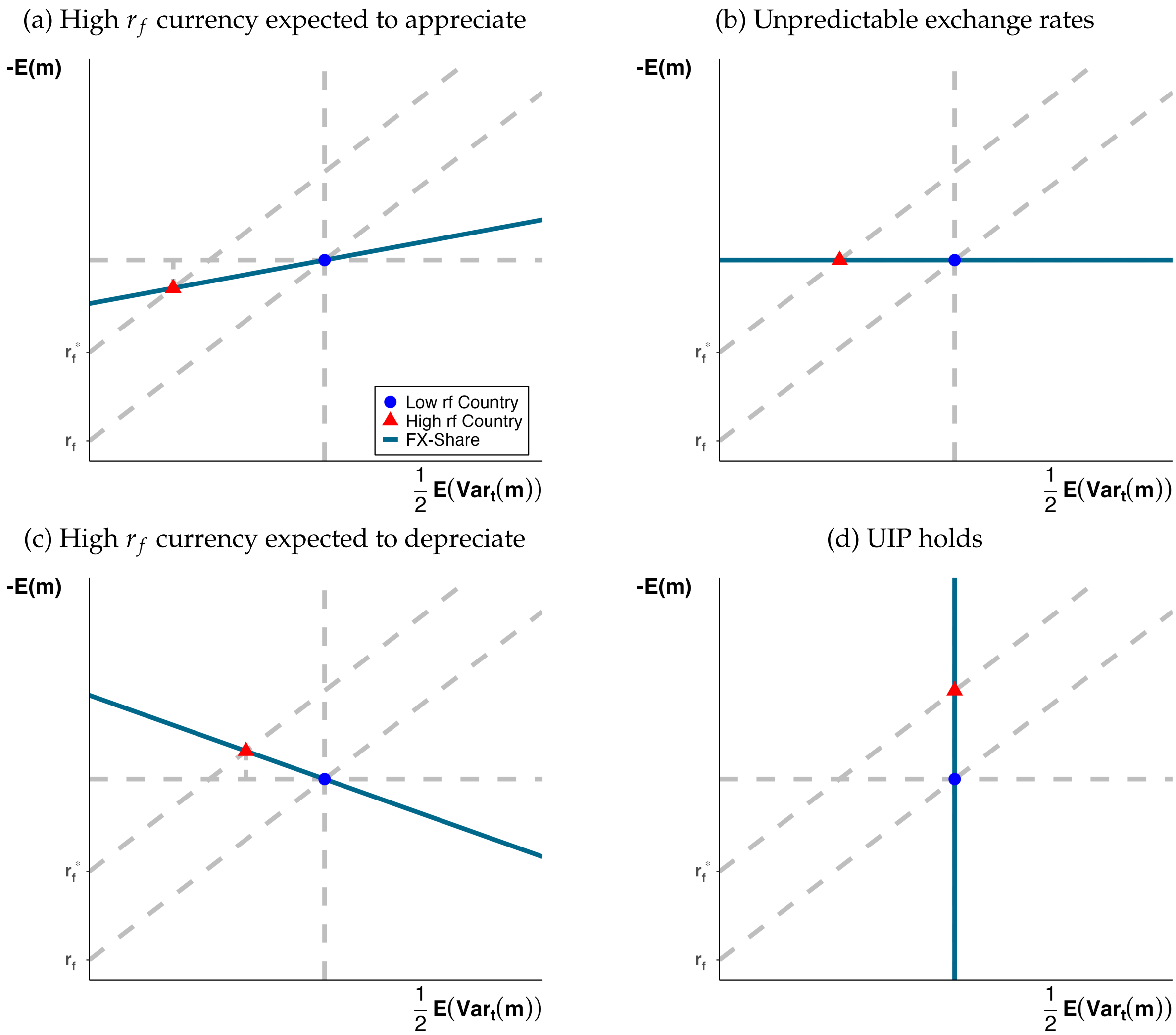
Note that we can decompose currency premia into an interest rate differential and an expected change in the exchange rate. The relative importance of these two terms is key to our analysis. To facilitate later discussion, we label the share of the expected change in the exchange rate in the currency premium, $\frac{\mathbb{E}(\Delta s_{t+1})}{\mathbb{E}(rx_{t+1})}$, the "FX-share." The FX-share measures the fraction of the currency premium that comes from predicted changes in the exchange rate, rather than the difference in interest rates. In particular, a positive FX-share means investors expect the high-interest-rate currency to appreciate on average.

Equations (2) and (4) show exchange rates and currency premia are tightly linked to means and variances of log SDFs. Each country's SDF is characterized by one mean-variance pair $(\frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1})), -\mathbb{E}(m))$, which we represent as a point in (unconditional) SDF space. Data on exchange rates and currency premia between any two countries provide information on the relative position of their corresponding points. In particular, expected changes in the exchange rate determine the vertical differences (equation (2)) and currency premia govern horizontal differences (equation (3)).

Figure 2 visualizes properties of exchange rates and currency premia in SDF space under various configurations. Panel (a) shows the high-interest-rate currency appreciating on average relative to the low-interest-rate currency. The high-interest-rate foreign currency (red triangle) lies on a higher iso-rf line and is expected to appreciate ($\mathbb{E}(\Delta s_{t+1}) = [-\mathbb{E}_t(m_{t+1})] - [-\mathbb{E}_t(m_{t+1}^\star)] > 0$) relative to the low-interest-rate home currency (blue dot). The slope of the blue line connecting the two dots measures the FX-share – the vertical distance between two countries (the expected rate of appreciation) divided by their horizontal distance (the currency

[8] In the literature, this return is sometimes referred to as "currency risk premia" or simply "currency returns." We call it currency premia, analogously to equity premium.

Figure 2: Currency Premium in the SDF Space



This figure plots four scenarios of currency premia in the SDF space. In each panel, grey dashed lines represent iso-rf lines and vertical/horizontal lines. The blue dot represents the low-interest-rate country, and the red triangle represents the high-interest-rate country. Panel (a) represents the case when the high-interest-rate currency is expected to appreciate; panel (b) represents the case when exchange rates are unpredictable; panel (c) represents the case when the high-interest-rate currency is expected to depreciate; and panel (d) represents the case when uncovered interest rate parity holds.

premium).

In panel (b), exchange rates are unpredictable, namely, $\mathbb{E}(\Delta s) = 0$, and the means of the log SDFs are consequently equalized across countries, so that the blue line is horizontal, with an FX-share of zero. Panel (c) shows the situation where the high-interest-rate currency is expected to depreciate — the blue line has a negative slope. If uncovered interest rate parity holds, interest-rate differences are exactly offset by expected change in exchange rates, and currency premia (differences in variances) vanish, so that the blue line is vertical (panel (d)).

The main takeaway from Figure 2 is that the slope of the solid blue line that connects the two countries measures the FX-share:

$$\begin{aligned} \text{slope} &= \frac{[-\mathbb{E}(m_{t+1})] - [-\mathbb{E}(m^{\star}_{t+1})]}{\frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1})) - \frac{1}{2}\mathbb{E}(\text{var}_t(m^{\star}_{t+1}))} \\ &= \frac{\mathbb{E}(\Delta s)}{\mathbb{E}(rx)} = \text{FX-share.} \end{aligned} \tag{5}$$

It pins down the split between expected depreciations and interest-rate differentials. As we discuss below, the currency-premium puzzle states that long-run risk and habit models generate an FX-share that is at odds with the data, i.e., they imply a currency premium that is of the wrong composition.

What do the data tell us about the FX-share? A voluminous literature going back to Meese and Rogoff (1983) has documented that exchange rates are largely unpredictable. That is, we already know from this literature that the FX-share must be close to zero in the presence of currency premia.

To estimate the FX-share more directly, we build on a key result in Hassan and Mano (2019), who estimate the share of (conditional and unconditional) currency returns that are attributable to predictable depreciations. Table 1 summarizes their results for a baseline sample of countries.

The top row shows the returns on a simple (unconditional) version of the carry trade that goes long currencies that generally have high interest rates (such as New Zealand) and goes short currencies that have persistently low interest rates (such as Japan).

$$\mathbb{E}[rx^{st}] \equiv \sum_t \sum_i rx_{i,t+1}(\bar{r}_i - \bar{r})$$

The portfolio weights each currency i in proportion to its average interest differential with the US in the pre-sample (the 15 years prior to 1995, $\bar{r}_i - \bar{r}$), without allowing any adjustments of weights thereafter.[9] Hassan and Mano (2019) refer to this strategy as the "Static Trade," because it mimics the famous conditional carry trade, but without allowing for rebalancing of the portfolio in response to short-term fluctuations in the interest rate.

The table shows two main insights. First, unconditional differences in currency returns are large, with a mean return to this strategy of 3.46% annually and a Sharpe ratio of 0.39. Second, the vast majority of

[9]The data range from 1983–2010. Our results are robust to including newer data. We keep our sample identical to Hassan and Mano (2019) to facilitate comparison. The table shows results from their "1 Rebalance" sample, which consists of 16 countries/regions: Australia, Canada, Switzerland, Denmark, Hong Kong SAR, Japan, Kuwait, Malaysia, Norway, New Zealand, Saudi Arabia, Sweden, Singapore, United Kingdom, South Africa, and the US. The empirical results below are robust to alternative means of portfolio construction, and the general pattern is confirmed in many related works (e.g., Lustig, Roussanov, and Verdelhan (2011)).

these returns are due to long-lasting differences in interest rates between countries (4.76 percentage points), whereas, if anything, high-interest-rate currencies depreciate on average, deducting 1.30 percentage points from the overall return. Hassan and Mano (2019) show how to estimate this share more formally using regression analysis and how to construct appropriate standard errors, which are given in square brackets below.[10] Moreover, they show both features of the data (large interest-rate differentials and a slightly negative FX-share) are robust to a wide range of samples and estimation techniques. Appendix C gives details and also updates their estimates using more recent data.

Table 1: FX-Share in the Data

	Return (%)	Change in FX (%)	Interest Rate Diff (%)	FX-share
	$\mathbb{E}(rx^{st})$	$\mathbb{E}(\Delta s^{st})$	$\mathbb{E}(r^{\star,st} - r^{st})$	$\frac{\mathbb{E}(\Delta s^{st})}{\mathbb{E}(rx^{st})}$
Static Trade	3.46 [1.18,5.54]	-1.30 [-3.82,0.60]	4.76 [1.30,8.46]	-0.37 [-1.16,0.23]
	$\mathbb{E}(rx^{ct})$	$\mathbb{E}(\Delta s^{ct})$	$\mathbb{E}(r^{\star,ct} - r^{ct})$	$\frac{\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$
Conditional Carry Trade	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]

We use the 1-rebalance sample of Hassan and Mano (2019). All moments are annual. Static trade returns are calculated by first sorting the unconditional forward premia (interest rates), and then always shorting a weighted portfolio of the currencies with below-average unconditional forward premia, and longing a weighted portfolio of the rest. Details on the construction of portfolios can be found in Appendix C. Confidence intervals are reported in brackets and are obtained by bootstrapping over countries. Alternative regression-based estimations can be found in Appendix C and in Hassan and Mano (2019).

On average, investing in high-interest-rate currencies makes money on the interest-rate differential and loses money on the exchange rate. The FX-share is negative (-37%), as in panel (c) in Figure 2, with a confidence interval of (-1.16,0.23), which includes a horizontal relationship (unpredictable exchange rates as in panel (b)). However, the data clearly exclude the possibility that investors on average expect high-interest-rate currencies to appreciate (panel (a)) or that UIP might hold (panel (d)).

The bottom row in Table 1 shows the same metrics for an alternative trading strategy, the conditional carry trade, which aims at exploiting time-varying differences in interest rates across countries. The conditional carry trade goes long high-interest-rate currencies and short low-interest-rate currencies, just like the static

[10]Formally, and using the notation in their paper, the FX-share in the unconditional SDF space is $1-1/\beta^{stat}$, whereas the equivalent FX-share in conditional SDF space is $1-1/\beta^{ct}$.

trade. It differs, however, in that the conditional carry trade compares risk-free rates period by period and rebalances the portfolio accordingly. Because of this period-by-period rebalancing, conditional carry trade returns also contain conditional information, which we use to discipline the FX-share in models that allow for time variation in the variance of the log SDF below.[11]

$$\mathbb{E}[rx^{ct}] \equiv \sum_t \sum_i rx_{i,t+1}(r_{it} - \bar{r}_t),$$

where $\bar{r}_t$ denotes the average interest rate across countries at time t.

In sum, a complete-markets model that rationalizes the large and persistent differences in interest rates that we see in the data must satisfy two key properties:

Property 1. *A large difference in the variances of log SDFs*[12]

$$\mathbb{E}(var_t(m) - var_t(m^\star)) \geq 0.07$$

Property 2. *The FX-share is non-positive*

$$\frac{[-\mathbb{E}(m_{t+1})] - [-\mathbb{E}(m^\star_{t+1})]}{\frac{1}{2}\mathbb{E}(var_t(m) - var_t(m^\star))} \leq 0$$

which means the high-interest-rate currency depreciates on average.

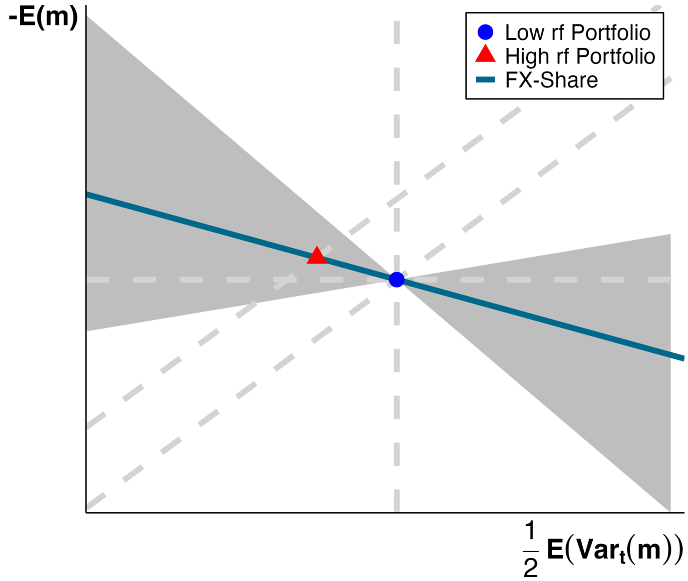
We incorporate these empirical results in the graphical representation of SDFs in Figure 3. The FX-share (slope of the blue line) is weakly negative, with the shaded region representing a 95% confidence interval.

In the following sections, we show risk premia generated in models with canonical long-run risk and habit models cannot satisfy both properties simultaneously. They either generate unpredictable exchange rates with small currency returns (Property 2 but not Property 1) or generate large currency returns through predictable changes in exchange rates (Property 1 but not Property 2); that is, risk premia from both models are incompatible with large interest rate differentials that are the primary drivers of currency returns. We show theoretically that the fundamental reason these models fail to account for the two properties is that they introduce a negative functional relationship between means and variances of log SDFs.

[11]In these models, the FX-share measures the relative position of the portfolio that we long each period and the portfolio that we short each period, as we explain below.

[12]To put things into perspective, the Hansen and Jagannathan (1991) bound implies $\text{var}_t(m) \geq 0.25$.

Figure 3: Static Trade in SDF Space



This figure plots the relative position of the low-interest-rate portfolio and the high-interest-rate portfolio implied by the static trade returns in the data, as well as the confidence intervals of the FX-share. The position of the low-interest-rate portfolio is arbitrarily chosen because only the relative positions of these dots matter for our discussion. The position of the high-interest-rate portfolio is inferred from the data using equations (2) and (4). The shaded area represents confidence intervals for the FX-share.

2 Long-Run Risk Models

In this section, we set up and analyze a canonical long-run risk model under complete markets. We derive in closed form the relationship between the first two moments of the log SDFs and argue such a relationship is at odds with data from currency markets. We then numerically solve and simulate a number of prominent versions of the long-run risk model and show the currency premium puzzle quantitatively for each of them.

A representative agent derives utility according to

$$U_t = \left((1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}} . \tag{6}$$

U_t denotes utility at time t and C_t denotes consumption. δ is the subjective time discount factor, ψ governs the elasticity of intertemporal substitution, and γ scales risk aversion. Following the long-run risk approach, we further assume consumption follows

$$\Delta c_{t+1} = \mu + z_t + \sigma \varepsilon_{t+1}$$

$$z_t = \rho z_{t-1} + \sigma_{LR} \varepsilon_{LR,t},$$

where μ is the mean growth rate of consumption, z_t is a long-run process that moves the mean of consumption growth, and ρ denotes its persistence. σ governs the volatility of short-run shocks and σ_{LR} governs the volatility of long-run shocks. ε_{t+1} and $\varepsilon_{LR,t}$ are short-run and long-run shocks, respectively. For ease of exposition, we set $\sigma = 0$ for our derivation in the main text. Appendix A.1.2 shows the full solution including short-run shocks, and Appendix A.2 generalizes our results to settings with stochastic volatility.

Under these preferences and with complete markets, the log SDF is given by

$$\begin{aligned} m_{t+1} &= \log(\delta) - \frac{1}{\psi}\Delta c_{t+1} \\ &+ \left(\frac{1}{\psi} - \gamma\right)\left(u_{t+1} - \frac{1}{1-\gamma}\log\left(\mathbb{E}_t[\exp((1-\gamma)u_{t+1})]\right)\right). \end{aligned} \tag{7}$$

Assuming u_{t+1} is normally distributed, we solve for the first and second moment of the log SDF as

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)\mathbb{E}(\mathrm{var}_t(u_{t+1})) \tag{8}$$

$$\frac{1}{2}\,\mathbb{E}(\mathrm{var}_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathbb{E}(\mathrm{var}_t(u_{t+1})). \tag{9}$$

These two equations show the first and second moments of the log SDF are tightly linked whenever utility is not time separable ($\gamma \neq \frac{1}{\psi}$). In fact, when we substitute out $\mathbb{E}(\mathrm{var}_t(u_{t+1}))$ from these conditions, we get a functional relationship between the mean and variance:[13]

$$-\,\mathbb{E}(m_{t+1}) = \frac{1}{2}\frac{1-\gamma}{\frac{1}{\psi} - \gamma}\,\mathbb{E}(\mathrm{var}_t(m_{t+1})) + \left(\frac{1}{\psi}\mu - \log(\delta)\right) \tag{10}$$

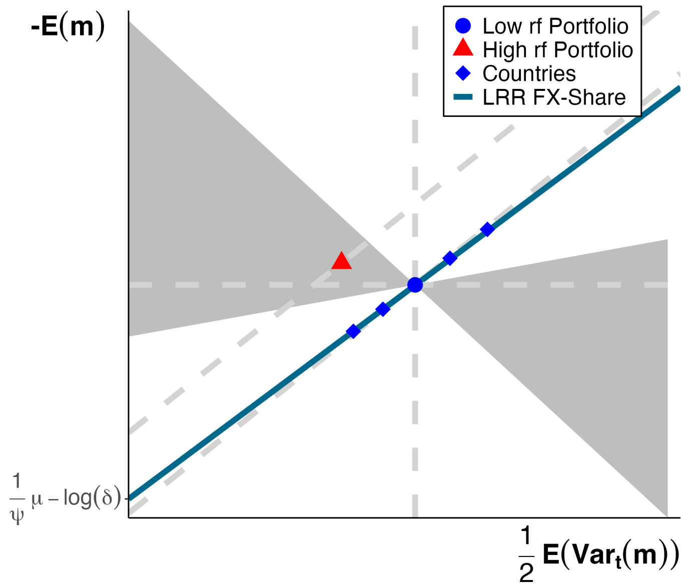
Standard calibrations assume a preference for early resolution of uncertainty, where $\gamma > 1$ and $\gamma > \frac{1}{\psi}$. In this case, The EZ utility function (8) implies a positive coefficient on the second moment of the log SDF. As a result, any shock that increases the variance lowers the mean log SDF, as shown in (10) above.

Importantly, all SDFs that satisfy the relationship in equation (10) lie on a line in SDF space. We add this line to Figure 4 as the blue line with a common calibration of $\gamma = 6.5$ and $\psi = 1.6$. Note the blue line has a slope of 0.94 and thus has a positive slope very similar to that of the iso risk-free lines (a slope of 1). For a given country, this feature of the long-run risk model helps resolve both the equity premium puzzle and the risk-free rate puzzle because it ensures a stable risk-free rate even when the variance of the SDF is large or

[13] We put $-\,\mathbb{E}(m_{t+1})$ on the left-hand side to facilitate graphical representation in SDF space.

changes over time.

Figure 4: The LRR FX-share in the SDF Space



This figure plots the long-run risk FX-share with $\gamma = 6.5$ and $\psi = 1.6$. The red triangle represents the high-interest-rate portfolio in the data. The blue dot represents the low-interest-rate portfolio in the data. The shaded area represents confidence intervals inferred from the data. Blue squares are examples of countries that differ in the variance of their log SDFs. These differences could be driven by heterogeneous loadings on a global shock, country size, trade centrality, or any other differences in the economic environment.

However, if all countries share the same preference parameters, all SDFs have to lie on the same blue line. The slope of this line represents the long-run risk model implied FX-share (LRR FX-share) across countries. Given that the LRR FX-share is close to 1, no specification of the model exists that would lead to a large difference in interest rates without generating substantial exchange rate predictability. Specifically, the model cannot generate a high-interest-rate portfolio with little or no exchange rate predictability, such as the red triangle in the figure. Put differently, the LRR FX-share is clearly rejected by the data (outside of the admissible grey cone).

To illustrate this point, in Figure 4, we plot five representative countries (blue rhomboids) that differ in the variances of their log SDFs. These differences in variances could be driven by various forms of cross-country heterogeneities in risk characteristics proposed in the literature (different loadings on a global shock, country size, trade centrality, etc.). As long as all countries share the same preference parameters, they lie on the same line and no linear combination of them would match the empirical pattern established in section 1.2 (the red triangle).[14]

[14] For example, we confirm that in the calibrated long-run risk model of Colacito et al. (2018a) (a heterogeneous country general equilibrium model where countries differ in their loading on a global endowment shock), all countries indeed lie on the same LRR line. We confirm this by solving their model using risk-adjusted-affine approximation (log-linearization with risk-adjustments. See,

The long-run risk model thus pins down the slope of the relationship between means and variances of log SDFs via equation (10), and thus the FX-share, whenever $\psi \neq 1/\gamma$.

Proposition 1. *Let γ, ψ, μ, and δ be identical across countries and assume absence of short-run risk ($\sigma = 0$). Further assume u_{t+1} is normally distributed. Then, for any two countries, the long-run risk model implies*

$$FX\text{-}share = \frac{\mathbb{E}(\Delta ex_{t+1})}{\mathbb{E}(rx_{t+1})} = \frac{1-\gamma}{\frac{1}{\psi}-\gamma}. \tag{11}$$

If agents prefer early resolution of uncertainty so that $\gamma > 1/\psi$, and we assume $\gamma > 1$, the FX-share is positive and the model cannot satisfy Property 2. In particular, if $\gamma > 2 - \frac{1}{\psi}$, FX-share $\geq \frac{1}{2}$, so that the expected change in the exchange rate $\mathbb{E}(\Delta ex_{t+1})$ accounts for more than 50% of the currency premium.

Proof. Implied by equations (2), (4), and (10). □

Equation (11) shows the LRR FX-share is determined purely by preference parameters. It is thus deeply connected with the assumption of recursive preferences: when agents prefer an early (or late) resolution of uncertainty ($\gamma \neq \frac{1}{\psi}$), the expected continuation utility enters non-linearly in (6), so that its higher-order moments (in the case of log-normal shocks, its second moment) move not just the variance but also the conditional mean of the log SDF. The FX-share, and thus the composition of currency premia, directly follows from the choice of preferences and is, in this sense, independent of the economic environment. As we show in Appendix A.2 and section 4, this logic is very general, and this relationship is robust to adding volatility shocks and relaxations of the log-normality assumption.

To show the implications of the theoretical analysis, we solve and simulate five state-of-the-art long-run risk models in the literature and summarize the results in Table 2. These models are replications of the seminal works in Colacito et al. (2018a), Colacito et al. (2018b), Bansal and Shaliastovich (2013), and Colacito and Croce (2013), as well as the original long-run risk model in Bansal and Yaron (2004).[15] The most recent of the five models (Colacito et al. (2018a)) allows countries to differ in their loadings on global productivity shocks, so that some countries (currencies) are permanently riskier than others in equilibrium. Through this channel, the model is able to generate sizable currency premia, matching our empirical Property 1. However, consistent with the currency premium puzzle, more than 90% of these currency premia arise from

e.g., Chen and Palomino (2019), Malkhozov (2014) and Lopez, Lopez-Salido, and Vazquez-Grande (2015)). We use the calibration in Table II (in particular, $\gamma = 6.5$, $\psi = 1.6$), and the heterogeneous loadings on the global shock are evenly spaced between [-0.65,0.65], as in the paper's calibration on page 18. We then numerically solve for the unconditional variances and means of each country's SDF.

[15] We thank Ric Colacito, Max Croce, Federico Gavazzoni, and Robert Ready for providing their code.

Table 2: Static Trade Returns under LRR Models

	Return (%) $\mathbb{E}(rx^{st})$	Change in FX (%) $\mathbb{E}(\Delta s^{st})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,st} - r^{st})$	FX-share $\frac{\mathbb{E}(\Delta s^{st})}{\mathbb{E}(rx^{st})}$	P1	P2
Data	3.46 [1.18,5.54]	-1.30 [-3.82,0.60]	4.76 [1.30,8.46]	-0.37 [-1.16,0.23]	-	-
Colacito, Croce, Gavazzoni and Ready (2018) JF	7.10	5.98	1.12	0.93	Yes	No
Colacito, Croce, Ho and Howard (2018) AER	0.00	0.00	0.00	0.98*	No	No
Bansal and Shaliastovich (2013) RFS	0.00	0.00	0.00	0.96*	No	No
Colacito and Croce (2013) JF	0.00	0.00	0.00	0.95*	No	No
Bansal and Yaron (2004) JF	-	-	-	0.94*	No	No

This table shows the simulation results of four long-run risk models. For Colacito et al. (2018a), we simulate the model in their section II, with calibrations in their Table II (exactly the same model as the heterogeneous EZ model in their Table III). For Colacito et al. (2018b), we simualte the model in their section II, with calibrations in their Table 2 (the same model as EZ-BKK in their Table 3). For Bansal and Shaliastovich (2013), we simulate the model in their Section 2, with calibrations in their Table 4 and section 4.4 (the same model as Table 7), we report the simulation results for the real model to be consistent with other models. For Colacito and Croce (2013), we simulate the model in their sections II and III, with calibrations in their Table II, Panel A (the same model as model (1) in their Table II, Panel B). For Colacito et al. (2018a), we follow their approach and simulate 100 economies, each with 320 periods and 100 burn-in periods (discarded). FX-share are calculated for each sample and averaged across different simulations. For all the other models, we simulate 100 economies of 10,000 periods. Static trade returns are computed using the same method in our empirical analysis. All moments are averaged across periods and simulations. Because Colacito et al. (2018b), Bansal and Shaliastovich (2013) and Colacito and Croce (2013) feature symmetric countries and currency premia are 0, we show the theoretical FX-shares for these models using Proposition 1 instead. We mark these theoretical FX-shares with asterisks. We also show the theoretical FX-share for the calibration of Bansal and Yaron (2004) for comparison (with $\gamma = 10$, $\psi = 1.5$). The last two columns summarize whether the simulated results can match our empirical Properties 1 and 2, respectively. All moments are annual.

predictable appreciation of the high-interest-rate currencies, implying an FX-share close to 1 and contradicting our empirical Property 2. All of the remaining models do not explicitly model asymmetries across countries. (In this sense, they fail to match Property 1.) However, the table shows they would suffer from the currency premium puzzle even if they did. In each of the four models, going back to Bansal and Yaron (2004), the FX-share is very close to 1 (the slope of the iso-rf line), suggesting predictable appreciation would again make up well over 90% of any currency returns the models could produce.

To be clear, the composition of currency premia was not the object of any of the papers, which are each highly successful on matching the data in a number of other dimensions. They are the state-of-the art in terms of matching both prices and quantities in open economies. However, our analysis reveals that the standard long-run risk setup struggles to match the additional empirical fact central to this paper.

Table 3: Conditional Carry Trade Returns under LRR Models

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	-	-
Colacito, Croce, Gavazzoni and Ready (2018) JF	4.47	2.76	1.71		Yes	No
Colacito, Croce, Ho and Howard (2018) AER	-0.09	-0.52	0.44	6.11	No	No
Bansal and Shaliastovich (2013) RFS	-0.03	-0.26	0.23	9.54	No	No
Colacito and Croce (2013) JF	0.05	-0.35	0.41	-7.02	No	No

This table shows the simulation results for the four long-run risk models of Table 2. Instead of the static trade, this table focuses on the conditional carry trade return. The last two columns summarize whether the simulated results can match our empirical Properties 1 and 2, respectively.

Table 3 displays a similar results for the conditional carry trade that allows investors to re-sort their portfolios every month. With the formation of portfolios based on current interest rates, the models are able to produce interest rate differentials, although they are still smaller than the ones in the data. At the same time, currency premia are very small. Countries with temporarily high expected consumption growth have a temporarily high interest rate which does not translate into the currency's return. The reason is that the realizations of long-run shocks only appear in the conditional mean of log SDFs but not the conditional variance.

$$\mathbb{E}_t(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{\psi}z_t - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\,\mathrm{var}_t(u_{t+1})$$

$$\frac{1}{2}\,\mathrm{var}_t(m_{t+1}) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathrm{var}_t(u_{t+1}).$$

As a consequence, UIP holds almost exactly in response to these shocks. We will return to this effect in

Section 6 where we show that any heterogeneity that affects the mean but not the variance of log SDFs cannot resolve the currency premium puzzle.

In the models in the bottom three rows of Table 3, currency returns are non-zero due to changes in volatility depending on the state of the economy. This stochastic volatility results in temporary differences in the variance of log SDFs across countries and thus produces small currency premia.

As a consequence of sizable expected exchange rate movements and small currency returns, the models produce very large FX-shares. In keeping with our discussion above, these small conditional currency returns are predominantly generated by predictable variation in exchange rates, generating FX shares in excess of 100%.

To summarize, in this section, we derived a closed-form functional relationship between the mean and the variance of the log SDFs under long-run risk models with (recursive) EZ preferences. We show that under standard long-run risk calibrations (Bansal and Yaron, 2004), where agents prefer early resolution of uncertainty, the model's risk premia can only generate a positive FX-share and cannot satisfy Property 2. Most of the generated currency returns are therefore accounted for by an appreciation of the high-interest-rate currency, in contrast to the data.

3 External Habit Models

Next, we show external habit models display similar difficulties in matching the data as long-run risk models. We illustrate these challenges using the pioneering work of Verdelhan (2010) which extends the the classical habit model in Campbell and Cochrane (1999) to an open economy framework and allows for a closed-form solution. We study more involved variations of the habit model numerically below.

Agents feature habit utilities of the form

$$\mathbb{E}\sum_{t=0}^{\infty}\delta^t\frac{(C_t-H_t)^{1-\gamma}-1}{1-\gamma},$$

where H_t is an externally given habit level. δ denotes the time discount factor and γ governs risk aversion as before. With the surplus consumption ratio

$$X_t\equiv\frac{C_t-H_t}{C_t},$$

the pricing kernel becomes

$$M_{t+1} = \delta \left(\frac{X_{t+1}}{X_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma}.$$

Log consumption follows a random walk with drift given by

$$\Delta c_{t+1} = \mu + \sigma \varepsilon_{t+1},$$

where μ is the mean growth rate and σ is the conditional volatility. ε_{t+1} is an i.i.d. normal shock. Moreover, Campbell and Cochrane (1999) and Verdelhan (2010) assume the following process for the log surplus consumption ratio:

$$x_{t+1} = (1-\phi)\bar{x} + \phi x_t + \lambda(x_t)(\Delta c_{t+1} - \mu),$$

where ϕ governs persistence, the sensitivity function $\lambda(x_t)$ is specified as

$$\lambda(x_t) = \begin{cases} \frac{1}{\bar{X}}\sqrt{1-2(x_t-\bar{x})} - 1 & \text{when } x < x_{max} \\ 0 & \text{elsewhere,} \end{cases}$$

and the logarithm of the upper bound x_{max} is given by

$$x_{max} = \bar{x} + \frac{1-(\bar{X})^2}{2}.$$

The steady state of the surplus consumption ratio is

$$\bar{X} = \sigma \sqrt{\frac{\gamma}{1-\phi-B/\gamma}}.$$

Note $\gamma(1-\phi) - B > 0$ by construction.[16] Using the process for the log surplus consumption ratio x_t, one can derive the log SDF as

$$m_{t+1} = \log(\delta) - \gamma(\Delta c_{t+1} + \Delta x_{t+1}).$$

[16]The specification of the $\lambda()$ function strictly follows Campbell and Cochrane (1999). As they point out, the specific functional form is designed to keep the risk-free rates stable. Parameter B nests different SDFs from the literature. $B < 0$ in Verdelhan (2010), $B = 0$ in Campbell and Cochrane (1999), and $B > 0$ in Wachter (2006).

This specification implies the first two moments of the log SDF are

$$\mathbb{E}_t(m_{t+1}) = \log(\delta) - \gamma\mu + \gamma(1-\phi)(x_t - \bar{x}) \tag{12}$$

$$\begin{aligned} \frac{1}{2}\text{var}_t(m_{t+1}) &= \frac{1}{2}\gamma^2(1+\lambda(x_t))^2\sigma^2 \\ &= \frac{1}{2}(\gamma(1-\phi) - B) - (\gamma(1-\phi) - B)(x_t - \bar{x}). \end{aligned} \tag{13}$$

Both the conditional mean and the variance of the log SDF depend on the surplus consumption ratio $x_t - \bar{x}$. This state dependence is critical to the habit model's ability to match asset pricing moments. Since unconditional moments are constant, we derive the model in terms of conditional moments and study the returns to the conditional carry trade.

Substituting the surplus consumption ratio out unveils a linear functional relationship between the first two moments, which are now conditioned on time t information:

$$-\mathbb{E}_t(m_{t+1}) = \frac{1}{2}\frac{\gamma(1-\phi)}{\gamma(1-\phi)-B}\text{var}_t(m_{t+1}) - (\log(\delta) - \gamma\mu) - \frac{1}{2}\gamma(1-\phi). \tag{14}$$

Equation (14) again implies a line in the—now conditional—SDF space. We plot this line in Figure 5 with a calibration of $\gamma = 2$, $\phi = 0.995$, and $B = -0.01$, taken directly from Verdelhan (2010). The blue line again has a positive slope, and all SDFs admissible by these preferences must line on this line. The slope is close to one, so that the blue line is again almost on top of an iso-rf line, effectively immobilizing the risk-free rate while allowing for high risk premium (high variance of log SDF).

As in the case of long-run risk models, all countries that share the same preferences must have SDFs on this blue line, so that the preference parameters chosen again fully determine the FX-share of any currency premium the model can produce.

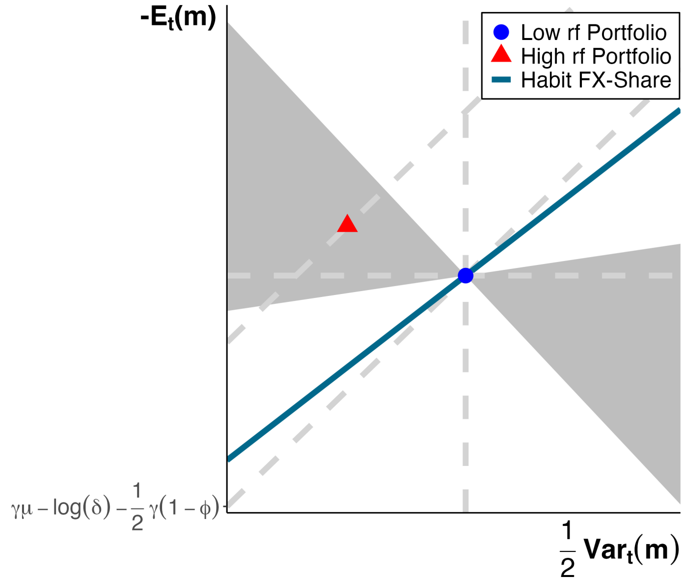
Proposition 2. *If γ, δ, ϕ, B and μ are symmetric across countries, the FX-share is given by*

$$\textit{FX-share} = \frac{\mathbb{E}_t(\Delta s_{t+1})}{\mathbb{E}_t(rx_{t+1})} = \frac{\gamma(1-\phi)}{\gamma(1-\phi)-B}.$$

Because $\gamma(1-\phi) - B > 0$ is required for stationarity, the FX-share is always strictly positive and the model cannot satisfy Property 2. Furthermore, if $\gamma(1-\phi) > -B$, then FX-share $> \frac{1}{2}$. Thus, an appreciation of the high-interest-rate currency accounts for more than 50% of the currency premium.

Proof. Implied by equations (2), (4), and (14). □

Figure 5: The Habit FX-share in the SDF Space



This figure plots the FX-share of the habit model with $\gamma = 2$, $\phi = 0.995$, and $B = -0.01$. The red triangle represents the high-interest-rate portfolio in the data. The blue dot represents the low-interest-rate portfolio in the data. The shaded area shows the confidence intervals inferred from the data.

To show that the currency premium puzzle applies to a wider set of habit models, we numerically solve and simulate several prominent and state-of-the-art models in the literature. We summarize our results in Table 4. The habit model of Verdelhan (2010) closely matches the conditional carry trade returns in the data (Property 1). Almost half of these returns, however, arise from expected appreciations of the high-interest-rate currency, contradicting Property 2. The FX-share is strongly positive at +0.48 in this model,[17] whereas it is negative in the data (-0.43).

In the continuous-time habit model of Stathopoulos (2017), the high-interest-rate currency does depreciate on average.[18] But it depreciates so much that the expected depreciation exceeds the interest rate differential, and the resulting currency premium is negative, at odds with the data and contradicting both our empirical properties. The deep habit model of Heyerdahl-Larsen (2014) features an FX-share of 0.88 and thus also fails to match Property 2.[19] We further report the FX-share for Campbell and Cochrane (1999), which is 1 exactly. In fact, the authors explicitly parameterize the model to fully immobilize the risk-free rate, so that it also allows for no variation in interest rates across countries.

As for the literature on long-run risks, it is worth noting that none of these models aimed to fit the

[17]This is consistent with the prediction of Proposition 2, which predicts an FX-share of 0.5 under the caliberation of Verdelhan (2010).

[18]We thank Andreas Stathopoulos for providing us with his code.

[19]We thank Christian Heyerdahl-Larsen for providing us with his simulated samples.

Table 4: Conditional Carry Trade Returns under Habit Models

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	-	-
Verdelhan (2010) JF	4.54	2.19	2.35	0.48	Yes	No
Stathopoulos (2017) RFS	-1.23	-2.40	1.17	1.95	No	No
Heyerdahl-Larsen (2014) RFS	3.48	3.05	0.43	0.88	Yes	No
Campbell and Cochrane (1999) JPE	-	-	-	1.00*	No	No

This table shows the simulation results of three habit models. For Verdelhan (2010), we simulate the model in section I, with calibrations in Table II. For Stathopoulos (2017), we simulate the model in section 1, with calibrations in Table 1. For Verdelhan (2010) and Heyerdahl-Larsen (2014), we simulate 100 economies of 10,000 periods. All moments are averaged across periods and simulations. Results for Heyerdahl-Larsen (2014) are obtained by using the same simulated samples that generated his Table 10. Details on conditional carry trade return construction can be found in Appendix B, and the same method is used for both the empirical and the simulation results. We also show the theoretical FX-share for the calibration of Campbell and Cochrane (1999) for comparison. Numbers with an asterisk are obtained from Proposition 2 rather than simulated. The last two columns summarize whether the simulated results can match our empirical properties 1 and 2, respectively. All moments are annual.

composition of currency premia, and instead focused on other salient features of the data. Our analysis shows that the persistent differences in interest rates combined with unpredictable exchange rates pose a tough challenge for a wide range of models in international finance.

We should note that unlike in long-run risk models, no clear separation exists between preference parameters and the economic environment (endowment process) in habit models, so that they may allow for more degrees of freedom. By construction, habit models allow changes in the endowment process to directly affect agents' risk aversion (preferences). One possible path forward might be to allow for cross-country variation in σ, which governs the volatility of the surplus consumption ratio. However, this parameter has no effect on interest rates in the standard formulation of the model. One might also consider allowing B, and thus the long-run habit level, to differ across countries to generate unconditional differences in variance of log SDFs (see equations (12) and (13)),[20] though one might argue doing so might amount again to simply modifying preferences in arbitrary ways to fit the data. More research is needed to establish how such variations could

[20]For example, Wang (2021) uses different Bs as a reduced-form way to generate large cross-country variations in risk-free rates, quantitatively linking currency premia to capital-output ratios.

be disciplined by the data.[21]

To summarize, in this section, we derived a closed-form functional relationship between the conditional mean and variance of the log SDFs under external habit models. We showed that under standard calibrations, standard models in this literature generate a positive FX-share and cannot satisfy Property 2, thereby running into the currency premium puzzle.

4 Relaxing Log-Normality

In addition to long-run risk and habit models, a large number of authors have considered rare disasters and other departures from log-normal shocks as possible explanations for the equity premium puzzle and other closed-economy asset pricing phenomena. In this section, we show that, perhaps surprisingly, the currency premium puzzle also applies in this broader class of models.

We first generalize our basic framework from section 1 to a non-normal distribution and show the tension between the equity premium puzzle, the risk-free rate, and exchange rate predictability exists regardless of log-normality. We then examine two existing prominent international asset pricing models that feature disaster risk and find both are subject to the currency premium puzzle.

For general distributions, the risk-free rate can be written as (see Backus, Foresi, and Telmer (2001))

$$
\begin{aligned}
r_t &= -\log(\mathbb{E}_t M_{t+1}) = -\mathbb{E}_t(m_{t+1}) - \left[\log(\mathbb{E}_t(M_{t+1})) - \mathbb{E}_t(m_{t+1})\right] \\
&= -\mathbb{E}_t(m_{t+1}) - \Xi_t(m_{t+1}),
\end{aligned}
$$

where $\Xi_t(m_{t+1}) = \log(\mathbb{E}_t(M_{t+1})) - \mathbb{E}_t(m_{t+1})$ denotes the entropy of the SDF.

We investigate whether the composition of currency premia also puts restrictions on models with higher moments in the distribution. In this generalized case, the expected change in exchange rates and currency premia are now given by

$$
\begin{aligned}
\mathbb{E}_t(\Delta s_{t+1}) &= [-\mathbb{E}_t(m_{t+1})] - [-\mathbb{E}_t(m^\star_{t+1})] \\
\mathbb{E}_t(rx_{t+1}) &= \Xi_t(m_{t+1}) - \Xi_t(m^\star_{t+1}).
\end{aligned}
$$

The expressions are identical to the ones in section 1 except that the entropy $\Xi_t(m_{t+1})$ takes the place of what

[21] In fact, studies have tied B to slopes of term structure (Wachter (2006), Verdelhan (2010)), but to our knowledge, no study has tied B to a specific economic source.

used to be the variance of the log SDF, $\frac{1}{2}\operatorname{var}_t(m_{t+1})$. The relationship between risk-free rates, exchange rates, currency premia, and the mean, and now the entropy of the SDFs is, however, preserved. As a result, the discussion in section 1 still applies. The log-normal model emerges as a special case, and the tension between a high equity premium, a low and stable risk-free rate, and the composition of currency premium extends to the generalized setup.

As an illustration, we first consider the framework of Gourio, Siemer, and Verdelhan (2013), which focuses on disaster risk and thus higher moments of the distribution of the SDF. Agents feature EZ preferences[22] as in (6), and their log SDF is given by (7). The first moment and the entropy are then given by[23]

$$\begin{aligned}
\mathbb{E}_t(m_{t+1}) &= \log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\Delta c_{t+1}) + \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\mathbb{E}_t\left((1-\gamma)u_{t+1}\right) - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right) \\
\Xi_t(m_{t+1}) &= \log\mathbb{E}_t(M_{t+1}) - \mathbb{E}_t(m_{t+1}) \\
&= \Xi_t(-\frac{1}{\psi}\Delta c_{t+1}) + \log\left(\operatorname{cov}_t\left[\frac{C_{t+1}^{-\frac{1}{\psi}}}{C_t^{-\frac{1}{\psi}}}, U^{\frac{1}{\psi}-\gamma} \Big/ \mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\right]\right) - \mathbb{E}_t\left(\left(\frac{1}{\psi}-\gamma\right)u_{t+1}\right) + \log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right)
\end{aligned}$$

To simplify the exposition, we assume the entropy of next period's consumption growth $\Xi_t(-\frac{1}{\psi}\Delta c_{t+1}) \approx 0$; that is, all higher moments of consumption growth are approximately equal to 0.[24] We relax this assumption in our quantitative exercise and show simulation results are consistent with our theoretical predictions. Note the covariance term in the second equation above also equals zero under this assumption, and the unconditional mean and the entropy of the SDF are given by

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mathbb{E}(\Delta c_{t+1}) + \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\mathbb{E}\left[((1-\gamma)u_{t+1}) - \log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right)\right] \tag{15}$$

$$\mathbb{E}(\Xi_t(m_{t+1})) = -\mathbb{E}\left[\left(\frac{1}{\psi}-\gamma\right)u_{t+1} - \log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right)\right]. \tag{16}$$

Mean and entropy are again tightly linked since both depend on continuation utility (the terms in square brackets), implied by EZ preferences.[25]

[22]The model in Gourio, Siemer, and Verdelhan (2013) features labor supply. Thus, leisure shows up in the utility function. We abstract away from this setup for simplicity. We also suppress the production side of the economy because all our results do not depend on the specific setup of the economic environment. We confirm this with numerical exercises.

[23]For a detailed derivation, see Appendix B.

[24]Entropy is a function of all the higher moments. Thus, this is similar to the "no-short-run-shock" assumption in the log-normal case of section 2. Empirically, aggregate consumption growth is quite smooth.

[25]Although the defining feature of disaster models is the existence of disaster risk, EZ preferences are commonly used. See Barro (2009), Gourio (2012), for example. The reason is that standard CRRA preferences, in which the intertemporal elasticity of substitution equals the reciprocal of risk aversion, gives rise to the equity premium puzzle and other counterfactual predictions for

We can make further headway in two special cases. We first consider the special case where the elasticity of intertemporal substitution ψ is equal to 1. In this case, we obtain a straightforward functional link between the mean of the log SDF and its entropy:

$$-\mathbb{E}(m_{t+1}) = \mathbb{E}(\Xi_t(m_{t+1})) - \log(\delta) + \mathbb{E}(\Delta c_{t+1}).$$

This linear functional relationship shows an FX-share of 1 in a generalized SDF space, where the x-axis is now entropy instead of variance of SDF. That is, countries with identical preference parameters and $\psi = 1$ must always have the same interest rate.[26]

Looking beyond the case of a unit elasticity of intertemporal substitution, we can make further progress by limiting countries to differ in only one higher-order moment of the distribution of their shocks. For concreteness, we may capture the key ingredient in the disaster risk literature by allowing for country-specific skewness in the distribution of consumption. To highlight the role of higher moments, we exploit the recursiveness of preferences to recover the relationship between the first moment and the entropy of the SDF. From equations (15) and (16), using cumulant generating functions (Backus, Foresi, and Telmer (2001)), we obtain

$$\mathbb{E}(m_{t+1}) = \left(\log(\delta) - \frac{1}{\psi}\mathbb{E}(\Delta c_{t+1})\right) - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)\mathbb{E}(\kappa_{2,t}(u_{t+1})) - \frac{1}{6}(1-\gamma)^2\left(\frac{1}{\psi} - \gamma\right)\mathbb{E}(\kappa_{3,t}(u_{t+1})) + \dots,$$

where $\kappa_{i,t}(u_{t+1})$ is the ith cumulant of u_{t+1}.[27] A similar expansion applies to the entropy

$$\mathbb{E}(\Xi_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2\mathbb{E}(\kappa_{2,t}(u_{t+1})) + \frac{1}{6}\left(\frac{1}{\psi} - \gamma\right)^3\mathbb{E}(\kappa_{3,t}(u_{t+1})) + \dots$$

With skewness ($\kappa_{3,t}(u_{t+1})$) differing across countries and setting all other cumulants to be identical, we get

$$-\mathbb{E}(m_{t+1}) = \left(\frac{1-\gamma}{\frac{1}{\psi} - \gamma}\right)^2\mathbb{E}(\Xi_t(m_{t+1})) + \text{constant}. \tag{17}$$

asset prices. For example, when risk aversion is larger than 1 under CRRA, an increase in uncertainty raises the price-dividend ratio, and an increase in growth rate lowers it, and risk premia are procyclical, all of which contradicts the data. Also see Bansal and Yaron (2004) for the benefits of separating risk aversion and IES.

[26] One path forward might be to combine heterogeneity in disaster risk to an off-setting source of variation in the conditional mean of the log SDF. We discuss this possibility below.

[27] Cumulants are functions of the usual central moments.

We again see a tight relationship between the entropy and the first moment. In particular, (17) implies

$$\text{FX-share} = \left(\frac{1-\gamma}{\frac{1}{\psi} - \gamma} \right)^2 ,$$

which is always positive and close to 11 under standard calibrations ($\gamma > \frac{1}{\psi}$, $\gamma > 1$ and $\psi > 1$). For example, with $\gamma = 8.5$ and $\psi = 2$, the FX-share is 0.88.

Both special cases thus strongly suggest departures from log-normality do not fundamentally change the challenge of addressing the currency premium puzzle, as long as (symmetric) EZ preferences are involved.

We next show quantitatively that this intuition holds for influential models in the existing literature. To this end, we consider a highly successful contribution to the international finance literature, as well as a special case of the closed-economy model by Gourio (2012), and compare them to the data.[28] Closely related to our theoretical results, Gourio, Siemer, and Verdelhan (2013) features a model with both EZ utility and heterogeneous exposures to global disaster risk.

Table 5: Conditional Carry Trade Returns under Disaster Models

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	P1 $-\frac{1}{\text{FX-share}}$	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	-	-
Gourio, Siemer and Verdelhan (2013) JIE	2.36	1.81	0.55	0.77	Yes	No
Gourio (2012) AER	-	-	-	1.00*	No	No

The return, FX-share, and FX-share for Gourio, Siemer, and Verdelhan (2013) are calculated from their Tables 2 and 4. We also show the theoretical FX-share for a special case of the closed-economy model of Gourio (2012) for comparison, assuming an IES of 1 ($\psi = 1$). We mark this theoretical prediction with an asterisk. Alternatively, if we use Gourio's preferred calibration ($\psi = 2, \gamma = 3.8$) and use the approximation in equation (17), we end up with FX-share = 0.72. The last two columns summarize if the simulated results can match our empirical properties 1 and 2, respectively. All moments are annual.

As Table 5 shows, the currency premium puzzle is also evident in these disaster models. The conclusions are the same as for long-run risk and habit models: with FX-shares above 50%, the models do not match the composition of currency premia. [29]

[28] Barro (2009) is another closed-economy disaster model with EZ preferences. Given that our theoretical results only rely on the use of these preferences, his setup is also likely to be subject to the currency premium puzzle. Ready, Roussanov, and Ward (2017a) constructs a two-country disaster model with commodity and producer countries without using EZ preferences. They generate a conditional carry trade return of 4.15% with an interest rate difference of 3.41 (FX-share = 0.2). However, their framework inherits the usual implications of CRRA preferences: risk-free rates are too high, and other problems mentioned in footnote 25.

[29] Perhaps more surprisingly, virtually identical results hold for Farhi and Gabaix (2016), who build a disaster model with constant

To summarize, the inability of standard models to simultaneously generate a high equity premium, a low and stable risk-free rate, and largely unpredictable exchange rates extends beyond the case of log-normality. In particular, existing disaster models are also subject to the currency premium puzzle.[30] That said, allowing multiple higher cumulants or moments to differ across countries at the same time may offer more degrees of freedom. How such a model should be constructed and what would drive these differences in cumulants is not obvious and requires further research.

5 Incomplete Spanning

Can market incompleteness help resolve the currency premium puzzle? In this section, we relax the assumption of complete markets and explore incomplete spanning as a potential avenue for matching the empirical patterns in interest rates and exchange rates. We find that this form of market incompleteness does not offer a clear-cut resolution of the currency premium puzzle.

Specifically, we consider a scenario where agents do not have full access to foreign financial markets: agents can buy and sell their own country's (domestic) risk-free assets, but cannot trade financial assets across borders. In this case, the expected change in the exchange rate is no longer fully determined by the ratio of expected SDFs across countries. In other words, (2) no longer holds. Following Lustig and Verdelhan (2019), we summarize incomplete spanning in international financial markets by a wedge, η_{t+1}, between changes in exchange rates and log SDFs so that [31]

$$\mathbb{E}(\Delta s_{t+1}) = \mathbb{E}(m_{t+1}) - \mathbb{E}(m^*_{t+1}) - \mathbb{E}(\eta_{t+1}). \tag{18}$$

(As before, all unconditional equations in this section also hold conditionally.)

$\mathbb{E}(m^\star_{t+1}) + \mathbb{E}(\eta_{t+1})$ is then the expectation of a foreign log SDF adjusted for the incomplete-market wedge.

relative risk aversion and tradeable and nontradeable goods. Their conditional carry trade simulation results show a FX-share of 0.69 (calculated from Table III in their paper). Although the time-separable preferences in this model do not force a negative functional relationship between the mean and higher-order moments of the log SDF, this relationship appears to assert itself nevertheless when calibrating the model to reflect low and stable risk-free rates given that countries are symmetric in terms of disaster resilience. We discuss the case when countries differ permanently in their disaster resilience in Section 6.

[30] Relatedly, Jurek (2014) and Farhi et al. (2009) find that even when disaster risk is hedged using options, conditional carry trade returns remain large, suggesting disasters are not the sole explanation for systematic variation in currency returns.

[31] Lustig and Verdelhan (2019) assume agents can trade all the risk-free assets across borders. For what follows, we only need the weaker assumption that each agent has access to her own domestic risk-free asset.

Plugging (1) and (18) into equation (4) yields the currency premium

$$\mathbb{E}(rx_{t+1}) = \frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1})) - \left(\frac{1}{2}\text{var}_t(m^*_{t+1})) - \mathbb{E}(\eta_{t+1})\right). \tag{19}$$

Equations (18) and (19) show the expected incomplete market wedge, $\mathbb{E}(\eta_{t+1})$, provides an extra degree of freedom. The question is whether this degree of freedom can help resolve the currency premium puzzle.

The answer is it likely cannot. To see why, recall that as long as agents can freely buy and sell their own country's risk-free asset, equation (1) holds, so that the wedge $\mathbb{E}(\eta_{t+1})$ affects currency returns and predicted depreciations in (18) and (19), but not the size of the interest rate differential. Even when spanning is incomplete, the interest differential thus depends only on the mean and variance of each country's log SDF, as before:

$$\begin{aligned}\mathbb{E}(r^\star_t - r_t) &= \mathbb{E}(m_{t+1}) - \mathbb{E}(m^\star_{t+1}) \\ &\quad - \frac{1}{2}\,\mathbb{E}(\text{var}_t(m^\star_{t+1}) - \text{var}_t(m_{t+1})).\end{aligned}$$

In this sense, incomplete spanning cannot address the core problem that all the canonical models in the sections above force FX-shares close to 1, and thus interest rate differentials that are far too small relative to the data.

Another way of seeing the same point is that modulating $\mathbb{E}(\eta_{t+1})$ for a given country pair allows movement away from the now familiar blue lines in SDF space, but only along the 45-degree line.[32] Figure 6, panel (a) shows this effect graphically, using as an example the same LRR model and parameters as in Figure 4 with $\gamma = 6.5$ and $\psi = 1.6$. As in our prior example, each dot on the blue line represents a country—indicating the mean and variance of its log SDF. The blue dots depict the situation under complete markets, where differences in countries' risk characteristics determine where on the blue line each country lies.

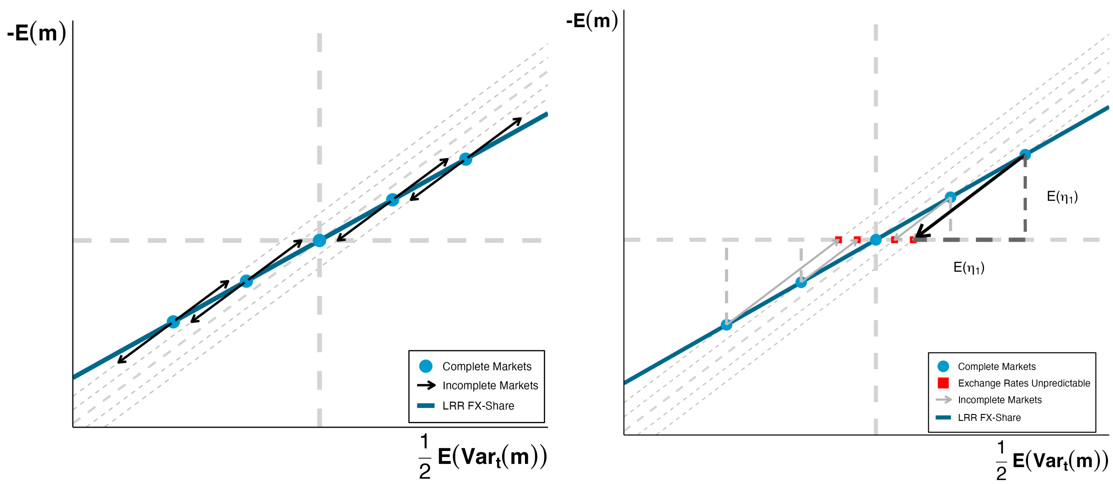
To the extent that the FX-share in the long run risk model is different from 1, that is, if $\psi \neq 1$, the wedge induced by incomplete spanning now allows movement off the blue line, but again only along a given iso-rf line. By altering a country i's expected wedge $\mathbb{E}(\eta^i_{t+1})$, we can then increase or decrease its currency's expected rate of depreciation, and the part of the currency return determined by it, but not the interest differential. Panel (a) of Figure 6 illustrates these possible movements along the 45 degree line intersecting at each point

[32] We are slightly abusing the notation in the sense that we fix the home country point $(\frac{1}{2}\,\mathbb{E}(\text{var}_t(m), -\mathbb{E}(m))$, and treat the foreign point as $(\frac{1}{2}\,\mathbb{E}(\text{var}_t(m^\star) - \mathbb{E}(\eta), \mathbb{E}(m^\star) + \mathbb{E}(\eta))$, as implied by (18) and (19).

along the blue line.

In the best possible case, one might come up with a mechanism that generates an off-setting wedge for each country that is exactly large enough to render its exchange rate unpredictable, that is, $\mathbb{E}(\Delta s_{t+1}) = 0$. We plot this special case as red squares in Figure 6, panel (b). However, even if one had such a mechanism, as long as the FX-share under complete markets is close one (as in the canonical long-run risk and habit models), the same wedge would then again wipe out the vast majority of the expected currency return, leaving us with the same (small) interest rate differentials we obtained in the complete-markets model—the horizontal distances between the red squares.

Figure 6: Incomplete Markets vs. Complete Markets



(a) Effect of Incomplete-market Wedge (b) Achieving Unpredictable Exchange Rates

This figure plots a long-run risk FX-share (blue line) with five countries. The five countries must lie on the same LRR line under complete markets (blue dots). Panel (a) shows how the incomplete market wedge could move each country around in the SDF space; panel (b) shows what the incomplete market wedge of each country needs to do to achieve unpredictable exchange rates (red squares).

Two observations from this analysis stand out. First, as shown in Figure 6, panel (b), the incomplete market wedges $\mathbb{E}(\eta^i)$ have to differ significantly across countries and would have to line up in an off-setting pattern to achieve exchange rate unpredictability across country pairs. What type of mechanism would bring about such a constellation of wedges so as to exactly render each country's exchange rate unpredictable is unclear.

Second, even if one were to achieve such an off-setting pattern, the expected wedge lowers exchange rate predictability and currency premia by the same amount (as implied by equations (18) and (19)). Consequently, incomplete spanning, in combination with wedges that reduce exchange rate predictability, make it more difficult for any model to satisfy empirical Property 1, which states that currency premia need to be large.

Table 6: Implied Wedges in Canonical LRR Model

Country	Return (%)		Change in FX (%)		Interest Rate Diff (%)	Implied Wedge
	Complete	Incomplete	Complete	Incomplete		
1	2.91	0.16	-2.75	0.00	0.16	-2.75
2	1.52	0.08	-1.44	0.00	0.08	-1.44
3	0.00	0.00	0.00	0.00	0.00	0.00
4	-1.64	-0.07	1.57	0.00	-0.07	1.57
5	-3.41	-0.12	3.28	0.00	-0.12	3.28

This table shows currency premia, expected depreciation of the high-interest-rate currency, and interest rate differences for the five representative countries under both complete and incomplete markets. The currency of the middle, country 3, is used as the base currency. The implied incomplete-market wedge is set to equal differences in the means of log SDFs to match the fact that exchange rates are unpredictable.

To illustrate these two observations quantitatively, Table 6 shows currency premia and exchange rate appreciations for five representative countries under both complete and incomplete markets, using country 3 (the middle country) as the base currency. The currency premium puzzle is again evident across all country pairs, with counterfactually small interest rate differences and large currency premia driven mostly by expected appreciations. As in Figure 6, the incomplete market wedge can be utilized to achieve unpredictable exchange rates as in the data. In particular, by setting $\mathbb{E}(\Delta s_{t+1})$ in (18) to 0, we can back out the required wedges to eliminate predictability from exchange rates for each of the country pairs. We report these implied wedges in the last column. Setting the incomplete wedge to these values improves the model's performance in matching the composition of currency premia (Property 2), because all currency premia arise entirely from interest rate differences. The implied wedges strongly differ across country pairs.

As an example, country 1 (perhaps New Zealand) produces a currency premia of 2.91% relative to country 3 under complete markets. If we set the incomplete market wedge to -2.75% so that exchange rates are unpredictable, we end up with a currency premium of merely 0.16%.[33] In other words, whereas variation in the wedge across countries can ensure Property 2 holds, Property 1, which says currency premia are large, will fail in those cases.[34]

This argument applies more broadly. Recall that in all models we have considered, changes in exchange rates account for more than 50%—in many cases, for more than 90%—of currency premia. Fixing the

[33]Similarly, consider our simulation results in Table 2: the simulated difference in variances of log SDFs is 7.10%, whereas the difference in means is -5.98%. If we believe exchange rates are unpredictable as the data suggest and set $\mathbb{E}(\eta_{t+1}) = 5.98\%$, we can indeed generate a $\mathbb{E}(\Delta s_{t+1}) = 0$ and $\mathbb{E}(rx_{t+1}) = 1.12\%$. Now, all currency premia are accounted for by differences in risk-free rates: incomplete markets helps the long-run risk model of Colacito et al. (2018a) to match Property 2, that is, to get the right composition of currency risk premia. But it does so by shrinking the currency premia by more than 90%, significantly weakening the model's ability to match Property 1.

[34]In principle, one could design a long-run risk (or habit) model with even larger differences in the variance of SDFs and the right incomplete market wedge to match the data, but how such extreme heterogeneity can be justified, and how such strong market frictions can prevent exchange rates to move in such models, remain unclear.

composition of currency premia in these models using the incomplete spanning wedge implies a significant decrease in currency premia, weakening their ability to match the empirically large currency premia. This fundamental issue is again embedded in preferences chosen to keep risk-free rate stable by imposing a tight functional relationship between means and variances of log SDFs.

6 Off-setting Differences in $\mathbb{E}(m)$

Finally, we explore connecting risk-based models, which have been the focus of this article and the recent literature, with the traditional macroeconomic view of cyclical interest rate differentials.

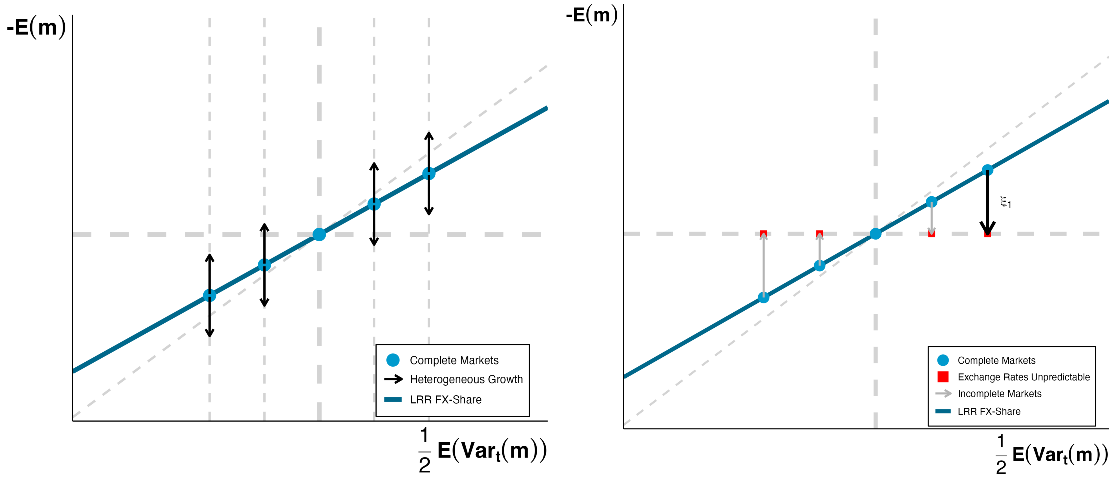
The premise of risk-based models is that countries differ in their risk characteristics (var(m) in equation (4)) so that some countries' currencies (such as the US dollar and the Euro) are safer than others and pay lower returns to foreign investors. This risk-based view of currency returns has become the dominant paradigm in the literature based on two key empirical observations: First, interest rate differentials between countries are large and long-lasting, so that uncovered interest parity fails and investors can make money by borrowing in the low interest rate country and lending in the high interest rate country (Fama, 1984; Hassan and Mano, 2019). Second, there is clear evidence of risk factors in currency markets, where currencies with low interest rates tend to appreciate at times of stress in the global economy (Lustig and Verdelhan, 2007; Campbell, Serfaty-De Medeiros, and Viceira, 2010; Lustig, Roussanov, and Verdelhan, 2011; Lettau, Maggiori, and Weber, 2014).

As we have shown above, however, under canonical long-run risk and habit preferences, because of the functional relationship between $\mathbb{E}(m)$ and var(m) at the core of the theory, heterogeneity in risk characteristics forces risk premia to manifest as predictable appreciations (equation (2)), which contradicts the data.

One possible avenue to addressing the currency premium puzzle might thus be adding an additional source of heterogeneity that moves the means of the log SDFs to off-set these predictable exchange rate movements.

For example, permanent differences in consumption growth rates across countries would induce permanent differences in interest rates that are reversed by predictable depreciations, moving countries vertically off the iso-rf line, without affecting return differentials. Heterogeneity in long-run growth rates individually

Figure 7: Heterogeneous Growth vs Homogeneous Growth



(a) Effect of Heterogeneous Growth (b) Achieving Unpredictable Exchange Rates

This figure plots the FX-share from a long-run risk model (blue line) with five countries. The five countries must lie on the same blue line under complete markets (blue dots). Panel (a) shows how an additional source of heterogeneity in $\mathbb{E}(m)$ can move each country horizontally in the SDF space; panel (b) shows what the specific level of $\mathbb{E}(m)$ of each country needs to do to achieve unpredictable exchange rates (red squares).

shifts each country's intercept of the relation between mean and variance

$$-\mathbb{E}(m_{i,t+1}) = \frac{1}{2}\frac{1-\gamma}{\frac{1}{\psi}-\gamma}\mathbb{E}(\mathrm{var}_t(m_{i,t+1})) - \left(\log(\delta) - \frac{1}{\psi}(\mu+\xi_i)\right), \tag{20}$$

where ξ_i can be interpreted as a country-specific shift to the long-term growth rate.

To see that such long-run growth differentials affect interest rates but not returns, note that differences in $\mathbb{E}(m)$ affect interest rates in equation (3) but not expected returns in equation (4). Panel (a) of Figure 7 shows this contrast graphically. Differences in $\mathbb{E}(m)$ across countries move them along the vertical arrows in the figure. While these movements induce variation in interest rates, this additional variation is fully reversed by predictable depreciations of the exchange rate, so that there is no movement in the horizontal direction.

This lack of an effect of (permanent or transitory) growth differentials on returns is precisely why uncovered interest parity holds in standard macro models that abstract from differences in currency risk (Backus, Kehoe, and Kydland, 1992; Galí and Monacelli, 2005). In this traditional macroeconomic view, countries with temporarily high expected consumption growth have a temporarily high interest rate, but this increase in the interest rate has no effect on the currency's return.

Thus, heterogeneity in risk characteristics can rationalize differences in expected returns, whereas heterogeneity in $\mathbb{E}(m)$ cannot.

However, a promising avenue might be to combine the two forces, for example by pairing heterogeneity in risk characteristics with off-setting differences in long-term growth rates: Heterogeneity in risk characteristics produce differences in expected returns that transmit themselves through expected appreciations of high-interest-rate currencies (as required by the canonical long-run risk and habit models). But if countries with risky currencies also grow at faster rates, then these expected appreciations may be transformed into large interest rate differentials by off-setting differences in the means of countries log SDFs. Countries with high $\text{var}(m)$ would then also have persistently high $\mathbb{E}(m)$'s, so that the expected appreciation of the riskier country is offset by an expected depreciation due to its high $\mathbb{E}(m)$. Panel (b) of Figure 7 shows an extreme case, where the two forces exactly offset each other, producing persistent differences in interest rates and fully unpredictable exchange rates.

Of course, one would have to propose some theory of why and how heterogeneity in the two moments of the log SDF might be linked. One challenge that arises is stationarity because of long-run differences in the expected growth rate of consumption.

Two papers in the literature suggest avenues for doing so. Andrews et al. (2024) propose a particularly fruitful partial equilibrium long-run risk model where risky countries exogenously have higher consumption growth rates for long periods of time. They start from a long-run risk model very similar to the one we discuss in section 2, where currencies vary in their expected returns, because some countries are more exposed to global risk than others. Following our discussion above, their model would ordinarily would produce an FX-share close to one. However, they point out that, in the data, countries with higher risk exposure also exhibit lower consumption growth rates over long periods of time. Effectively, they show that the link between differences in growth rates and differences in expected returns is well approximated by

$$\xi^i = -0.206 \left(\frac{1}{2}\, \mathbb{E}(\text{var}_t(m^i_{t+1})) - \frac{1}{2}\, \mathbb{E}(\text{var}_t(m_{t+1})) \right),$$

where ξ is again the difference between country i's long-run consumption growth rate and the world average, and $\mathbb{E}(\text{var}_t(m_{t+1}))$ refers to the world average variance of the log SDF.[35] Although their focus is on other features of the data, this link between long-lasting heterogeneity in the first and second moments of log SDFs significantly reduces the FX-share of currency returns: The slope of this relation translates into the FX-share by adding one, such that the slope of -0.21 in their setup results in an FX-share of 0.79 for the static trade. For

[35]See Appendix A.3 for a detailed derivation.

the conditional carry trade, they get FX-Shares as low as 0.47 in some variations of their model. These are still numbers far above the FX-Share of −0.43 we measure in the data, but nevertheless a significant advancement.

Another approach might be to make differences in the expected log SDF unobservable in-sample, for example by postulating differences in countries' resilience to disasters. This possibility is hinted at in Farhi and Gabaix (2016). More work is needed to determine the potential of this approach.

7 Conclusion

In this paper, we highlight a fundamental tension between canonical asset pricing models, which have been notably successful in explaining closed-economy puzzles, and the empirical observations in open economies. This tension, which we term the 'currency premium puzzle', manifests in the inability of these models to reconcile large and persistent interest rate differentials with the unpredictability of exchange rates.

Standard models generate currency premia that predominantly manifest as predictable exchange rate movements. This theoretical prediction sharply contrasts with empirical observations, where exchange rates are notoriously difficult to predict and interest rate differentials are the primary source of currency returns.

This failure is inherent to the preference structures in these models: the same preference structures that reconcile large equity premia with low and stable risk-free rates in the context of a closed economy also require that the vast majority of any differences in currency premia across countries must transmit themselves through predicted depreciations of the exchange rate—as long as markets are complete.

More research is needed to assess to what extent incomplete markets could contribute to resolving the currency premium puzzle. Our own exploration has focused on one specific type of market incompleteness—incomplete spanning. Incomplete spanning tends to simultaneously diminish the predictability of exchange rates and the magnitude of currency returns, thus falling short in explaining the significant interest rate differentials observed in the data. This finding underscores that moving away from the assumption of complete markets is far from being a straightforward solution.

The implications of our findings are two-fold. First, they underscore a significant limitation of the current generation of asset pricing models when applied to open economies. Second, the currency premium puzzle highlights the necessity for new models that can simultaneously account for large interest rate differentials and unpredictable exchange rates. This need is not merely academic; it is crucial for building models that can assess the effect of global and local risk premia on capital flows and the allocation of capital across countries.

By pinpointing a crucial inconsistency between canonical asset pricing and international macroeconomic models, we hope to spur more work on the broader challenge of integrating these two areas. In our view, this integration is essential for developing a more comprehensive understanding of critical phenomena, including the violation of uncovered interest parity, contagion, the global financial cycle, flights to safety, capital retrenchments, and sudden stops. All these phenomena ultimately result from the interplay of international financial markets, risk premia, and allocations. They are critical to understand. The currency premium puzzle, as we have defined it, calls for innovative approaches to address this challenge.

References

Alvarez, Fernando, Andrew Atkeson, and Patrick J. Kehoe. 2007. "If Exchange Rates are Random Walks, Then Almost Everything We Say About Monetary Policy is Wrong." *American Economic Review* 97 (2):339–345. URL https://www.aeaweb.org/articles?id=10.1257/aer.97.2.339.

Andrews, Spencer, Riccardo Colacito, Mariano M. Croce, and Federico Gavazzoni. 2024. "Concealed carry." *Journal of Financial Economics* 159:103874. URL https://linkinghub.elsevier.com/retrieve/pii/S0304405X24000977.

Backus, D, S Foresi, and C Telmer. 2001. "Affine Term Structure Models and the Forward Primium Anomaly." *Journal of Finance* 56 (1):279–305.

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland. 1992. "International Real Business Cycles." *Journal of Political Economy* 100 (4):745–775.

Backus, David K and Gregor W Smith. 1993. "Consumption and real exchange rates in dynamic economies with non-traded goods." *Journal of International Economics* 35:297–316.

Bai, Yan, Patrick J. Kehoe, Pierlauro Lopez, and Fabrizio Perri. 2023. "World Financial Cycles." *Unpublished* .

Bansal, Ravi and Ivan Shaliastovich. 2013. "A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets." *Review of Financial Studies* 26 (1):1–33. URL https://academic.oup.com/rfs/article-lookup/doi/10.1093/rfs/hhs108.

Bansal, Ravi and Amir Yaron. 2004. "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles." *The Journal of Finance* 59 (4):1481–1509. URL https://onlinelibrary.wiley.com/doi/10.1111/j.1540-6261.2004.00670.x.

Barro, Robert J. 2009. "Rare Disasters, Asset Prices, and Welfare Costs." *American Economic Review* 99 (1):243–64. URL https://www.aeaweb.org/articles?id=10.1257/aer.99.1.243.

Becker, Gary S. 1976. *The Economic Approach to Human Behavior*. Chicago: University of Chicago Press.

Boehm, Christoph E and T Niklas Kroner. 2023. "The US, economic news, and the global financial cycle." Tech. rep., National Bureau of Economic Research.

Brandt, Michael W., John H. Cochrane, and Pedro Santa-Clara. 2006. "International risk sharing is better than you think, or exchange rates are too smooth." *Journal of Monetary Economics* 53 (4):671–698. URL http://linkinghub.elsevier.com/retrieve/pii/S0304393206000432.

Campbell, John Y. and John H. Cochrane. 1999. "By Force of Habit: A Consumption-Based Explanation of Plantation of Aggregate Stock Market Behavior." *Journal of Political Economy* 107 (205-250):205–251.

Campbell, John Y., Karine Serfaty-De Medeiros, and Luis M. Viceira. 2010. "Global currency hedging." *Journal of Finance* 65 (1):87–121.

Chari, Anusha, Karlye Dilts Stedman, and Christian Lundblad. 2020. "Capital Flows in Risky Times: Risk-on/Risk-off and Emerging Market Tail Risk." Working Paper 27927, National Bureau of Economic Research. URL http://www.nber.org/papers/w27927.

Chen, Andrew Y. and Francisco Palomino. 2019. "An Irrelevance Theorem for Risk Aversion and Time-Varying Risk." *SSRN Electronic Journal* .

Chernov, Mikhail, Valentin Haddad, and Oleg Itskhoki. 2023. "What do financial markets say about the exchange rate?" URL https://itskhoki.com/papers/FMER.pdf,Accessed2023-11-18.

Colacito, Ric, Mariano M. Croce, Federico Gavazzoni, and Robert Ready. 2018a. "Currency Risk Factors in a Recursive Multicountry Economy." *The Journal of Finance* 73 (6):2719–2756. URL http://doi.wiley.com/10.1111/jofi.12720.

Colacito, Ric, Max Croce, Steven Ho, and Philip Howard. 2018b. "BKK the EZ Way: International long-run growth news and capital flows." *American Economic Review* 108 (11):3416–3449.

Colacito, Riccardo and Mariano M Croce. 2011. "Risks for the Long Run and the Real Exchange Rate." *Journal of Political Economy* 119 (1):153–181.

Colacito, Riccardo and Mariano M. Croce. 2013. "International Asset Pricing with Recursive Preferences." *Journal of Finance* 68 (6):2651–2686.

di Giovanni, Julian, Şebnem Kalemli-Özcan, Mehmet Fatih Ulu, and Yusuf Soner Baskaya. 2022. "International Spillovers and Local Credit Cycles." *The Review of Economic Studies* 89 (2):733–773. URL https://doi.org/10.1093/restud/rdab044.

Epstein, Larry G. and Stanley E. Zin. 1989. "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." *Econometrica* 57 (4):937.

Fama, Eugene F. 1984. "Forward and spot exchange rates." *Journal of Monetary Economics* 14 (3):319–338. URL http://linkinghub.elsevier.com/retrieve/pii/0304393284900461.

Fang, Xiang. 2021. "Intermediary Leverage and Currency Risk Premium." *SSRN Electronic Journal* URL https://papers.ssrn.com/abstract=3290317.

Farhi, Emmanuel, Samuel Paul Fraiberger, Xavier Gabaix, Romain Ranciere, and Adrien Verdelhan. 2009. "Crash Risk in Currency Markets." Working Paper 15062, National Bureau of Economic Research. URL http://www.nber.org/papers/w15062.

Farhi, Emmanuel and Xavier Gabaix. 2016. "Rare disasters and exchange rates." *Quarterly Journal of Economics* 131 (1):1–52. URL https://academic.oup.com/qje/article-lookup/doi/10.1093/qje/qjv040.

Forbes, Kristin J. 2013. "The "Big C": Identifying and Mitigating Contagion." In *The Changing Policy Landscape. 2012 Jackson Hole Symposium*. Federal Reserve Bank of Kansas City, 23–87.

Forbes, Kristin J. and Francis E. Warnock. 2021. "Capital flow waves—or ripples? Extreme capital flow movements since the crisis." *Journal of International Money and Finance* 116:102394. URL https://doi.org/10.1016/j.jimonfin.2021.102394.

Gabaix, Xavier and Matteo Maggiori. 2015. "International Liquidity and Exchange Rate Dynamics." *The Quarterly Journal of Economics* 130 (3):1369–1420. URL https://academic.oup.com/qje/article-lookup/doi/10.1093/qje/qjv016.

Galí, Jordi and Tommaso Monacelli. 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy." *The Review of Economic Studies* 72 (3):707–734. URL https://doi.org/10.1111/j.1467-937X.2005.00349.x.

Gourio, François, Michael Siemer, and Adrien Verdelhan. 2013. "International risk cycles." *Journal of International Economics* 89 (2):471–484.

Gourio, François. 2012. "Disaster Risk and Business Cycles." *American Economic Review* 102 (6):2734–66. URL https://www.aeaweb.org/articles?id=10.1257/aer.102.6.2734.

Hansen, Lars Peter and Ravi Jagannathan. 1991. "Implications of Security Market Data for Models of Dynamic Economies." *Journal of Political Economy* 99 (2):225–262.

Hassan, Tarek A. 2013. "Country Size, Currency Unions, and International Asset Returns." *Journal of Finance* 68 (6):2269–2308.

Hassan, Tarek A and Rui C Mano. 2019. "Forward and Spot Exchange Rates in a Multi-Currency World*." *The Quarterly Journal of Economics* 134 (1):397–450. URL https://academic.oup.com/qje/advance-article/doi/10.1093/qje/qjy026/5144784.

Hassan, Tarek A. and Tony Zhang. 2021. "The Economics of Currency Risk." *Annual Review of Economics* 13:281–307. URL https://www.annualreviews.org/doi/abs/10.1146/annurev-economics-092220-103354.

Heyerdahl-Larsen, Christian. 2014. "Asset prices and real exchange rates with deep habits." *Review of Financial Studies* 27 (11):3280–3317. URL https://academic.oup.com/rfs/article-lookup/doi/10.1093/rfs/hhu043.

Itskhoki, Oleg and Dmitry Mukhin. 2021. "Exchange Rate Disconnect in General Equilibrium." *Journal of Political Economy* 129 (8):2183–2232. URL https://www.journals.uchicago.edu/doi/10.1086/714447.

Jiang, Zhengyang. 2021. "Fiscal Cyclicality and Currency Risk Premia." *The Review of Financial Studies* URL https://academic.oup.com/rfs/advance-article/doi/10.1093/rfs/hhab061/6279758.

———. 2023. "Market Incompleteness and Exchange Rate Spill-over." *SSRN Electronic Journal* URL https://papers.ssrn.com/abstract=3861805.

Jiang, Zhengyang, Arvind Krishnamurthy, and Hanno Lustig. 2023. "Implications of Asset Market Data for Equilibrium Models of Exchange Rates." Working Paper 31851, National Bureau of Economic Research, Cambridge, MA. URL http://www.nber.org/papers/w31851.pdf.

Jiang, Zhengyang, Arvind Krishnamurthy, Hanno N. Lustig, and Jialu Sun. 2022. "Beyond Incomplete Spanning: Convenience Yields and Exchange Rate Disconnect." *SSRN Electronic Journal* URL https://papers.ssrn.com/abstract=3852298.

Jurek, Jakub W. 2014. "Crash-neutral currency carry trades." *Journal of Financial Economics* 113 (3):325–347.

Kekre, Rohan and Moritz Lenel. 2024. "Exchange Rates, Natural Rates, and the Price of Risk." *Working Paper* .

Lettau, Martin, Matteo Maggiori, and Michael Weber. 2014. "Conditional risk premia in currency markets and other asset classes." *Journal of Financial Economics* 114 (2):197–225.

Lopez, Pierlauro, David Lopez-Salido, and Francisco Vazquez-Grande. 2015. "Risk-Adjusted Linearizations of Dynamic Equilibrium Models." *SSRN Electronic Journal* .

Lustig, Hanno, Nikolai Roussanov, and Adrien Verdelhan. 2011. "Common risk factors in currency markets." *Review of Financial Studies* 24 (11):3731–3777. URL https://academic.oup.com/rfs/article-lookup/doi/10.1093/rfs/hhr068.

Lustig, Hanno and Adrien Verdelhan. 2007. "The Cross Section of Foreign Currency Risk Premia and Consumption Growth Risk." *American Economic Review* 97 (1):89–117.

———. 2019. "Does Incomplete Spanning in International Financial Markets Help to Explain Exchange Rates?" *American Economic Review* 109 (6):2208–2244. URL https://pubs.aeaweb.org/doi/10.1257/aer.20160409.

Malkhozov, Aytek. 2014. "Asset prices in affine real business cycle models." *Journal of Economic Dynamics and Control* 45:180–193.

Martin, Ian. 2011. "The Forward Premium Puzzle in a Two-Country World." Tech. rep., National Bureau of Economic Research, Cambridge, MA. URL http://www.nber.org/papers/w17564.

Meese, Richard A. and Kenneth Rogoff. 1983. "Empirical exchange rate models of the seventies. Do they fit out of sample?" *Journal of International Economics* 14 (1-2):3–24.

Mehra, Rajnish and Edward C. Prescott. 1985. "The equity premium: A puzzle." *Journal of Monetary Economics* 15 (2):145–161.

Mendoza, Enrique G. 2010. "Sudden Stops, Financial Crises, and Leverage." *American Economic Review* 100 (5):1941–1966.

Menkhoff, Lukas, Lucio Sarno, Maik Schmeling, and Andreas Schrimpf. 2012. "Carry Trades and Global Foreign Exchange Volatility." *Journal of Finance* 67 (2):681–718.

Miranda-Agrippino, Silvia and Hélène Rey. 2020. "U.S. Monetary Policy and the Global Financial Cycle." *The Review of Economic Studies* 87 (6):2754–2776. URL https://academic.oup.com/restud/article/87/6/2754/5834728.

Morelli, Juan M., Pablo Ottonello, and Diego J. Perez. 2022. "Global Banks and Systemic Debt Crises." *Econometrica* 90 (2):749–798. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA17433.

Oskolkov, Aleksei. 2024. "Heterogeneous Impact of the Global Financial Cycle." Tech. rep., mimeo University of Chicago.

Ready, Robert, Nikolai Roussanov, and Colin Ward. 2017a. "After the tide: Commodity currencies and global trade." *Journal of Monetary Economics* 85:69–86.

———. 2017b. "Commodity Trade and the Carry Trade: A Tale of Two Countries." *Journal of Finance* 72 (6):2629–2684. URL http://doi.wiley.com/10.1111/jofi.12546.

Rey, Hélène. 2015. "Dilemma not Trilemma: The Global Financial Cycle and Monetary Policy Independence." *NBER Working Paper 21162* (21162). URL http://www.nber.org/papers/w21162.

Richmond, Robert J. 2019. "Trade Network Centrality and Currency Risk Premia." *Journal of Finance* 74 (3):1315–1361.

Samuelson, Paul A. 1948. *Consumption Theory in Terms of Revealed Preference*, vol. 15. Wiley.

Sandulescu, Mirela, Fabio Trojani, and Andrea Vedolin. 2021. "Model-Free International Stochastic Discount Factors." *Journal of Finance* 76 (2):935–976. URL https://onlinelibrary.wiley.com/doi/10.1111/jofi.12970.

Stathopoulos, Andreas. 2017. "Asset prices and risk sharing in open economies." *Review of Financial Studies* 30 (2):363–415. URL https://academic.oup.com/rfs/article-lookup/doi/10.1093/rfs/hhw074.

Verdelhan, Adrien. 2010. "A Habit-Based Explanation of the Exchange Rate Risk Premium." *Jounal of Finance* LXV (1):123–146.

Wachter, Jessica A. 2006. "A consumption-based model of the term structure of interest rates." *Journal of Financial Economics* 79 (2):365–399.

Wang, Jingye. 2021. "Currency Risk and Capital Accumulation." *SSRN Electronic Journal* URL https://papers.ssrn.com/abstract=3962478.

Weil, Philippe. 1989. "The equity premium puzzle and the risk-free rate puzzle." *Journal of Monetary Economics* 24 (3):401–421.

Wiriadinata, Ursula. 2021. "External Debt, Currency Risk, and International Monetary Policy Transmission." URL https://sites.google.com/site/uwiriadinata/research?authuser=0,Accessed2023-11-18.

A Details on the LRR Models

A.1 Derivation of moments of log SDF

Start from Epstein-Zin (EZ) preferences.

$$U_t = \left((1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}. \tag{A1}$$

The stochastic discount factor (SDF) is given by

$$M_{t+1} = \frac{\frac{\partial U_t}{\partial C_{t+1}}}{\frac{\partial U_t}{\partial C_t}} = \frac{\frac{\partial U_t}{\partial U_{t+1}} \frac{\partial U_{t+1}}{\partial C_{t+1}}}{\frac{\partial U_t}{\partial C_t}},$$

where the derivatives are

$$\begin{aligned}
\frac{\partial U_t}{\partial U_{t+1}} &= \delta \left((1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{\frac{1}{\psi}}{1-\frac{1}{\psi}}} \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} U_{t+1}^{-\gamma} \\
&= \delta U_t^{\frac{1}{\psi}} \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} U_{t+1}^{-\gamma} \\
\frac{\partial U_t}{\partial C_t} &= (1-\delta) \left((1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{\frac{1}{\psi}}{1-\frac{1}{\psi}}} C_t^{-\frac{1}{\psi}} \\
&= (1-\delta) U_t^{\frac{1}{\psi}} C_t^{-\frac{1}{\psi}}.
\end{aligned}$$

It immediately follows that the SDF is

$$M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{\left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}.$$

Taking logs on both side, we get the log SDF as

$$m_{t+1} = \log(\delta) - \frac{1}{\psi}\Delta c_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(u_{t+1} - \frac{1}{1-\gamma}\log(\mathbb{E}_t\left[\exp((1-\gamma)u_{t+1})\right])\right).$$

A.1.1 Model without short-run shocks

The consumption process is given by

$$\Delta c_{t+1} = \mu + z_t \tag{A2}$$

$$z_t = \rho z_{t-1} + \sigma\varepsilon_t. \tag{A3}$$

Assumption 1. *Assume u_{t+1} is normal.*

Plugging these expressions into the log SDF from the previous section results in

$$\begin{aligned}\mathbb{E}_t(m_{t+1}) &= \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{\psi}z_t - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\operatorname{var}_t(u_{t+1})\\ \frac{1}{2}\operatorname{var}_t(m_{t+1}) &= \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2\operatorname{var}_t(u_{t+1}).\end{aligned}$$

This implies

$$-\mathbb{E}_t(m_{t+1}) = \frac{1-\gamma}{\frac{1}{\psi}-\gamma}\cdot\frac{1}{2}\operatorname{var}_t(m_{t+1}) - (\log(\delta) - \frac{1}{\psi}\mu) + \frac{1}{\psi}z_t \tag{A4}$$

and the risk-free rate is given by

$$\begin{aligned}r_t &= -\mathbb{E}_t(m_{t+1}) - \frac{1}{2}\operatorname{var}_t(m_{t+1})\\ &= \frac{1}{\psi}z_t - (\log(\delta) - \frac{1}{\psi}\mu) + \left(\frac{1}{\psi} - \gamma\right)\left(1 - \frac{1}{\psi}\right)\operatorname{var}_t(u_{t+1}) \qquad \text{(A5)}\end{aligned}$$

Long-run shocks z_t move the risk-free rate. Equation (10) follows from unconditional moments

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\,\mathbb{E}(\operatorname{var}_t(u_{t+1})) \tag{A6}$$

$$\frac{1}{2}\mathbb{E}(\operatorname{var}_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2\mathbb{E}(\operatorname{var}_t(u_{t+1})). \tag{A7}$$

A.1.2 Model with short-run shocks

Assuming the presence of short-run shocks $\varepsilon_{SR,t}$

$$\Delta c_{t+1} = \mu + z_t + \varepsilon_{SR,t+1},$$

$$z_t = \rho z_{t-1} + \sigma \varepsilon_t.$$

To facilitate the computation of the second moment, we define $V_t = \frac{U_t}{C_t}$. Then

$$V_t = \left((1-\delta) + \delta \left\{ \mathbb{E}_t \left[\left(\frac{U_{t+1}}{C_t} \right)^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}} = \left((1-\delta) + \delta \left\{ \mathbb{E}_t \left[\left(V_{t+1} \frac{C_{t+1}}{C_t} \right)^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}.$$

Note that the conditional covariance of v_{t+1} and Δc_{t+1} is zero. To see this, suppose that after some terminal date T, all shocks are zero and $z_T = 0$. Then, we have

$$\begin{aligned} v_T &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_T \left[\exp(1-\gamma) \left(v_{T+1} + \Delta c_{T+1} \right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right) \\ &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_T \left[\exp(1-\gamma) \left(v_{T+1} + \mu \right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right). \end{aligned}$$

Since $v_T = v_{T+1}$, we can solve for v_T as a constant. At $T-1$, we have

$$\begin{aligned} v_{T-1} &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-1} \left[\exp(1-\gamma) \left(v_T + \Delta c_T \right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right) \\ &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-1} \left[\exp(1-\gamma) \left(v_T + \mu + z_{T-1} + \varepsilon_{SR,T} \right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right). \end{aligned}$$

$\mathbb{E}_{T-1} \left[\exp(1-\gamma) \left(v_T + \mu + z_{T-1} + \varepsilon_{SR,T} \right) \right]$ is only a function of z_{T-1}, and thus is v_{T-1}. Similarly,

$$\begin{aligned} v_{T-2} &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-2} \left[\exp(1-\gamma) \left(v_{T-1} + \Delta c_{T-1} \right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right) \\ &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-2} \left[\exp(1-\gamma) \left(v_{T-1} + \mu + z_{T-2} + \varepsilon_{SR,T-1} \right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}}. \right) \end{aligned}$$

As a result, v_{T-2} is also only a function of z_{T-2}. Using backward induction, we conclude that for any t, v_{t+1} is a function of z_{t+1}, and thus,

$$\mathrm{cov}_t(v_{t+1}, \Delta c_{t+1}) = 0.$$

The reason is that consumption growth depends on z_t, which is known at time t.

The stochastic discount factor takes the form

$$M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{\left\{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]\right\}^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}\frac{C_{t+1}}{C_t}}{\left\{\mathbb{E}_t\left[\left(V_{t+1}\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]\right\}^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}.$$

Taking logs results in

$$m_{t+1} = \log(\delta) - \frac{1}{\psi}\Delta c_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(v_{t+1} + \Delta c_{t+1} - \frac{1}{1-\gamma}\log \mathbb{E}_t\left[\exp((1-\gamma)(v_{t+1} + \Delta c_{t+1}))\right]\right).$$

Now, assuming v_{t+1} is log-normal, we have

$$\begin{aligned}
\mathbb{E}_t(m_{t+1}) &= \log(\delta) - \frac{1}{\psi}\,\mathbb{E}_t(\Delta c_{t+1}) + \left(\frac{1}{\psi} - \gamma\right)\left(\mathbb{E}_t(v_{t+1}) - \mathbb{E}_t(v_{t+1}) - \frac{1}{2}(1-\gamma)\,\mathrm{var}_t(v_{t+1} + \Delta c_{t+1})\right)\\
&= \log(\delta) - \frac{1}{\psi}\,\mathbb{E}_t(\Delta c_{t+1}) - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)(\mathrm{var}_t(v_{t+1} + \Delta c_{t+1}).)
\end{aligned}$$

And the conditional variance of the log SDF is given by

$$\begin{aligned}
\frac{1}{2}\,\mathrm{var}_t(m_{t+1}) &= \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \mathrm{var}_t(\Delta c_{t+1}) + \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathrm{var}_t(v_{t+1} + \Delta c_{t+1}) - \frac{1}{\psi}\left(\frac{1}{\psi} - \gamma\right)\mathrm{cov}_t(\Delta c_{t+1}, v_{t+1} + \Delta c_{t+1})\\
&= \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \mathrm{var}_t(\Delta c_{t+1}) + \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathrm{var}_t(v_{t+1} + \Delta c_{t+1}) - \frac{1}{\psi}\left(\frac{1}{\psi} - \gamma\right)\mathrm{var}_t(\Delta c_{t+1}).
\end{aligned}$$

The second equality uses the fact that $\mathrm{cov}_t(v_{t+1}, \Delta c_{t+1}) = 0$. Substituting in the time-series processes, we get

$$\begin{aligned}
\mathbb{E}_t(m_{t+1}) &= \log(\delta) - \frac{1}{\psi}\,\mathbb{E}_t(\mu + z_t) - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)(\mathrm{var}_t(v_{t+1} + \Delta c_{t+1}))\\
\frac{1}{2}\,\mathrm{var}_t(m_{t+1}) &= \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \sigma_{SR}^2 + \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathrm{var}_t(v_{t+1} + \Delta c_{t+1}) - \frac{1}{\psi}\left(\frac{1}{\psi} - \gamma\right)\sigma_{SR}^2.
\end{aligned}$$

Unconditional moments are thus

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)\mathbb{E}\left(\text{var}_t(v_{t+1}+\Delta c_{t+1})\right)$$

$$\frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi}\right)^2\sigma_{SR}^2 + \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2\mathbb{E}(\text{var}_t(v_{t+1}+\Delta c_{t+1})) - \frac{1}{\psi}\left(\frac{1}{\psi}-\gamma\right)\sigma_{SR}^2.$$

The variance of short-run shocks shifts the intercept in this relation between the mean and the variance. Keeping short-run shock volatilities identical across countries produces the same results as in the main text; if we do allow them to differ across countries, resulting currency premia are given by

$$\frac{1}{\psi}\left(-\frac{1}{2}\frac{1}{\psi}+\gamma\right)(\sigma_{SR}^2-(\sigma_{SR}^\star)^2).$$

Given that consumption growth volatility is low in the data, differences in the variance of short-run shocks are small and this term is small. We confirm this claim with our quantitative results in Section 2.

A.2 Long-run risk model with stochastic volatility

A.2.1 Main results

In this section, we explore if adding time variation in the second moments might help in mitigating the currency premium puzzle. Section 2 abstracts from this mechanism. Following Bansal and Yaron (2004), many long-run risk models feature stochastic volatility to generate time-varying risk premia. Here, we examine the role of stochastic volatility on the FX-share.

With stochastic volatility, the endowment process for the long-run risk model is given by

$$\begin{aligned}
\Delta c_{t+1} &= \mu + z_t + \sigma_{SR}\varepsilon_{SR,t+1} \\
z_t &= \rho z_{t-1} + w_{t-1}\varepsilon_{LR,t} \\
w_t^2 &= (1-\phi)w_0^2 + \phi w_{t-1}^2 + \sigma_w\varepsilon_{w,t},
\end{aligned}$$

where w_t^2 induces stochastic volatility and ϕ governs its persistence and σ_w its volatility. The remaining setup is identical to the model in section 2. While we keep short-run shocks in the setup for now, we ultimately abstract from them by setting $\sigma_{SR} = 0$.

When including stochastic volatility, a closed-form solution is only available for approximations of the model. Using a log-linearization with risk adjustments to solve the model, we show that[36]

$$\mathbb{E}(\hat{m}_{t+1}) = -\frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)(A_{vw}^2\sigma_w^2 + A_{vz}^2 w_0^2) \quad \text{(A8)}$$

$$\frac{1}{2}\,\mathbb{E}(\text{var}_t(\hat{m}_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 (A_{vw}^2\sigma_w^2 + A_{vz}^2 w_0^2). \quad \text{(A9)}$$

The various coefficients labeled A are constants and the variables with a hat denote deviations from the deterministic steady state. Substituting out the right-hand side shows the tight link between first and second moments of the log SDFs:

$$-\,\mathbb{E}(\hat{m}_{t+1}) = \frac{1-\gamma}{\frac{1}{\psi} - \gamma} \cdot \frac{1}{2}\,\mathbb{E}(\text{var}_t(\hat{m}_{t+1})).$$

As a consequence, adding stochastic volatility to the model does not help to resolve the currency premium puzzle. There remains a tight link between the mean and the variance of the log SDF.

A.2.2 Detailed derivation

To derive the link between the first and second moments of the log SDF under stochastic volatility, we use risk-adjusted affine approximation.Dividing both sides of the EZ preference by C_t and defining $V_t = U_t/C_t$, we have

$$\begin{aligned}
\exp\left(\left(1-\frac{1}{\psi}\right)v_t\right) &= 1-\delta+\delta\exp\left(\left(1-\frac{1}{\psi}\right)q_t\right)\\
q_t &= \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(v_{t+1}+\Delta c_{t+1})\right)^{1-\gamma}\right]\right\}\\
m_{t+1} &= \log(\delta) - \frac{1}{\psi}\Delta c_{t+1} + \left(\frac{1}{\psi}-\gamma\right)\left(v_{t+1} - q_t + \Delta c_{t+1}\right).
\end{aligned}$$

[36]See a detailed proof in section A.2.2.

Log-linearizing the system around the deterministic steady state, we get

$$
\begin{aligned}
\Delta\hat{c}_{t+1} &= z_t + \sigma_{SR}\varepsilon_{SR,t+1} \\
z_t &= \rho z_{t-1} + w_{t-1}\varepsilon_{LR,t} \\
w_t^2 &= (1-\phi)w_0^2 + \phi w_{t-1}^2 + \sigma_w \varepsilon_{w,t} \\
\hat{v}_t &= \delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)\hat{q}_t \\
\hat{m}_{t+1} &= -\frac{1}{\psi}\Delta\hat{c}_{t+1} + \left(\frac{1}{\psi}-\gamma\right)\left(\hat{v}_{t+1} - \hat{q}_t + \Delta\hat{c}_{t+1}.\right)
\end{aligned}
$$

Using the log SDF and the link between $\hat{q}_t$ and $\hat{v}_t$, we obtain the following equation

$$
\hat{q}_t = \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(\hat{v}_{t+1} + \Delta\hat{c}_{t+1})\right)^{1-\gamma}\right]\right\}.
$$

We guess the functional form of the solution as

$$
\hat{v}_t = A_{v0} + A_{vz}z_t + A_{vw}w_t^2
$$

and plug it into the equation immediately above to obtain

$$
\begin{aligned}
&\frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}\left(A_{v0} + A_{vz}z_t + A_{vw}\hat{w}_t^2\right) = \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(A_{v0} + A_{vz}z_{t+1} + A_{vw}\hat{w}_{t+1}^2 + z_t + \sigma_{SR}\varepsilon_{SR,t+1})\right)^{1-\gamma}\right]\right\} \\
&= \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(A_{v0} + A_{vz}\rho z_t + A_{vz}\hat{w}_t\varepsilon_{LR,t+1})\right)^{1-\gamma}\right]\right\} \\
&\quad + \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(A_{vw}(1-\phi)w_0^2 + A_{vw}\phi w_t^2 + A_{vw}\sigma_w\varepsilon_{w,t+1})\right)^{1-\gamma}\right]\right\} \\
&\quad + \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(z_t + \sigma_{SR}\varepsilon_{SR,t+1})\right)^{1-\gamma}\right]\right\} \\
&= A_{v0} + A_{vz}\rho z_t + \frac{1}{2}(1-\gamma)A_{vz}^2 w_t^2 + A_{vw}(1-\phi)w_0^2 + A_{vw}\phi w_t^2 + \frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2 + z_t + \frac{1}{2}(1-\gamma)\sigma_{SR}^2 \\
&= A_{v0} + A_{vw}(1-\phi)w_0^2 + \frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2 + \frac{1}{2}(1-\gamma)\sigma_{SR}^2 + (1 + A_{vz}\rho)z_t + \left(\frac{1}{2}(1-\gamma)A_{vz}^2 + A_{vw}\phi\right)w_t^2.
\end{aligned}
$$

Matching coefficients results in

$$\begin{aligned}
\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)} A_{v0} &= A_{v0} + A_{vw}(1-\phi)w_0^2 + \frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2 + \frac{1}{2}(1-\gamma)\sigma_{SR}^2 \\
\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)} A_{vz} &= 1 + A_{vz}\rho \\
\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)} A_{vw} &= \frac{1}{2}(1-\gamma)A_{vz}^2 + A_{vw}\phi.
\end{aligned}$$

As a result, the three coefficients are given by

$$\begin{aligned}
A_{vz} &= \frac{1}{\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)} - \rho} \\
A_{vw} &= \frac{\frac{1}{2}(1-\gamma)A_{vz}^2}{\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)} - \phi} \\
A_{v0} &= \frac{A_{vw}(1-\phi)w_0^2 + \frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2 + \frac{1}{2}(1-\gamma)\sigma_{SR}^2}{\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)} - 1}.
\end{aligned}$$

We can then solve for the pricing kernel:

$$\begin{aligned}
\hat{m}_{t+1} &= -\frac{1}{\psi}\Delta\hat{c}_{t+1} + \left(\frac{1}{\psi}-\gamma\right)\left(\hat{v}_{t+1} - \hat{q}_t + \Delta\hat{c}_{t+1}\right) = -\gamma\Delta\hat{c}_{t+1} + \left(\frac{1}{\psi}-\gamma\right)\left(\hat{v}_{t+1} - \hat{q}_t\right) \\
&= -\gamma(z_t + \sigma_{SR}\varepsilon_{SR,t+1}) + \left(\frac{1}{\psi}-\gamma\right)\left(\hat{v}_{t+1} - \frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}\hat{v}_t\right) \\
&= -\gamma(z_t + \sigma_{SR}\varepsilon_{SR,t+1}) + \left(\frac{1}{\psi}-\gamma\right)\left(A_{v0} + A_{vz}(\rho z_t + w_t\varepsilon_{LR,t+1}) + A_{vw}((1-\phi)w_0^2 + \phi w_t^2 + \sigma_w\varepsilon_{w,t+1})\right) \\
&\quad - \left(\frac{1}{\psi}-\gamma\right)\left(\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}(A_{v0} + A_{vz}z_t + A_{vw}w_t^2)\right)
\end{aligned}$$

This expression simplifies to

$$
\begin{aligned}
\hat{m}_{t+1} = &-\left(\frac{1}{\psi}-\gamma\right)\left(\frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2+\frac{1}{2}(1-\gamma)\sigma_{SR}^2\right)\\
&-\frac{1}{\psi}z_t+\left(\frac{1}{\psi}-\gamma\right)\left(-\frac{1}{2}(1-\gamma)A_{vz}^2\right)w_t^2+\left(\frac{1}{\psi}-\gamma\right)A_{vz}w_t\varepsilon_{LR,t+1}\\
&+\left(-\gamma\sigma_{SR}\varepsilon_{SR,t+1}+\left(\frac{1}{\psi}-\gamma\right)(A_{vw}\sigma_w\varepsilon_{w,t+1})\right).
\end{aligned}
$$

The conditional mean of the log SDF is thus

$$
\mathbb{E}_t(\hat{m}_{t+1}) = -\frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)(1-\gamma)A_{vw}^2\sigma_w^2-\frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)(1-\gamma)\sigma_{SR}^2-\frac{1}{\psi}z_t-\frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)(1-\gamma)A_{vz}^2w_t^2
$$

and the conditional variance

$$
\frac{1}{2}\operatorname{var}_t(\hat{m}_{t+1}) = \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2A_{vz}^2w_t^2+\frac{1}{2}\gamma^2\sigma_{SR}^2+\frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2A_{vw}^2\sigma_w^2.
$$

Taking unconditional expectations, we get a link between the mean and variance of the log SDF

$$
\begin{aligned}
\mathbb{E}(\hat{m}_{t+1}) &= -\frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)(1-\gamma)A_{vw}^2\sigma_w^2-\frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)(1-\gamma)\sigma_{SR}^2-\frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)(1-\gamma)A_{vz}^2w_0^2\\
\frac{1}{2}\mathbb{E}(\operatorname{var}_t(\hat{m}_{t+1})) &= \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2A_{vz}^2w_0^2+\frac{1}{2}\gamma^2\sigma_{SR}^2+\frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2A_{vw}^2\sigma_w^2\\
&= \frac{1}{\psi}\left(-\frac{1}{2}\frac{1}{\psi}+\gamma\right)\sigma_{SR}^2-\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\mathbb{E}(\hat{m}_{t+1}).
\end{aligned}
$$

Setting $\sigma_{SR}=0$ yields (A8) and (A9).

A.3 Long-run Risk Models with Heterogeneous Growth

A.3.1 Theoretical Results

In this subsection, we study the effect of allowing countries to feature different growth rates in addition to heterogeneous risk characteristics. In particular, we study the model in Andrews et al. (2024).

The model is largely identical to the one in A.1.1[37], except for the following two features: First, countries differ in their loadings on a global long-run process; second, different countries feature different growth

[37] Andrews et al. (2024) do feature short-run shocks, but the volatilities are assumed to be the same across countries, thus do not affect currency premium, exchange rates, or interest rate differentials. One can thus safely set them to 0 for simplicity.

rates. With these changes, consumption growth rates are given by

$$\Delta c_{i,t+1} = \mu_c^i + \beta_c^i x_{c,t}$$
$$x_{c,t} = \rho_c x_{c,t-1} + \sigma_{x,c}\varepsilon_{c,t},$$

where $x_{c,t}$ is the time-varying component in expected global growth (or a global long-run process) that is common to all countries. Different countries' consumption growth rates have different loadings β_c^i on this component.[38] Country-specific growth rates are related to the loadings via the following equation:

$$\mu_c^i = \mu_c + \xi^i \tag{A10}$$

where $\xi^i = \mu_c(1-\beta_c^i)$ so that higher loading countries (e.g., Japan) also grow slower.

The relationship between mean and variance of log SDF is now given by

$$-\mathbb{E}(m_{i,t+1}) = \frac{1-\gamma}{\frac{1}{\psi}-\gamma} \cdot \frac{1}{2}\mathbb{E}(\mathrm{var}_t(m_{t+1})) - (\log(\delta) - \frac{1}{\psi}(\mu + \xi^i)). \tag{A11}$$

If we set $\xi^i = 0$ for all countries, (A11) implies the familiar FX-share of $\frac{1-\gamma}{\frac{1}{\psi}-\gamma}$. In fact, if we use the calibration of Andrews et al. (2024) and set $\psi = 1$, symmetric growth rates imply identical risk-free rates across countries and an FX-share of 1 such that all currency premia are accounted for by expected changes in exchange rates.

With hetergeneous ξ^i, the FX-share is given by

$$\begin{aligned}\text{FX-share}^{\text{LRR}}_{\text{Heter Growth}} &= \frac{(-\mathbb{E}(m^i)) - (-\mathbb{E}(m))}{\frac{1}{2}\mathbb{E}(\mathrm{var}_t(m^i)) - \frac{1}{2}\mathbb{E}(\mathrm{var}_t(m))} \\ &= \frac{1-\gamma}{\frac{1}{\psi}-\gamma} + \frac{1}{\psi}\frac{\xi^i}{\frac{1}{2}\mathbb{E}(\mathrm{var}_t(m^i_{t+1})) - \frac{1}{2}\mathbb{E}(\mathrm{var}_t(m_{t+1}))}\end{aligned}$$

where the country without a superscript denotes the average country with $\xi^i = 0$. This implies that one can pin down the FX-share by examining the relationship between ξ^i and the variances (currency premia) in the model. To see this, note that re-arranging yields

$$\xi^i = \psi\left(\text{FX-share} - \frac{1-\gamma}{\frac{1}{\psi}-\gamma}\right) \cdot \left(\frac{1}{2}\mathbb{E}(\mathrm{var}_t(m^i_{t+1})) - \frac{1}{2}\mathbb{E}(\mathrm{var}_t(m_{t+1}))\right). \tag{A12}$$

[38] Andrews et al. (2024) also feature an inflation process similar to that of consumption growth. We discuss this more complicated model in our quantitative analysis and focus on the real model for simplicity for now.

If we set $\psi = 1$ as in Andrews et al. (2024), the above equation further simplifies to

$$\xi^i = (\text{FX-share} - 1) \cdot \left(\frac{1}{2}\,\mathbb{E}(\text{var}_t(m^i_{t+1})) - \frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1})) \right) \tag{A13}$$

Thus, one can approximate the model implied FX-share by running a regression of ξ^i on $\frac{1}{2}\,\mathbb{E}(\text{var}_t(m^i_{t+1})) - \frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1}))$. Using the calibration of Andrews et al. (2024), the estimated coefficient is 0.206, which implies an FX-share of 0.79. As we will show in our quantitative exercises, this approximation is quite accurate.

A.3.2 Quantitative Results

We solve and simulate the full model in this subsection. The model is partial equilibrium and can be solved analytically. We thus show theoretical results along with simulation results whenever possible. Theoretical results are obtained by solving the object in closed form and then plugging in the calibration.

Table 7: ACCG Results: Static Trade

	Return (%) $\mathbb{E}(rx^{st})$	Change in FX (%) $\mathbb{E}(\Delta s^{st})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,st} - r^{st})$	FX-share $\frac{\mathbb{E}(\Delta s^{st})}{\mathbb{E}(rx^{st})}$	P1	P2
Data	3.46 [1.18,5.54]	-1.30 [-3.82,0.60]	4.76 [1.30,8.46]	-0.37 [-1.16,0.23]	-	-
Real Simulation	5.76	4.87	0.89	0.85	Yes	No
Nominal Simulation	3.95	2.61	1.34	0.66	Yes	No
Real: Theory	4.41	3.49	0.92	0.79	Yes	No
Nominal: Theory	3.90	2.26	1.64	0.58	Yes	No

This table shows the static trade returns for the model in Andrews et al. (2024), using calibrations in their Tables 6 and F.1. For the simulation results, shocks are chosen so that $x_{c,0} = x_{\pi,0} = 0.13\%$ and $x_{c,t^\star} = x_{\pi,t^\star} = -0.54\%$, as in the paper. We simulate 1000 different samples with 100 periods, with $t^\star = 51$. Real Simulation shows the results for real variables, Nominal Simulation for nominal variables. We also report the real and nominal theoretical returns. Portfolios are constructed in the same way as our data (see Appendix C). All moments are annual.

Table 8: ACCG Results: Conditional Carry Trade

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	-	-
Real Simulation	5.63	4.74	0.89	0.84	Yes	No
Nominal Simulation	3.74	1.76	1.98	0.47	Yes	No

This table shows the conditional carry trade returns for the model in Andrews et al. (2024), using calibrations in their Tables 6 and F.1. For the simulation results, shocks are chosen so that $x_{c,0} = x_{\pi,0} = 0.13\%$ and $x_{c,t^\star} = x_{\pi,t^\star} = -0.54\%$, as in the paper. We simulate 1000 different samples with 100 periods, with $t^\star = 51$. Real Simulation shows the results for real variables, Nominal Simulation for nominal variables. Portfolios are constructed in the same way as our data (see Appendix C). All moments are annual.

Agents feature the following preference

$$U_{i,t} = (1-\delta)\log C_{i,t} + \delta \frac{1}{1-\gamma} \log \mathbb{E}_t\, e^{(1-\gamma)U_{i,t+1}}$$

The elasticity of substitution is equal to one, i.e., $\psi = 1$. Absent differences in growth rates or inflation, these preferences imply that all countries share the same unconditional risk-free rate (see the theoretical results in Section 2) and the FX-share is one.

There is a long-run component in both consumption and inflation, and we have

$$\begin{bmatrix} x_{\pi,t} \\ x_{c,t} \end{bmatrix} = \begin{bmatrix} \rho_\pi & 0 \\ \rho_{c\pi} & \rho_c \end{bmatrix} \cdot \begin{bmatrix} x_{\pi,t-1} \\ x_{c,t-1} \end{bmatrix} + \begin{bmatrix} \sigma_{x,\pi} & 0 \\ 0 & \sigma_{x,c} \end{bmatrix} \begin{bmatrix} \varepsilon_{\pi_t} \\ \varepsilon_{c,t} \end{bmatrix}$$

Consumption and inflation follow

$$\Delta c_{i,t+1} = \mu_c^i + \beta_i^c x_{c,t} + \sigma_c \eta_{i,t+1}^c$$
$$\pi_{i,t+1} = \mu_\pi^i + \beta_i^\pi x_{\pi,t} + \sigma_\pi \eta_{i,t+1}^\pi$$

All shocks are i.i.d. normal. Country-specific mean growth rates and inflation rates

$$\mu_c^i = \bar{\mu}_c + \bar{\mu}_c(1-\beta_c^i)$$
$$\mu_\pi^i = \bar{\mu}_\pi - \bar{\mu}_\pi(1-\beta_\pi^i)$$

These equations highlight the assumed link between growth rates and loadings on shocks.

The real log SDF is

$$m_{i,t+1} = \bar{m}_i^{real} - \beta_c^i x_{c,t} - k_{\varepsilon c}^i \sigma_{x,c}\varepsilon_{c,t+1} + k_{\varepsilon\pi}^i \sigma x,\pi\varepsilon_{\pi,t+1} - \gamma\sigma_c\eta_{i,t+1}^c,$$

from which we can compute the unconditional expectation

$$\mathbb{E}(m_{i,t+1}^{real}) = \log(\delta) - \frac{1}{2}(1-\gamma)^2\sigma_c^2 - \mu_c^i - \frac{1}{2}\left[(k_{\varepsilon c}^i\sigma_{x,c})^2 + (k_{\varepsilon\pi}^i\sigma_{x,\pi})^2\right],$$

where

$$k_{\varepsilon c}^i = (\gamma-1)\beta_c^i\left(\frac{\delta}{1-\delta\rho_c}\right)$$
$$k_\varepsilon\pi^i = -\rho_{c\pi}k_{\varepsilon c}^i\left(\frac{\delta}{1-\delta\rho_\pi}\right).$$

The unconditional expectation of the nominal SDF is linked to the expectation of the real log SDF via

$$\mathbb{E}(m_{i,t+1}^{nominal}) = \mathbb{E}(m_{i,t+1}^{real}) - \mu_\pi^i.$$

From these expressions, we get an average risk-free rate of

$$\mathbb{E}(r^{i,real}) = \mu_c^i - \log(\delta) - \left(\frac{1}{2}-\frac{1}{\theta}\right)\sigma_c^2.$$

The unconditional expectation of nominal risk-free rates is given by

$$\mathbb{E}(r^{i,nominal}) = \mu_c^i + \mu_\pi^i - \log(\delta) - \left(\frac{1}{2}-\frac{1}{\theta}\right)\sigma_c^2 - \frac{1}{2}\sigma_\pi^2.$$

Using these relationships, the currency premium (note that σ_π are the same across countries) is

$$\begin{aligned}\mathbb{E}(rx) &= r^{i,real} - r^{US,real} + m^{i,real} - m^{US,real} \\ &= r^{i,nominal} - r^{US,nominal} + m^{i,nominal} - m^{US,nominal} \\ &= \frac{1}{2}\left[(k^{US}_{\varepsilon c}\sigma_{x,c})^2 + (k^{US}_{\varepsilon\pi}\sigma_{x,\pi})^2\right] - \frac{1}{2}\left[(k^{i}_{\varepsilon c}\sigma_{x,c})^2 + (k^{i}_{\varepsilon\pi}\sigma_{x,\pi})^2\right].\end{aligned}$$

The expected change in real exchange rates is

$$\begin{aligned}\mathbb{E}(\Delta s) &= \mathbb{E}(m^{real}_{US}) - \mathbb{E}(m^{real}_{i}) \\ &= \mu^{US}_c - \mu^i_c + \frac{1}{2}\left[(k^{US}_{\varepsilon c}\sigma_{x,c})^2 + (k^{US}_{\varepsilon\pi}\sigma_{x,\pi})^2\right] - \frac{1}{2}\left[(k^{i}_{\varepsilon c}\sigma_{x,c})^2 + (k^{i}_{\varepsilon\pi}\sigma_{x,\pi})^2\right],\end{aligned}$$

and the expected change in nominal exchange rates

$$\begin{aligned}\mathbb{E}(\Delta s) &= \mathbb{E}(m^{nominal}_{US}) - \mathbb{E}(m^{nominal}_{i}) \\ &= \mu^{US}_c - \mu^i_c + \mu^{US}_\pi - \mu^i_\pi + \frac{1}{2}\left[(k^{US}_{\varepsilon c}\sigma_{x,c})^2 + (k^{US}_{\varepsilon\pi}\sigma_{x,\pi})^2\right] - \frac{1}{2}\left[(k^{i}_{\varepsilon c}\sigma_{x,c})^2 + (k^{i}_{\varepsilon\pi}\sigma_{x,\pi})^2\right].\end{aligned}$$

Using the model to pin down real interest rate differentials

$$r^{i,nominal} - r^{US,nominal} = \mu^i_c - \mu^{US}_c$$

and nominal interest rate differentials

$$r^{i,nominal} - r^{US,nominal} = \mu^i_c - \mu^{US}_c + \mu^i_\pi - \mu^{US}_\pi,$$

we get that both nominal and real interest rates are identical when growth rates and inflation are the same across countries. This confirms our results in Section 2.

Because the model is solved in closed form, we show both theoretical and simulation results for the static trade in Table 7. The discrepancy between the two originates from the fact that when doing simulation, we follow Andrews et al. (2024) where the consumption and inflation news processes are not purely random (see the table notes for details). Allowing different countries to feature different growth rates reduces the FX-share, but it is not enough to match the empirical result. Note that the theoretical FX-share of 0.79 is well approximated by our regression (A13). Adding inflation processes further reduces FX-share, but again results in FX-shares much higher than in the data. In fact, in Andrews et al. (2024), inflation processes and growth rates are virtually equivalent in terms of their effects on the FX-share, it is thus not surprising that adding more heterogeneity would alleviate the problem.

Table 8 shows the simulation results for conditional carry trade. A similar pattern emerges: Allowing growth rates and inflation to differ reduces the FX-share, but not by enough to match the data.[39]

B Details on the Disaster Models

The Gourio, Siemer, and Verdelhan (2013) model features Epstein-Zin preferences, and thus, the SDF is given by

$$M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{\left\{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]\right\}^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}.$$

As a result, the mean log SDF is given by

$$\mathbb{E}_t(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\Delta c_{t+1}) + \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\mathbb{E}_t\left((1-\gamma)u_{t+1}\right) - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right). \tag{A14}$$

To derive the entropy $\Xi_t(m_{t+1}) = \log \mathbb{E}_t(M_{t+1}) - \mathbb{E}_t(m_{t+1})$, we compute $\log \mathbb{E}_t(M_{t+1})$ via

$$\begin{aligned}\log \mathbb{E}_t(M_{t+1}) &= \log \mathbb{E}_t\left(\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\right)\\ &= \log(\delta) + \log\left(\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\right]\mathbb{E}_t\left[\left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\right] + \mathrm{cov}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}, \left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\right]\right).\end{aligned}$$

We consider the special case where $\frac{C_{t+1}}{C_t}$ is known at time t. That is, the entropy of consumption growth is zero. This assumption is a direct extension of the "no-short-run-shocks" assumption in the long-run risk framework. We show in Table 5 that this assumption does not drive the results. With this simplification, we

[39] In their table 11, Andrews et al. (2024) reports an FX-share of $\frac{0.85}{2.75} = 0.31$. This is due to a different weighting scheme (GDP-weighted vs forward-premium-weighted).

have

$$\log \mathbb{E}_t(M_{t+1}) = \log(\delta) + \log \mathbb{E}_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\right) + \log \mathbb{E}_t\left(\left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\right)$$

$$= \log(\delta) - \frac{1}{\psi}(\Delta c_{t+1}) + \left[\log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right) - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\log\left(\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]\right)\right].$$

Combining with (A14), the entropy is

$$\Xi_t(m_{t+1}) = \log \mathbb{E}_t(M_{t+1}) - \mathbb{E}_t(m_{t+1}) = -\mathbb{E}_t\left(\left(\frac{1}{\psi}-\gamma\right)u_{t+1}\right) + \log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right). \tag{A15}$$

By comparing the two terms on the right-hand side of equations (A14) and (A15), we can already see a tight link between the two. In particular, if $\psi = 1$, we have

$$\Xi_t(m_{t+1}) = -\mathbb{E}_t(m_{t+1}) + \log(\delta) - \mathbb{E}_t(\Delta c_{t+1}).$$

Taking unconditional expectations, we have

$$\mathbb{E}(\Xi_t(m_{t+1})) = -\mathbb{E}(m_{t+1}) + \log(\delta) - \mathbb{E}(\Delta c_{t+1}).$$

This is a line with a slope of -1 in the generalized SDF space with entropy on the y-axis. Consequently, as long as all countries feature the same preference and consumption growth rate, they all lie on the same line, which coincides with an iso-rf line. That is, when $\psi = 1$, all countries feature the same risk-free rates.

In standard calibrations, ψ is typically close to but larger than 1. In this case, we can use cumulant generating functions and obtain

$$
\begin{aligned}
\Xi_t(m_{t+1}) &= \log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right) - \mathbb{E}_t\left(\left(\frac{1}{\psi}-\gamma\right)u_{t+1}\right) \\
&= \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 \kappa_{2,t}(u_{t+1}) + \frac{1}{6}\left(\frac{1}{\psi}-\gamma\right)^3 \kappa_{3,t}(u_{t+1}) + \frac{1}{24}\left(\frac{1}{\psi}-\gamma\right)^4 \kappa_{4,t}(u_{t+1}) \dots \\
\mathbb{E}_t(m_{t+1}) &= \left(\log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\Delta c_{t+1})\right) - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\left(\log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right) - \mathbb{E}_t\left((1-\gamma)u_{t+1}\right)\right) \\
&= \left(\log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\Delta c_{t+1})\right) \\
&\quad - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)\kappa_{2,t}(u_{t+1}) - \frac{1}{6}(1-\gamma)^2\left(\frac{1}{\psi}-\gamma\right)\kappa_{3,t}(u_{t+1}) - \frac{1}{24}(1-\gamma)^3\left(\frac{1}{\psi}-\gamma\right)\kappa_{4,t}(u_{t+1}) + \dots.
\end{aligned}
$$

Taking unconditional expectations, we end up with

$$
\begin{aligned}
\mathbb{E}(m_{t+1}) &= \left(\log(\delta) - \frac{1}{\psi}\mathbb{E}(\Delta c_{t+1})\right) - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)\mathbb{E}(\kappa_{2,t}(u_{t+1})) - \frac{1}{6}(1-\gamma)^2\left(\frac{1}{\psi}-\gamma\right)\mathbb{E}(\kappa_{3,t}(u_{t+1})) + \dots, \\
\mathbb{E}(\Xi_t(m_{t+1})) &= \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 \mathbb{E}(\kappa_{2,t}(u_{t+1})) + \frac{1}{6}\left(\frac{1}{\psi}-\gamma\right)^3 \mathbb{E}(\kappa_{3,t}(u_{t+1})) + \dots
\end{aligned}
$$

If we only allow skewness ($\mathbb{E}(\kappa_{3,t}(u_{t+1}))$) to differ across countries and set all other cumulants to be identical, we get

$$
\begin{aligned}
\mathbb{E}(m_{t+1}) &= -\frac{1}{6}(1-\gamma)^2\left(\frac{1}{\psi}-\gamma\right)\mathbb{E}(\kappa_{3,t}(u_{t+1})) + \text{constant}, \\
\mathbb{E}(\Xi_t(m_{t+1})) &= \frac{1}{6}\left(\frac{1}{\psi}-\gamma\right)^3 \mathbb{E}(\kappa_{3,t}(u_{t+1})) + \text{constant}
\end{aligned}
$$

Substituting $\mathbb{E}(\kappa_{3,t}(u_{t+1}))$ out, we end up with

$$
-\mathbb{E}(m_{t+1}) = \left(\frac{1-\gamma}{\frac{1}{\psi}-\gamma}\right)^2 \mathbb{E}(\Xi(m_{t+1})) + \text{constant}
$$

The mean-entropy pairs of all countries should satisfy the above equation. What's more, the FX-share is now given by

$$
\text{FX-share} = \frac{\mathbb{E}(\Delta s)}{\mathbb{E}(\Delta rx)} = \frac{[-\mathbb{E}(m_{t+1})] - [-\mathbb{E}(m^{\star}_{t+1})]}{\mathbb{E}(\Xi_t(m_{t+1}) - \Xi_t(m^{\star}_{t+1}))} = \left(\frac{1-\gamma}{\frac{1}{\psi}-\gamma}\right)^2
$$

If only unconditional skewness are allowed to differ across countries, the FX-share is again directly pinned down by preference parameters and is very close to 1 under standard calibrations.

C Portfolio Construction and Alternative Estimates

In this section, we discuss the construction of static trade and conditional carry trade portfolios in this paper, as well as alternative estimates of the FX-share. We follow the procedure and estimates in Hassan and Mano (2019). Our results are robust to alternative portfolio construction methods, for example, the equal-weighted method utilized in Lustig, Roussanov, and Verdelhan (2011).[40] We use the Hassan and Mano (2019) portfolios because of their close connection to regression-based results, on which we would base our alternative estimates of the FX-share in this section. [41]

Static trade and conditional carry trade returns are constructed by forming a portfolio of currencies weighted by their forward premium relative to the US (equivalently, their risk-free rates). Letting $f_{i,t}$ be the log one-period forward exchange rate of currency i at time t, and let $s_{i,t}$ be the spot rate, both quoted in units of currency i per US dollar, by covered interest-rate parity, we have

$$r_{i,t} - r_{US,t} = f_{i,t} - s_{i,t} = fp_{i,t}.$$

Sorting currencies by their forward premia ($fp_{i,t}$) is thus equivalent to sorting currencies on their risk-free rates. Letting $rx_{i,t+1} = f_{i,t} - s_{i,t+1} = r_{i,t} - r_{US,t} - \Delta s_{i,t+1}$ be the currency premium of currency i relative to the US dollar, our static trade return is then given by

$$\Sigma_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)],$$

where $\overline{fp}_i^e$ denotes the estimated[42] forward premium of currency i over time and $\overline{fp}^e = \frac{1}{N}\Sigma_i \overline{fp}_i$. Intuitively, investors conducting the static trade would weight the currencies using their long-term forward premium, longing high-interest-rate currencies and shorting low-interest-rate ones. They fix their portfolio (the weights, $\overline{fp}_i^e - \overline{fp}^e$, do not change over time), thus conducting a "static" carry trade.

[40] The broad pattern that carry traders lose money on the exchange rate is also evident in Table 1 of Lustig, Roussanov, and Verdelhan (2011).

[41] We only provide essential information in this section. Interested readers should refer to Hassan and Mano (2019) for details

[42] From an investor's perspective, a currency's forward premium over the whole sample period is unknown. We assume investors simply expect $\overline{fp}_i$ to be equal to the mean of $\overline{fp}_{i,t}$ across all available data prior to the investment period. We do the same thing when running all our simulations in the paper.

The conditional carry trade return is given by

$$\Sigma_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_t)],$$

where $\overline{fp}_t = \frac{1}{N}\Sigma_i fp_{i,t}$. The only difference from static trade is that the weights now change over time. In each period, investors would weight currencies based on their forward premium in that period.

For both of these trades, we can view the currency portfolio that investors long as one representative country, and the currency portfolio that investors short as another representative country. Using the static trade portfolio as an example, mathematically, the currency return for the representative high-interest-rate country relative to the US is given by

$$\Sigma_{i\in\{\forall i \text{ s.t. } \overline{fp}_i - \overline{fp} > 0\},t}[rx_{i,t+1}(\overline{fp}_i - \overline{fp})].$$

The risk-free rate (or forward premium) and exchange rate of this representative country are defined in a similar manner, simply replacing $rx_{i,t+1}$ with $fp_{i,t+1}$ and $-\Delta s_{i,t+1}$, respectively. Using the unconditional moments of these returns, as well as their composition, we could infer FX-shares for these representative countries as in the main text of this paper.

As shown in Hassan and Mano (2019), one major advantage of forming portfolios in this way is one can easily relate these portfolios to regression results. For example, the static trade is closely related to β^{static} in the following regression:

$$rx_{i,t+1} - \overline{rx}_{t+1} = \beta^{static}(\overline{fp}_i^e - \overline{fp}^e) + \epsilon_{i,t+1}^{static}.$$

More importantly, estimates of β^{static} are closely related to the FX-share:

$$\text{FX-share} = 1 - \frac{1}{\hat{\beta}^{static}}.$$

To see this, note $\hat{\beta}^{static} = \frac{\Sigma_{i,t}[(rx_{i,t+1} - \overline{rx}_{t+1})(\overline{fp}_i^e - \overline{fp}^e)]}{\Sigma_{i,t}[(\overline{fp}_i^e - \overline{fp}^e)^2]} = \frac{\Sigma_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]}{\Sigma_{i,t}[(\overline{fp}_i^e - \overline{fp}^e)^2]}$, so we have

$$1 - \frac{1}{\hat{\beta}^{static}} = \frac{\Sigma_{i,t}[\Delta s_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]}{\Sigma_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]} = \text{FX-share}.$$

Table 9: Estimation of FX-Share

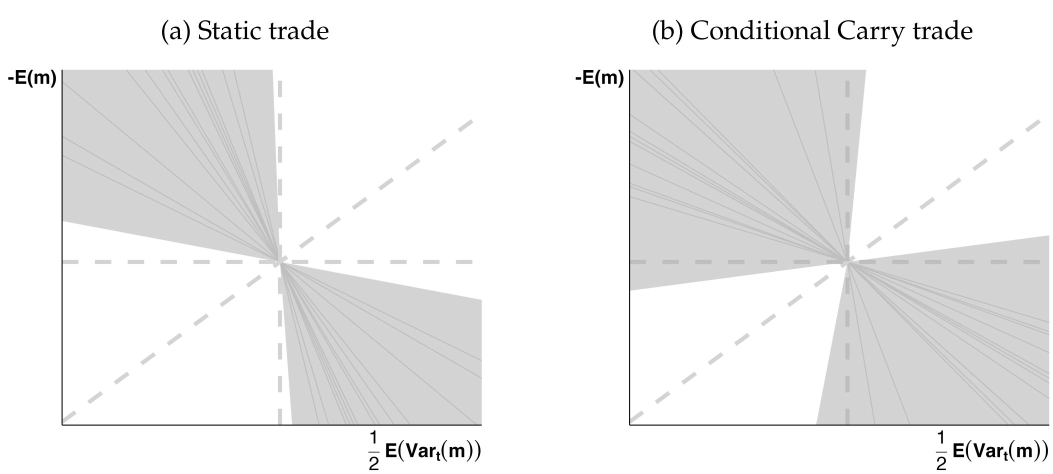
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Horizons (months)	1	1	6	12	1	1	6	12
Sample	1 Reblance				3 Reblance			
Static T: β^{stat}	0.47	0.37	0.56	0.60	0.26	0.18	0.26	0.25
	[0.31, 0.63]	[0.19, 0.55]	[0.36, 0.76]	[0.40, 0.80]	[0.16, 0.36]	[0.08, 0.28]	[0.18, 0.34]	[0.13, 0.37]
Static T: FX-share	-1.13	-1.70	-0.79	-0.67	-2.85	-4.56	-2.85	-3.00
	[-2.19, -0.60]	[-4.17, -0.83]	[-1.75, -0.32]	[-1.48, -0.26]	[-5.17, -1.79]	[-11.20, -2.60]	[-4.51, -1.96]	[-6.55, -1.72]
Carry T: β^{ct}	0.68	0.55	0.62	0.71	0.57	0.45	0.42	0.43
	[0.15, 1.21]	[0.04, 1.06]	[0.05, 1.19]	[0.20, 1.22]	[0.20, 0.94]	[0.10, 0.80]	[0.01, 0.83]	[0.06, 0.80]
Carry T: FX-share	-0.47	-0.82	-0.61	-0.41	-0.75	-1.22	-1.38	-1.33
	[-5.63,0.17]	[-23.75,0.06]	[-18.38,0.16]	[-3.99,0.18]	[-4.06, -0.06]	[-9.29, -0.25]	[-118.05, -0.20]	[-16.36, -0.25]
Sample	6 Reblance				12 Reblance			
Static T: β^{stat}	0.23	0.15	0.25	0.24	0.34	0.23	0.31	0.30
	[0.13, 0.33]	[0.05, 0.25]	[0.17, 0.33]	[0.14, 0.34]	[0.18, 0.50]	[0.05, 0.41]	[0.15, 0.47]	[0.14, 0.46]
Static T: FX-share	-3.35	-5.67	-3.00	-3.17	-1.94	-3.35	-2.23	-2.33
	[-6.58, -2.05]	[-18.23, -3.03]	[-4.83, -2.05]	[-6.04, -1.96]	[-4.46, -1.01]	[-17.66, -1.46]	[-5.53, -1.14]	[-5.98, -1.19]
Carry T: β^{ct}	0.56	0.45	0.45	0.11	0.67	0.52	0.57	0.22
	[0.21, 0.91]	[0.12, 0.78]	[0.08, 0.82]	[-0.16, 0.38]	[0.36, 0.98]	[0.21, 0.83]	[0.26, 0.88]	[-0.11, 0.55]
Carry T: FX-share	-0.79	-1.22	-1.22	-8.09	-0.49	-0.92	-0.75	-3.55
	[-3.83,-0.10]	[-7.56,-0.28]	[-11.89,-0.22]	$[7.08,+\infty) \cup (-\infty,-1.60]$	[-1.81,-0.02]	[-3.84,-0.20]	[-2.90,-0.13]	$[9.83,+\infty) \cup (-\infty,-0.81]$

Point estimates are taken from Table III in Hassan and Mano (2019). Confidence intervals are calculated using the corresponding standard errors.

A similar result can be obtained for carry-trade returns. We then use the estimates of β^{static} and β^{ct} in Hassan and Mano (2019) to construct alternative estimates of FX-share.

Table 9 lists all the estimates using different samples. We illustrate all these estimates in our SDF space in Figure 8. Each of the lines represent an estimate of the FX-share, with the shaded area showing the widest confidence interval. One can clearly see the results are consistent with our bootstrap-based estimates: high-interest-rate currency tends to depreciate, and the FX-share tends to be negative or, at most, approximately horizontal.

Figure 8: Alternative Estimations of the FX-share



This figure plots the point estimates and confidence intervals of the FX-shares inferred from estimates of β^{static} and β^{carry}. Solid grey lines represent point estimates across different samples. The shaded grey area represents the widest confidence interval.