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German Inflation-Linked Bonds: Overpriced, yet Undervalued

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Abstract

We document that German inflation-linked government bond yields contain a convenience or safety premium averaging 0.33 percent. Yet, the German Federal Finance Agency decided to cease all future issuance of these bonds in November 2023. We examine the market response to this announcement and find that neither the safety premia nor the trading conditions of these bonds have been negatively impacted. Hence, this bond market remains a rich source of information on real rates in the euro area in addition to offering investors a safe inflation-protected asset.

JEL Classification: C32, E43, E52, G12

Keywords: affine arbitrage-free term structure model, financial market frictions, convenience premium, safety premium, rstar

The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of San Francisco or the Federal Reserve System, or those of the Banque de France or the Eurosystem.

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1 Introduction

This paper is among the first to provide a comprehensive analysis of the important, yet little known and mostly overlooked market for German inflation-linked government bonds. This sense of under-appreciation was further reinforced by the German Federal Finance Agency's recent decision to cease all future inflation-linked bond issuance. This neglect stands in sharp contrast to the immense attention paid to the market for standard fixed-coupon German government bonds, widely known as bunds, which represent the benchmark class of safe assets in the euro area and is second only to U.S. Treasuries in terms of investor base, liquidity, and market depth.

Given the large and well-documented flight-to-safety effects in the German bund market, we conjecture that investors might also be willing to pay a premium, albeit somewhat smaller, for safely storing their wealth in German inflation-linked government bonds despite their lack of market attention. Hence, the main purpose of our inquiry is to examine whether there are any convenience premia embedded in the prices of German inflation-linked government bonds as little is known about the pricing in this market.

To estimate any such bond-specific convenience premia along with conventional real term premia, we use an arbitrage-free dynamic term structure model of real yields augmented with a bond-specific risk factor. The identification of the bond-specific risk factor comes from its unique loading for each individual bond security as in Andreasen et al. (2021, henceforth ACR). Our analysis uses prices of individual bonds rather than the more usual input of yields from fitted synthetic curves. The underlying mechanism assumes that, over time, an increasing proportion of the outstanding inventory is locked up in buy-and-hold investors' portfolios. Given forward-looking investor behavior, this lock-up effect means that a particular bond's sensitivity to the market-wide bond-specific risk factor will vary depending on how seasoned the bond is and how close to maturity it is. In a careful study of nominal U.S. Treasuries, Fontaine and Garcia (2012) find a pervasive bond-specific factor that affects all bond prices, with loadings that vary with the maturity and age of each bond. By observing a cross section of bond prices over time—each with a different time-since-issuance and time-to-maturity—we can identify the overall bond-specific risk factor and each bond's loading on that factor. This technique is particularly useful for analyzing inflation-linked debt when only a limited sample of bonds may be available as in our case.[1]

Using this modeling tool, we are the first to document the existence of large and time-varying convenience premia in this market that average 0.33 percent for our sample period that runs from October 2007 to December 2024. Given that the German inflation-linked bonds are much less liquid than standard bunds in terms of bid-ask spreads, we follow Christensen

[1] Finlay and Wende (2012) examine prices from a limited number of Australian inflation-linked bonds but do not account for bond-specific liquidity or convenience premia.

and Mirkov (2022) and refer to these convenience premia as safety premia.

We first compare our estimates with the average convenience premium for French inflation-linked government bonds, known as OAT€s, reported in Christensen and Mouabbi (2024, henceforth CM). Their average estimated series is daily covering the period from October 2002 to December 2022 with a mean of 0.01 percent. Hence, there is a notable and persistent level difference in the real yields observed across the French and German inflation-linked bond markets. This result also means that we advise against pooling prices from these two markets, a practice used by economists at the European Central Bank (ECB) in the early years of these markets when the number of outstanding inflation-linked bonds in each indeed was limited; see Ejsing et al. (2007) for an example.[2]

We then compare our estimates with the safety premia of standard German bunds estimated by Christensen et al. (2025).[3] Their estimated series average 1.24 percent over the 1999-2021 period. Hence, our estimated safety premia for the inflation-linked market are smaller as anticipated and only about one quarter the size of those in the standard bund market.

Our findings also contrast with the results of ACR, who report an average estimated liquidity discount premium for U.S. TIPS yields of 34 basis points for the 1997-2013 period. We speculate that the scarcity of both French and German inflation-linked bonds explains their high prices whereas the U.S. TIPS market is much larger, more well-established, and hence more likely to face the inherent illiquidity challenges of inflation-linked bonds discussed at length in Cardozo and Christensen (2024).

Similar to CM, we employ regression analysis to examine the determinants of the safety premia in the German inflation-linked government bond market. Using a large battery of explanatory variables, the results suggest that these premia behave less like a liquidity premium and more like a safety premium, which accords well with the regression results reported by CM for the convenience premia of French OAT€s.

We then dedicate a section to a detailed analysis of the market reaction to the announcement by the German Federal Finance Agency to cease all future issuance of inflation-linked debt made public on November 22, 2023. We find that neither the safety premia nor the trading conditions of the inflation-linked bonds were negatively impacted by this decision. Thus, it does not appear as if investors positioned themselves for somewhat slower inflation-linked market trading going forward. Overall, though, the market reaction was tempered, and the inflation-linked market has continued to function on par with the past through the end of our sample. Hence, for now, inflation-linked trading remains active despite no new issuance

[2]We note that these differences may have mattered little for their sample as yield spreads across a wide variety of markets were very compressed in the years ahead of the global financial crisis.

[3]Christensen at al. (2025) provide estimates of safety premia for an international panel of government bond prices, including those of German bunds.

has come to market since before November 2023. These findings also support our usage of the inflation-linked data through the end of our sample, and presumably will support their usage well into the future given that the longest dated outstanding German inflation-linked bond can be expected to continue to trade until 2046. That said, we do feel compelled to caution that the usefulness of this market information will inevitably decline over time as the remaining inflation-linked bonds reach maturity. However, at this point, it remains a rich source of information for both policy and trading analysis as we demonstrate in this paper.

As a final exercise, we again follow CM, but this time focus on the market-based estimate of the natural rate r_t^* that can be produced using the German inflation-linked bond price data.[4] In comparing our results with those reported by CM using prices from the larger French OAT€ market, we find that our German market-based r_t^* is less persistent, more stable, and operate at a higher level. Instead, the persistent trend in the observed GBi yields is explained by trends in the residual real term premium. We take the lack of persistence in the model-implied real rate expectations to be a consequence of both the shorter available sample—the German data start in October 2007 versus October 2002—and the much smaller universe of bonds with generally shorter maturities compared to the French OAT€ market. These shortcomings that apply across a range of specifications and implementations suggest that our German models' estimated dynamics suffer notably from the finite-sample bias problem discussed at length in Bauer et al. (2012). Overall, we take this evidence to imply that the GBi market is less well suited for this kind of longer-term analysis, and the decision by the German Federal Finance Agency to phase out inflation-linked debt is not helpful in addressing these data-related limitations. In our view, the simple remedy would be for the German Federal Finance Agency to resume its issuance of inflation-linked debt and tilt it towards bonds with longer maturities up to 30 years. In addition to preserving the usefulness of this data, that strategy would also send a strong signal to investors and other stakeholders that the German government is committed to continuing to offer this unique class of safe inflation-protected securities to the public.

The analysis in this paper relates to several important literatures. Most directly, our results relate to research on financial market liquidity and convenience premia. Second, our estimates of the real yield curve that would prevail without trading frictions have implications for asset pricing analysis on the true slope of the real yield curve. Third, the paper is among the first to document what happens to trading and market dynamics when a government decides to terminate issuance of inflation-linked debt. Furthermore, it speaks to the burgeoning literature on measurement of the natural rate of interest. Finally, the paper contributes to the rapidly growing literature on the economic consequences of the COVID-19 pandemic and

[4] As in Christensen and Rudebusch (2019, henceforth CR), we take a longer-run perspective and define r_t^* as the average real short-term interest rate expected to prevail over a five-year period that starts five years ahead.

its aftermath.

The remainder of the paper is organized as follows. Section 2 contains a description of the German inflation-linked government bond data, while Section 3 details the no-arbitrage term structure models we use and presents the empirical results. Section 4 describes the estimated German inflation-linked safety premia, including an analysis of their empirical determinants, while Section 5 examines the German inflation-linked bond market response to the surprise cancellation of all future inflation-linked bond issuance. Section 6 analyzes our market-based estimates of the natural rate along with a comparison with other measures. Finally, Section 7 concludes.

2 The German Inflation-Linked Government Bond Data

This section briefly describes the available data downloaded from Bloomberg for the market for German inflation-linked bonds referencing the harmonized index for consumer prices (HICP) and officially known as Bund/€i's.[5] Here, we will refer to them throughout as GBi's, and their nominal fixed-coupon equivalents as simply bunds consistent with the literature.

To give a sense of the size of the German government bond market, we note up front that, as of the end of December 2024, the total outstanding notional amount of tradeable securities issued by the German Federal government was €1.882 trillion, of which €66,25 billion, or 3.5 percent, represented inflation-linked bonds.[6] Given the small size of the German government bond market relative to the German economy—with nominal GDP of €4.186 trillion in 2023—it is not surprising that the German government holds a triple-A rating with a stable outlook from all major rating agencies. Thus, there is effectively no credit risk to account for in the bond price data, and these bonds can be considered truly safe assets.

The German Federal government issued its first inflation-linked GBi bond referencing the HICP on March 15, 2006, several years after France and Italy, which issued their first such government bonds in 2001 and 2003, respectively.[7] Moreover, the German inflation-linked market is characterized by a very limited number of bonds most of which have had ten years or less time to maturity at issuance. These relatively short maturities set the German market apart from other inflation-linked bond markets—with the exception of Japan where

[5] We stress that these bonds are indexed using the euro area HICP index (ex. tobacco) without any seasonal adjustment, which is the benchmark for government bonds indexed to euro area inflation and the standard reference index for other financial products, most notably euro area inflation-linked swaps; see Ejsing et al. (2007).

[6] This information is available at
https://www.deutsche-finanzagentur.de/en/federal-securities/trading/tradeable-securities and
https://www.deutsche-finanzagentur.de/en/federal-securities/types-of-federal-securities/inflation-linked-federal-securities

[7] All auction information for German federal government securities back to January 1999 is available at: https://www.deutsche-finanzagentur.de/en/federal-securities/issuances/issuance-results

Inflation-linked bond	Number of obs.		Issuance		Total uplifted
	Daily	Monthly	Date	amount	notional amount
(1) 1.50% 4/15/2016	1,943	90	3/15/2006	5,500	15,000
(2) 2.25% 4/15/2013	1,162	54	10/26/2007	4,000	11,000
(3) 1.75% 4/15/2020	2,560	118	6/12/2009	3,000	16,000
(4) 0.75% 4/15/2018	1,562	72	4/15/2011	3,000	15,000
(5) 0.10% 4/15/2023	2,616	121	3/23/2012	2,000	16,500
(6) 0.50% 4/15/2030	2,787	129	4/10/2014	2,000	22,150
(7) 0.10% 4/15/2026	2,547	118	3/12/2015	2,000	19,200
(8) 0.10% 4/15/2046	2,477	115	6/16/2015	500	14,250
(9) 0.10% 4/15/2033	1,008	47	2/11/2021	1,500	10,650

Table 1: **Sample of German Inflation-Linked Government Bonds**
The table reports the characteristics, first issuance date and amount, and total uplifted notional amount outstanding either at maturity or as of September 30, 2024, in millions of euros for the sample of German inflation-linked government bonds. Also reported are the number of daily and monthly observations for each bond during the sample period from October 26, 2007, to December 30, 2024.

the government solely issues ten-year inflation-linked bonds; see Christensen and Spiegel (2022).

Table 1 contains the contractual details of all nine GBi's as well as the number of daily and monthly observations for each, while the time-varying maturity distribution of the nine GBi's in our sample is illustrated in Figure 1. Here, each security is represented by a downward-sloping line showing its remaining years to maturity at each date.

The limited set of bonds poses some challenges in modeling the term structure of interest rates in this market, but we use recently developed tools to deal with this technical complication. Moreover, these features combined also explain why few papers have examined this market in detail.

Figure 2 shows the yields to maturity for all nine German GBi bonds in our sample at daily frequency from October 26, 2007, to December 30, 2024.[8] Note the following regarding these yield series. First, we highlight the significant persistent decline in real yields over the first fifteen years of the sample as well as the notable sharp partial reversal during the last two years of the sample. Long-term real yields in the euro area were close to 2 percent in late 2007 and had dropped below -2 percent by late 2021 before retracing more than half of that decline by the end of our sample. Second, business cycle variation in the shape of the yield curve is pronounced around the lower trend. The yield curve tends to flatten ahead of recessions and steepen during the initial phase of economic recoveries. These characteristics are the practical motivation behind our choice of using a three-factor model for the frictionless part of the euro-area real yield curve, adopting an approach similar to what is standard for

[8]Our model estimation requires at least two observed bond prices for each observation date. This determines the start date for our sample.

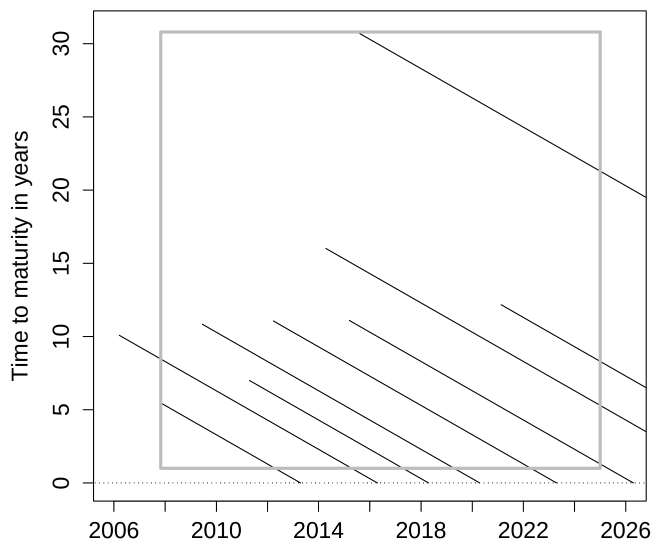


Figure 1: **Maturity Distribution of German Inflation-Linked Government Bonds**
Illustration of the maturity distribution of the available universe of German inflation-linked government bonds. The solid grey rectangle indicates the sample used in the empirical analysis, where the sample is restricted to start on October 26, 2007, and end on December 30, 2024, and limited to bond prices with more than one year to maturity.

U.S. and U.K. nominal yield data; see Christensen and Rudebusch (2012).

In unreported results, we note that the inflation index ratios for all nine German GBi bonds in our sample are all well above 1. Hence, none of these bonds have been exposed to any prolonged period of deflation, defined as periods with inflation index ratios below one. Indeed, thanks to the generally positive inflation environment in the euro area, the ratios tend to relatively quickly become significantly positive. This suggests that the deflation protection offered by these bonds is likely to be of modest value, similar to what Christensen and Mouabbi (2023) find for French government bonds indexed using the French CPI and known as OATi's. We therefore disregard this component in our analysis and leave it for future research to assess its value.

Before turning to our models and their estimation, we examine the bid-ask spreads of the German inflation-linked government bonds to provide support for the ACR approach to identify the bond-specific risk premia. The spreads are constructed by converting the bid and ask prices into the corresponding yield to maturity and calculating the difference with all data downloaded from Bloomberg. Figure 3 shows the bid-ask spread series for all nine German GBi bonds since February 2011, when the data become available on Bloomberg. All series

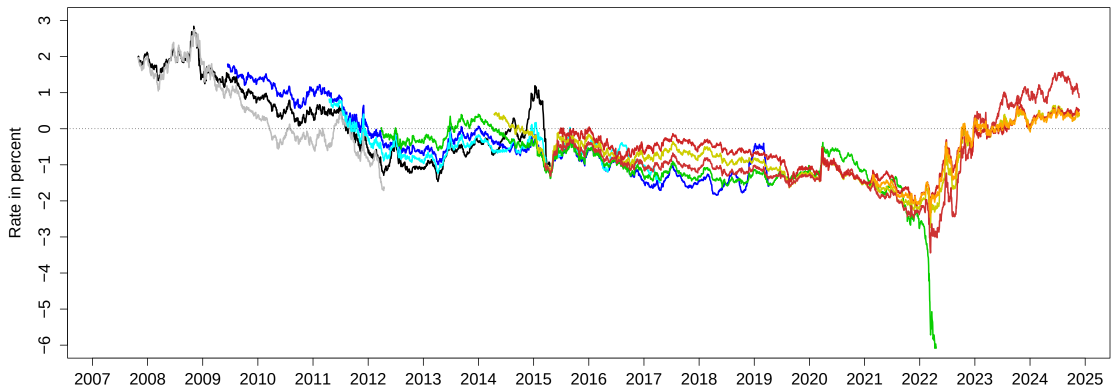


Figure 2: **Yield to Maturity of German Inflation-Linked Government Bonds**
Illustration of the yield to maturity implied by the German inflation-linked government bond prices considered in this paper, which are subject to two sample choices: (1) sample limited to the period from October 26, 2007, to December 30, 2024; (2) censoring of a bond's price when it has less than one year to maturity. Each bond yield series is shown with its own colored line.

are smoothed four-week moving averages and measured in basis points. Similar to what ACR document for U.S. TIPS, the German GBi bid-ask spreads are systematically wider for more seasoned bonds than for recently issued bonds. Rational, forward-looking investors are aware of these dynamics and the fact that future market liquidity of a given bond is likely to be below its current market liquidity. This gives rise to bond-specific premia in the bond prices. This pattern in observed measures of *current* market liquidity of German GBi's is consistent with the factor loading of the bond-specific risk factor in our approach that is intended to model the effects on current GBi prices of expected *future* market demand conditions. Although the natural interpretation of these premia would be to think of them as liquidity discounts, we note that, given the high credit quality of these bonds, they may be viewed by investors as very safe assets and hence trade at a safety premium; see Christensen and Mirkov (2022). We stress that the model we use is flexible enough to accommodate either of these outcomes.

3 Model Estimation and Results

In this section, we first describe how we model yields in a world without any frictions to trading. This model of frictionless dynamics is fundamental to our analysis. We then detail the augmented model that accounts for the bond-specific premia in the inflation-linked bond yields. This is followed by a description of the restrictions imposed to achieve econometric identification of this model and its estimation. We end the section with a brief summary of

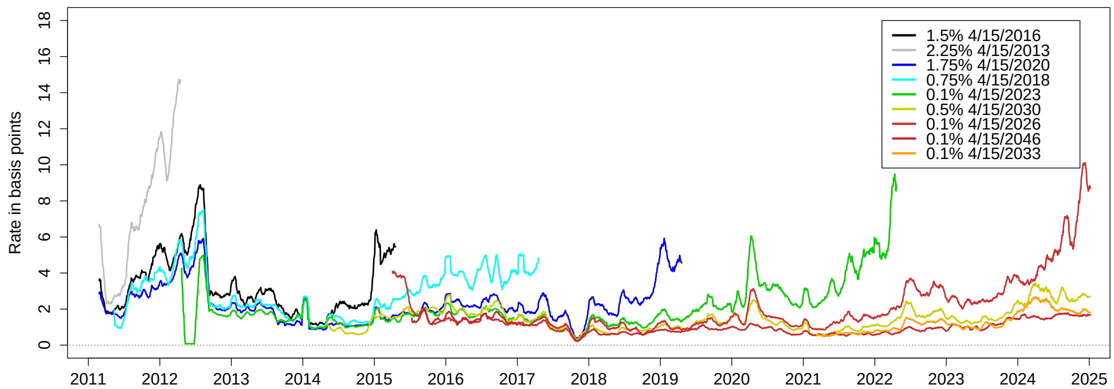


Figure 3: **Bid-Ask Spreads of German Inflation-Linked Government Bonds**
Illustration of the four-week moving average of bid-ask spreads of German inflation-linked government bonds constructed as explained in the main text. The series are daily covering the period from February 22, 2011, to December 30, 2024.

our estimation results.

3.1 A Frictionless Arbitrage-Free Model of Real Yields

To capture the fundamental or frictionless factors operating the German GBi real yield curve, we choose to focus on the tractable affine dynamic term structure model introduced in Christensen et al. (2011).[9]

In this arbitrage-free Nelson-Siegel (AFNS) model, the state vector is denoted by $X_t = (L_t, S_t, C_t)$, where L_t is a level factor, S_t is a slope factor, and C_t is a curvature factor. The instantaneous risk-free real rate is defined as

$$r_t = L_t + S_t. \quad (1)$$

The risk-neutral (or $\mathbb{Q}$-) dynamics of the state variables used for pricing are given by the

[9] Although the model is not formulated using the canonical form of affine term structure models introduced by Dai and Singleton (2000), it can be viewed as a restricted version of the canonical Gaussian model; see Christensen et al. (2011) for details.

stochastic differential equations[10]

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda & \lambda \\ 0 & 0 & -\lambda \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{Q}} \\ dW_t^{S,\mathbb{Q}} \\ dW_t^{C,\mathbb{Q}} \end{pmatrix}, \tag{2}$$

where Σ is the constant covariance (or volatility) matrix that is assumed to be diagonal, as recommended by Christensen et al. (2011).[11] Based on this specification of the $\mathbb{Q}$-dynamics, real zero-coupon bond yields preserve the Nelson-Siegel factor loading structure as

$$y_t(\tau) = L_t + \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t - \frac{A(\tau)}{\tau}, \tag{3}$$

where $A(\tau)$ is a convexity term that adjusts the functional form in Nelson and Siegel (1987) to ensure absence of arbitrage (see Christensen et al. (2011)).

To complete the description of the model and to implement it empirically, we will need to specify the risk premia that connect these factor dynamics under the $\mathbb{Q}$-measure to the dynamics under the real-world (or physical) $\mathbb{P}$-measure. It is important to note that there are no restrictions on the dynamic drift components under the empirical $\mathbb{P}$-measure beyond the requirement of constant volatility. To facilitate empirical implementation, we use the essentially affine risk premium specification introduced in Duffee (2002). In the Gaussian framework, this specification implies that the risk premia Γ_t depend on the state variables; that is,

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^3$ and $\gamma^1 \in \mathbf{R}^{3\times 3}$ contain unrestricted parameters.

Thus, the resulting unrestricted three-factor AFNS model has $\mathbb{P}$-dynamics given by

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} \kappa_{11}^{\mathbb{P}} & \kappa_{12}^{\mathbb{P}} & \kappa_{13}^{\mathbb{P}} \\ \kappa_{21}^{\mathbb{P}} & \kappa_{22}^{\mathbb{P}} & \kappa_{23}^{\mathbb{P}} \\ \kappa_{31}^{\mathbb{P}} & \kappa_{32}^{\mathbb{P}} & \kappa_{33}^{\mathbb{P}} \end{pmatrix} \left(\begin{pmatrix} \theta_1^{\mathbb{P}} \\ \theta_2^{\mathbb{P}} \\ \theta_3^{\mathbb{P}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} \right) dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{P}} \\ dW_t^{S,\mathbb{P}} \\ dW_t^{C,\mathbb{P}} \end{pmatrix}.$$

This is the transition equation in the Kalman filter estimation.

3.2 An Arbitrage-Free Model of Real Yields with Bond-Specific Risk

In this section, we augment the frictionless AFNS model introduced above to account for any bond-specific risk premia embedded in the GBi prices. To do so, let $X_t = (L_t, S_t, C_t, X_t^R)$

[10] As discussed in Christensen et al. (2011), with a unit root in the level factor, the model is not arbitrage-free with an unbounded horizon; therefore, as is often done in theoretical discussions, we impose an arbitrary maximum horizon.

[11] As per Christensen et al. (2011), $\theta^{\mathbb{Q}}$ is set to zero without loss of generality.

denote the state vector of the four-factor AFNS-R model with bond-specific risk premium adjustment. As in the non-augmented model, we let the frictionless instantaneous real risk-free rate be defined by equation (1), while the risk-neutral dynamics of the state variables used for pricing are given by

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \\ dX_t^R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda & -\lambda & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \kappa_R^{\mathbb{Q}} \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ \theta_R^{\mathbb{Q}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \\ X_t^R \end{pmatrix} \right] dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{Q}} \\ dW_t^{S,\mathbb{Q}} \\ dW_t^{C,\mathbb{Q}} \\ dW_t^{R,\mathbb{Q}} \end{pmatrix},$$

where Σ continues to be a diagonal matrix.

In the augmented model, GBi yields are sensitive to bond-specific risks because the net present value of their future cash flow is calculated using the following discount function:

$$\overline{r}^i(t, t_0^i) = r_t + \beta^i(1 - e^{-\lambda^{R,i}(t-t_0^i)})X_t^R = L_t + S_t + \beta^i(1 - e^{-\lambda^{R,i}(t-t_0^i)})X_t^R. \qquad (4)$$

CR show that the net present value of one unit of consumption paid by GBi bond i at time $t+\tau$ has the following exponential-affine form

$$\begin{aligned} P_t(t_0^i, \tau) &= E^{\mathbb{Q}}\left[e^{-\int_t^{t+\tau} \overline{r}^i(s,t_0^i)ds}\right] \\ &= \exp\Big(B_1(\tau)L_t + B_2(\tau)S_t + B_3(\tau)C_t + B_4(t, t_0^i, \tau)X_t^R + A(t, t_0^i, \tau)\Big). \end{aligned}$$

This result implies that the model belongs to the class of Gaussian affine term structure models. Note also that, by fixing $\beta^i = 0$ for all i, we recover the AFNS model.

Now, consider the whole value of GBi bond i issued at time t_0^i with maturity at $t + \tau^i$ that pays an annual coupon C^i. Its price is given by[12]

$$\begin{aligned} \overline{P}_t(t_0^i, \tau^i, C^i) &= C^i(t_1 - t)E^{\mathbb{Q}}\left[e^{-\int_t^{t_1} \overline{r}^{R,i}(s,t_0^i)ds}\right] + \sum_{j=2}^{N} C^i E^{\mathbb{Q}}\left[e^{-\int_t^{t_j} \overline{r}^{R,i}(s,t_0^i)ds}\right] \\ &\quad + E^{\mathbb{Q}}\left[e^{-\int_t^{t+\tau^i} \overline{r}^{R,i}(s,t_0^i)ds}\right]. \end{aligned}$$

There are only two minor omissions in this bond pricing formula. First, it does not account for the lag in the inflation indexation of the GBi bond payoff. The potential error from this omission should be modest (see Grishchenko and Huang 2013), especially as we exclude bonds from our sample when they have less than one year of maturity remaining. Second, we do not account for the value of deflation protection offered by GBi bonds, as already noted. However,

[12]This is the clean price that does not account for any accrued interest and maps to our observed bond prices.

Christensen and Mouabbi (2023) find these values to be very small for French OATi indexed to the French consumer price index, and, given that HICP inflation has run quite a bit above French CPI inflation during our sample, the value of this protection for GBi bonds is likely to be entirely negligible.

Finally, to complete the description of the AFNS-R model, we again specify an essentially affine risk premium structure, which implies that the risk premia Γ_t take the form

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^4$ and $\gamma^1 \in \mathbf{R}^{4\times4}$ contain unrestricted parameters. Thus, the resulting unrestricted four-factor AFNS-R model has $\mathbb{P}$-dynamics given by

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \\ dX_t^R \end{pmatrix} = \begin{pmatrix} \kappa_{11}^{\mathbb{P}} & \kappa_{12}^{\mathbb{P}} & \kappa_{13}^{\mathbb{P}} & \kappa_{14}^{\mathbb{P}} \\ \kappa_{21}^{\mathbb{P}} & \kappa_{22}^{\mathbb{P}} & \kappa_{23}^{\mathbb{P}} & \kappa_{24}^{\mathbb{P}} \\ \kappa_{31}^{\mathbb{P}} & \kappa_{32}^{\mathbb{P}} & \kappa_{33}^{\mathbb{P}} & \kappa_{34}^{\mathbb{P}} \\ \kappa_{41}^{\mathbb{P}} & \kappa_{42}^{\mathbb{P}} & \kappa_{43}^{\mathbb{P}} & \kappa_{44}^{\mathbb{P}} \end{pmatrix} \left(\begin{pmatrix} \theta_1^{\mathbb{P}} \\ \theta_2^{\mathbb{P}} \\ \theta_3^{\mathbb{P}} \\ \theta_4^{\mathbb{P}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \\ X_t^R \end{pmatrix} \right) dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{P}} \\ dW_t^{S,\mathbb{P}} \\ dW_t^{C,\mathbb{P}} \\ dW_t^{R,\mathbb{P}} \end{pmatrix}.$$

This is the transition equation in the Kalman filter estimation.

3.3 Model Estimation and Econometric Identification

Due to the nonlinear relationship between the state variables and the bond prices, the model cannot be estimated with the standard Kalman filter. Instead, we use the extended Kalman filter as in Kim and Singleton (2012); see CR for details. Furthermore, to make the fitted errors comparable across bonds of various maturities, we scale each bond price by its duration. Thus, the measurement equation for the bond prices takes the following form

$$\frac{P_t^i(t_0^i, \tau^i)}{D_t^i(t_0^i, \tau^i)} = \frac{\widehat{P}_t^i(t_0^i, \tau^i)}{D_t^i(t_0^i, \tau^i)} + \varepsilon_t^i,$$

where $\widehat{P}_t^i(t_0^i, \tau^i)$ is the model-implied price of bond i and $D_t^i(t_0^i, \tau^i)$ is its duration, which is calculated before estimation. See Andreasen et al. (2019) for evidence supporting this formulation of the measurement equation.

Furthermore, since the bond-specific risk factor is a latent factor that we do not observe, its level is not identified without additional restrictions. As a consequence, we let the first GBi bond, which was issued on March 15, 2006, before the start of our sample, have a unit loading on this factor, that is, this ten-year bond maturing on April 15, 2016, with 1.50 percent coupon has $\beta^i = 1$. This choice implies that the β^i sensitivity parameters measure bond-specific risk sensitivity relative to that of the ten-year 2016 GBi bond.

Finally, we note that the $\lambda^{R,i}$ parameters can be hard to identify if their values are too

GBi bond	Pricing errors				Estimated parameters			
	AFNS		AFNS-R		AFNS-R			
	Mean	RMSE	Mean	RMSE	β^i	SE	$\lambda^{R,i}$	SE
(1) 1.50% 4/15/2016	0.46	3.26	0.36	2.08	1	n.a.	0.1736	0.0375
(2) 2.25% 4/15/2013	-0.40	2.95	-0.05	2.17	0.9194	0.1342	3.1314	3.3620
(3) 1.75% 4/15/2020	0.27	2.74	-0.02	1.33	79.9708	6.2587	0.0008	0.0000
(4) 0.75% 4/15/2018	0.25	3.16	0.02	1.73	0.5612	0.0523	8.6256	5.5970
(5) 0.10% 4/15/2023	-0.32	7.12	-0.08	6.80	42.6115	4.8361	0.0018	0.0001
(6) 0.50% 4/15/2030	-0.54	2.81	0.73	1.53	0.6769	0.2152	0.5032	0.2326
(7) 0.10% 4/15/2026	1.22	3.70	0.92	1.65	0.9452	0.1916	0.1970	0.0615
(8) 0.10% 4/15/2046	1.32	3.16	0.85	1.46	0.1755	0.4254	10.0000	6.8645
(9) 0.10% 4/15/2033	1.77	3.17	0.78	1.92	0.5360	0.2604	10.0000	6.0023
All yields	0.39	3.94	0.41	2.99	-	-	-	-
Max $\mathcal{L}^{EKF}$	4,434.62		4,698.04		-		-	

Table 2: **Pricing Errors and Estimated Bond-Specific Risk Parameters**
This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of German inflation-linked bonds in the AFNS and AFNS-R models estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ. The errors are computed as the difference between the German GBi bond market price expressed as yield to maturity and the corresponding model-implied yield. All errors are reported in basis points. Standard errors (SE) are not available (n.a.) for the normalized value of β^1.

large or too small. As a consequence, we follow ACR and impose the restriction that they fall within the range from 0.0001 to 10, which is without practical consequences, as demonstrated by Christensen and Mouabbi (2023). Also, for numerical stability during model optimization, we impose the restriction that the β^i parameters fall within the range from 0 to 250, which turns out not to be a binding constraint for any of the nine bonds in our sample. Hence, this constraint is also without practical consequences.

3.4 Estimation Results

This section presents our benchmark estimation results. In the interest of simplicity, in this section we focus on a version of the AFNS-R model where $K^{\mathbb{P}}$ and Σ are diagonal matrices. As shown in ACR, these restrictions have hardly any effects on the estimated bond-specific risk premium for each inflation-linked bond, because it is identified from the model's $\mathbb{Q}$-dynamics, which are independent of $K^{\mathbb{P}}$ and only display a weak link to Σ through the small convexity adjustment in the bond yields. Furthermore, we stress that we relax this assumption in Section 6 when we analyze estimates of r_t^*, which are indeed sensitive to the specification of the models' $\mathbb{P}$-dynamics.

Table 2 reports the summary statistics for the fitted errors of individual GBi bonds as well as for all bonds combined. Note that there is uniform improvement in model fit from incorporating the bond-specific risk factor into the AFNS model. Still, it is worth noting that the AFNS model is able to deliver a root mean-squared fitted error of 3.9 basis points

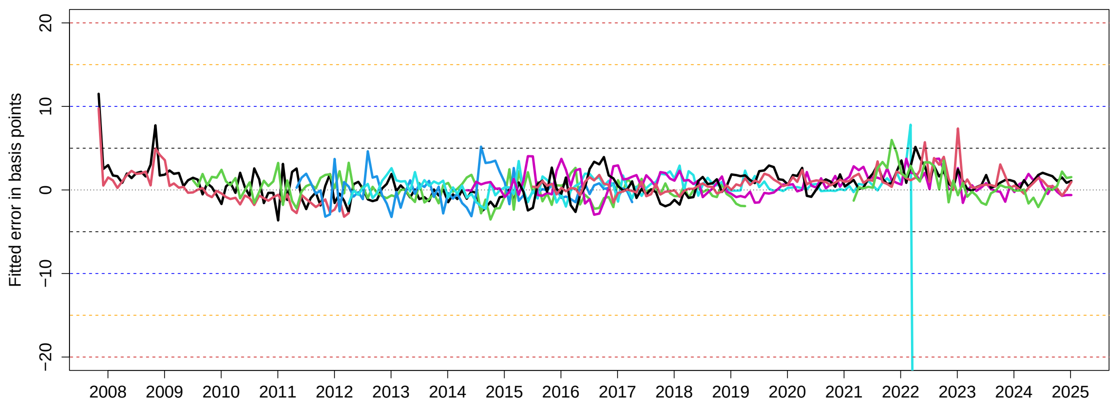


Figure 4: **Fitted Errors of GBi Bond Yields**
Illustration of the fitted errors of GBi yields to maturity implied by the AFNS-R model estimated at monthly frequency for the period from October 31, 2007, to December 30, 2024.

across all bonds combined, which in general could be characterized as a satisfactory fit, but obviously not as good as the RMSE of 3.0 basis points for all bonds combined achieved by the AFNS-R model, which represents a really good fit to the entire cross section of yields. This salient fit is also on display in Figure 4, which shows the individual fitted error series from the AFNS-R model. With the exception of a single observation from a single bond, the model consistently provides a very accurate fit to the cross section of bonds.

Table 3 contains the estimated dynamic parameters. Note that the dynamics of the first three factors are rather different across the two estimations. Although the estimated mean parameters are comparable for the first three factors, their estimated mean-reversion and volatility parameters are notably larger for the slope and curvature factor in the AFNS model. Hence, the frictionless dynamics of the state variables within the AFNS-R model are broadly somewhat more persistent and less volatile. Furthermore, λ is smaller in the AFNS-R model. This implies that the yield loadings of the slope factor decays toward zero more slowly as the maturity increases. At the same time, the peak of the curvature yield loadings is located at a later maturity compared with its loading in the AFNS model. As a consequence, slope and curvature matter more for longer-term yields in the AFNS-R model. This helps explain part of the better fit to the entire cross section of bonds within that model. Finally, the bond-specific risk factor is the least persistent and most volatile factor in the AFNS-R model. Moreover, it is even more stationary under the risk-neutral $\mathbb{Q}$-measure used for pricing and with a negative mean equal to -0.0060. The high value of $\kappa_R^{\mathbb{Q}}$ combined with the negative value of $\theta_R^{\mathbb{Q}}$ suggests that the bond-specific risk premia are likely to be mostly negative, a

Parameter	AFNS		AFNS-R	
	Est.	SE	Est.	SE
$\kappa_{11}^{\mathbb{P}}$	0.0433	0.0257	0.2046	0.1696
$\kappa_{22}^{\mathbb{P}}$	0.8603	0.0286	0.3856	0.3094
$\kappa_{33}^{\mathbb{P}}$	1.4287	0.0279	0.4500	0.2864
$\kappa_{44}^{\mathbb{P}}$	-	-	1.1776	0.5324
σ_{11}	0.0045	0.0001	0.0072	0.0004
σ_{22}	0.0215	0.0014	0.0132	0.0014
σ_{33}	0.0274	0.0019	0.0143	0.0018
σ_{44}	-	-	0.2434	0.1656
$\theta_1^{\mathbb{P}}$	0.0224	0.0142	0.0182	0.0087
$\theta_2^{\mathbb{P}}$	-0.0083	0.0073	-0.0074	0.0094
$\theta_3^{\mathbb{P}}$	-0.0341	0.0063	-0.0226	0.0097
$\theta_4^{\mathbb{P}}$	-	-	0.0279	0.0666
λ	0.3987	0.0060	0.3055	0.0294
$\kappa_R^{\mathbb{Q}}$	-	-	9.7329	6.5325
$\theta_R^{\mathbb{Q}}$	-	-	-0.0060	0.0015
σ_y	0.0005	1.71×10^{-5}	0.0002	1.76×10^{-5}

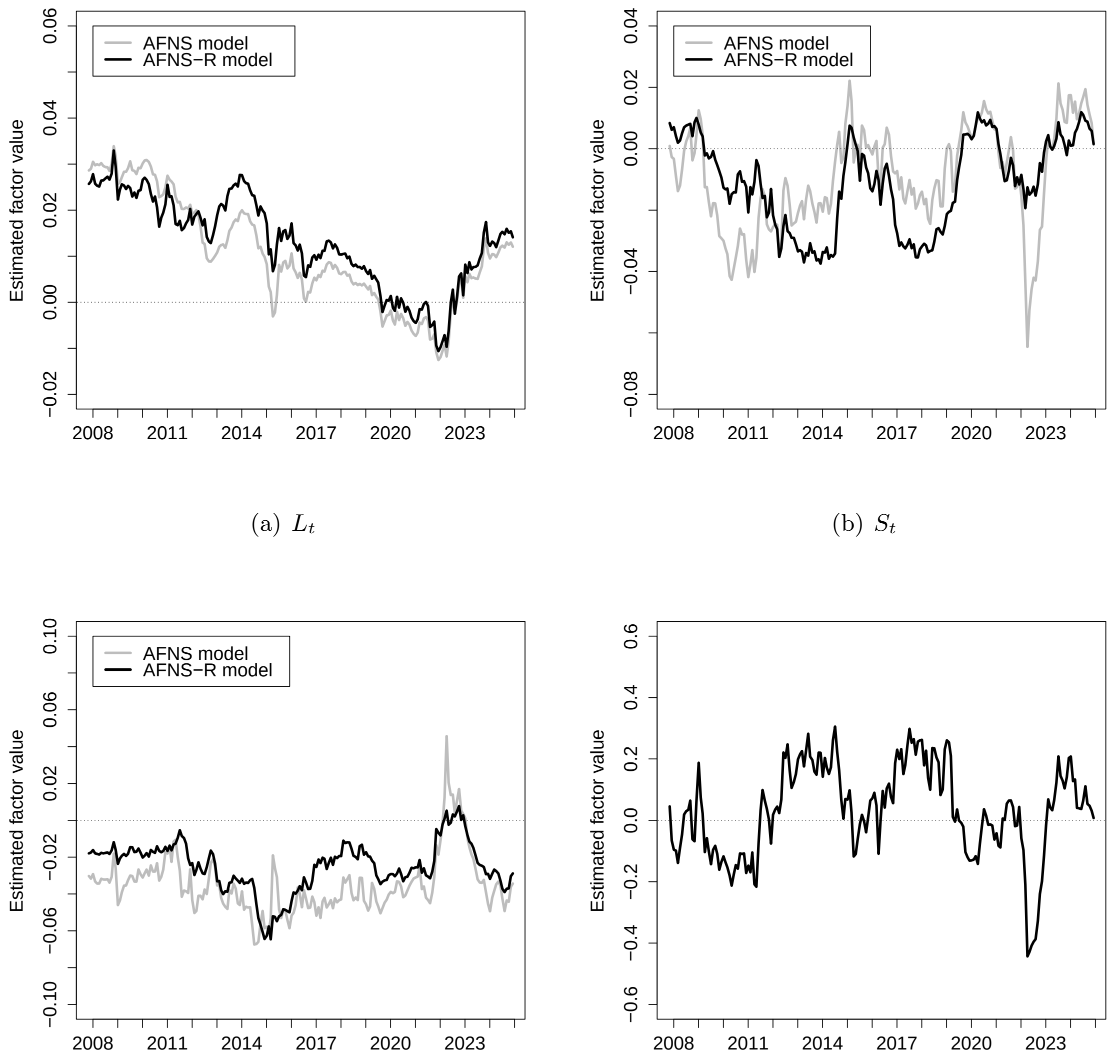
Table 3: **Estimated Dynamic Parameters**
The table shows the estimated dynamic parameters for the AFNS and AFNS-R models estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ.

conjecture we confirm in the following section.

The estimated paths of the level, slope, and curvature factors from the two models are shown in Figure 5. While the two models' level factors are fairly close to each other most of the time, their slope and curvature factors tend to have wedges between them. However, they generally operate at similar levels and frequently move in tandem. Hence, the roles of these three factors within each model can be characterized as broadly similar. Importantly, there are some sharp spikes in the data that are ascribed to the slope and curvature factors within the AFNS model, while they appear to be captured by the bond-specific factor within the AFNS-R model. This also helps explain why the slope and curvature factors are less volatile within the AFNS-R model compared to the AFNS model.

4 The GBi Bond-Specific Risk Premium

In this section, we analyze the German GBi bond-specific risk premia implied by the estimated AFNS-R model described in the previous section. First, we formally define the bond-specific risk premium, study its historical evolution, and compare it with other estimates from the literature before we briefly assess its sensitivity to the assumed factor dynamics and the data frequency used in the model estimation. We end the section with a regression analysis that



(a) L_t

(b) S_t

(c) C_t

(d) X_t^R

Figure 5: **Estimated State Variables**
Illustration of the estimated state variables from the AFNS and AFNS-R models.

examines the determinants of the GBi bond-specific risk premia.

4.1 The Estimated GBi Bond-Specific Risk Premia

We now use the estimated AFNS-R model to extract the bond-specific risk premium in the GBi market. To compute these premia, we first use the estimated parameters and the filtered states $\left\{X_{t|t}\right\}_{t=1}^{T}$ to calculate the fitted GBi prices $\left\{\hat{P}_t^i\right\}_{t=1}^{T}$ for all outstanding GBi securities in our sample. These bond prices are then converted into yields to maturity $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^{T}$ by

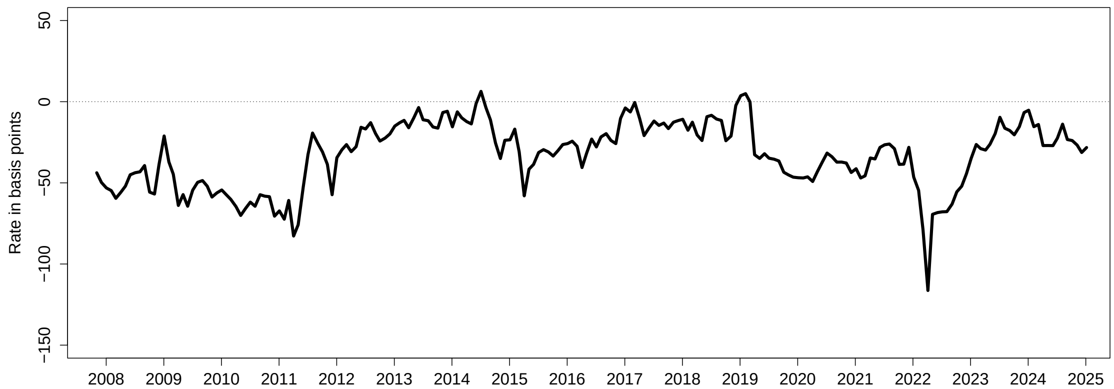


Figure 6: **Average Estimated GBi Bond-Specific Risk Premium**
Illustration of the average estimated bond-specific risk premium of GBis for each observation date implied by the AFNS-R model estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ. The bond-specific risk premia are measured as the estimated yield difference between the fitted yield to maturity of individual GBi's and the corresponding frictionless yield to maturity with the bond-specific risk factor turned off. The data are monthly and cover the period from October 31, 2007, to December 30, 2024.

solving the fixed-point problem

$$\begin{aligned}\hat{P}_t^i &= C(t_1 - t)\exp\left\{-(t_1 - t)\hat{y}_t^{c,i}\right\} + \sum_{k=2}^{n} C\exp\left\{-(t_k - t)\hat{y}_t^{c,i}\right\} \\ &\quad + \exp\left\{-(T - t)\hat{y}_t^{c,i}\right\},\end{aligned} \tag{5}$$

for $i = 1, 2, ..., n_{GBi}$, meaning that $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^{T}$ is approximately the real rate of return on the ith GBi bond if held until maturity (see Sack and Elsasser 2004). To obtain the corresponding yields with correction for the bond-specific risk premia, we compute a new set of model-implied bond prices from the estimated AFNS-R model using only its frictionless part, i.e., using the constraints that $X_{t|t}^R = 0$ for all t as well as $\sigma_{44} = 0$ and $\theta_R^Q = 0$. These prices are denoted $\left\{\tilde{P}_t^i\right\}_{t=1}^{T}$ and converted into yields to maturity $\tilde{y}_t^{c,i}$ using equation (5). They represent estimates of the prices that would prevail in a world without any financial frictions or special demands for certain bonds. The bond-specific risk premium for the ith GBi bond is then defined as

$$\Psi_t^i \equiv \hat{y}_t^{c,i} - \tilde{y}_t^{c,i}. \tag{6}$$

Figure 6 shows the average estimated GBi bond-specific risk premium $\bar{\Psi}_t$ across the outstanding GBi bonds at each point in time. Note that a negative value means that the fitted

GBi bond price is *above* the model-implied frictionless price, i.e., GBi bond prices are higher than they should be in a world without any frictions. Importantly, the mean of the series is -0.33 percent. Thus, on average, GBi bond prices are higher than they are likely to be in a frictionless world without any excess demand for safe assets. Given the relatively low liquidity of these bonds, this convenience premium cannot be a consequence of their moneyness as it is challenging to trade these bonds in large volumes on short notice. Instead, we follow Christensen and Mirkov (2022) and interpret it as a safety premium investors are willing to pay thanks to the high credit quality of these bonds. Moreover, there are some detectable trends and time variation in the series, which explains its standard variation of 9.46 basis points. These safety premia were increasing and approaching zero on average during the European Sovereign Debt Crisis in the 2011-2013 period. This suggests that even German government bonds were perceived as less safe investments during that challenging period. The average premium then trended sideways slightly below zero until 2019. It then experienced a pronounced and persistent decline that lasted until spring 2022. Hence, according to our model, a notable part of the decline in GBi yields during the pandemic years reflected declines in the bond-specific safety premia. We take this to indicate that GBi's regained their status as a very safe class of bonds. Notably, the average safety premium got a boost and dropped below -100 basis points in March 2022 when HICP inflation was highly elevated. We interpret this drop as an added convenience premium arising from the fact that, when inflation is highly elevated, inflation-linked bonds like GBi's become convenient assets to hold. This boost was short-lived, though, as ECB and other central banks responded forcefully to the inflation spike by tightening monetary policy significantly. We take the relatively quick normalization of the estimated safety premium series as a sign that investors did not expect the high inflation to last for very long, an indicator of central bank credibility of sorts. As a consequence, during the remaining part of our sample, the average estimated bond-specific safety premium was close to its historical average.

To summarize, we feel that the average estimated GBi bond-specific safety premium broadly follows a reasonable time series pattern that aligns well with the safety premium interpretation that we adopt henceforth.

As an additional validation exercise and to put our average estimated safety premium from the market for German GBi's into an international context, we compare it with similar estimates from two other major bond markets, specifically the market for French OAT€s with cash flows also adjusted to the HICP examined by CM and the much larger market for standard German bunds studied in Christensen et al. (2025). Figure 7 shows the respective average estimated bond-specific safety premium series from these three major euro area bond markets.

As already noted, the estimated bond-specific safety premia for German GBi's average -33

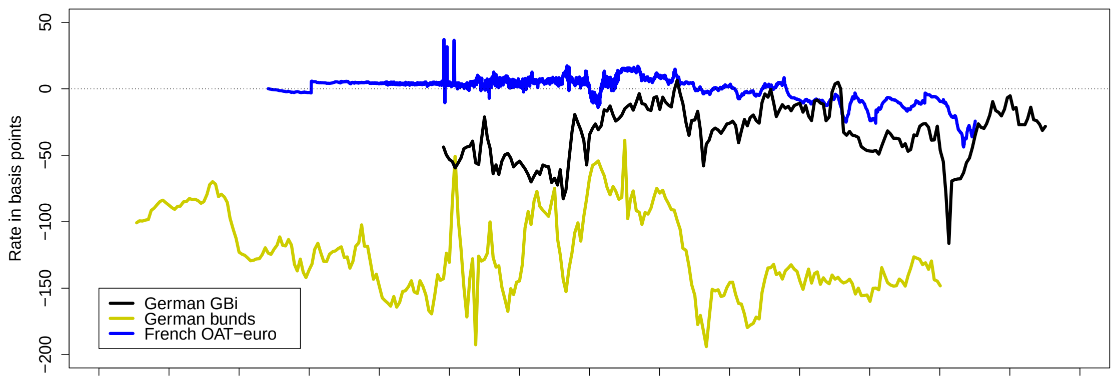


Figure 7: **Comparison of Average Estimated Bond-Specific Safety Premia**
Illustration of the average estimated bond-specific safety premium of German GBi bonds implied by the AFNS-R model estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ. Also shown are the average estimated bond-specific safety premium of standard German bunds reported by Christensen et al. (2025) and the average estimated bond-specific safety premium of French OAT€s reported by CM.

basis points over our sample period from October 2007 to December 2024, while the estimated safety premia of French OAT€s average -1 basis points for the shown period from October 2002 to December 2022. Hence, our results contrast with those reported by CM for French OAT€s in that the latter market contains no detectable safety premium. This sizable and persistent wedge in the pricing of bonds across these two otherwise seemingly similar markets implies that each market should be analyzed separately, just like nominal French OATs and German bunds are analyzed in solation and not pooled. More importantly, we note that the estimated safety premia of German bunds average -124 basis points. Thus, the safety premia of nominal bunds are about four times larger than those estimated for the much smaller and less liquid market for GBi's.

To contrast these results for the euro area with those reported in the literature for the United States, we note that U.S. TIPS prices contain a sizable liquidity premium discount, which is well documented in the literature; see ACR, D'Amico et al. (2018), and Pflueger and Viceira (2016), among many others. Cardozo and Christensen (2024) offer a rationale for the illiquidity of inflation-linked bonds like TIPS. By being protected against inflation, indexed bonds are inherently less traded than nominal bonds. In addition, foreigners not exposed to the domestic price index do not benefit from owning them. Combined this significantly reduces their trading volumes and make the market for these bonds be dominated by patient domestic buy-and-hold investors. This drives up the search frictions in the over-the-counter

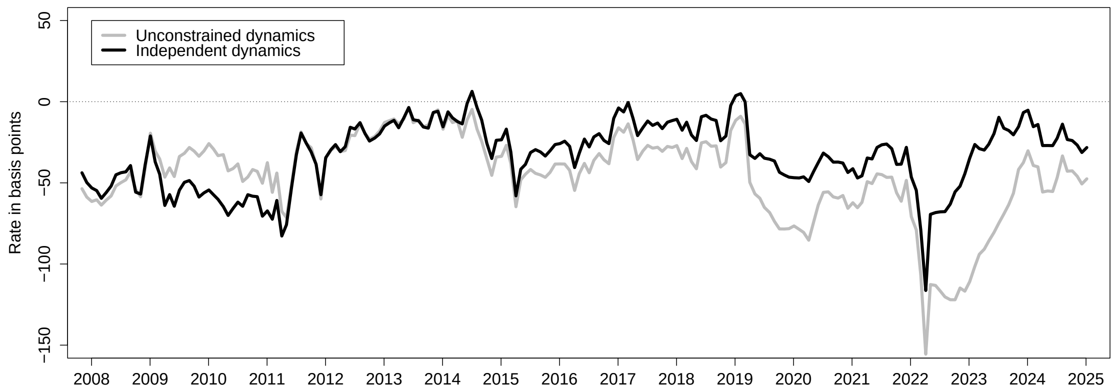


Figure 8: **Average Estimated GBi Bond-Specific Safety Premium: Alternative $\mathbb{P}$ Dynamics**
Illustration of the average estimated bond-specific safety premium of German GBi bonds for each observation date implied by the AFNS-R model when estimated with unconstrained dynamics as detailed in the text as well as independent factor dynamics with a diagonal specification of $K^{\mathbb{P}}$ and Σ. In both cases, the bond-specific safety premia are measured as the estimated yield difference between the fitted yield to maturity of individual GBi bonds and the corresponding frictionless yield to maturity with the bond-specific risk factor turned off.

market for these bonds and leads to a steady-state outcome with their prices containing a large liquidity discount. Here, we find that, thanks to their high credit quality, GBi bonds are able to overcome this inherent illiquidity by offering euro-area investors a really safe asset to store their wealth.

4.2 Robustness Analysis

This section examines the robustness of the average safety premium reported in the previous section to some of the main assumptions imposed so far. Throughout the section, the AFNS-R model with diagonal $K^{\mathbb{P}}$ and Σ matrices serves as the benchmark model.

First, we assess whether the specification of the dynamics within the AFNS-R model matters for the estimated GBi bond-specific safety premium. To do so, we estimate the AFNS-R model with unconstrained dynamics, that is, the AFNS-R model with unrestricted $K^{\mathbb{P}}$ matrix and lower triangular Σ matrix. Figure 8 shows the estimated GBi bond-specific safety premium from this estimation and compares it to the series produced by our benchmark model. Note that they are highly positively correlated. Thus, we conclude that the specification of the dynamics within the AFNS-R model only play a minor role for the estimated bond-specific safety premia, which is consistent with the findings of ACR in the context of U.S. TIPS.

Second, we assess whether the data frequency plays any role for our results. To do so, we

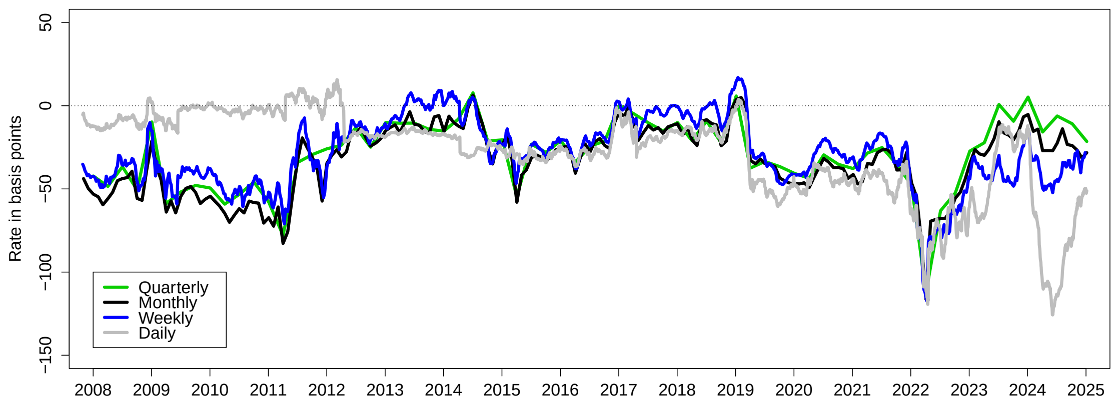


Figure 9: **Average Estimated GBi Bond-Specific Safety Premium: Data Frequency** Illustration of the average estimated bond-specific safety premium of German GBi bonds for each observation date implied by the AFNS-R model with a diagonal specification of $K^{\mathbb{P}}$ and Σ when estimated using daily, weekly, monthly, and quarterly data. In all cases, the bond-specific safety premia are measured as the estimated yield difference between the fitted yield to maturity of individual GBi bonds and the corresponding frictionless yield to maturity with the bond-specific risk factor turned off.

estimate the AFNS-R model using daily, weekly, monthly, and quarterly data, and based on the results above it suffices to focus on the most parsimonious AFNS-R model with diagonal $K^{\mathbb{P}}$ and Σ matrices. Figure 9 shows the average estimated GBi bond-specific safety premium series from all four estimations. In the ten-year period from 2012 to 2022 when there are consistently four or more GBi bonds outstanding, the state variables within the AFNS-R model are all well identified and not sensitive to the data frequency. As a consequence, the average estimated GBi bond-specific safety premia series are all very similar during this period. In contrast, prior to 2012 and towards the end of our sample when there are less than four GBi's trading, not all state variables in the model are fully identified. In that case, the estimated GBi bond-specific safety premia become sensitive to the shocks in the data, which vary with the data frequency. This explains the wider dispersion among the estimates in the early and later parts of our sample.

4.3 Determinants of the GBi Safety Premium

In this section, we explore which factors matter for the size of the bond-specific safety premia in the GBi bond prices. To explain the variation of the average estimated safety premium series, we run regressions with it as the dependent variable and a wide set of explanatory variables that are thought to play a role for the bond-specific safety premia as explained in the following.

To begin, we are interested in the role of factors that are believed to matter for GBi market liquidity specifically or bond market liquidity more broadly as they could matter for the estimated bond-specific safety premia. First, we include the average GBi bond age and the one-month realized volatility of the ten-year GBi bond yield as proxies for GBi bond liquidity following the work of Houweling et al. (2005).[13] Inspired by the analysis of Hu et al. (2013), we also include a noise measure of German bund prices to control for variation in the amount of arbitrage capital available in the German government bond market.[14] Finally, we add the euro overnight interbank rate to proxy for the opportunity cost of holding money and the associated liquidity premia of German government bonds, including GBi's, as explained in Nagel (2016). Combining these four explanatory variables tied to market liquidity and functioning produces the results reported in regression (1) in Table 4. We note a relatively modest adjusted R^2 of 0.36. The average GBi bond age, the one-month realized volatility of the ten-year GBi yield, and the noise measure all have statistically significant negative coefficients. This implies that an increase in the liquidity risk of GBi bonds is associated with *lower* average estimated bond-specific safety premia. In contrast, the overnight rate, which serves as a proxy for the opportunity cost of holding money, has a positive coefficient consistent with the mechanisms detailed in Nagel (2016). Overall, we take these results to show that our average estimated bond-specific safety premia in the GBi prices do not behave like traditional liquidity premium discounts, which seems reasonable given the fact that they are significantly negative on average and hence represent convenience safety premia.

After having explored the role of liquidity factors, we examine the effects of factors reflecting risk sentiment domestically and globally on the average estimated GBi bond-specific safety premia. This set of variables includes the VIX, which represents near-term uncertainty about the general stock market as reflected in options on the Standard & Poor's 500 stock price index and is widely used as a gauge of investor fear and risk aversion. The set also contains the yield difference between seasoned (off-the-run) U.S. Treasury securities and the most recently issued (on-the-run) U.S. Treasury security of the same ten-year maturity. This on-the-run (OTR) premium is a frequently used measure of financial frictions in the U.S. Treasury market. To control for factors related to the uncertainty about the interest rate environment, we include the MOVE index. The fourth variable is the U.S. TED spread, which is calculated as the difference between the three-month U.S. LIBOR and the three-month U.S. T-bill interest rate. This spread represents a measure of the perceived general credit risk

[13] The ten-year OAT€ bond yield is the ten-year fitted real yield implied by the estimated AFNS model.

[14] The noise measure is the mean absolute fitted error from an estimated arbitrage-free generalized Nelson-Siegel (AFGNS) model of German bund prices; see Christensen et al. (2009). Note that each error is calculated as the difference between the observed bund price converted into yield to maturity and the fitted bund price also converted into yield to maturity.

Explanatory variables	(1)	(2)	(3)
Avg. bond age (yrs)	-2.978*		-0.836
	(1.532)		(1.581)
One-month realized volatility of ten-year real yield (bps)	-0.876***		-0.668*
	(0.294)		(0.382)
Bund noise measure (bps)	-9.106***		-6.062***
	(1.678)		(2.300)
Overnight rate (%)	8.298***		6.261**
	(1.894)		(2.469)
VIX (%)		-1.049***	-0.703**
		(0.216)	(0.327)
Ten-year OTR premium (bps)		0.0696	0.209
		(0.207)	(0.604)
MOVE index (bps)		-0.233***	-0.230**
		(0.0750)	(0.116)
TED spread (bps)		0.124***	0.193*
		(0.0382)	(0.112)
Composite credit risk measure (bps)		0.190***	0.157**
		(0.0403)	(0.0730)
Ten-year US Treasury yield (%)		4.950***	3.042
		(1.897)	(3.156)
WTI ($)		-0.278***	-0.0962
		(0.0615)	(0.116)
Constant	16.74	0.705	11.88
	(10.28)	(6.135)	(10.57)
Observations	186	207	186
R^2	0.363	0.383	0.482

Table 4: **Regression Results for Average Estimated GBi Bond-Specific Safety Premium**
The table reports the results of regressions with the average estimated bond-specific safety premium of German GBi's as the dependent variable and 11 explanatory variables. Standard errors computed by the Newey-West estimator (with 3 lags) are reported in parentheses. Asterisks *, ** and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels, respectively.

in global financial markets. As an additional indicator of credit risk and credit risk sentiment across core euro area government bond markets, we use the composite measure of the credit risk of French inflation-linked government bonds vis-à-vis German inflation-linked government bonds constructed by CM. The next variable in the set is the ten-year U.S. Treasury yield from the Federal Reserve's H.15 database, which is included to control for reach-for-yield effects in advanced economies. This may be particularly relevant for our sample during the period between December 2008 and December 2015 and again in the 2020-2021 period when U.S. short-term interest rates were constrained by the zero lower bound. Finally, we include the West Texas Intermediate (WTI) Cushing crude oil price to proxy for energy prices, which represent a significant risk to the inflation outlook in many countries around the world, including many euro area member states. The results of the regression with these seven explanatory variables are reported in regression (2) in Table 4. This produces a slightly better adjusted R^2 of 0.38. We note that six of the seven variables have explanatory power as their estimated coefficients are statistically significant.

To assess the robustness of the results from the first two regressions, we include all 11 variables with the results reported in column (3) in Table 4. This joint regression produces a high adjusted R^2 of 0.48. The notable increase in the adjusted R^2 suggests that there is only modest overlap between the two sets of explanatory variables. The first set is squarely focused on the liquidity in the GBi market, while the second set represents global risk sentiment and flight-to-safety effects.

In the following, we elaborate on the interpretation of the estimated regression coefficients based on the results for the joint regression model reported in the last column of Table 4.

First, the negative coefficients on the bond age, the GBi ten-year yield volatility, and the noise measure point to some flight-to-safety effects whereby spells of heightened bond market liquidity risk seem to benefit the pricing of GBi's through lower and hence even more negative safety premia. The same interpretation applies to the significantly negative coefficients on the VIX and the MOVE index, i.e., increased risk aversion in the U.S. stock market as captured by the VIX and increased uncertainty about the interest rate environment as reflected in the MOVE index both correlate with more negative safety premia in GBi bond yields, equivalent to higher bond prices.

In contrast, the positive coefficients on the TED spread and the composite credit risk measure indicate that perceptions about credit risk in financial markets more broadly as represented by the TED spread or in sovereign bond markets in the euro area more narrowly as captured by our composite credit risk measure both push the estimated GBi safety premium series higher. Based on these results we conclude that some part of the estimated bond-specific safety premia seems to reflect compensation for credit risk, but we leave it for future research to examine that conjecture further.

Still, we take these findings to imply that systemic questions or fears about the solvency of the global financial system or that of core euro area governments as reflected in the TED spread and our composite credit risk measure are associated with diminishing safety premia in GBi prices. These observations are also consistent with the findings of Christensen and Mirkov (2022), who document a lasting upward shift in the safety premia of Danish and Swiss government bonds following the introduction of the euro in January 1999, meaning that these bonds became more valuable after the euro started circulating. The authors argue that the launch of the euro raised the prospect of potential scenarios with either a breakup of the euro area or some countries, presumably France or Germany, bailing out one or more other euro member states. Either way, such scenarios would bring into question the safe asset status of government bonds even in core euro-area countries and diminish their safety premia. The European Sovereign Debt Crisis represents an example of such outcomes as it involved an outright default by Greece along with material risks to the solvency of multiple other euro member states and even questions about the survival of the euro itself. Our results indicate that the safety premia of GBi bonds were greatly diminished during this period and its aftermath. In Figure 7, we also note a similar change to the safety premia of German bunds based on the estimates reported by Christensen et al. (2025).

In contrast, during spells of non-systemic shocks that merely reflect elevated liquidity risks in financial markets or heightened risk aversion among global investors as captured by several of our explanatory variables,[15] the GBi safety premia indeed behave like safety premia. We take these results to imply that GBi bonds behave like safe assets in the sense that investors are not (fire) selling these bonds during such spells of financial market turmoil. Instead, they seem to value these bonds even more highly as their safety premia tend to increase under those market conditions. However, we are reluctant to refer to these effects as flight-to-safety effects because GBi's are hard to buy and sell on short notice, and the involved volumes are most likely modest in comparison with the trading volumes in the standard bund market.

Returning to the remaining variables, higher U.S. Treasury yields are likely to make euro-area safe assets less attractive in a relative sense all else being equal. This seems to account for the estimated positive coefficient on the U.S. Treasury yield with higher yields entailing an increase in our estimated safety premium series, meaning a reduction in the excess price GBi's can command in the market. However, we note that this effect is insignificant in the joint regression model.

Finally, if energy prices increases as measured by the WTI oil price, inflation is likely to go up. In that case, inflation-indexed bonds become more convenient assets to hold. In our analysis, this shows up as a more negative safety premium and explains the negative

[15] These variables include the average bond age, the GBi ten-year yield volatility, the noise measure, the VIX, and the MOVE index.

coefficient on the WTI price series. However, we stress that this effect is not significant in the joint regression.

With the systematic negative coefficients on the liquidity risk variables—and even on the VIX and the MOVE index—we feel that we can confidently reject the conjecture that our average estimated bond-specific risk premia in the GBi prices should represent liquidity premia. Hence, the trading dynamics in the GBi market seem to be fundamentally different from those prevailing in the U.S. TIPS market, where liquidity premium discounts are a well-documented phenomenon. This is also consistent with the results reported by CM for the OAT€ market.

5 Termination of the GBi Program

In this section, we examine the market response to the German government's decision to terminate the GBi program announced on November 22, 2023.

5.1 The Announcement

The decision to cease issuance of GBi was made by the German Federal Finance Agency and communicated to the public through a simple press release on November 22, 2023. It contained the following two brief paragraphs:[16]

"*The Federal government has decided to withdraw from the market for inflation-linked bonds: From 2024, no further inflation-linked Federal securities will be issued, nor will already outstanding securities be reopened.*
The currently outstanding inflation-linked Federal securities will continue to be tradable on the market. The remaining programme comprises four securities with a current total volume of €66.25 billion and remaining maturities between 2.5 to 22.5 years."

The announcement caught investors and market observers by surprise as there had been no prior indication of any changes to the GBi program. Moreover, conversations with staff at the German Bundesbank indicate that Bundesbank was not consulted in this matter.

On its web site, the German Federal Finance Agency states:[17]

"*At €106 billion, the trading volume of inflation-linked Federal securities in 2023 was significantly lower than in the previous year – at €151 billion. Their share of the total trading volume of all Federal securities remained at the 2 percent recorded in 2022.*"

[16] The press release is available at: https://www.deutsche-finanzagentur.de/fileadmin/user_upload/Pressemitteilung/en/2023/2023_11_22_PM_08_Federal_government_discontinues_programme_for_inflation-linked_bonds.pdf.

[17] See https://www.deutsche-finanzagentur.de/en/federal-securities/types-of-federal-securities/inflation-linked-federal-securities.

GBi	0.10% 4/15/2026	0.50% 4/15/2030	0.10% 4/15/2033	0.10% 4/15/2046
Mat. (yrs)	2.40	6.40	9.40	22.40
11/21-2023	110.7	39.2	37.0	43.5
11/22-2023	116.1	41.7	38.4	40.7
11/23-2023	112.5	34.1	30.9	28.8
1-day change	5.4	2.5	1.4	-2.8
2-day change	1.8	-5.1	-6.1	-14.7

Table 5: **Response of German Inflation-Indexed Government Bond Yields to GBi Termination Announcement**

The table reports the one- and two-day responses of the outstanding German inflation-indexed bond yields to the announcement to terminate all future GBi issuance on November 22, 2023. All numbers are measured in basis points. The data are mid-market quoted yields to maturity at market close downloaded from Bloomberg.

5.2 The Market Reaction

In the following, we assess the impact of the termination of all future GBi issuance on both GBi yields directly and on our estimated GBi bond-specific risk premia.

Table 5 reports the one- and two-day yield changes for the GBi's outstanding at the time of the announcement. The results indicate a notable market reaction with a two-day change in the observed RRB yields of almost -15 basis points at the 22-year maturity. Importantly, though, the yield declines on November 23, 2023, were driven by investors digesting the minutes of the European Central Bank's October 2023 governing council meeting where policymakers were cautiously optimistic about inflation falling in the euro zone. Thus, we can only rely on the one-day responses as a guide to investors' reactions. These responses were all very timid. We interpret the yield increase for short- to medium-term GBi's as reflecting somewhat weaker expected liquidity going forward. In contrast, the yield decline for the single long-term GBi we take to reflect the positive impact of no competing supply coming to market for these long-term securities.

Since the responses in Table 5 reflect changes in yields to maturity, they are sensitive to both the bond coupon sizes and the shape of the underlying real yield curve and therefore hard to interpret and compare across bonds. For a cleaner read, Table 6 reports the responses of fitted real zero-coupon yields where we focus on the important five- to ten-year maturity range that is commonly used in the construction of breakeven inflation measures. Note the initial one-day reactions between 0 and 2 basis points, which turn negative with a two-day event window for the reasons already described earlier.

Maturity	5-year	6-year	7-year	8-year	9-year	10-year
11/21/2023	49.66	42.35	38.85	37.60	37.66	38.41
11/22/2023	51.76	43.96	40.03	38.44	38.20	38.70
11/23/2023	46.00	37.89	33.76	32.03	31.69	32.11
1-day change	2.10	1.60	1.18	0.84	0.54	0.29
2-day change	-3.66	-4.46	-5.08	-5.58	-5.97	-6.30

Table 6: **Response of German Real Zero-Coupon Yields to GBi Termination Announcement**
The table reports the two-day response of German real government zero-coupon bond yields to the announcement by the German Federal Finance Agency to permanently terminate its issuance of GBi bonds on November 22, 2023. All numbers are measured in basis points.

5.3 Impact on GBi Market Conditions

In this section, we explore how the decision affected the trading conditions and the functioning of the GBi market.

First, we examine bid-ask spreads of the outstanding set of GBi bonds. We think of these spreads as representative measures of the current trading conditions in the market for these bonds. Figure 3 shows four-week moving averages of the bid-ask spreads for each GBi. We first note the general upward trend in the bid-ask spread series caused by the fact that the bonds become more seasoned and less liquid as time passes. Importantly, though, there is no major change in the general bid-ask spread levels in the period following the November 22, 2023, announcement. Thus, the GBi trading conditions do not seem to have fundamentally changed.

Second, we assess whether there seems to have been any impact on the performance of the AFNS-R model and its ability to fit the GBi bond prices. To that end, we estimate the model using daily data instead of the monthly frequency considered so far. Figure 10 shows the resulting average estimated GBi bond-specific safety premium series since the start of 2020 through the end of December 2024. Importantly, there is hardly any reaction in the average estimated GBi bond-specific safety premium in the days following the announcement to cease all future GBi issuance. Thus, based on our model results, investors did not seem to worry much about the future liquidity in the GBi market despite no new supply being issued. Interestingly, though, the average estimated GBi bond-specific safety premium was clearly well below its historical average throughout 2022 and well into 2023, meaning that GBi's were trading at particularly high prices during this period that preceded the decision to cease future issuance. Normally, this would be the market signal to *increase* issuance. Hence, it seems ironic based on these results that it is precisely after an extended period with below average bond-specific safety premia that the German Federal Finance Agency decides

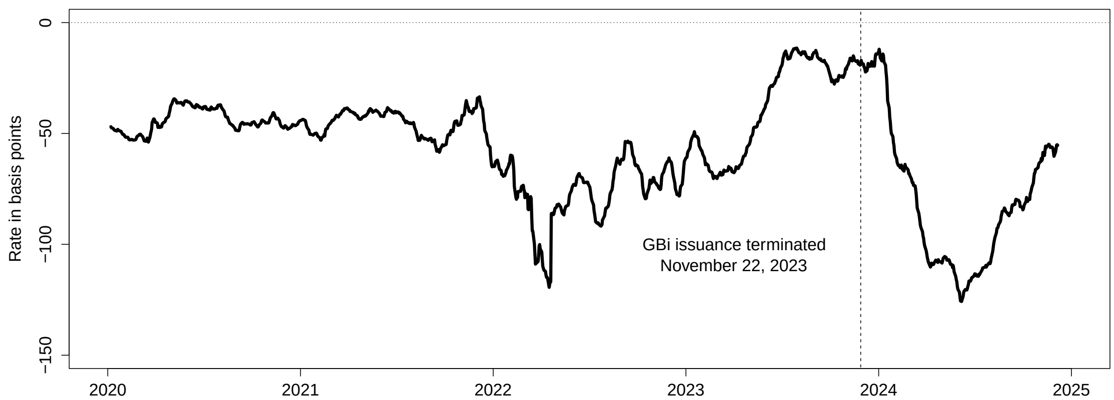


Figure 10: **Average Daily Estimated Bond-Specific Safety Premium since 2020**
Illustration of the average estimated GBi bond-specific safety premium for each observation date implied by the AFNS-R model estimated at daily frequency with a diagonal specification of $K^{\mathbb{P}}$ and Σ. The GBi bond-specific safety premia are measured as the estimated yield difference between the fitted yield-to-maturity of individual GBi bonds and the corresponding frictionless yield-to-maturity with the bond-specific risk factor turned off. The shown data cover the period from January 2, 2020, to November 29, 2024.

to terminate the GBi program.

As for the model fit specifically, Figure 4 shows the monthly fitted error series for all nine GBi bonds in our sample going back to October 2007. We note that there is no discernable tendency for larger or more volatile fitted errors since late 2023. Thus, the AFNS-R model has clearly maintained its ability to fit the cross section of GBi prices really well.

In related research, Christensen et al. (2024) analyze the announcement by the Canadian Finance Department on November 2, 2022, to permanently cease issuance of its inflation-linked bonds, known as Real Return Bonds (RRB). They also find a very modest response similar to our results above. This leads them to conclude that the decision seems to be premature and not rooted in any detailed analysis, and they encourage the Canadian government to relaunch its RRB program and increase the outstanding amount of these bonds.

To go beyond the analysis so far and that of Christensen et al. (2024), we gauge to what extent GBi's are priced differently than standard German bunds. To do so, we contrast the pricing of each of these two classes of bonds with their respective French counterpart. If bunds are priced fundamentally different than GBi's, which could be the case based on the notable difference between the average safety premia in the two markets documented in Figure 7, it should show up as differences in the yield spreads relative to their French counterparts. Specifically, as a representative measure of such spreads, we consider the five-year forward

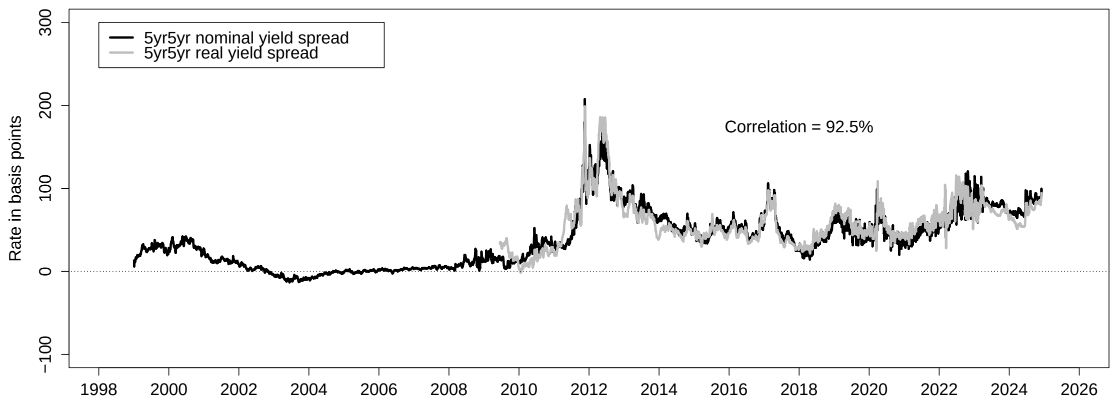


Figure 11: **Franco-German Government Bond Yield Spreads**

yield spread between French and German inflation-linked bond yields for a period starting five years ahead. Due to the late launch of the German inflation-linked government bond program, we can only construct this 5yr5yr Franco-German real yield spread starting in June 12, 2009. The available series since then through the end of our sample is shown with a solid grey line in Figure 11. We follow CM and interpret the 5yr5yr Franco-German real yield spread as mainly reflecting differences in credit risk premia rather than differences in liquidity risk premia.

We then calculate the matching 5yr5yr yield spread between nominal French OATs and German bunds, which is available back to January 1999 and shown with a solid black line in Figure 11. Given the high liquidity of German bunds and their associated flight-to-safety status during spells of elevated financial market illiquidity, we acknowledge that this spread likely reflects elements of both the liquidity and safety premium advantages of the bund market relative to the French market for OATs. Against that background the similarity between the 5yr5yr nominal and real Franco-German bond yield spreads is both evident and striking. For the overlapping period since June 12, 2009, with a total of 4,023 daily observations, the correlation in levels is an astonishing 92.5 percent, while the correlation in first differences at daily frequency is 76.2 percent. Equally importantly, the spreads are of nearly identical magnitudes with means of 58.8 basis points and 58.4 basis points, respectively, for the overlapping period, while their corresponding standard deviations are 27.6 basis points and 28.4 basis points, respectively. The visual and statistical similarity implies that any statistical test will suggest that these two spread series are statistically indistinguishable from each other.

The first immediate important implication of this finding is that it allows us to conclude that the Franco-German yield spread does not represent liquidity risk or flight-to-safety as nobody in their right mind would "fly" to safety in a GBi—in particular not after November 2023 when it is known that no new GBi supply will be issued. That said, the spreads do show spikes during flight-to-safety episodes: The European Sovereign Debt Crisis, the first Trump presidential victory in November 2016, and the peak of the COVID-19 pandemic, to name a few. However, these are all short-lived in nature, and the yield spreads do not revert to zero afterwards, but remain elevated with meaningful variation even in normal times. These observations combined lead us to conclude that these spreads reflect safety premia of the kind described in Christensen and Mirkov (2022) and Christensen et al. (2025). Based on this interpretation, we also note that the safety of German government bonds have increased relative to that of French government bonds since 2018 as both spread series have trended higher in tandem.

For the topic of this section about the benefit to the German government of issuing inflation-linked bonds, there is an equally important takeaway, namely that GBi's appear to be as beneficial to issue when benchmarked against their French inflation-linked equivalents as regular nominal German bunds vis-à-vis their French nominal OAT equivalents. The yield spread saved is statistically indistinguishable effectively at all times for the available fifteen-year period. As a consequence, it is nearly impossible to pinpoint a time when it would have made a material difference to only issue nominal bunds as opposed to the mix of bunds and GBi's that happened to actually be issued during our sample period. If anything, in the counterfactual with greater bund issuance and fewer GBi's outstanding, the spreads might have been favorable towards greater GBi issuance due their increased scarcity. This is effectively the scenario the German Finance Ministry has decided to realize going forward. As a result, it will be interesting to follow how these spreads evolve in the future.

To summarize, we find that neither the safety premia nor the trading conditions of the inflation-linked bonds were negatively impacted by the decision to cease future issuance of GBi bonds. Hence, for now, inflation-linked trading remains active despite no new issuance has come to market since before November 2023. However, as time goes by and the few remaining GBi's start to reach maturity, the available number of bonds will inevitably shrink. This will exacerbate the problems with weak identification of the state variables within the AFNS-R model identified in Section 4.2. Over time, this will cause the model results to be overall less robust and reliable. Hence, although the GBi market appears to be well functioning at this point, the eventual shrinking market for these bonds is a cause for concern and makes us caution against relying too heavily on this market in the longer run. However, obviously, for historical analysis of key events in the euro area during our sample period, this market is and will remain a rich and useful source.

6 Natural Rate Estimates Using GBi Prices

In this section, we first go through a careful model selection process to find a preferred specification of the AFNS-R model's objective $\mathbb{P}$-dynamics. We then use this AFNS-R model to account for bond-specific safety and standard term premia in the GBi prices and obtain expected real short-term interest rates and the associated measure of the natural rate.

6.1 Definition of the Natural Rate

Our working definition of the natural rate of interest r_t^* is taken from CR and given by

$$r_t^* = \frac{1}{5}\int_{t+5}^{t+10} E_t^{\mathbb{P}}[r_s^R]ds, \tag{7}$$

that is, the average expected real short-term interest rate over a five-year period starting five years ahead, where the expectation is with respect to the objective $\mathbb{P}$-probability measure. This 5yr5yr forward average expected real short rate should be little affected by short-term transitory shocks. Alternatively, r_t^* could be defined as the expected real short-term interest rate at an infinite horizon. However, this quantity will depend crucially on whether the factor dynamics exhibit a unit root. As is well known, the typical spans of time series data that are available do not distinguish strongly between highly persistent stationary processes and nonstationary ones. Our model follows the finance literature and adopts the former structure, so strictly speaking, our infinite-horizon steady-state expected real short-term interest rate is constant. However, we view our data sample as having insufficient information in the ten-year to infinite horizon range to definitively pin down that steady state, so we prefer the definition above with a medium- to long-run horizon.

6.2 Model Selection

For estimation of the natural rate and associated real term premia, the specification of the mean-reversion matrix $K^{\mathbb{P}}$ is crucial as noted earlier. To select the best-fitting specification of the model's real-world dynamics, we use a general-to-specific modeling strategy in which the least significant off-diagonal parameter of $K^{\mathbb{P}}$ is restricted to zero and the model is re-estimated. This strategy of eliminating the least significant coefficient is carried out down to the most parsimonious specification, which has a diagonal $K^{\mathbb{P}}$ matrix. The final specification choice is based on the value of the Bayesian information criterion (BIC), as in Christensen et al. (2014).[18]

[18] The Bayesian information criterion is defined as $\text{BIC} = -2\log L + k\log T$, where k is the number of model parameters and $T = 207$ is the number of monthly data observations.

Alternative specifications	Goodness of fit statistics			
	$\log L$	k	p-value	BIC
(1) Unrestricted $K^{\mathbb{P}}$	4,712.32	45	n.a.	-9,184.676
(2) $\kappa_{43}^{\mathbb{P}} = 0$	4,712.25	44	0.69	-9,189.85
(3) $\kappa_{43}^{\mathbb{P}} = \kappa_{14}^{\mathbb{P}} = 0$	4,712.02	43	0.50	-9,194.74
(4) $\kappa_{43}^{\mathbb{P}} = \kappa_{14}^{\mathbb{P}} = \kappa_{12}^{\mathbb{P}} = 0$	4,711.83	42	0.53	-9,199.68
(5) $\kappa_{43}^{\mathbb{P}} = \ldots = \kappa_{21}^{\mathbb{P}} = 0$	4,711.53	41	0.44	-9,204.43
(6) $\kappa_{43}^{\mathbb{P}} = \ldots = \kappa_{13}^{\mathbb{P}} = 0$	4,710.73	40	0.20	-9,208.15
(7) $\kappa_{43}^{\mathbb{P}} = \ldots = \kappa_{32}^{\mathbb{P}} = 0$	4,709.92	39	0.20	-9,211.86
(8) $\kappa_{43}^{\mathbb{P}} = \ldots = \kappa_{34}^{\mathbb{P}} = 0$	4,707.78	38	0.04	-9,212.91
(9) $\kappa_{43}^{\mathbb{P}} = \ldots = \kappa_{42}^{\mathbb{P}} = 0$	4,707.52	37	0.47	-9,217.73
(10) $\kappa_{43}^{\mathbb{P}} = \ldots = \kappa_{41}^{\mathbb{P}} = 0$	4,705.90	36	0.07	-9,219.81
(11) $\kappa_{43}^{\mathbb{P}} = \ldots = \kappa_{31}^{\mathbb{P}} = 0$	4,705.18	35	0.23	**-9,223.72**
(12) $\kappa_{43}^{\mathbb{P}} = \ldots = \kappa_{23}^{\mathbb{P}} = 0$	4,701.24	34	< 0.01	-9,221.16
(13) $\kappa_{43}^{\mathbb{P}} = \ldots = \kappa_{24}^{\mathbb{P}} = 0$	4,698.04	33	0.01	-9,220.09

Table 7: **Evaluation of Alternative Specifications of the AFNS-R Model**
There are 13 alternative estimated specifications of the AFNS-R model. Each specification is listed with its maximum log likelihood ($\log L$), number of parameters (k), the p-value from a likelihood ratio test of the hypothesis that it differs from the specification above with one more free parameter, and the Bayesian information criterion (BIC). The period analyzed covers monthly data from October 31, 2007, to December 30, 2024.

The summary statistics of the model selection process are reported in Table 7. The BIC is minimized by specification (11), which has a $K^{\mathbb{P}}$-matrix given by

$$K_{BIC}^{\mathbb{P}} = \begin{pmatrix} \kappa_{11}^{\mathbb{P}} & 0 & 0 & 0 \\ 0 & \kappa_{22}^{\mathbb{P}} & \kappa_{23}^{\mathbb{P}} & \kappa_{24}^{\mathbb{P}} \\ 0 & 0 & \kappa_{33}^{\mathbb{P}} & 0 \\ 0 & 0 & 0 & \kappa_{44}^{\mathbb{P}} \end{pmatrix}.$$

The estimated parameters of the preferred specification are reported in Table 8. The estimated $\mathbb{Q}$-dynamics used for pricing and determined by (Σ, λ, $\kappa_R^{\mathbb{Q}}$, $\theta_R^{\mathbb{Q}}$) are very close to those reported in Table 3 for the AFNS-R model with diagonal $K^{\mathbb{P}}$. This implies that both model fit and the estimated GBi safety premia from the preferred AFNS-R model are very similar to those already reported and therefore not shown. Furthermore, the estimated objective $\mathbb{P}$-dynamics in terms of $\theta^{\mathbb{P}}$ and Σ are also qualitatively similar to those reported in Table 3.

Still, to understand the role played by the mean-reversion matrix $K^{\mathbb{P}}$ for estimates of the natural rate, we will later analyze the most flexible model with unrestricted mean-reversion matrix $K^{\mathbb{P}}$ and the most parsimonious model with diagonal $K^{\mathbb{P}}$, in addition to our preferred specification described above.

$K^{\mathbb{P}}$	$K^{\mathbb{P}}_{\cdot,1}$	$K^{\mathbb{P}}_{\cdot,2}$	$K^{\mathbb{P}}_{\cdot,3}$	$K^{\mathbb{P}}_{\cdot,4}$	$\theta^{\mathbb{P}}$		Σ
$K^{\mathbb{P}}_{1,\cdot}$	0.2339	0	0	0	0.0159	σ_{11}	0.0071
	(0.1836)				(0.0073)		(0.0004)
$K^{\mathbb{P}}_{2,\cdot}$	0	1.4883	0.8089	0.1249	-0.0070	σ_{22}	0.0132
		(0.5515)	(0.4128)	(03276)	(0.0107)		(0.0015)
$K^{\mathbb{P}}_{3,\cdot}$	0	0	0.4611	0	-0.0268	σ_{33}	0.0150
			(0.2798)		(0.0092)		(0.0017)
$K^{\mathbb{P}}_{4,\cdot}$	0	0	0	0.7940	-0.0339	σ_{44}	0.2865
				(0.4375)	(0.1102)		(0.7565)

Table 8: **Estimated Dynamic Parameters of the Preferred AFNS-R Model**
The table shows the estimated parameters of the $K^{\mathbb{P}}$ matrix, $\theta^{\mathbb{P}}$ vector, and diagonal Σ matrix for the preferred AFNS-R model according to the BIC. The estimated value of λ is 0.3164 (0.0259), while $\kappa^{\mathbb{Q}}_R$ = 10.43 (27.45), and $\theta^{\mathbb{Q}}_R$ = -0.0069 (0.0033). The maximum log likelihood value is 4,705.18. The numbers in parentheses are the estimated parameter standard deviations.

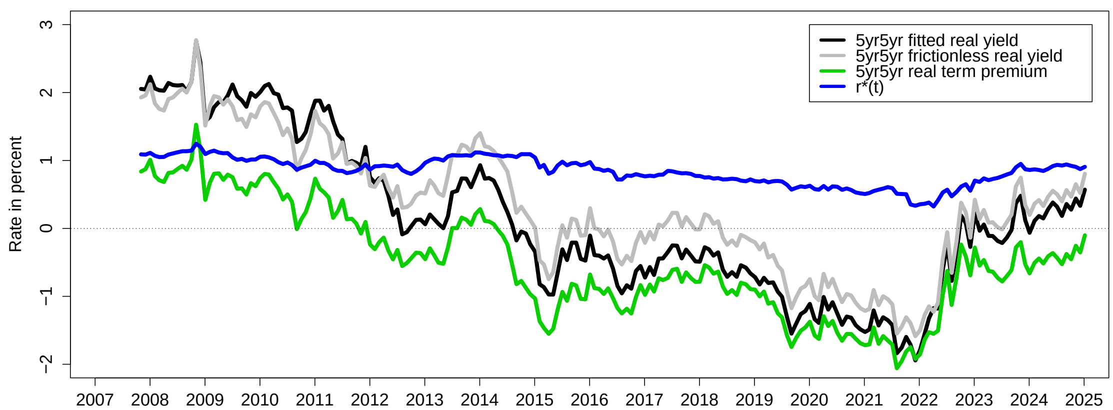


Figure 12: **AFNS-R Model 5yr5yr Real Yield Decomposition**

6.3 Estimates of the Natural Rate

Our market-based measure of the natural rate is the average expected real short rate over a five-year period starting five years ahead. This 5yr5yr forward average expected real short rate should be little affected by short-term transitory shocks and well positioned to capture the persistent trends in the natural real rate.

To illustrate the decomposition underlying our definition of r^*_t, recall that the real term premium is defined as

$$TP_t(\tau) = y_t(\tau) - \frac{1}{\tau}\int_t^{t+\tau} E^{\mathbb{P}}_t[r_s]ds.$$

That is, the real term premium is the difference in expected real returns between a buy-and-

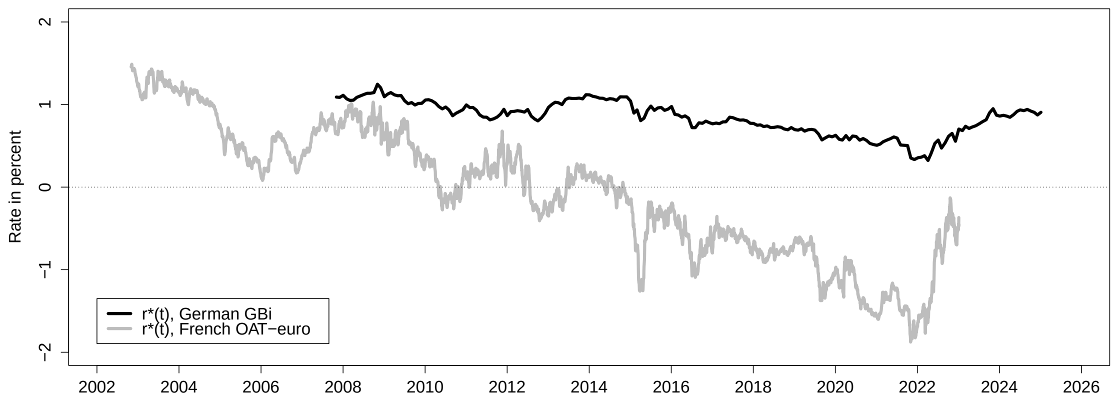


Figure 13: **Comparison with Another Market-Based Estimate of r***

hold strategy for a τ-year real bond and an instantaneous rollover strategy at the risk-free real rate r_t.

Figure 12 shows the AFNS-R model decomposition of the 5yr5yr forward frictionless real yield based on this equation. First, we note the fitted 5yr5yr real yield is mostly below the frictionless 5yr5yr real yield. The difference reflect the safety premia in the GBi prices. Second, we note that both real yield series are characterized by a persistent declining trend between 2007 and 2021 followed by a perstent increasing trend since then. The AFNS-R model decomposition suggests that these large gyrations in euro area long-term real yields reflect persistent fluctuations in real term premia, while the estimate of r_t^* is much more stable and appears to be stationary.

As a validation exercise, we compare our estimate of the natural rate to another existing market-based estimate of the natural rate in the euro area taken from the literature. Specifically, we compare our r_t^* estimate to the estimate reported by CM based on the prices of French OAT€bonds. These two market-based estimates of the natural rate are shown in Figure 13. As already noted, the estimate based on GBi's is very stable and essentially without any trends. This seems at odds with the observed GBi yields shown in Figure 2, which indeed do have a notable declining trend between 2007 and 2021 followed by an equally notable sharp reversal since 2022. As documented below, this result for r_t^* estimates based on GBi's is pervasive and not sensitive to either the assumed model dynamics or the data frequency. In contrast, the market-based r_t^* estimate based on the French data is much more realistic in that it indeed shares the persistent trends visible in both French and German inflation-linked bond data.

To assess the sensitivity of our r_t^* estimate to the specification of the mean-reversion

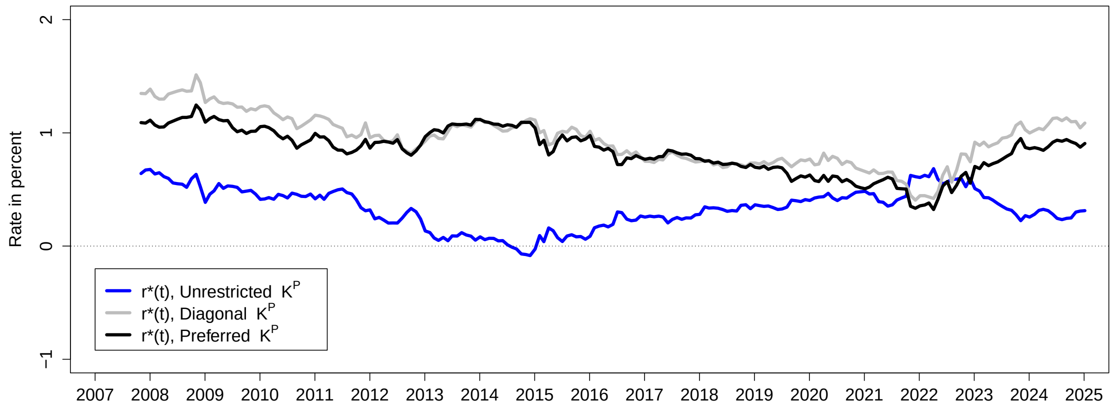


Figure 14: **The Sensitivity of r* Estimates to $K^{\mathbb{P}}$ Specification**

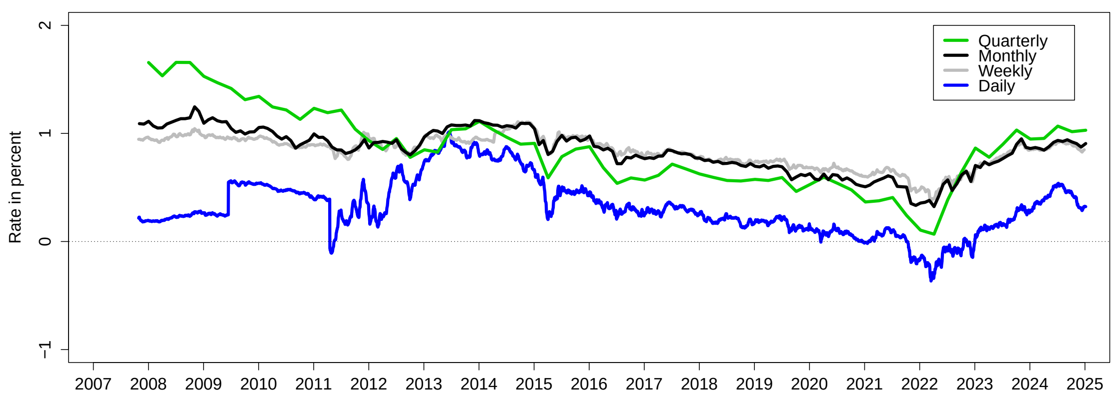


Figure 15: **The Sensitivity of r* Estimate to Data Frequency**

matrix $K^{\mathbb{P}}$, we compare it in Figure 14 to the estimates from the AFNS-R models with unrestricted and diagonal $K^{\mathbb{P}}$ matrix, respectively. As noted in the figure, our r_t^* estimate is indeed very sensitive to this model choice, but parsimonious specifications like our preferred AFNS-R model specification favored by the data tend to give fairly similar r_t^* estimates. Still, these results demonstrate how insignificant off-diagonal parameters in the specification of the mean-reversion $K^{\mathbb{P}}$ matrix can materially distort estimates of r_t^*. Hence, the results underscore the importance of our careful model selection procedure needed to identify appropriate specifications of $K^{\mathbb{P}}$ supported by the bond price data.

The role of the data frequency is examined in Figure 15, which shows the r_t^* estimates implied by our preferred AFNS-R model estimated at daily, weekly, monthly, and quarterly frequency. The results show that our estimate has little sensitivity to our choice to focus on monthly data.

To summarize, we find the AFNS -R model decomposition of the 5yr5yr real yield to be overly stable and stationary in its expectations component. We take this as a sign that the estimated model dynamics suffer significantly from the finite-sample bias problem discussed at length in Bauer et al. (2012). Combined with the fact that any future issuance of the bonds has been cancelled by the German Federal Finance Agency, this implies that the GBi market is not well suited for this type of longer-run analysis. In this particular regard, it comes across as inferior to the much larger and more well-established French market for OAT€s. Thus, unless GBi issuance is resumed, we advise against using this market to decompose real yields into their various expectations and risk premium components.

7 Conclusion

In this paper, we provide an in-depth analysis of the little known German market for inflation-linked government bonds. As the first, we document the existence of large convenience premia in the prices of these bonds that average 0.33 percent over our sample, meaning that investors are willing to overpay to own these bonds. Given their relatively low liquidity and related lack of moneyness, we refer to these convenience premia as safety premia. Regression analysis with a large battery of explanatory variables supports this interpretation.

Despite being overpriced, the German Federal Finance Agency decided in November 2023 to cease all future issuance of such bonds as well as any reopening of existing ones. We examine the market reaction to this consequential announcement and find that neither the trading of these bonds nor their safety premia have been negatively affected by the fact that no new supply will come to market going forward. Hence, this overlooked and understudied market remains a rich source of information about real rates in the euro area. This also makes it a promising candidate for the construction of breakeven inflation for the euro area, although we leave that task for future research. Unfortunately, this rosy outlook is likely to change eventually as the few remaining bonds reach maturity. Thus, we caution against putting too much weight on this market in the longer run.

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